Type-II seesaw dominance in nonsupersymmetric and split-supersymmetry SO(10) models and proton lifetime

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Type II Seesaw Dominance in Non-supersymmetric and Split Susy $SO(10)$ and Proton Life Time

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Recently type II seesaw in a supersymmetric SO(10) framework has been found useful in explaining large solar and atmospheric mixing angles as well as a larger value of θ_{13} while unifying quark and lepton masses. An important question in these models is whether there exists consistency between coupling unification and type II seesaw dominance. Scenarios where this consistency can be demonstrated have been given in a SUSY framework. In this paper we give examples where type II dominance occurs in SO(10) models without supersymmetry but with additional TeV scale particles and also in models with split-supersymmetry. Grand unification is realized in a two step process via breaking of $SO(10)$ to $SU(5)$ and then to a TeV scale standard model supplemented by extra fields and an SU(5) Higgs multiplet 15_H at a scale of about 10^{12} GeV to give type II seesaw. The predictions for proton lifetime in these models are in the range $\tau_p^0 = 2 \times 10^{35}$ yrs. to 6×10^{35} yrs. A number of recent numerical fits to GUT-scale fermion masses can be accommodated within this model.

PACS numbers:

I. I. INTRODUCTION

Supersymmetric (SUSY) grand unification has long been considered a very attractive paradigm for physics beyond the standard model since it solves the gauge hierarchy problem and leads to coupling unification while at the same time providing a candidate for dark matter. The discovery of neutrino masses has added an extra appeal to them since understanding small neutrino masses via seesaw mechanism requires the seesaw (or $B-L$) scale close to the scale of grand unification (or GUT scale). GUT theories based on $SO(10)$ [1] are just right for this purpose since they unify all fermions of each generation including the right-handed (RH) neutrino, needed in the seesaw mechanism (and no extra fields) into a single spinor representation . It also provides a spontaneous origin of P (=Parity) and CP violations. Furthermore, it was pointed out some years ago that there is a class of renormalizable SUSY SO(10) models [2] that use **10** and **126** Higgs fields contributing to fermion masses which have a small number of parameters in the Yukawa sector and can therefore be quite predictive for neutrino masses and mixings, making them testable using neutrino oscillation data. We focus on a sub-class of these models in this paper.

It is well known that there are two kinds of seesaw contributions to neutrino masses in SO(10) models : the type I seesaw contribution [3] which uses heavy right handed neutrinos whose masses arise from $B-L$ breaking and type II seesaw [4] which uses a heavy SM triplet Higgs field, in both cases with masses close to the GUT scale. Because of their predictive power, a large number of SUSY $SO(10)$ models with **10** and **126** Higgs fields have been constructed over the recent years using type-I or type-II dominance, or a mixture of both [5–10]. Of these models, the ones that assume type II seesaw dominance [6] stand out for a very special reason that the diverse mixing patterns between quark and lepton sectors at low energies are explained from the fact that the bottom and tau masses become nearly equal near the GUT scale [6, 7] and without any need for additional symmetries. These models also automatically predict a "larger" θ_{13} [7] which seems to be indicated by recent data [11]. They have since been the focus of many investigations [7–10] and have clearly defined a distinct approach to the quark-lepton flavor problem.

An immediate question for this class of models is whether type II domiance is indeed consistent with the constraints of grand unification. In this paper, we will discuss this question. To define the issue clearly, we note that the neutrino

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mass formula in generic SO(10) models has the form

