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Abstract

We investigate the gluon distribution functions and their contributions to the Higgs boson production in pp collisions in the transverse momentum dependent factorization formalism. In addition to the usual azimuthal symmetric transverse momentum dependent gluon distribution, we find that the azimuthal correlated gluon distribution also contributes to the Higgs boson production. This explains recent findings on the additional contribution in the transverse momentum resummation for the Higgs boson production as compared to that for electroweak boson production processes. We further examine the small-x naive k_t-factorization in the dilute region and find that the azimuthal correlated gluon distribution contribution is consistently taken into account, and the result agrees with the transverse momentum dependent factorization formalism. We comment on the possible breakdown of the naive k_t-factorization in the dense medium region, due to the unique behaviors for the gluon distributions.

I. INTRODUCTION

Recently, several studies have found that the transverse momentum resummation for the gluon-fusion processes differ from those for the quark-antiquark annihilation (electroweak boson/the Drell-Yan lepton pair production) processes in the Collins-Soper-Sterman (CSS) framework [1, 2], in particular, in the Higgs boson production [3] and di-photon production [4] processes. Similar results have been found in the context of the soft-collinear-effective theory formalism for the Higgs boson production [5]. These results have raised some concern on the derivation of the CSS formalism and the associated factorization argument for the gluon-gluon fusion processes. In this paper, we re-examine the transverse momentum dependent (TMD) factorization for Higgs boson production in pp collisions. We find that there is an additional contribution in the leading power in the TMD factorization from the azimuthal correlated transverse momentum dependent gluon distribution [6]. With the complete TMD factorization results, the CSS resummation stands for the gluon-gluon fusion process.

Meanwhile, the azimuthal correlated gluon distribution, also referred as the linearly polarized gluon distribution, has been recently discussed in the context of the transverse momentum dependent factorization formalism in, for example, the dijet-correlation in deep inelastic scattering process [7], di-photon production in pp collisions [8]. This gluon distribution will lead to the azimuthal asymmetries in these reactions, and the experimental observation shall provide important information on it. Moreover, in Ref. [9] it was found that the azimuthal correlated gluon distribution has an unique behavior at small-x from the saturation model [10–12] calculations. This property emphasizes its special role in the study of the small-x gluon saturation in the high energy scattering processes, in particular, at the planed electron-ion collider [13, 14]. In this paper, we will also examine the TMD factorization for the Higgs boson production at small-x, by taking into account the azimuthal correlated gluon distribution contribution, and compare to the result obtained from the well-known small-x k_t-factorization formalism in the dilute region [15, 16], where the two gluon distribution functions are the same at small-x limit. The consistency between these two frameworks shed important insights on the factorization property for the hard processes at small-x. However, in the dense medium and small transverse momentum region, the azimuthal correlated gluon distribution is different from the usual gluon distribution function, and we can not write the Higgs boson production as the simple naive

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k_t-factorization formalism. This indicates that the naive k_t-factorization breaks down even for the color-neutral particle production in the dense region in hadronic scattering processes as also found in Ref. [17], similar to the situation for the heavy quark-antiquark pair production process [18].

The rest of the paper is organized as follows. In Sec. II, we introduce the leading order transverse momentum dependent gluon distributions, including the usual one and the azimuthal correlated gluon distribution function. We will also discuss the Collins-Soper evolution for these functions, which are important for the transverse momentum resummation. We will show that the azimuthal correlated gluon distribution also contributes to the Higgs boson production in pp collisions. The CSS resummation is provided in Sec.III, where the gluon distributions are calculated in terms of the integrated parton distributions at large transverse momentum (small $b_\perp$). In Sec.IV, we extend our discussions to the small-x region, where the TMD factorization and k_t-factorization formalisms are compared. We conclude our paper in Sec. V.

