Ultrahigh energy predictions of proton-air cross sections from accelerator data: An update

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Ultra-high Energy Predictions of Proton-Air Cross Sections from Accelerator Data: an Update

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At the pp center of mass energy $\sqrt{s} = 57 \pm 7$ TeV, the Pierre Auger Observatory (PAO) collaboration has recently measured the proton-air inelastic production cross section $\sigma_{p-\rm air}^{\rm prod}$, using a cosmic ray beam consisting mainly of protons, with some helium contamination. Assuming a helium contamination of 25%, they subtracted 30 mb from their measured $\sigma_{p-\rm air}^{\rm prod}$, resulting in a p-air inelastic production cross section, $\sigma_{p-\rm air}^{\rm prod} = 475 \pm 22$ (stat.) $\pm_{15}^{20}$ (syst.) mb, where (stat.) is the statistical error and (syst.) is the systematic error, exclusive of helium contamination. Using this result in a Glauber calculation to obtain the pp inelastic cross section, at 57 TeV they found the inelastic pp cross section $\sigma_{\rm inel} = 90 \pm 7$ (stat.) $\pm_{11}^{9}$ (syst.) ± 1.5 (Glaub.) mb, where (syst.) is the systematic and (Glaub.) is the error associated with the Glauber calculation needed to convert $\sigma_{p-\rm air}^{\rm prod}$ to pp $\sigma_{\rm inel}$. Parameterization of the $\bar{p}p$ and pp cross sections incorporating analyticity constraints and unitarity has allowed us to make accurate extrapolations to ultra-high energies, and, using Glauber calculations, also accurately predict cosmic ray results for $\sigma_{p-\rm air}^{\rm prod}$. In this update for 57 TeV, we predict i) a pp total cross section, $\sigma_{\rm tot} = 133.4 \pm 1.6$ mb, using high energy predictions from a saturated Froissart bound parameterization of accelerator data on forward $\bar{p}p$ and pp scattering amplitudes and ii) a p-air inelastic production cross section, $\sigma_{p-\rm air}^{\rm prod} = 483 \pm 3$ mb, by using $\sigma_{\rm tot}$ together with Glauber theory. Using the PAO estimates of the variation of their measured $\sigma_{p-\rm air}^{\rm prod}$ with helium contamination, we were able to determine independently that the helium contamination was 19%, in reasonable agreement with their estimate of 25%. Our predictions agree with all available cosmic ray extensive air shower measurements, both in magnitude and in energy dependence. Further, by using our value for the pp total cross section at 57 TeV, Block and Halzen [1] have predicted that the pp inelastic cross section is $\sigma_{\rm inel} = 92.9 \pm 1.6$ mb, in agreement with the measured PAO value.

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Introduction. The PAO collaboration [2] has recently published the value for the proton-air inelastic production cross section $\sigma_{p-\rm air}^{\rm prod} = 505 \pm 22$ (stat.) $^{+20}_{-15}$ (syst.) mb, at $\sqrt{s} = 57 \pm 7$ GeV, where $\sqrt{s}$ is the pp center of mass (cms) energy, if the cosmic ray beam consists exclusively of protons. If the beam is contaminated by helium nuclei, the PAO collaboration [2] states that the measured cross section is reduced by 0, -12, -30 and -80 mb for a 0, 10, 25 and 50% contamination, respectively. They also measured [3] the inelastic pp cross section $\sigma_{\rm inel} = 90 \pm 7$(stat.) $\pm_{11}^{9}$ (syst.) ± 1.5 (Glaub.) mb, assuming a 25% helium contamination.

This note is a very short update of Block's [4] paper, "Ultra-high Energy Predictions of proton-air Cross Sections from Accelerator Data", which has made very accurate predictions at cosmic ray energies for the total pp cross section, σ_{pp}, from fits [6] to accelerator data that used adaptive data sifting algorithms [7] and analyticity constraints [8] that were not available in the earlier work of Block, Halzen and Stanev [9]. Using these results and the available cosmic ray measurements, an excellent fit to the then-available cosmic ray $\sigma_{p-\rm air}^{\rm prod}$ measurements was made. In this note, we discuss our predictions for the pp cms energy of 57 TeV at which the PAO results were obtained.