$$M_\nu = f v_L \; - \frac{m_D^2}{f v_{B-L}}, \tag{1}$$

where $v_L = v^2/\mathrm{M_T}$, $\mathrm{M_T} =$ is the mass of the triplet Higgs field. The first term in the above expression is the type II seesaw term whereas the second is the type I contribution. Note that strictly speaking the type II term has the form $\frac{v^2 \lambda v_{B-L}}{\mathrm{M_T^2}}$; however, to keep the self energy contribution to the mass of the **15**-plet (or the triplet Higgs) of the same order as M_T, it is natural to require $\lambda v_{B-L} \sim M_T$, which then gives the form for the type II term in Eq.1. In minimal renormalizable SO(10) models of the type in [2], the coupling matrix f that determines the relative sizes of the two contributions also contributes to charged fermion masses and is therefore constrained by fits to the quark and lepton masses as well as the CKM mixings. Detailed numerical fits [7] show that its largest element has the value $(f_{ij})_{max} \sim 10^{-3}$ making the type-I seesaw term dominate. This would apparently suggest a serious tension between type II seesaw dominance and grand unification and this issue must be resolved before models with type II seesaw dominance can be taken seriously. First attempt at solving this problem was made in [12], where it was noted that the following two conditions prove sufficient for a solution to this problem :

- (i) $SO(10)$ breaks to SM in two stages with $SO(10)$ first breaking to $SU(5)$ at a scale much larger than the canonical GUT scale e.g. $v_{B-L} \geq 10^{17}$ GeV. This makes $f v_{B-L}$ larger making the type I term in Eq. (1) smaller than the type II term which is independent of v_{B-L}.

- (ii) without upsetting grand unification, the complete multiplet 15_H of $SU(5)$ containing the left-handed (LH) triplet Δ_L is made lighter with masses around $10^{12} - 10^{13}$ GeV.

These two conditions can be successfully implemented in SUSY $SO(10)$ models [12] if the Higgs system consists of **10**, **126**, **54** and a **210** fields. This puts the neutrino mass discussions that use type II dominance on sound theoretical footing. A second scenario which also resolves this problem has been given in a more recent paper [13]. The question we investigate here is: what happens to type II seesaw dominance in theories without supersymmetry or with high scale SUSY breaking and in particular, is it compatible with coupling unification [14]. We are motivated to look at this question by the fact that lower limits on some of the superpartner masses keep going up at LHC with no trace of supersymmetry anywhere else and also that neither coupling unification nor seesaw mechanisms per se depend on the existence of supersymmetry.

The first challenge in implementing the above two conditions in a non-SUSY SO(10) framework is that coupling unification is known not to work for SM field content and therefore two step unification of the type we are contemplating, will have to require new physics below the GUT scale. Secondly while adding additional fields to restore coupling unification, one must not run into conflict with proton decay constraints. In this paper, we isolate two classes of models where the above strategy works: (i) non-SUSY SO(10) models with extra fermions and an extra Higgs doublet at the TeV scale and a SU(5) **15**-plet at an intermediate scale and (ii) a split-SUSY model [16] where SUSY is broken at a much higher scale than TeV scale. The particular models we present here lead to a proton life time that may be accessible in planned experiments such as HyperK [15], when they probe proton life time above 10^{35} yrs.

This paper is organized as follows: in sec. 2 and sec. 3, we outline the non-SUSY and split-SUSY frameworks where two step SO(10) breaking with type II seesaw dominance is realized; in sec.4, we discuss fermion mass fits in these models. In sec. 5, we present our conclusions.

II. II. COUPLING UNIFICATION IN NON-SUSY SO(10) AND PROTON LIFETIME

As noted, one way to establish type-II seesaw dominance in non-SUSY GUT SO(10) is to consider the symmetry breaking pattern

$$SO(10) \rightarrow^{M_U^{(10)}} SU(5) \rightarrow^{M_U^{(5)}} SM, \tag{2}$$

where the Higgs representations 210_H and/or other multiplets along with $\overline{126}_H$ implement the first step of breaking. The right-handed (RH) triplet Δ_R Higgs in $\overline{126}_H$ carrying $B-L=2$ through its high scale VEV $v_{B-l} \sim M_U^{(10)}$ generates right-handed neutrino mass to drive type-I seesaw mechanism. If this scale is much larger than the canonical GUT scale of 2×10^{16} GeV, type I contribution to neutrino masses will be small. To achieve precision gauge coupling unification in this case, we will clearly need new TeV scale fields. In this example, we choose the non-standard TeV scale particles from the well known MSSM spectrum minus its superpartners i.e.

