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Phys. Rev. D **84**, 101502 — Published 8 November 2011

DOI: 10.1103/PhysRevD.84.101502

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(Dated: October 11, 2011)

In this paper, we show that the spin-0 gravitons appearing in Horava-Lifshitz gravity without the projectability condition can be eliminated by extending the gauge symmetries of the foliation-preserving diffeomorphisms to include a local U(1) symmetry. As a result, the problems of stability, ghost and different speeds in the gravitational sector are automatically resolved. In addition, with the detailed balance condition softly breaking, the number of independent coupling constants can be significantly reduced (from more than 70 down to 15), while the theory is still UV complete and possesses a healthy IR limit, whereby the prediction powers of the theory are considerably improved. The strong coupling problem can be cured by introducing an energy scale M_*, so that $M_* < \Lambda_\omega$, where M_* denotes the suppression energy of high order derivative terms, and Λ_ω the would-be strong coupling energy scale.

PACS numbers: 04.60.-m; 04.50.Kd

I. INTRODUCTION

Quantization of gravitational fields has been one of the main deriving forces in Physics in the past decades by following several different paths [1]. Recently, Horava proposed a theory of quantum gravity in the framework of quantum field theory, with the spacetime metric as the elementary field in the language of the standard path-integral formulas [2]. Applied to cosmology, it results in various remarkable features and has attracted a great deal of attention [3]. In particular, high order spatial derivative terms can give rise to a bouncing universe [4, 5]; the anisotropic scaling solves the horizon problem and leads to scale-invariant perturbations without inflation [6, 7]; the lack of the local Hamiltonian constraint leads to "dark matter as an integration constant" [8]; the dark sector can have its purely geometric origins [9]; and the inclusion of a U(1) symmetry (with the projectability condition) [10, 11] not only eliminates the spin-0 gravitons but also leads to a flat universe [12].

With the perspective that Lorentz symmetry may appear only as an emergent one at low energies, but can be fundamentally absent at high energies, Horava considered a gravitational system whose scaling at short distances exhibits a strong anisotropy between space and time,

$$\mathbf{x} \to b^{-1}\mathbf{x}, \quad t \to b^{-z}t. \tag{1}$$

This is quite similar to Lifsitz's scalar field [13], so the theory is often referred to as the Horava-Lifshitz (HL) gravity. In $(d+1)$-dimensions, in order for the theory to be power-counting renormalizable, the critical exponent z must be $z \geq d$ [2, 14]. The associated gauge symmetry now is broken from the general diffeomorphisms down to the foliation-preserving ones,

$$\delta t = -f(t), \quad \delta x^i = -\zeta^i(t, \mathbf{x}), \tag{2}$$

to be denoted by Diff(M, $\mathcal{F}$).

Abandoning the general diffeomorphisms, on the other hand, gives rise to a proliferation of independently coupling constants [2, 15], which could potentially limit the prediction powers of the theory. In particular, only the sixth-order spatial derivative terms are more than 60 [16]. To reduce the number of the independent coupling constants, two conditions were introduced, the *projectibility* and *detailed balance* [2]. The former requires that the lapse function N be a function of t only, while the latter requires that the gravitational potential should be obtained from a superpotential W_g, where W_g is given by an integral of the gravitational Chern-Simons term over a 3-dimensional space, $W_g \sim \int_\Sigma \omega_3(\Gamma)$. With these two conditions, the general action now contains only five independent coupling constants [2]. The detailed balance condition has several remarkable features [2, 17]. For example, it is in the same spirit of the AdS/CFT correspondence [18], where a string theory and gravity defined on one space is equivalent to a quantum field theory without gravity defined on the conformal boundary of this space, which has one or more lower dimension(s). Yet, in the non-equilibrium thermodynamics, the counterpart of the superpotential W_g plays the role of entropy, while $\delta W_g/\delta g_{ij}$ represents the corresponding entropic force [19]. This might shed further lights on the nature of gravitational forces, as proposed recently by Verlinde [20].