The purpose of this update is to i) make an *accurate* prediction of $\sigma_{p-\rm air}^{\rm prod}$, the 57 TeV cosmic ray p-air total cross section , and ii) from this prediction, estimate the helium contamination of the PAO result, iii) predict the 57 TeV $\sigma_{\rm tot}$, the pp total cross section and finally, iv) compare our prediction of $\sigma_{\rm inel}$, the pp inelastic cross section, with the PAO result [3]. This note is purposely intended to be very brief; the relevant details of the fundamental calculations can be found in the original paper of Block [4] and will not be presented here.

Determination of $\sigma_{pp}(s)$. Block and Halzen [6] have made an analytic amplitude fit that saturates the Froissart bound [10] to data for both the high energy total cross section and the ρ-value, where ρ is defined as the ratio of the real to the imaginary portion of the forward scattering amplitude, for both $\bar{p}p$ and pp interactions. Their high-energy behavior is parameterized using the analytic amplitude form

$$\sigma^{\pm}(\nu) = c_0 + c_1 \ln\left(\frac{\nu}{m}\right) + c_2 \ln^2\left(\frac{\nu}{m}\right) + \beta_{\mathcal{P}'}\left(\frac{\nu}{m}\right)^{\mu-1} \pm \delta\left(\frac{\nu}{m}\right)^{\alpha-1}, \quad (1)$$

$$\rho^{\pm}(\nu) = \frac{1}{\sigma^{\pm}(\nu)}\left\{\frac{\pi}{2}c_1 + c_2\pi\ln\left(\frac{\nu}{m}\right) - \beta_{\mathcal{P}'}\cot\left(\frac{\pi\mu}{2}\right)\left(\frac{\nu}{m}\right)^{\mu-1} + \frac{4\pi}{\nu}f_+(0) \pm\delta\tan\left(\frac{\pi\alpha}{2}\right)\left(\frac{\nu}{m}\right)^{\alpha-1}\right\}, \quad (2)$$

where the upper sign is for pp and the lower for $\bar{p}p$ scattering Here ν is the laboratory energy, m the proton mass, $\mu = 0.5$, and $f_+(0)$ is a dispersion relation subtraction constant. The 7 real constants $c_0, c_1, c_2, \beta_{\mathcal{P}'}, \delta, \alpha$ and $f_+(0)$ are parameters of the fit. At high energies, s, the square of the cms energy, approaches $2m\nu$; hence, we see from Eq. (1) that the cross sections behave as $\ln^2 s$ at high energies, thus saturating the Froissart bound [10]. From Eq. (2), we see that $\rho \to 0$ as $1/\ln s$ as $s \to \infty$.

Using 4 analyticity constraints[8], resulting from finite energy sum rules that used very high accuracy low-energy cross section measurements ($2 \le \sqrt{s} \le 4$ GeV), they anchored both the cross sections $\sigma_{\bar{p}p}$ and σ_{pp} and their laboratory energy derivatives to data at $\sqrt{s} = 4$ GeV, thus reducing the number of parameters to from 7 to 4. The fit was to data with $6 \le \sqrt{s} \le 1800$ GeV. This use of analyticity constraints resulted in an excellent fit that, in turn, constrained pp cross sections at cosmic ray energies to an accuracy $\sim 1-2\%$, even though (conflicting) Tevatron data provided the highest energy input. The Block and Halzen fits [6] to the pp and $\bar{p}p$ cross sections are shown in Fig. 1. The PAO energy of 57 TeV is indicated by the dotted line crossing the fit. Our predicted pp total cross section at 57 TeV is $\sigma_{\rm tot} = 133.4 \pm 1.6$ mb For brevity, we have not shown the fit to ρ; see Block [4].