$$\chi(2,-1/2,1), F_\phi(2,1/2,1), F_\chi(2,-1/2,1), F_\sigma(3,0,1), F_b(1,0,1), F_C(1,0,8), \tag{3}$$

where F_i's denote fermions . This spectrum may be recognized to be the same as in split-SUSY models [16] except for the additional presence of χ. In the split-SUSY case discussed later we show how the TeV-scale spectrum of eq.(3) with a second Higgs doublet can be realized. In non-SUSY $SO(10)$ the non-standard fermions of eq.(3) can be shown to originate from the adjoint and vectorial matter representations, 45_F and 10_F of $SO(10)$ from GUT-scale Lagrangian by suitable tuning of parameters [17] where the $54_H \subset SO(10)$ representation has also been added. Once the fermions are made light they could be protected by corresponding global symmetries like a Z_2 discrete symmetry group e.g. matter parity, $P_M = (-1)^{3(B-L)}$, under which all standard fermions (Higgs scalars) are odd (even). In the context of non-SUSY $SO(10)$ all fermions in eq.(3) have even parity. At around 10^{13} GeV, we add the $SU(5)$ **15**-scalar representation to implement the type II seesaw mechanism.

Using the SM particle masses and $m_{F_\phi} \simeq m_{F_\chi} \simeq m_{F_\sigma} \simeq m_\chi \simeq 1$ TeV and $m_{F_C} \simeq 6-10$ TeV, the resulting precision unification of gauge couplings in the non-SUSY theory occurs close to the MSSM GUT scale with $\mathrm{M}_U = 10^{15.96}$ GeV and $\alpha_G^{-1} = 35.5$ and the unification scale is identical to the direct breaking of SO(10) in [17]. The departure from the well known MSSM GUT scale, $\mathrm{M}_U^{\mathrm{MSSM}} = 2\times 10^{16}$ GeV has arisen because of our choice of TeV scale masses and the requirement of better precision in gauge coupling unification than the MSSM. We treat this as the non-SUSY $SU(5)$ unification scale. In the next step by tuning the GUT-scale parameters in the Higgs potential, we make all components of the Higgs representation $15_H \supset \Delta_L$ to remain at the scale $\mathrm{M}_{\Delta_L} = \mathrm{M}_{(15)} = 10^{13}$ GeV, as noted. Being a complete representation of $SU(5)$, the introduction of all its components with degenerate masses at the lower scale does not change the unification scale , $\mathrm{M}_U = 10^{15.96}$ GeV, although now we have $\alpha_G^{-1} = 34.3$. This pattern of gauge coupling unification is shown in Fig.1 which satisfies the desired condition for type-II dominance.

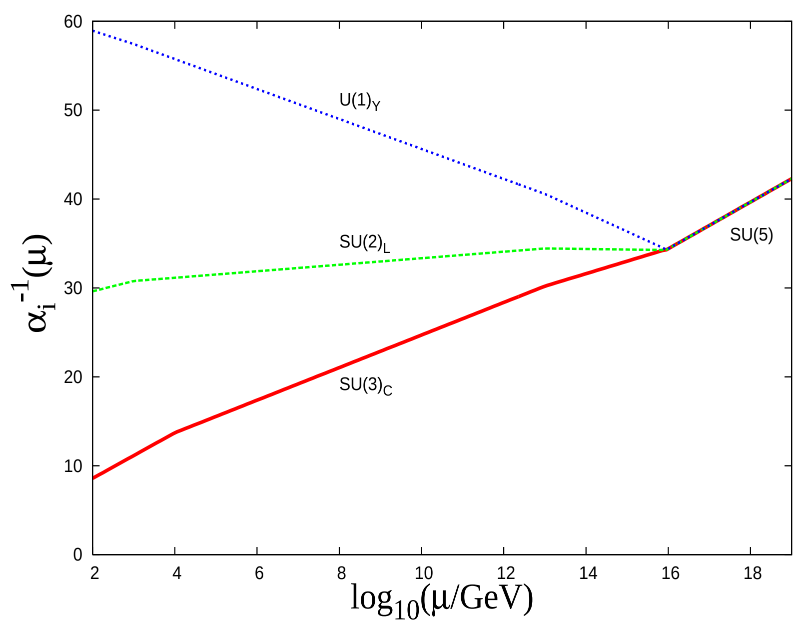

FIG. 1: Unification of gauge couplings in the high scale non-SUSY $SO(10)$ model with the $SU(5)$ unification scale at $\mathrm{M}_U^{(5)} = 10^{15.96}$ GeV and the complete $15_H \subset SU(5)$ at the type-II seesaw scale $M_{15} = 10^{13}$ GeV. The high scale of $SO(10)$ breaking has been taken as $\mathrm{M}_U^{(10)} > 10^{17}$ GeV.