Despite of all the above remarkable features, the theory (with these two conditions) is plagued with several problems [3], including the instability, ghost [2, 21], and

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strong coupling [16, 22]. However, all of them are closely related to the existence of the spin-0 gravitons [3]. Because of their presence, another important question also raises: Their speeds are generically different from those of gravitational waves, as they are not related by any symmetry. This poses a great challenge for any attempt to restore Lorentz symmetry at low energies where it has been well tested experimentally. In particular, one needs a mechanism to ensure that in those energy scales all species, including gravitons, have the same effective speed and light cones. With these in mind, Horava and Melby-Thompson (HMT) [10] extended the symmetry (2) to include a local $U(1)$,

$$U(1) \ltimes \text{Diff}(M,\ \mathcal{F}). \tag{3}$$

With this enlarged symmetry, the spin-0 gravitons are eliminated [10, 23, 24], and the theory has the same long-distance limit as that of general relativity. This was initially done in the special case $\lambda = 1$, and soon generalized to the case with any λ [11], where λ is a coupling constant defined below. Although the spin-0 gravitons are also eliminated in the general case, the strong coupling problem raises again [12]. However, it can be solved by the Blas-Pujolas-Sibiryakov (BPS) mechanism [15], in which a new energy scale M_* is introduced, so that $M_* < \Lambda_\omega$, where M_* denotes the suppression energy of high order derivative terms of the theory, and Λ_ω the would-be strong coupling energy scale [25].

Note that in [10, 11] the projectability condition was assumed. In this paper, we shall study the case without it, and our goals are twofold: (i) Extend the symmetry (3) to the case without the projectability condition, so that the spin-0 gravitons are eliminated. (ii) Reduce significantly the number of the coupling constants by the detailed balance condition, whereby the prediction powers of the theory can be improved considerably. To have a healthy IR, we allow it to be broken softly by adding all the low dimensional relevant terms [17]. We also note that in [26] a U(1) extension of F(R) HL gravity was recently studied.

II. THE MODEL

Under the U(1) transformations, the metric coefficients transform as [10],

$$\delta_\alpha N = 0, \quad \delta_\alpha N_i = N\nabla_i\alpha, \quad \delta_\alpha g_{ij} = 0, \tag{4}$$

where α is the U(1) generator, N the lapse function, N_i the shift vector, and ∇_i the covariant derivative of g_{ij}. Since [15]

$$\hat{S}_g = \zeta^2 \int dtdx^3 N\sqrt{g}\left(\mathcal{L}_K - \mathcal{L}_V(g_{ij}, a_k)\right), \tag{5}$$

where $a_i \equiv (\ln N)_{,i}$, $\mathcal{L}_K \equiv K_{ij}K^{ij} - \lambda K^2$ and $K_{ij} = \left(-\dot{g}_{ij} + \nabla_i N_j + \nabla_j N_i\right)/(2N)$, the potential $\mathcal{L}_V$ is invariant under (4), while the kinetic part transforms as

$$\delta_\alpha S_K = \zeta^2 \int dtd^3x N\sqrt{g}\Big\{\left(\dot{\alpha} - N^i\nabla_i\alpha\right)\frac{R}{N} + 2\alpha G_{ij}K^{ij} + 2K_{ij}\hat{\mathcal{G}}^{ijlk}a_{(l}\nabla_{k)}\alpha + 2(1-\lambda)K\left(\nabla^2\alpha + a^k\nabla_k\alpha\right)\Big\}, \tag{6}$$

where $\hat{\mathcal{G}}^{ijkl} = \mathcal{G}^{ijkl}\big|_{\lambda=1}$, $\mathcal{G}^{ijkl}(\lambda) \equiv (g^{ik}g^{jk} + g^{il}g^{jk})/2 - \lambda g^{ij}g^{lk}$, $G_{ij} \equiv R_{ij} - Rg_{ij}/2$ and $f_{(ij)} = (f_{ij} + f_{ji})/2$. R_{ij} (R) is the Ricci tensor (scalar) of g_{ij}. To have the $U(1)$ symmetry, we introduce the gauge field A [10], which transforms as

$$\delta_\alpha A = \dot{\alpha} - N^i\nabla_i\alpha. \tag{7}$$

Then, adding [1]

$$S_A = -\zeta^2 \int dtd^3x\sqrt{g}A(R - 2\Lambda_g), \tag{8}$$

to $\hat{S}_g$, one finds that its variation (with $\Lambda_g = 0$) with respect to α exactly cancels the first term in Eq.(6). To repair the rest, we introduce the Newtonian prepotential φ [10], which transforms as