II. TRANSVERSE MOMENTUM DEPENDENT GLUON DISTRIBUTIONS AND HIGGS BOSON PRODUCTION

The transverse momentum dependent factorization is an important step to derive the CSS resummation for the Higgs boson production in pp collisions [1]. Following the Drell-Yan example, in our previous calculations [19], we have studied the factorization for the Higgs boson production in terms of the transverse momentum dependent gluon distributions, where however the azimuthal correlated gluon distribution function was not considered. In the following, we will find that its contribution is at the same order as the usual gluon distribution in the TMD factorization formalism. We consider, in general case, the Higgs boson production in pp collisions,

$$P_A + P_B \to H_0 + X \ , \tag{1}$$

where H_0 represents the scalar-Higgs boson, and the hadron A is moving $+\hat{z}$ direction and B along the $-\hat{z}$. Let us first introduce the spin-independent TMD gluon distributions, which

can be defined through the following matrix [19–22],

$$\mathcal{M}^{\mu\nu}(x, k_\perp, \mu, x\zeta, \rho) = \int \frac{d\xi^- d^2\xi_\perp}{P^+(2\pi)^3} e^{-ixP^+\xi^- + i\vec{k}_\perp \cdot \vec{\xi}_\perp}$$
$$\times \left\langle P | F_a^{+\mu}(\xi^-, \xi_\perp) \mathcal{L}_{vab}^\dagger(\xi^-, \xi_\perp) \mathcal{L}_{vbc}(0, 0_\perp) F_c^{\nu+}(0) | P \right\rangle , \qquad (2)$$

where $F_a^{\mu\nu}$ is the gluon field strength tensor. The light-cone components are defined as $k^\pm = (k^0 \pm k^3)/\sqrt{2}$. In the above equation, $P^+ = P_A^+$ represent the light-cone momentum of the hadron A, and x is the longitudinal momentum fraction carried by the gluon, whereas $k_\perp$ is the transverse momentum. For the TMD parton distributions, the gauge link $\mathcal{L}_v$ depends on the process [23]. In this paper, we focus on the Higgs boson production, and the gauge link is from the past: $\mathcal{L}_v(\xi^-, \xi_\perp; -\infty) = P\exp\left(-ig\int_{-\infty}^0 d\lambda v \cdot A(\lambda v + \xi)\right)$ in the covariant gauge, where $A^\mu = A_c^\mu t^c$ is the gluon potential in the adjoint representation, with $t_{ab}^c = -if_{abc}$. In a singular gauge, a transverse gauge link at the spatial infinity has to be included as well. Four-velocity v^μ is an off-light-cone vector $v^\mu = (v^-, v^+, v_\perp = 0)$ with $v^- \gg v^+$ to regulate the light-cone singularity for the TMD parton distributions. With this parametrization, the TMD parton distributions will depend on $\zeta^2 = (2v \cdot P)^2/v^2$. (In later part, we also use ζ_1^2 for the TMD gluon distribution from hadron A: $\zeta_1^2 = (2v \cdot P_A)^2/v^2$.) An evolution equation respect to ζ^2 is called the Collins-Soper evolution equation, and can be used to resum the large logrithms [1, 2].

In the context of the TMD factorization, we take into account the leading power contribution in terms of $P_\perp/M$ where $P_\perp$ and M are the transverse momentum and mass for the Higgs particle. To obtain the full result in the TMD factorization, we have to take into account the contributions from all the leading power gluon distribution functions. The leading power expansion of the matrix $\mathcal{M}^{\mu\nu}$ in the unpolarized nucleon contains two independent TMD gluon distributions [7, 8, 21, 22],

$$\mathcal{M}^{\mu\nu}(x, k_\perp) = \frac{1}{2}\left[xg(x, k_\perp)g_\perp^{\mu\nu} + xh_g(x, k_\perp)\left(\frac{2k_\perp^\mu k_\perp^\nu}{k_\perp^2} - g_\perp^{\mu\nu}\right)\right] , \qquad (3)$$

where $g_\perp^{\mu\nu}$ is defined as $g_\perp^{\mu\nu} = -g^{\mu\nu} + (P_A^\mu P_B^\nu + P_A^\nu P_B^\mu)/P_A \cdot P_B$. In the above parameterizations, $g(x, k_\perp)$ is the usual azimuthal symmetric TMD gluon distribution, and $h_g(x, k_\perp)$ the azimuthal correlated TMD gluon distribution function. h_g vanishes when integrating over transverse momentum for the matrix $\mathcal{M}^{\mu\nu}$, which means there is no integrated gluon distribution $h_g(x)$. In the literature, this function was also called "linearly polarized" gluon distribution. However, in order to differ from the true linearly polarized gluon distribution

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defined for the spin-1 hadrons [24, 25], we prefer to use the name of "azimuthal correlated" gluon distribution for the spin-1/2 hadrons, following the notation of Ref. [3]. Similar functional form has been discussed in the generalized parton distribution for the gluons as well (see, for example, Ref. [26]).