The $X^{\pm 4/3}, Y^{\pm 1/3}$ gauge boson mediated decay width for $p \to e^+\pi^0$ can be expressed as

$$\begin{aligned}\Gamma(p\to e^+\pi^0) &= \frac{m_p}{64\pi f_\pi^2}(\frac{{g_G}^4}{{M_U}^4}){A_L}^2\bar{\alpha_H}^2(1+D+F)^2[A_{SR}^2+A_{SL}^2 \\ &\times\ (1+|V_{ud}|^2)^2].\end{aligned} \tag{4}$$

In eq.(4) M_U represents degenerate mass of 12 superheavy gauge bosons and g_G is their coupling to quarks and leptons ($\alpha_G = g_G^2/4\pi$) at the GUT scale $\mu = M_U$. Here $\bar{\alpha}_H$= hadronic matrix elements, m_p =proton mass= 938.3

MeV, f_π =pion decay constant = 139 MeV and the chiral Lagrangian parameters are $D = 0.81$ and $F = 0.47$. V_{ud} represents the CKM- matrix element $(V_{CKM})_{12}$ for quark mixings.
The d = 6 operator when evolved from the GeV scale, short-distance renormalization factor from $\mu = M_U - M_Z$ turns out to be $A_{SL} \simeq A_{SR} \simeq A_{SD} \simeq 2.566$ and the long distance renormalization factor is $A_L \simeq 1.25$. These are estimated using values of gauge couplings in the relevant mass ranges, the anomalous dimensions and the one-loop beta-function coefficients. Using $A_R = A_L A_{SD} \simeq 3.20$, $F_q = 1 + (1 + |V_{ud}|^2)^2 \simeq 4.8$, we express inverse decay width for $p \to e^+\pi^0$ as

$$\Gamma^{-1}(p \to e^+\pi^0) = 1.01 \times 10^{34} yrs. \left[\frac{0.012\ GeV^3}{\alpha_H}\right]^2 \left[\frac{3.2}{A_R}\right]^2 \times \left[\frac{1/35.3}{\alpha_G}\right]^2 \left[\frac{4.8}{F_q}\right] \left[\frac{M_U}{3.2 \times 10^{15}}\right]^4, \quad (5)$$

where we have used $\alpha_H = \bar{\alpha_H}(1 + D + F) \simeq 0.012$ GeV^3 from recent lattice theory estimations. Using the one-loop values, $M_U = M_U^{(5)} = 10^{15.96}$ GeV $\alpha_G = 1./34.3$ and all other parameters as specified in eq.(5) gives

$$\tau_p^0 = 6.3 \times 10^{35} \quad yrs., \quad (6)$$

which is nearly 62.4 times longer than the current experimental limit and may be accessible to measurements by next generation proton decay searches. However two-loop effects combined with threshold effects from TeV-scale spectrum, and the Type-II seesaw scale in addition to small GUT-threshold effects [18] are likely to bring the predicted lifetime within the accessible range of future searches [15].