$$\delta_\alpha\varphi = -\alpha. \tag{9}$$

Under Eq.(4), the variation of the term,

$$\begin{aligned} S_\varphi = \zeta^2 \int dtd^3x\sqrt{g}N\Big\{&\varphi\mathcal{G}^{ij}\left(2K_{ij} + \nabla_i\nabla_j\varphi + a_i\nabla_j\varphi\right) \\ &+ (1-\lambda)\Big[\left(\nabla^2\varphi + a_i\nabla^i\varphi\right)^2 + 2\left(\nabla^2\varphi + a_i\nabla^i\varphi\right)K\Big] \\ &+ \frac{1}{3}\hat{\mathcal{G}}^{ijlk}\Big[4\left(\nabla_i\nabla_j\varphi\right)a_{(k}\nabla_{l)}\varphi + 5\left(a_{(i}\nabla_{j)}\varphi\right)a_{(k}\nabla_{l)}\varphi \\ &+ 2\left(\nabla_{(i}\varphi\right)a_{j)(k}\nabla_{l)}\varphi + 6K_{ij}a_{(l}\nabla_{k)}\varphi\Big]\Big\}, \end{aligned} \tag{10}$$

will exactly cancel the rest in Eq.(6) as well as the one from the term $\Lambda_g A$ in Eq.(8), where $\mathcal{G}_{ij} \equiv G_{ij} + \Lambda_g g^{ij}$ and $a_{ij} \equiv \nabla_i a_j$. Hence, the total action

$$S_g = \hat{S}_g + S_A + S_\varphi, \tag{11}$$

is invariant under (3).

Equation (3) does not uniquely fix $\mathcal{L}_V$. To our second goal, we introduce the "generalized" detailed balance condition,

$$\hat{\mathcal{L}}_V \;=\; E_{ij}\mathcal{G}^{ijkl}E_{kl} - g_{ij}A^iA^j, \tag{12}$$

where

$$\begin{aligned} E^{ij} \;&=\; \frac{1}{\sqrt{g}}\frac{\delta W_g}{\delta g_{ij}}, \quad A^i = \frac{1}{\sqrt{g}}\frac{\delta W_a}{\delta a_i}, \quad W_g = \frac{1}{w^2}\int_\Sigma \omega_3(\Gamma), \\ W_a \;&=\; \int d^3x\sqrt{g}a^i\sum_{n=0}^{n=1} b_n\Delta^n a_i, \end{aligned} \tag{13}$$

[1] Note the difference between the notations used here and the ones used in [10, 11]. In particular, we have $K_{ij} = -K^{HMT}_{ij}$, $\Lambda_g = \Omega^{HMT}$, $\varphi = -\nu^{HMT}, \mathcal{G}_{ij} = \Theta^{HMT}_{ij}$, where quantities with the superindices "HMT" were used in [10, 11].

with $\Delta \equiv g^{ij}\nabla_i\nabla_j$ and b_n being arbitrary constants. $\omega_3(\Gamma)$ denotes the gravitational Chern-Simons term, given by

$$\begin{aligned}\omega_3(\Gamma) &\equiv \mathrm{Tr}\Big(\Gamma\wedge d\Gamma + \frac{2}{3}\Gamma\wedge\Gamma\wedge\Gamma\Big)\\ &= \frac{e^{ijk}}{\sqrt{g}}\Big(\Gamma^m_{jl}\partial_j\Gamma^l_{km} + \frac{2}{3}\Gamma^n_{il}\Gamma^l_{jm}\Gamma^m_{kn}\Big), \quad (14)\end{aligned}$$

where Γ^i_{jk} are the Christoffel symbols built out of g_{ij}, and $e^{123} = 1$. Some comments now are in order. First, the term $a_i\Delta^{1/2}a^i$ in principle can be included into W_a, which will give rise to fifth order derivative terms. In this paper we shall discard them by parity. Second, it is well-known that with the detailed balance condition a scalar field is not UV complete [4], and the Newtonian limit does not exist [27]. Following [17], we break it softly by adding all the low dimensional relevant terms to $\hat{\mathcal{L}}_V$, so that the potential finally takes the form,