As mentioned above, the TMD parton distribution functions depend on the energy of the parenting hadrons, through the variable $\zeta^2 = (2v \cdot P)^2/v^2 \approx 2(P^+)^2 v^-/v^+$. This equation is better presented in the impact parameter space [20],

$$\zeta \frac{\partial}{\partial \zeta} g(x, b_\perp, x\zeta, \mu, \rho) = (K_g + G_g) g(x, b_\perp, x\zeta, \mu, \rho) , \tag{4}$$

where the gluon distribution in the impact parameter space $g(x, b)$ is the Fourier transform of the TMD distribution: $g(x, b_\perp) = \int d^2 k_\perp e^{ik_\perp \cdot b_\perp} g(x, k_\perp)$, and K_g and G_g are soft and hard evolution kernels, respectively. It is straightforward to extend the above equation to that for the azimuthal correlated gluon distribution h_g [27, 28],

$$\zeta \frac{\partial}{\partial \zeta} \tilde{h}_g^{\mu\nu}(x, b, x\zeta, \mu, \rho) = (K_g + G_g) \tilde{h}_g^{\mu\nu}(x, b, x\zeta, \mu, \rho) , \tag{5}$$

where $\tilde{h}_g$ is defined as

$$\tilde{h}_g^{\mu\nu}(x, b_\perp) = \frac{1}{2} \int d^2 k_\perp e^{ik_\perp \cdot b_\perp} \left(\frac{2k_\perp^\mu k_\perp^\nu}{k_\perp^2} - g_\perp^{\mu\nu} \right) h_g(x, k_\perp) . \tag{6}$$

In particular, we find that the sum of $K_g + G_g$ are the same for the above two equations and, at one-loop order, read,

$$K_g + G_g = -\frac{\alpha_s C_A}{\pi} \ln \frac{x^2 \zeta^2 b^2}{4} e^{2\gamma_E - \frac{3}{2}} . \tag{7}$$

Furthermore, the K_g and G_g obey the following renormalization group equation,

$$\mu \frac{\partial K_g}{\partial \mu} = -\mu \frac{\partial G_g}{\partial \mu} = -\gamma_{Kg} = -2 \frac{\alpha_s C_A}{\pi} . \tag{8}$$

The above evolution equations will be used to perform the transverse momentum resummation for the Higgs boson production in pp collisions.

For the TMD gluon distribution from hadron B, we can formulate similarly. We will also introduce an off-light-cone vector $\bar{v}^\mu = (\bar{v}^-, \bar{v}^+, \bar{v}_\perp = 0)$ with $\bar{v}^+ \gg \bar{v}^-$ to regulate the associated light-cone singularity, and energy dependent variable $\zeta_2^2 = 4(\bar{v} \cdot P_B)^2/\bar{v}^2$. The same Collins-Soper evolution equations can be derived as well.

The Higgs boson production in the gluon-fusion process can be calculated from the effective lagrangian,

$$\mathcal{L}_{eff} = -\frac{1}{4} g_\phi \Phi F^a_{\mu\nu} F^{a\mu\nu} , \tag{9}$$

