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Nonzero θ_{13} for Neutrino Mixing in a Supersymmetric $B-L$ Gauge Model with T_7 Lepton Flavor Symmetry

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We discuss how $\theta_{13} \neq 0$ is accommodated in a recently proposed renormalizable model of neutrino mixing using the non-Abelian discrete symmetry T_7 in the context of a supersymmetric extension of the Standard Model with gauged $U(1)_{B-L}$. We predict a correlation between θ_{13} and θ_{23}, as well as the effective neutrino mass m_{ee} in neutrinoless double beta decay.

In a recent paper [1], a supersymmetric $B-L$ gauge model with T_7 lepton flavor symmetry is proposed with the following desirable features. (1) Neutrino tribimaximal mixing is achieved in a renormalizable theory, without the addition of auxiliary symmetries and particles. (2) The resulting neutrino mass matrix depends on only two complex parameters, and is of the same form already considered some time ago [2], using the discrete symmetry A_4 [3, 4]. (3) The charged-lepton Yukawa sector exhibits a residual discrete Z_3 symmetry, i.e. lepton flavor triality [5, 6], under which $e, \mu, \tau \sim 1, \omega^2, \omega$, where $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$. (4) There are physical scalar doublets transforming as ω, ω^2 which will decay exclusively into leptons such that lepton flavor triality is conserved. (5) If the new gauge boson Z' corresponding to the spontaneous symmetry breaking of $B-L$ [7] has a mass around 1 TeV, its production and decay into these exotic scalars may be observable at the Large Hadron Collider (LHC).

Recently, the T2K Collaboration has announced that a new measurement [8] has yielded a nonzero θ_{13} for neutrino mixing, i.e.

$$0.03\ (0.04) \leq \sin^2 2\theta_{13} \leq 0.28\ (0.34) \tag{1}$$

for $\delta_{CP} = 0$ and normal (inverted) hierarchy of neutrino masses. This indicates a possibly significant deviation from tribimaximal mixing [9] where $\theta_{13} = 0$, $\tan^2\theta_{12} = 1/2$, and $\sin^2 2\theta_{23} = 1$ are predicted. Whereas the tribimaximal pattern has an elegant theoretical interpretation [4] in terms of the simplest application of A_4 [3], deviations from it are expected [4, 10]. In this paper, we present a variation of our previous T_7 proposal [1] and show how a different choice of the residual symmetry of the soft terms of this model will lead to a four-parameter neutrino mass matrix with nonzero θ_{13} and predicts a strong correlation between θ_{13} and θ_{23} as well as the effective neutrino mass m_{ee} in neutrinoless double beta decay.

The tetrahedral group A_4 (12 elements) is the smallest group with a real $\underline{3}$ representation. The Frobenius group T_7 (21 elements) is the smallest group with a pair of complex $\underline{3}$ and $\underline{3}^*$ representations. It is generated by

$$a = \begin{pmatrix} \rho & 0 & 0 \\ 0 & \rho^2 & 0 \\ 0 & 0 & \rho^4 \end{pmatrix}, \quad b = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \tag{2}$$

where $\rho = \exp(2\pi i/7)$, so that $a^7 = 1$, $b^3 = 1$, and $ab = ba^4$. The character table of T_7 (with $\xi = -1/2 + i\sqrt{7}/2$) is given by

class	n	h	χ_1	$\chi_{1'}$	$\chi_{1''}$	χ_3	χ_{3^*}
C_1	1	1	1	1	1	3	3
C_2	7	3	1	ω	ω^2	0	0
C_3	7	3	1	ω^2	ω	0	0
C_4	3	7	1	1	1	ξ	ξ^*
C_5	3	7	1	1	1	ξ^*	ξ

TABLE I: Character table of T_7.

The group multiplication rules of T_7 include

$$\underline{3} \times \underline{3} = \underline{3}^*(23, 31, 12) + \underline{3}^*(32, 13, 21) + \underline{3}(33, 11, 22), \tag{3}$$

$$\begin{aligned} \underline{3} \times \underline{3}^* &= \underline{3}(21^*, 32^*, 13^*) + \underline{3}^*(12^*, 23^*, 31^*) \\ &+ \underline{1}(11^* + 22^* + 33^*) + \underline{1}'(11^* + \omega 22^* + \omega^2 33^*) \\ &+ \underline{1}''(11^* + \omega^2 22^* + \omega 33^*). \end{aligned} \tag{4}$$

Note that $\underline{3} \times \underline{3} \times \underline{3}$ has two invariants and $\underline{3} \times \underline{3} \times \underline{3}^*$ has one invariant. These serve to distinguish T_7 from A_4 and $\Delta(27)$. We note that T_7 was first considered for quark and lepton masses some time ago [11].