$$\begin{aligned}\mathcal{L}_V &= \gamma_0\zeta^2 - \Big(\beta_0 a_i a^i - \gamma_1 R\Big) + \zeta^{-2}\Big(\gamma_2 R^2 + \gamma_3 R_{ij}R^{ij}\Big)\\ &\quad +\zeta^{-2}\Big[\beta_1\left(a_i a^i\right)^2 + \beta_2\left(a^i{}_i\right)^2 + \beta_3\left(a_i a^i\right)a^j{}_j\\ &\quad +\beta_4 a^{ij}a_{ij} + \beta_5\left(a_i a^i\right)R + \beta_6 a_i a_j R^{ij} + \beta_7 R a^i{}_i\Big]\\ &\quad +\zeta^{-4}\Big[\gamma_5 C_{ij}C^{ij} + \beta_8\left(\Delta a^i\right)^2\Big], \quad (15)\end{aligned}$$

where $\beta_0 \equiv b_0^2$, $\gamma_5 \equiv \zeta^4/w^4$, $\beta_8 \equiv -\zeta^4 b_1^2$, and C_{ij} denotes the Cotton tensor, defined as $C^{ij} \equiv \frac{e^{ikl}}{\sqrt{g}}\nabla_k\Big(R^j_l - \frac{1}{4}R\delta^j_l\Big)$. All the coefficients, β_n and γ_n, are dimensionless and arbitrary, except $\beta_0 \geq 0, \gamma_5 \geq 0$ and $\beta_8 \leq 0$, as indicated by their definitions. γ_0 is related to the cosmological constant by $\Lambda = \zeta^2\gamma_0/2$, while the IR limit requires $\gamma_1 = -1$, $\zeta^2 = 1/(16\pi G)$, where G is the Newtonian constant. From the above, we can see that with the "generalized" detailed balance condition softly breaking, the number of independent coupling constants is significantly reduced from more 70 to 15: $G,\ \Lambda,\ \lambda,\ \beta_n,\ \gamma_s,\ (n = 0,...,8;\ s = 2,3,5)$. In the following, we shall show that such a setup is UV complete and IR healthy.

III. ELIMINATION OF THE SPIN-0 GRAVITONS

To show this, it is sufficient to consider linear perturbations in the Minkowski background, given by

$$\begin{aligned}N &= 1+\phi, \quad N_i = \partial_i B, \quad g_{ij} = (1-2\psi)\delta_{ij} + 2E_{,ij},\\ A &= \delta A, \quad \varphi = \delta\varphi. \quad (16)\end{aligned}$$

Choosing the gauge, $E = 0 = \delta\varphi$, we find that

$$\begin{aligned}S^{(2)}_g &= \zeta^2\int dtd^3x\Big\{(1-3\lambda)(3\dot{\psi}^2 + 2\dot{\psi}\partial^2 B)\\ &\quad +(1-\lambda)(\partial^2 B)^2 - \left(\phi\eth + 4\beta_7\zeta^{-2}\partial^2\psi\right)\partial^2\phi\\ &\quad -2(\psi - 2\phi + 2A + \alpha_1\psi\partial^2)\partial^2\psi\Big\}, \quad (17)\end{aligned}$$

where $\alpha_1 \equiv \zeta^{-2}(8\gamma_2 + 3\gamma_3)$ and $\eth \equiv \beta_0 + \zeta^{-2}(\beta_2+\beta_4)\partial^2 - \zeta^{-4}\beta_8\partial^4$. Variations of $S^{(2)}_g$ with respect to A, ψ, B, and ϕ yield, respectively,

$$\partial^2\psi = 0, \quad (18)$$

$$\ddot{\psi} + \frac{1}{3}\partial^2\dot{B} + \frac{2\partial^2(A + \psi + \alpha_1\partial^2\psi)}{3(1-3\lambda)} = \frac{2\wp\partial^2\phi}{3(1-3\lambda)}, \quad (19)$$

$$\big(\lambda - 1\big)\partial^2 B = \big(1 - 3\lambda\big)\dot{\psi}, \quad (20)$$

$$\eth\phi = 2\wp\psi, \quad (21)$$

where $\wp \equiv 1 - \zeta^{-2}\beta_7\partial^2$. Eq.(18) shows that ψ is not propagating, and with proper boundary conditions, one can set $\psi = 0$. Then, Eqs.(19)-(21) show that B, A, and ϕ are also not propagating and can be set to zero. Hence, we obtain $\psi = B = A = \phi = 0$, that is, the scalar perturbations vanish identically in the Minkowski background, similar to that in general relativity. Thus, with the enlarged symmetry (3), the spin-0 gravitons are indeed eliminated even without the projectability condition.