in the heavy top quark limit, where Φ is the scalar field and g_ϕ is the effective coupling. We will use this effective lagrangian in the following calculations. We note the finite top quark mass effects will not change our general discussions. The leading order perturbative calculation produces Higgs particle with zero transverse momentum. Finite transverse momentum is generated by higher order gluon radiation contributions. However, at low transverse momentum these high order corrections introduces large logarithms of $\ln^2(M^2/P_\perp^2)$, which need to be resummed to make reliable predictions. The TMD factorization is a appropriate way to perform this resummation. In other words, at low transverse momentum $P_\perp \ll M$, the transverse momentum dependence can be factorized into various leading power TMD parton distributions, and the higher order corrections can be factorized into the relevant hard factors. The hard factors in the TMD factorization does not depend on the transverse momentum, and the resummation can be performed by solving the Collins-Soper evolution equations for the associated TMD parton distributions. The lesson from the recent studies [3] is that, in the TMD factorization, we have to include all leading power contributions. In particular, the azimuthal correlated gluon distribution h_g was completely ignored in the previous study [19]. It is straightforward to obtain this contribution in the TMD formula, and a similar factorization can be formulated as well. After adding this contribution, the Higgs boson production at low transverse momentum $P_\perp \ll M$ can be written as,

$$\begin{aligned}
\frac{d^3\sigma(M^2, P_\perp, y)}{d^2 P_\perp dy} = \sigma_0 \int d^2\vec{k}_{1\perp} d^2\vec{k}_{2\perp} d^2\vec{\ell}_\perp \delta^{(2)}(\vec{k}_{1\perp} + \vec{k}_{2\perp} + \vec{\ell}_\perp - \vec{P}_\perp) \\
\big\{ x_1 g(x_1, k_{1\perp}) x_2 g(x_2, k_{2\perp}) S(\ell_\perp, \mu, \rho) H(M^2, \mu, \rho) \\
+ \left(\frac{2(k_{1\perp} \cdot k_{2\perp})^2}{k_{1\perp}^2 k_{2\perp}^2} - 1 \right) x_1 h_g(x_1, k_{1\perp}) x_2 h_g(x_2, k_{2\perp}) \\
\times S_h(\ell_\perp, \mu, \rho) H_h(M^2, \mu, \rho) \big\} ,
\end{aligned} \tag{10}$$

where we follow the notations of Ref. [19], σ_0 is the leading-order scalar-particle production from two gluons, $\sigma_0 = \pi g_\phi^2/64$, and y and $P_\perp$ are Higgs particle's rapidity and transverse momentum, respectively. At low-transverse momentum, the longitudinal-momentum fractions x_1 and x_2 for the two incident gluons are related to the scalar particle's rapidity y through $x_1 = Me^y/\sqrt{S}$ and $x_2 = Me^{-y}/\sqrt{S}$, where S is the total center-of-mass energy squared

$S = (P_A + P_B)^2$. ζ_1 and ζ_2 are defined above as $\zeta_1^2 = 4(v \cdot P_A)/v^2$ and $\zeta_2^2 = 4(\bar{v} \cdot P_B)/\bar{v}^2$ and ρ is a scheme-dependent parameter to separate contributions to the soft and hard factors [19]. The above factorization result is accurate at leading power in $P_\perp^2/M^2$ at low transverse momentum. In particular, the interference between g and h_g is power suppressed in this limit.

The factorization for the first term with the usual gluon distribution follows the previous argument [19], and the relevant hard and soft factors have been calculated at one-loop order. Similar calculations can be done for the second term in Eq. (10). In particular, at one-loop order, we can factorize the gluon radiation contributions to the different factors in the factorization formula depending on the kinematic regions of the radiated gluon. For example, if the radiated gluon is parallel to the incoming hadron A, we factorize its contribution to the TMD gluon distribution h_g from A. If it is parallel to the hadron B, we include that contribution to the TMD gluon distribution h_g from B. When the gluon momentum is soft, we factorize its contribution to the soft factor. The hard factor is calculated from the hard gluon radiation in the virtual diagrams, because the real gluon radiation is power suppressed if all momentum components are hard in order of M. We have done the explicit calculations to show this factorization at one-loop order [29].