Under T_7, let $L_i = (\nu, l)_i \sim \underline{3}$, $l_i^c \sim \underline{1}, \underline{1}', \underline{1}''$, $i = 1,2,3$, $\Phi_i = (\phi^+, \phi^0)_i \sim \underline{3}$, and $\Phi'_i = (\phi'^0, -\phi'^-)_i \sim \underline{3}^*$. The Yukawa couplings $L_i l_j^c \Phi'_k$ generate the charged-lepton mass matrix

$$\begin{aligned} M_l &= \begin{pmatrix} f_1 v_1' & f_2 v_1' & f_3 v_1' \\ f_1 v_2' & \omega^2 f_2 v_2' & \omega f_3 v_2' \\ f_1 v_3' & \omega f_2 v_3' & \omega^2 f_3 v_3' \end{pmatrix} \\ &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_3 \end{pmatrix} v, \end{aligned} \tag{5}$$

if $v_1' = v_2' = v_3' = v'/\sqrt{3}$, as in the original A_4 proposal [3].

Let $\nu_i^c \sim \underline{3}^*$, then the Yukawa couplings $L_i \nu_j^c \Phi_k$ are allowed, with

$$M_D = f_D \begin{pmatrix} 0 & v_1 & 0 \\ 0 & 0 & v_2 \\ v_3 & 0 & 0 \end{pmatrix} = \frac{f_D v}{\sqrt{3}} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \tag{6}$$

for $v_1 = v_2 = v_3 = v/\sqrt{3}$ which is necessary for consistency since $v_1' = v_2' = v_3' = v'/\sqrt{3}$ has already been assumed for M_l. Note that Φ and Φ' have $B - L = 0$, and both are necessary because of supersymmetry.

Now add the neutral Higgs singlets $\chi_i \sim \underline{3}$ and $\eta_i \sim \underline{3^*}$, both with $B - L = -2$. Then there are two Yukawa invariants: $\nu_i^c \nu_j^c \chi_k$ and $\nu_i^c \nu_j^c \eta_k$ (which has to be symmetric in i, j). Note that $\chi_i^* \sim \underline{3^*}$ is not the same as $\eta_i \sim \underline{3^*}$ because they have different $B - L$. This means that both $B - L$ and the complexity of the $\underline{3}$ and $\underline{3^*}$ representations in T_7 are required for this scenario. The heavy Majorana mass matrix for ν^c is then

$$\begin{aligned} M_{\nu^c} &= h \begin{pmatrix} u_2 & 0 & 0 \\ 0 & u_3 & 0 \\ 0 & 0 & u_1 \end{pmatrix} + h' \begin{pmatrix} 0 & u_3' & u_2' \\ u_3' & 0 & u_1' \\ u_2' & u_1' & 0 \end{pmatrix} \\ &= \begin{pmatrix} A & C & B \\ C & D & C \\ B & C & D \end{pmatrix}, \end{aligned} \tag{7}$$

where $A = hu_2$, $B = h'u_2'$, $C = h'u_1' = h'u_3'$, and $D = hu_1 = hu_3$ have been assumed. This means that the residual symmetry in the singlet Higgs sector is Z_2, whereas that in the doublet Higgs sector is Z_3. This misalignment is different from that assumed previously [1], but is nevertheless achievable with suitably chosen soft terms, i.e. $\chi_2^* \chi_2$, ${\chi_2'}^* \chi_2'$, $\chi_2 \chi_2'$ + H.c., $\eta_2^* \eta_2$, ${\eta_2'}^* \eta_2'$, $\eta_2 \eta_2'$ + H.c., $\chi_1^* \chi_1 + \chi_3^* \chi_3$, ${\chi_1'}^* \chi_1' + {\chi_3'}^* \chi_3'$, $\chi_1 \chi_1' + \chi_3 \chi_3'$ + H.c., $\eta_1^* \eta_1 + \eta_3^* \eta_3$, ${\eta_1'}^* \eta_1' + {\eta_3'}^* \eta_3'$, $\eta_1 \eta_1' + \eta_3 \eta_3'$ + H.c., $\chi_2 \eta_2'$ + H.c., $\chi_2' \eta_2$ + H.c., $\chi_2(\eta_1' + \eta_3')$ + H.c., $\chi_2'(\eta_1 + \eta_3)$ + H.c., $(\chi_1 + \chi_3)\eta_2'$ + H.c., $(\chi_1' + \chi_3')\eta_2$ + H.c., $(\chi_1 + \chi_3)(\eta_1' + \eta_3')$ + H.c., $(\chi_1' + \chi_3')(\eta_1 + \eta_3)$ + H.c.