IV. STRONG COUPLING AND THE BLAS-PUJOLAS-SIBIRYAKOV MECHANISM

Since the spin-0 gravitons are eliminated, the ghost, instability, strong coupling and different speed problems in the gravitational sector do not exist any longer. But, the self-interaction of matter fields and the interaction between a matter field and a gravitational field can still lead to strong coupling, as shown recently in [25] for the theory with the projectability condition. In the following we shall show that this is also the case here. Since the proof is quite similar to that given in [25], in the following we just summarize our main results, and for detail, we refer readers to [25]. A scalar field χ with detailed balance condition softly breaking is described by Eqs.(3.11) and (3.12) of [17]. In the Minkowski background, we have $\bar{\chi} = 0 = V(0) = V'(0)$. Considering the linear perturbations $\chi = \delta\chi$, we find that the quadratic part of the scalar field action reads,

$$\begin{aligned}S^{(2)}_\chi = \int dtd^3x\Bigg[&\frac{1}{2}f\dot{\chi}^2 - \frac{1}{2}V''\chi^2 - \frac{1}{2}\left(1+2V_1\right)(\partial\chi)^2\\ &+c_1 A\partial^2\chi - V_2(\partial^2\chi^2)^2 - V_4'\chi\partial^4\chi + \sigma_3^2\partial^2\chi\partial^4\chi\Bigg], \quad (22)\end{aligned}$$

where f is a constant, usually chosen to be one [17]. Since $S^{(2)}_\chi$ does not depend on ψ, B and ϕ explicitly, the variations of the total action $S^{(2)} = S^{(2)}_\chi + S^{(2)}_g$ with respect to them will give the same Eqs.(19) - (21), while Eq.(18) is replaced by $\psi = c_1\chi/(4\zeta^2)$. Integrating out ϕ, ψ, B, A, we obtain

$$S^{(2)} = M^2\int dtd^3x\Big[\dot{\chi}^2 - \alpha_0\big(\partial\chi\big)^2 - m_\chi^2\chi^2$$

$$-M_A^{-2}\chi\partial^4\chi + M_B^{-4}\chi\partial^6\chi + \gamma\chi\partial^2\left(\frac{\wp^2\chi}{\eth}\right)\Bigg], \tag{23}$$

where $M^2 \equiv 2\pi G c_1^2/|c_\psi|^2 + f/2$, $\gamma \equiv 4\pi G c_1^2/M^2$, $\alpha_0 \equiv \left(1 + 2V_1 - 4\pi G c_1^2\right)/(2M^2)$, $M_A^2 \equiv M^2/\left(2\pi G\alpha_1 c_1^2 + V_2 + V_4'\right)$, $M_B^4 \equiv M^2/\sigma_3^2$, $m_\chi^2 \equiv V''/(2M^2)$, and $c_\psi^2 = (1-\lambda)/(3\lambda-1)$. Clearly, the scalar field is ghost-free for $f > 0$, and stable in the UV and IR. In fact, it can be made stable in all energy scales by properly choosing the coupling coefficients V_n. To consider strong coupling, one needs to calculate the cubic part of the total action, which takes the form,

$$\begin{aligned}
S^{(3)} = \int dt d^3x \Bigg\{ & \lambda_1\left(\frac{1}{\partial^2}\ddot{\chi}\right)\chi\partial^2\chi + \lambda_2\left(\frac{1}{\partial^2}\ddot{\chi}\right)\chi_{,i}\chi^{,i} \\
& + \lambda_3\dot{\chi}^2\left(\frac{2\wp}{\eth} - 1\right)\chi + \lambda_4\dot{\chi}\partial^i\left(\frac{2\wp}{\eth} - 1\right)\chi\partial_i\left(\frac{\dot{\chi}}{\partial^2}\right) \\
& + \lambda_5\left(\frac{\partial^i\partial^j}{\partial^2}\dot{\chi}\right)\left(\frac{\partial_i}{\partial^2}\dot{\chi}\right)\partial_j\left(\frac{2\wp}{\eth} + 3\right)\chi \\
& + \lambda_6\dot{\chi}\chi^{,i}\left(\frac{\partial_i}{\partial^2}\dot{\chi}\right) + ... \Bigg\},
\end{aligned} \tag{24}$$