It is easy to find that the soft factors for the above two terms are identical: $S_h(\ell_\perp) = S(\ell_\perp)$. This is because the soft gluon radiations do not depend on the spin/polarization, and are defined identically for these two terms. In the one-loop calculations, the hard factor are extracted from the virtual diagrams for the cross section and parton distribution calculations, and we find that the hard factors are also the same, $H_h(M^2) = H(M^2)$ [29],

$$
\begin{aligned}
H_h^{(1)}(M^2, \mu^2, \rho) &= H^{(1)}(M^2, \mu^2, \rho) \\
&= \frac{\alpha_s C_A}{\pi} \left[\ln \frac{M^2}{\mu^2} \left(2\beta_0 + \frac{1}{2} \ln \rho^2 - \frac{3}{2} \right) - \frac{3}{4} \ln \rho^2 + \frac{1}{8} \ln^2 \rho^2 + \pi^2 + \frac{7}{2} \right] ,
\end{aligned}
\tag{11}
$$

where a special coordinate system has been chosen in which $x_1^2 \zeta_1^2 = x_2^2 \zeta_2^2 = \rho M^2$. In a recent calculation for the single-spin dependent observable, a similar conclusion was also found [28], which may indicate that all the hard factors in the TMD factorization are independent of the spin/polarization.

It is convenient to write down the TMD factorization formula in the impact parameter

space,

$$\frac{d^3\sigma(M^2, P_\perp, y)}{d^2 P_\perp dy} = \sigma_0 \int \frac{d^2\vec{b}}{(2\pi)^2} e^{-iP_\perp \cdot b_\perp} W(x_1, x_2, b, M^2) \ , \qquad (12)$$

where W contains contribution from the two terms in the TMD factorization,

$$W(x_1, x_2, b, M^2) = W_g(x_1, x_2, b, M^2) + W_h(x_1, x_2, b, M^2) \ , \qquad (13)$$

and W_g and W_h represent the contributions from the usual gluon distribution $g(x, b)$ and the azimuthal correlated gluon distribution $h_g(x, b)$, respectively,

$$W_g(x_1, x_2, b, Q^2) = S(b, \mu, \rho) H(M^2, \mu, \rho) x_1 g(x_1, b, \mu, \rho M^2, \rho) x_2 g(x_2, b, \mu, \rho M^2, \rho) \qquad (14)$$

$$W_h(x_1, x_2, b, Q^2) = 2 S(b, \mu, \rho) H(M^2, \mu, \rho) x_1 \tilde{h}_g^{\mu\nu}(x_1, b, \mu, \rho M^2, \rho) x_2 \tilde{h}_g^{\mu\nu}(x_2, b, \mu, \rho M^2, \rho) \qquad (15)$$

where the W_h comes from the specific tensor structure in the factorization formula Eq. (10). The convolutions in the transverse-momentum space now reduce to products in the impact parameter b-space. In the factorization formula, the large logarithms will show up as $\ln M^2 b^2$ in the various factors in the above equations. We need to resum these large logarithms.

III. RESUMMATION

The large logarithms in the factorization formulas in the last section are resummed by following the Collins-Soper-Sterman method. The two terms in Eq. (13) satisfy the Collins-Soper evolution equation separately,

$$\frac{\partial W_{g,h}(x_i, b, M^2)}{\partial \ln M^2} = (K + G') W_{g,h}(x_i, b, M^2) \ , \qquad (16)$$

where K and G' are soft and hard evolution kernels. Since the two gluon distributions obey the same Collins-Soper evolution equation and the hard factors are the same, the evolution kernels are the same as well. Combining the Collins-Soper evolution equations for the TMD gluon distributions of Eqs. (5,7) and the hard factors at one-loop order of Eq.(11), we find that,

$$K + G' = -\frac{\alpha_s C_A}{\pi} \ln\left(\frac{M^2 b^2}{4} e^{2\gamma_E - 2\beta_0}\right) \ , \qquad (17)$$

where the ρ dependence between various terms cancels out. Solving the above evolution equations, we obtain, [1]

$$W_{g,h}(x_i, b, M^2) = e^{-\mathcal{S}_{Sud}^{g,h}(M^2, b, C_1/C_2)} W_{g,h}(x_i, b, C_1^2/C_2^2/b^2) \ , \qquad (18)$$

where the large logarithms are included in the Sudakov form factors,

$$\mathcal{S}_{Sud} = \int_{C_1^2/b^2}^{C_2^2 M^2} \frac{d\mu^2}{\mu^2} \left[\ln \left(\frac{C_2^2 M^2}{\mu^2} \right) A(C_1, \mu) + B(C_1, C_2, \mu) \right] \ . \tag{19}$$