The seesaw neutrino mass matrix is now

$$\begin{aligned} M_\nu &= -M_D M_{\nu^c}^{-1} M_D^T \\ &= \frac{-f_D^2 v^2}{3 \det(M_{\nu^c})} \begin{pmatrix} AD - B^2 & C(B-A) & C(B-D) \\ C(B-A) & AD - C^2 & C^2 - BD \\ C(B-D) & C^2 - BD & D^2 - C^2 \end{pmatrix}, \end{aligned} \tag{8}$$

where $\det(M_{\nu^c}) = A(D^2 - C^2) + 2BC^2 - D(B^2 + C^2)$. Redefining the parameters A, B, C, D to absorb the overall constant, we obtain the following neutrino mass matrix in the tribimaximal basis:

$$\begin{aligned} &\mathcal{M}_\nu^{(1,2,3)} \\ &= \begin{pmatrix} \frac{D(A+D-2B)}{2} & \frac{C(2B-A-D)}{\sqrt{2}} & \frac{D(A-D)}{2} \\ \frac{C(2B-A-D)}{\sqrt{2}} & AD - B^2 & \frac{C(D-A)}{\sqrt{2}} \\ \frac{D(A-D)}{2} & \frac{C(D-A)}{\sqrt{2}} & \frac{AD+D^2+2BD-4C^2}{2} \end{pmatrix}. \end{aligned} \tag{9}$$

This is achieved by first rotating with the 3×3 unitary matrix of Eq. (5), which converts it to the (e, μ, τ) basis, then by Eq. (10) below. Note that for $D = A$ and $C = 0$, this matrix becomes diagonal: $m_1 = A(A - B), m_2 = A^2 - B^2, m_3 = A(A + B)$, which is the tribimaximal limit. Normal hierarchy of neutrino masses is obtained if $B \simeq A$ and inverted hierarchy is obtained if $B \simeq -2A$.

The neutrino mixing matrix U has 4 parameters: s_{12}, s_{23}, s_{13} and δ_{CP} [12]. We choose the convention $U_{\tau 1}, U_{\tau 2}, U_{e3}, U_{\mu 3} \rightarrow -U_{\tau 1}, -U_{\tau 2}, -U_{e3}, -U_{\mu 3}$ to conform with that of the tribimaximal mixing matrix

$$U_{TB} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}. \tag{10}$$

then

$$\mathcal{M}_\nu^{(1,2,3)} = \begin{pmatrix} m_1 & m_6 & m_4 \\ m_6 & m_2 & m_5 \\ m_4 & m_5 & m_3 \end{pmatrix}$$
$$= U_{TB}^{-1} U \begin{pmatrix} m_1' & 0 & 0 \\ 0 & m_2' & 0 \\ 0 & 0 & m_3' \end{pmatrix} U^{-1} U_{TB}, \quad (11)$$

where $m'_{1,2,3}$ are the physical neutrino masses, with

$$m_2' = \pm\sqrt{{m_1'}^2 + \Delta m_{21}^2}, \quad (12)$$
$$m_3' = \pm\sqrt{{m_1'}^2 + \Delta m_{21}^2/2 + \Delta m_{32}^2} \text{ (normal hierarchy)}, \quad (13)$$
$$m_3' = \pm\sqrt{{m_1'}^2 + \Delta m_{21}^2/2 - \Delta m_{32}^2} \text{ (inverted hierarchy)}. \quad (14)$$

If U is known, then all $m_{1,2,3,4,5,6}$ are functions only of m_1'.