where "..." represents the terms that are independent of λ, so they are irrelevant to the strong coupling problem. λ_s are functions of λ, f, ζ and c_i. In particular, $\lambda_5 = c_1^3/(64\zeta^4|c_\psi|^4)$, which will yield the strong coupling energy Λ_ω given below. The exact expressions of other coefficients are not relevant to Λ_ω, so will not be given here. For a process with energy $E \ll M_A, M_B$, the first two terms in Eq.(23) are dominant, and $S^{(2)}$ is invariant under the relativistic rescaling $t \to b^{-1}t, x^i \to b^{-1}x^i, \chi \to b\chi$. Then, all the terms of λ_s in $S^{(3)}$ scale as b. As a result, when the energy of a process is greater than a certain value, say, Λ_ω, the amplitudes of these terms are greater than one, and the theory becomes nonrenormalizable [28]. In the present case, it can be shown that

$$\Lambda_\omega \simeq \left(\frac{M_{pl}}{c_1}\right)^{3/2} M_{pl}|c_\psi|^{5/2}. \tag{25}$$

However, if

$$M_* < \Lambda_\omega, \tag{26}$$

where $M_* = \mathrm{Min.}(M_A, M_B)$, one can see that before the strong coupling energy Λ_ω is reached, the high order derivative terms in Eq.(23) become large, and their effects must be taken into account. In particular, for $M_A \gtrsim M_B$, the sixth-order derivative terms become dominant for processes with $E \gtrsim M_*$, and the quadratic action (23) now is invariant only under the anisotropic rescaling,

$$t \to b^{-3}t, \quad x^i \to b^{-1}x^i, \quad \chi \to \chi. \tag{27}$$

Then, one finds that under the new rescaling all the terms of λ_s in Eq.(24) become scaling-invariant, while the rest scale as $b^{-\delta}$ with $\delta > 0$. The former are strictly renormalizable, while the latter are superrenormalizable [28]. Therefore Eq.(26) makes the strong coupling problem disappeared.

V. CONCLUSIONS

In this paper, we have shown that the spin-0 gravitons in the HL theory even without the projectability condition can be eliminated by the enlarged symmetry (3). An immediate result is that all the problems related to them disappear, including the ghost, instability, strong coupling and different speeds in the gravitational sector. In addition, the requirements [15], $|\lambda-1| \simeq \beta_0,\ 0 < \beta_0 < 2$, now become unnecessary, which might help to relax the observational constraints. Moreover, it is exactly because of this elimination that softly breaking detailed balance condition can be imposed, whereby the number of independent coupling constants is significantly reduced from more than 70 to 15. This considerably improves the prediction powers of the theory. Note that, without the elimination of the spin-0 gravitons, the strong coupling problem will appear in the gravitational sector and cannot be solved by the BPS mechanism, because this condition prevents the existence of sixth-order derivative terms in $S_g^{(3)}$.

These results put the HL theory with/without the projectability condition in the same footing, and provide a very promising direction to build a viable theory of quantum gravity, put forwards recently by HMT [10]. Certainly, many challenging questions [3] need to be answered before such a goal is finally reached.

Finally, we would like to note that, although the physics behind of it is not very clear, mathematically the existence of the enlarged symmetry (3) even without the projectability condition is because of the introduction of the Newtonian prepotential φ. As a matter of fact, it is exactly its presence that leads the enlarged symmetry to exist not only in the case with any coupling constant λ [11], but also in the case without the projectability condition, as one can see from Eq.(10).

Acknowledgements: This work was supported in part by DOE Grant, DE-FG02-10ER41692 (AW); NNSFC Grant, No. 11005165 (FWS); NNSFC Grant, No. 11047008 (QW, TZ); and NNSFC Grant, No. 11105120 (TZ).

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