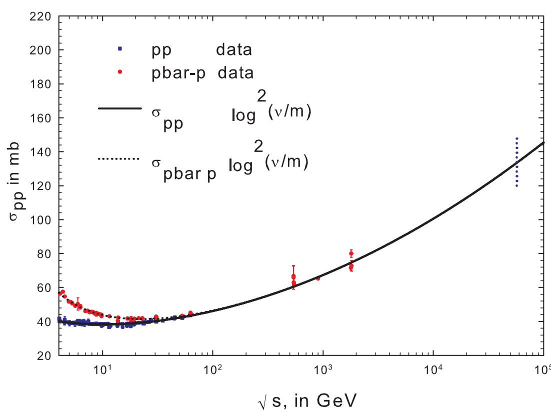

FIG. 1: The fitted total cross section, $\sigma_{\rm tot}$, for $\bar{p}p$ (dashed curve) and pp (dot-dashed curve) from Eq. (1), in mb vs. $\sqrt{s}$, the cms energy in GeV, taken from BH [6]. The $\bar{p}p$ data used in the fit are the (red) circles and the pp data are the (blue) squares. The fitted data were anchored by values of $\sigma_{\rm tot}^{\bar{p}p}$ and $\sigma_{\rm tot}^{pp}$, together with the energy derivatives $d\sigma_{\rm tot}^{\bar{p}p}/d\nu$ and $d\sigma_{\rm tot}^{pp}/d\nu$ at 6 GeV using FESR, as described in Ref. [6]. The vertical dotted line at 57000 GeV that intercepts the fit indicates the pp cms of the PAO cosmic ray experiment [2].

Comparison of $\sigma_{p-\rm air}$ with cosmic ray data. In Fig. 2 we have plotted all available cosmic ray data for $\sigma_{p-\rm air}^{\rm prod}$, the proton-air inelastic production cross section, in mb, as a function of $\sqrt{s}$, in GeV. The $\sigma_{p-\rm air}^{\rm prod}$ curve, derived from the total cross section fit of Eq. (1) and utilizing the elastic slope parameter $B \equiv d\left[\ln d\sigma_{\rm el}/dt\right]|_{t=0}$ in a Glauber calculation, is taken from Block [4]. The Glauber method [5], taking into account inelastic screening as described in Ref. [4], converted the total pp cross

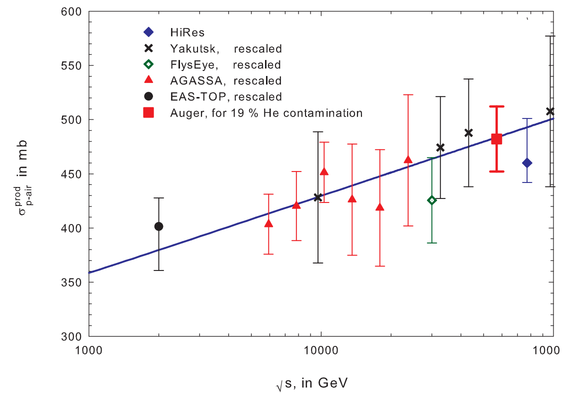

FIG. 2: A χ^2 fit of the renormalized AGASA, EASTOP, Fly's Eye and Yakutsk data for $\sigma_{p-\rm air}^{\rm prod}$, in mb, as a function of the energy, $\sqrt{s}$, in GeV. The HiRes point (solid diamond), at $\sqrt{s} = 57000$ GeV, is model-independent and has not been renormalized. The renormalized ARGO-YBJ data were not used in the fit. Details of the renormalization are given in Block [4], as are references to the cosmic ray data shown in the Figure. The large (red) square is the Auger measurement, adjusted for a 19% helium contamination (see text).

section into the *measured* p-air cross section $\sigma_{p-\rm air}^{\rm prod} = \sigma_{\rm p-air} - \sigma_{\rm p-air}^{\rm el} - \sigma_{\rm p-air}^{\rm q-el}$, where $\sigma_{\rm p-air}$ is the *total* p-air cross section obtained from the total pp cross section $\sigma_{\rm tot}$, and $\sigma_{\rm p-air}^{\rm el}$ and $\sigma_{\rm p-air}^{\rm q-el}$, are the elastic and quasi-elastic p-air cross sections, respectively—for details, see Block [4]. All of the cosmic ray data but the HiRes point were renormalized using a k-factor (k=1.264). For brevity, we omit the discussion of this renormalization procedure as well as detailed references to the cosmic data used in the Figure; again, for complete information, see Block [4].

The new point added in Fig. 2 is the large (red) square, whose central value is our prediction of $\sigma_{p-\rm air}^{\rm prod}$ for 57 TeV and whose error [2] is the total PAO experimental error excluding beam contamination uncertainty. As we will show in the next Section, this value, $\sigma_{p-\rm air}^{\rm prod} = 482 \pm 30$ mb, has a central value that corresponds to a 19% helium contamination.