In our model, the neutrino mass matrix has only 4 parameters A, B, C, D, so there are 2 conditions on $m_{1,2,3,4,5,6}$. They are given by

$$A = D + \frac{2m_4}{D}, \; B = D + \frac{m_4 - m_1}{D},$$
$$\frac{C}{D} = -\frac{m_6}{m_1\sqrt{2}} = -\frac{m_5}{m_4\sqrt{2}}, \quad (15)$$
$$D^2 = \frac{(m_1 - m_4)^2}{2m_1 - m_2} = \frac{m_1^2(m_3 + m_1 - 2m_4)}{2m_1^2 - m_6^2}. \quad (16)$$

We now input the allowed ranges of values for Δm_{21}^2, Δm_{32}^2, s_{12}, s_{23}, s_{13} assuming $\delta_{CP} = 0$. In that case, A, B, C, D can be chosen real. We then obtain $m_{1,2,3,4,5,6}$ as a function of m_1'. We now solve for m_1' using the condition $m_1 m_5 = m_4 m_6$ from Eq.(15). Using this value of m_1', we check Eq.(16) to see if the input values are allowed. In this way, we are able to find a strong correlation between s_{13} and s_{23} as shown in Fig. 1. It is very

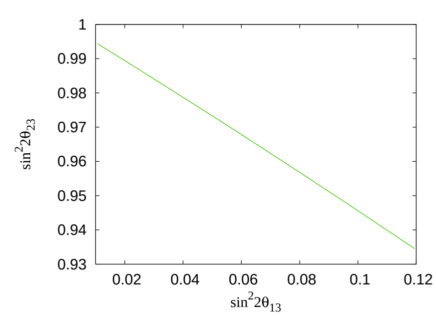

FIG. 1: $\sin^2 2\theta_{23}$ versus $\sin^2 2\theta_{13}$ for $\sin^2 2\theta_{12}$=0.87.

well approximated by

$$\sin^2 2\theta_{23} \simeq 1 - \frac{1}{2}\sin^2 2\theta_{13}, \quad (17)$$

for all solutions. Using [12] $\sin^2 2\theta_{13} < 0.135$, this implies $\sin^2 2\theta_{23} > 0.93$.

The effective neutrino mass m_{ee} in neutrinoless double beta decay is given by

$$m_{ee} = \frac{1}{3}|2m_1 + m_2 + 2\sqrt{2}m_6| \quad (18)$$
$$= \frac{1}{3}|2AD + D^2 - 2BD - B^2 + 2C(2B - A - D)|,$$

and the kinematic ν_e mass in nuclear beta decay is $m_{\nu_e} = \sum_i |U_{ei}^2 m_i'|$.

We find solutions for both normal and inverted hierarchies, using the central values of $\Delta m_{32}^2 = 2.40 \times 10^{-3}$ eV^2 and $\Delta m_{21}^2 = 7.65 \times 10^{-5}$ eV^2. We take 3 representative values of $\sin^2 2\theta_{12}$, i.e. 0.84, 0.87, 0.90. In Figures 2 to 4 we show the solutions for the physical neutrino masses as well as m_{ee} and m_{ν_e} as functions of $\sin^2 2\theta_{13}$ in the case of normal hierarchy. In Figures 5 to 7 we show these in the case of inverted hierarchy. For $\sin^2 2\theta_{12} = 0.87$ (corresponding to $\tan^2\theta_{12} = 0.47$), we plot in Figures 8 and 9 the T_7 parameters $(A + 2D)/3$, B, C, and $(A - D)/2$ in the case of normal and inverted hierarchies. It is clear that C and $(A - D)/2$ are small, showing that these solutions deviate only slightly from the tribimaximal limit. In particular, $C = 0$ exactly works for normal hierarchy, but it implies $\sin^2 2\theta_{12} > 8/9$, i.e. $\tan^2\theta_{12} > 1/2$ [4].

In conclusion, we have shown that a previously proposed [1] $T_7/B - L$ model of neutrino masses has a variation (supported by a Z_2 residual symmetry) which allows a nonzero θ_{13} and predicts the strong correlation $\sin^2 2\theta_{23} \simeq 1 - \sin^2 2\theta_{13}/2$ which is consistent with all data, including the recent T2K measurement [8]. We also predict the effective neutrino mass m_{ee} in neutrinoless double beta decay.