III. III. SPLIT-SUSY EXAMPLE WITH PRECISION UNIFICATION

As was noted in the original split-SUSY paper [16], the absence of the second Higgs doublet χ of eq.(3) in split-SUSY models distorts the one-loop precision unification. Here we suggest a way to restore precision unification which can be applied to all such split-SUSY models. The ultimate GUT scale theory in this model is supersymmetric. At first we carry out fine tuning in the GUT-scale superpotential [19] to have two Higgs doublets at the weak scale: one with mass in the hundreds of GeV range and a second one with mass in the TeV range. These two doublets can arise in $SO(10)$ models with two **10** fields at the GUT scale. The rest of the TeV scale spectrum is as in the split-SUSY models i.e. fermionic partners of Higgs fields and gluons. The super-partners of these fields are considered to be at 10^{11} GeV and the $SU(5)$ **15**-plet and its conjugate fields are at 10^{13} GeV, in the figure 2 below. We vary these scale later to study its effect on grand unification. They then lead to coupling unification as in Fig 2 below with $\alpha_{(5)}^{-1} = 24.3$ and $\alpha_{(10)}^{-1} = 20.2$ at $\mathrm{M}_U^{(5)} = 10^{15.96}$ GeV and $\mathrm{M}_U^{(10)} = 5 \times 10^{18}$ GeV, respectively.

Primarily due to increase in the GUT-gauge coupling, the proton lifetime for $p \to e^+\pi^0$ reduces by nearly 50% compared to the prediction in the non-SUSY case leading to

$$\tau_p^0 = 3.15 \times 10^{35} \quad yrs. \quad (7)$$

Although this prediction could possibly be probed by the next generation experiments [15], the situation could be more promising once two-loop and threshold effects are taken into account since they could pull this value downward e.g.in [18]. We have derived the above prediction for the SUSY scale of 10^{11} GeV. As we bring the SUSY scale downward (upward) the predicted lifetime decreases (increases).In particular we find that while the GUT scale would remain unchanged, the inverse fine structure constant at the GUT scale changes with $\alpha_{(5)}^{-1} = 19.3 - 25.8$ for $\mathrm{M}_{SUSY} = 10$ TeV -10^{13} GeV. The resulting variation of proton lifetime with M_{SUSY} is shown in Fig. 3. For $M_{SUSY} = 10$ TeV the predicted lifetime $\tau_p^0 = 2 \times 10^{35} yrs.$ which is nearly 20 times longer than the current experimental limit, is clearly accessible to future searches for the decay mode $p \to e^+\pi^0$.

IV. IV. FERMION MASSES AND MIXINGS

As is well known, fitting fermion masses in GUT theories starts with an extrapolation of known low scale masses to the GUT scale. This will depend on the nature of the theory from TeV scale to GUT scale. We perform this bottom-up extrapolation using the one-loop renormalization group equations [20] suitably modified for the model of sec. 2 where there is no SUSY till the GUT scale with the TeV scale spectrum of eq.(3) and the scalar **15**-plet at

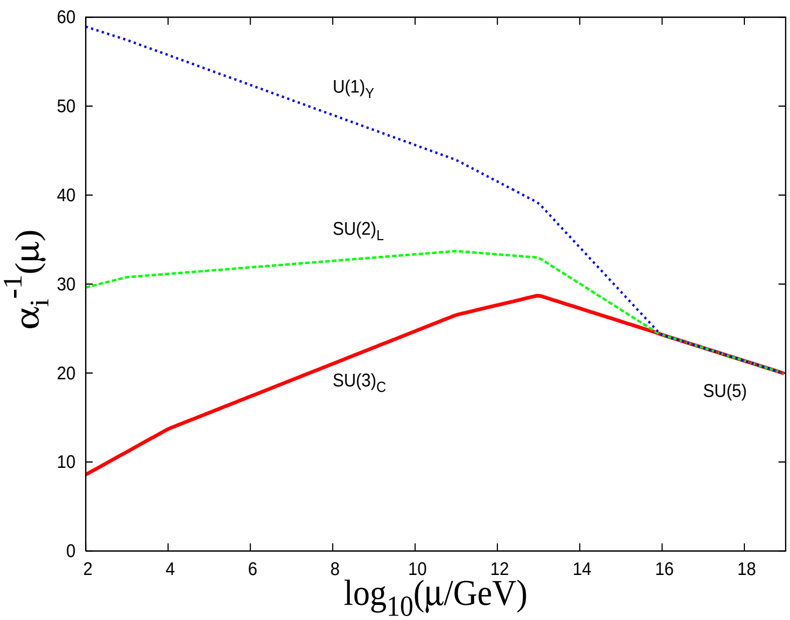

FIG. 2: Same as Fig.1 but in split-SUSY model with SUSY scale at 10^{11} GeV.