Here C_1 and C_2 are two parameters of order one. The functions A and B can be expanded perturbatively α_s, $A = \sum_{i=1}^{\infty} A^{(i)} \left(\frac{\alpha_s}{\pi} \right)^i$ and $B = \sum_{i=1}^{\infty} B^{(i)} \left(\frac{\alpha_s}{\pi} \right)^i$. Because the A coefficients come from soft factor which are the same for the two terms W_g and W_h, we expect A will be the same as well. On the other hand, B coefficients come from the hard factors in the TMD factorization formulas. Therefore, they could be different [30]. However, our one-loop calculations lead to the same hard factors and the same B coefficients for W_g and W_h. We expect that the effects discussed in Ref. [30] do not affect our calculations, and we conjecture that the B coefficients will be the same for these two terms at higher orders too. With this, we can combine the above two terms together as,

$$W(x_i, b, M^2) = e^{-\mathcal{S}_{Sud}(M^2, b, C_1/C_2)} \left[W_g(x_i, b, C_1^2/C_2^2/b^2) + W_h(x_i, b, C_1^2/C_2^2/b^2) \right] \ , \tag{20}$$

where $\mathcal{S}_{Sud}$ represents the universal Sudakov form factor for the Higgs boson production. Up to the one-loop order, we have verified this result.

The last step of the complete CSS resummation is to formulate the W_g and W_h of the right hand side of Eq. (20) at lower scale $C_1^2/C_2^2 b^2$ in terms of the integrated parton distributions,

$$W_g(x_i, b, C_1^2/C_2^2/b^2) = \sum_{ij} \int \frac{dx_1'}{x_1'} \frac{dx_2'}{x_2'} x_1' f_i(x_1', \mu) x_2' f_j(x_2', \mu)$$
$$\times C_{g/i}(x_1/x_1', C_1/C_2/b/\mu) C_{g/j}(x_2/x_2', C_1/C_2/b/\mu) \ , \tag{21}$$
$$W_h(x_i, b, C_1^2/C_2^2/b^2) = \sum_{ij} \int \frac{dx_1'}{x_1'} \frac{dx_2'}{x_2'} x_1' f_i(x_1', \mu) x_2' f_j(x_2', \mu)$$
$$\times C_{h/i}(x_1/x_1', C_1/C_2/b/\mu) C_{h/j}(x_2/x_2', C_1/C_2/b/\mu) \ , \tag{22}$$

where $f_{i,j}$ reprent the integrated quark/gluon distribution functions. For integrated gluon distribution, there is only the usual one, whereas the counterpart of h_g does not exist. The $C = \sum_{i=0} C^{(i)} \left(\frac{\alpha_s}{\pi} \right)^i$ coefficient functions can be calculated perturbatively. The coefficients $C_{g/i}$ have been calculated up to two-loop order [3], where $C_{h/i}$ are also calculated up to α_s order. In their calculations, the cross section $W(b) = W_g(b) + W_h(b)$ in the impact parameter ($b_\perp$) space is written as perturbative expansion of α_s, from which the relevant coefficients are extracted by comparing with the Eqs. (21,22). In the following, we will show how we can calculate $C_{h/i}$ from the TMD factorization formula Eq. (15).