Determination of the helium contamination in the cosmic ray 'proton' air showers. The PAO collaboration [2] notes that "We recognise (sic) and identify the unknown mass composition of cosmic rays as the major source of systematic uncertainty for the proton-air cross-section analysis and we evaluate its impact on the final result." They estimated that their best value for helium contamination was 25%. In this Section, we obtain an independent confirmation of this estimate.

We plot in Fig. 3 the helium fraction of the cosmic ray beam, in %, vs. the PAO collaboration's [2] corrected measurements for $\sigma_{p-\rm air}^{\rm prod}$, in mb. The large cross marks the point on the curve that is our prediction for the p-air inelastic production cross section, $\sigma_{p-\rm air}^{\rm prod} = 482$ mb, which corresponds to a $19 \pm 1\%$ helium contamination, in

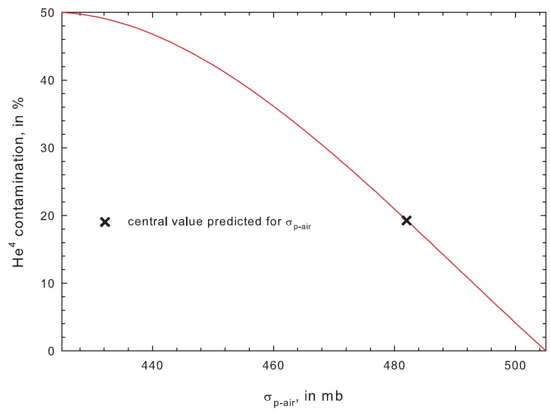

FIG. 3: Helium contamination of the cosmic ray beam, in %, vs. $\sigma_{p-\mathrm{air}}^{\mathrm{prod}}$, the inelastic p-air cross section, in mb, for the PAO experiment [2]. The large cross is our prediction for $\sigma_{p-\mathrm{air}}^{\mathrm{prod}}$, corresponding to a 19% helium contamination (see text).

qualitative agreement with the Auger estimate of 25%.

Predictions of the pp inelastic cross section, σ_{inel}, at very high energies. As mentioned earlier, the PAO collaboration [3] also measured the inelastic *pp* scattering at 57 TeV. By using a Glauber calculation, they converted their measured value of $\sigma_{p-\mathrm{air}}^{\mathrm{prod}}$ (after allowing for a 25 % helium contamination) into a *pp* inelastic cross section. They found $\sigma_{\mathrm{inel}} = 90\pm7$ (stat.)$\pm^{9}_{11}$ (syst.)±1.5 (Glaub.) mb. Using our predicted *pp* total cross section $\sigma_{\mathrm{tot}} = 133.4 \pm 1.6$ mb, Block and Halzen [1] determined that the *pp* inelastic cross section at 57 TeV was given by $\sigma_{\mathrm{inel}} = 92.9 \pm 1.6$ mb, which is in excellent agreement with the PAO value.

Further, we note that the 7 TeV LHC inelastic cross sections measurements of Atlas [11] and CMS [12], $\sigma_{\mathrm{inel}} = 69.1 \pm 7.4$ mb and 68.0 ± 5.1 mb, respectively, are also in excellent agreement with the predicted value of $\sigma_{\mathrm{inel}} = 69.0 \pm 1.3$ mb of Ref. [13], which uses the same scheme employed by Block and Halzen [1] that predicted the 57 TeV *pp* inelastic cross section. Since these inelastic cross section predictions employ the *same pp* total cross section predictions [1, 13] used in this note to determine $\sigma_{p-\mathrm{air}}^{\mathrm{prod}}$, this consistency at the highest available energies is most encouraging.

Conclusions. At $\sqrt{s} = 57$ TeV, we conclude that: i) the total *pp* cross section is $\sigma_{\mathrm{tot}} = 133.4 \pm 1.6$ mb, ii) the PAO p-air inelastic production cross section—after correction for a 19% helium contamination—is given by $\sigma_{p-\mathrm{air}}^{\mathrm{prod}} = 482 \pm 30$ mb. and iii) our prediction for the *pp* total cross section, σ_{tot} taken from Ref. [1], yields a *pp* inelastic cross section $\sigma_{\mathrm{inel}} = 92.9 \pm 1.6$ mb, compared to the PAO result [3] of $\sigma_{\mathrm{inel}} = 90 \pm 7$ (stat.) $\pm^{9}_{11}$ (syst.) $\pm$ 1.5 (Glaub.) mb.

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