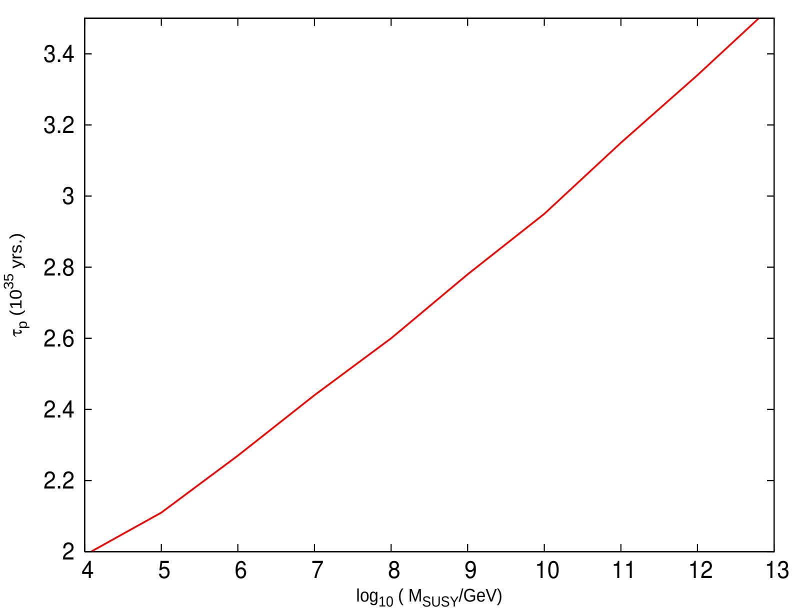

FIG. 3: Proton lifetime prediction as a function of SUSY scale in split-SUSY model with type-II seesaw dominance as described in the text.

$\mu = 10^{13}$ GeV. The extrapolated values for $\tan\beta = 10$ and $\tan\beta = 55$ are given in Table I which are similar to those obtained in [20]. In the split-SUSY case with $\tan\beta = 2-10$ we also obtain values similar to those obtained in [20] for $\tan\beta = 10$.
Also the extrapolated values of the CKM matrix elements are

$$\begin{aligned} V_{us} &= 0.2243 \pm 0.0016; \quad V_{ub} = 0.0032 \pm 0.0005, \\ V_{cb} &= 0.0351 \pm 0.0013, \quad J = (2.2 \pm 0.6) \times 10^{-5}. \end{aligned} \tag{8}$$

For estimating the model parameters, the mass-squared differences and mixing angles obtained from global fits to the

TABLE I: The renormalization group-extrapolated values of fermion masses at the GUT scale $\sim 10^{16}$ GeV in the new two-Higgs doublet model described in the text.

$\tan\beta$	55	10
m_e(MeV)	0.4402 ± 0.001	0.4395 ± 0.0003
m_μ(MeV)	92.9532 ± 0.058	92.7679 ± 0.12
m_τ(GeV)	1.7831 ± 0.037	1.5808 ± 0.0013
m_d(MeV)	1.6283 ± 0.41	1.5874 ± 0.6
m_s(MeV)	32.4341 ± 6	31.6157 ± 5.2
m_b(GeV)	1.2637 ± 0.6	$1.0734^{+0.14}_{-0.09}$
m_u(MeV)	0.6917 ± 0.2	0.6740 ± 0.25
m_c(MeV)	201.0028 ± 35	195.8392 ± 21
m_t(GeV)	66.3612 ± 38	$61.3505^{+30.3}_{-14.8}$

neutrino oscillation data are also essential

$$\begin{aligned}
\Delta m^2_{21} &= (7.65 \pm 0.23) \times 10^{-5}\text{eV}^2, \\
\Delta m^2_{31} &= (2.40 \pm 0.12) \times 10^{-3}\text{eV}^2, \\
\sin^2\theta_{12} &= 0.304 \pm 0.022, \\
\sin^2\theta_{23} &= 0.500 \pm 0.070, \\
\sin^2\theta_{13} &= 0.013 \pm 0.016. \qquad (9)
\end{aligned}$$

We have assumed normal or inverted hierarchial light neutrino masses so that renormalization group evolution has negligible effects on their masses and mixings.