To calculate the W_h in Eq. (22), we compute the azimuthal correlated gluon distribution $\tilde{h}_g^{\mu\nu}$ in terms of the integrated quark/gluon distribution functions and substitute into the factorization formula Eq. (15). First, we write down a similar factorization form for $\tilde{h}_g^{\mu\nu}(b_\perp)$,

$$\tilde{h}_g^{\mu\nu}(x, b_\perp) = \frac{1}{2}\left(g_\perp^{\mu\nu} - \frac{2b_\perp^\mu b_\perp^\nu}{b_\perp^2}\right) \int \frac{dx'}{x'} \tilde{C}_{h/i}(x/x', b_\perp, \mu) x' f_i(x', \mu) , \tag{23}$$

where the pre-factor of $\frac{1}{2}\left(g_\perp^{\mu\nu} - \frac{2b_\perp^\mu b_\perp^\nu}{b_\perp^2}\right)$ comes from the basic Lorentz structure for this function[1]. We know that there is no integrated h_g gluon distribution, which immediately leads to the zeroth order of α_s expansion of the above equation vanishes. As a consequence, the zeroth order of $C_{h/i}$ in Eq. (22) vanish as well,

$$C_{h/q}^{(0)} = C_{h/g}^{(0)} = 0 . \tag{24}$$

At order of α_s, we can generate the azimuthal correlated gluon distribution from the integrated quark/gluon distribution functions. For example, the contribution from the integrated gluon distribution is,

$$h_g(x, k_\perp) = \frac{\alpha_s}{\pi^2} C_A \frac{1}{k_\perp^2} \int \frac{dx'}{x'} \frac{1-\xi}{\xi} g(x') , \tag{25}$$

where $\xi = x/x'$. The Fourier transform into the impact parameter space leads to,

$$\tilde{h}_g^{\mu\nu}(x, b) = \frac{1}{2}\left(g_\perp^{\mu\nu} - \frac{2b_\perp^\mu b_\perp^\nu}{b_\perp^2}\right) \frac{\alpha_s}{\pi} C_A \int \frac{dx'}{x'} \frac{1-\xi}{\xi} g(x') . \tag{26}$$

This Fourier transform does not generate any divergence, which is consistent with the factorization formula of Eq. (23). Because the non-zero leading order expansion of Eq. (23) is at order α_s, the right hand side is associated with the leading order gluon distribution, and there is no collinear divergence. An interesting consequence is that the non-zero leading order coefficients do not depend on the factorization scale [3]. However, from the factorization formula Eq. (23), at order of α_s^2, we will find out the Fourier transform will lead to a collinear divergence which shall be absorbed into α_s order splitting of the integrated gluon distribution function. This indicates that order α_s^2 coefficients $\tilde{C}_{h/i}^{(2)}$ (and consequently the following $C_{h/i}^{(2)}$) will depend on the factorization scale.

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[1] In order to keep this factor traceless in the dimensional regulation calculations, the factor 2 in the bracket should be replaced by $d - 2$ where $d = 4 - \epsilon$ denotes the dimension. In our following calculations of Eqs. (26,27), since there are no divergence in the Fourier transform, we use $d = 4$.

Similar results are obtained for the azimuthal correlated gluon distribution in terms of the integrated quark distribution with the color-factor C_F instead of C_A,

$$\tilde{h}_g^{\mu\nu}(x,b) = \frac{1}{2}\left(g_\perp^{\mu\nu} - \frac{2b_\perp^\mu b_\perp^\nu}{b_\perp^2}\right)\frac{\alpha_s}{\pi}C_F\int\frac{dx'}{x'}\frac{1-\xi}{\xi}q(x') \ . \tag{27}$$

Combining Eqs. (26,27) with Eq. (15), we obtain,

$$C_{h/q}^{(1)} = C_F(1-\xi), \quad C_{h/g}^{(1)} = C_A(1-\xi) \ , \tag{28}$$

which reproduces the relevant resummation formula in Ref. [3].

From the above derivation, we find that the different resummation formalism for the gluon-gluon fusion processes as compared to that for the Drell-Yan lepton pair production process comes from the fact that there are two independent TMD gluon distribution functions at the leading order which contribute to the Higgs boson production at the same order in the limit of $P_\perp \ll M$. Although there are perturbative at different order in terms of the integrated quark/gluon distribution functions, we have to take into account the contributions from both functions in order to completely describe the Higgs boson production at low transverse momentum $P_\perp \ll M$. In particular, in certain kinematic region such as small-x region discussed in next section, the azimuthal correlated gluon distribution is as important as the usual one, where we have to include its contribution.