Here we confine to the non-SUSY model and exploit the $SO(10)$ invariant Yukawa interaction using Higgs fields a complex $\mathbf{10_H}$ $(\equiv H)$, $\overline{\mathbf{126}}_\mathbf{H}(\equiv \bar{\Delta})$, and $\mathbf{120_H}$ $(\equiv \Sigma)$

$$\mathcal{L}_{\text{Yuk.}} = h\psi\psi H + f\psi\psi\bar{\Delta} + h'\psi\psi\Sigma, \qquad (10)$$

where the spinorial representation **16** for each generation has been denoted by ψ and generation indices have been suppressed. Not including the Yukawa coupling of $\mathbf{10}^*$ may be justified by the possible return of supersymmetry at the GUT scale or by a PQ symmetry. Near the GUT scale the formulae for Dirac type Yukawa matrices for quarks, charged leptons, and the neutrinos are expressed as [21]

$$\begin{aligned}
Y_u &= \bar{h} + r_2\bar{f} + r_3\bar{h}', \\
Y_d &= \frac{r_1}{\tan\beta}(\bar{h} + \bar{f} + \bar{h}'), \\
Y_l &= \frac{r_1}{\tan\beta}(\bar{h} - 3\bar{f} + c_l\bar{h}'), \\
Y_\nu &= \bar{h} - 3r_2\bar{f} + c_\nu\bar{h}', \qquad (11)
\end{aligned}$$

where $\bar{h}, \bar{h}'$, and $\bar{f}$ are related to the h, h', and f by suitable factors of vacuum expectation values and $r_i (i = 1, 2, 3)$ are mixing parameters which relate the two up and down type doublets (H_u, H_d) at low scale to the corresponding doublets in various GUT multiplets. In the case where **120** is replaced by another **10** field, we have $c_l = 1$ and $c_\nu = r_3$. In the context of type-II dominance, recently an interesting connection to the well known tri-bi-maximal mixing (TBM) pattern in the neutrino sector has been noted. Without the introduction of any additional symmetry but simply by choice of basis, the type-II seesaw formula permits TBM form for neutrino mass matrix; possible corrections to such TBM form can then arise from the charged lepton mass matrix [10]. In $SO(10)$ such mixings arise from generalized GUT scale constraints on quark masses and mixings.

From a comparison of fermion masses at the GUT scale given in Table I and other experimental data given in eq.(8) and eq.(9) with those used in refs. [10], it is noted that these input values at the GUT scale used to fit the model parameters are quite similar. Particularly the GUT scale masses in all parametrizations are similar to the one obtained in ref. [20]. As a result, the fitted values of the present model parameters including v_L are expected to be similar to those already obtained in [10]. We utilize these v_L values and estimate the scalar triplet mass (M_T) defined through $v_L = v^2/M_T$ in eq.(1). We obtain $M_T = 10^{11} - 10^{13}$ GeV. These values are easily accommodated by the two models discussed here without distorting their respective precision unification.

V. V. SUMMARY AND OUTLOOK

In summary, we have given two scenarios of gauge coupling unification in non-supersymmetric as well as split-SUSY $SO(10)$ models via $SU(5)$ route which helps to incorporate manifest type-II seesaw dominance. This puts the neutrino mass discussions in SO(10) with type II dominance on a sound footing and supplements the already known result [12] for the SUSY SO(10) case. The predictions for proton decay depend on the unification scenario and we find that for the two models, proton lifetime is about 20 to 60 times longer than the current lower bound on the $p \to e^+ + \pi^0$ mode [22] and can be within the reach of planned proton decay search experiments such as Hyper-K. The two-loop effects are expected to reduce the life time somewhat. Our work also suggests a way to restore precision unification in split-SUSY theories which can be adopted in all such models with high scale supersymmetry. In this case, the GUT scale derived is not only independent of the SUSY scales, but it is also identical to the corresponding non-SUSY GUT scale. The models predict TeV scale spectrum rich in cold dark matter candidates as well as new TeV mass particles which can be directly searched for at LHC.

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