IV.   SMALL-x k_t-FACTORZATION

The two TMD gluon distribution functions at small-x have unique properties as recently discussed in Ref. [9]. Applying these results into the factorization formula of Eq. (10), we will be able to study the factorization of Higgs boson production in small-x region, where the well-known k_t-factorization formalism [10] has been applied too. In this section, we discuss the Higgs boson production in the small-x region, and examine the so-called naive k_t-factorization approach in this kinematic region.

From Eqs. (26), we notice that the gluon splitting contribution to the azimuthal correlated gluon distribution has the same small-x enhancement as the usual gluon distribution. Therefore, we expect the similar BFKL evolution for $h_g(x,k_\perp)$ at small-x in the dilute regime [14, 31–33], since the operator definition of $h_g(x,k_\perp)$ at low-x is also related to the quadrupole. As a consequence, the azimuthal correlated gluon distribution will be as important as the azimuthal symmetric one in this kinematic limit. In particular, from the

saturation model calculations of Ref. [9], we know that the azimuthal correlated gluon distribution function is the same as the usual gluon distribution function at small-x in the dilute region with $k_\perp \gg Q_s$, where Q_s is the characteristic scale in the saturation model. This is also consistent with the expectation from the BFKL evolution for these two functions [14]. Therefore, in this region, we can combine the two contributions in the factorization formula Eq. (10) into one,

$$\frac{d^3\sigma}{dyd^2P_\perp} = \sigma_0 \int d^2k_{1\perp}d^2k_{2\perp}\delta^{(2)}(P_\perp - k_{1\perp} - k_{2\perp})x_1g(x_1, k_{1\perp})x_2g(x_2, k_{2\perp})\frac{2(k_{1\perp} \cdot k_{2\perp})^2}{k_{1\perp}^2 k_{2\perp}^2} \ , \quad (29)$$

where we have used $g(x, k_\perp)$ to represent both g and h_g distribution functions, and neglected higher order corrections from the hard and soft factors in Eq. (10). The above result is exactly the same as that obtained in the naive-k_t factorization [15, 16] by taking the small transverse momentum limit $P_\perp \ll M$ [2]. By using the proper physical gluon polarization [10], one automatically takes into account the contribution from the azimuthal correlated gluon distribution in the naive k_t-factorization approach.

However, in the dense medium (large nucleus or extremely small-x), and in particular when $k_\perp \sim Q_s$, the azimuthal correlated gluon distribution is different from the usual gluon distribution [9] if they follow the definitions in Eq. (2,3). They are appropriate definitions for the gluon distribution in the Higgs boson production process [23]. Therefore, we can not combine these two terms into one universal structure as suggested in the naive k_t-factorization at small-x. This indicates that the naive-k_t factorization breaks down even for the color-neutral particle production in the dense medium in the hadronic scattering processes. Similar conclusion has also been drawn for the η' particle production in pA collisions in the saturation model calculations [17]. However, because of the large Higgs mass, we should be able to modify the naive-k_t factorization to establish an effective k_t-factorization for its production at low transverse momentum $P_\perp \ll M$, following the similar study in Ref. [23]. This will lead to consistent results as the TMD factorization of Eq. (10) with the small-x gluon distributions calculated in the dense region. An explicit calculation, including high order corrections, will be very important to investigate the QCD factorization property for the hard processes at small-x.

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[2] The difference between the results in Refs. [15, 16] vanishes in this limit.

V. CONCLUSIONS

In summary, in this paper, we have investigated the transverse momentum dependent gluon distribution functions and the Higgs boson production in pp collisions in the transverse momentum dependent factorization approach. We found that the azimuthal correlated gluon distribution contributes to the Higgs boson production in the leading power of P_T/M. After taking into account this contribution, we will be able to explain recent findings on the resummation for the Higgs boson production at moderate transverse momentum. It will be interesting to extend this study to the di-photon production process and the associated resummation formalism [4, 8].

We further extended our discussion to the small-x region, where we compared the TMD factorization result with the well-known naive k_t-factorization result, and found that they are consistent in the dilute region. We expect they will differ in the dense region, which may indicate the naive k_t-factorization is violated even for the neutral particle production at small-x region.

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