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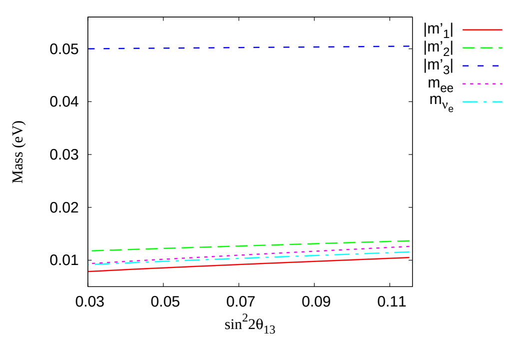

FIG. 2: Normal hierarchy solution in case of $\sin^2 2\theta_{12}$=0.84.

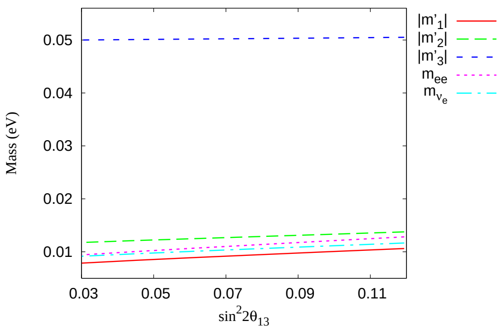

FIG. 3: Normal hierarchy solution in case of $\sin^2 2\theta_{12}$=0.87.

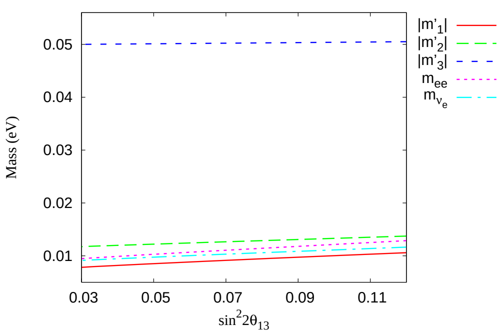

FIG. 4: Normal hierarchy solution in case of $\sin^2 2\theta_{12}$=0.90.

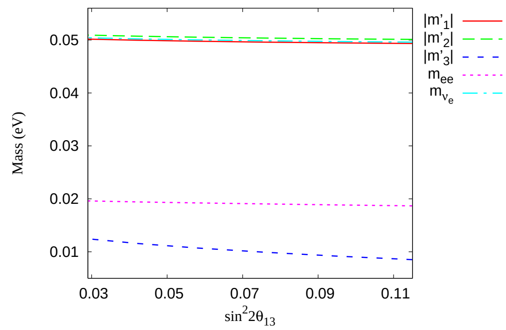

FIG. 5: Inverted hierarchy solution in case of $\sin^2 2\theta_{12}$=0.84.

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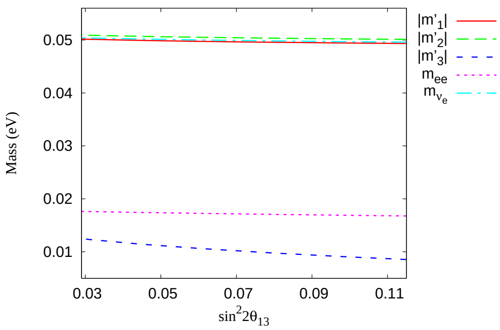

FIG. 6: Inverted hierarchy solution in case of $\sin^2 2\theta_{12}$=0.87.

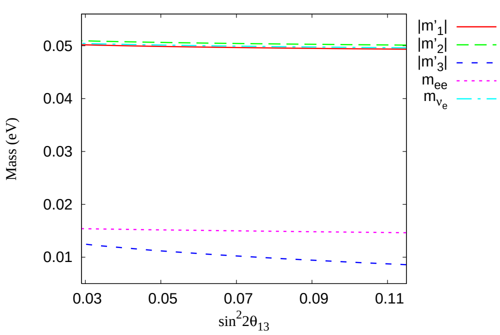

FIG. 7: Inverted hierarchy solution in case of $\sin^2 2\theta_{12}$=0.90.

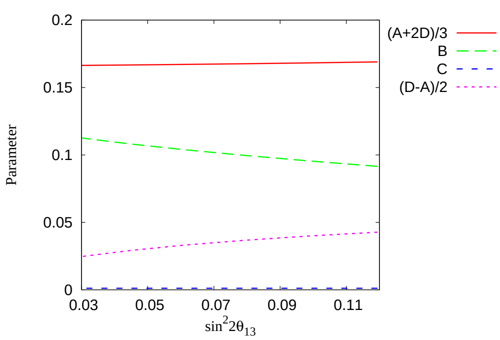

FIG. 8: T_7 parameters for normal hierarchy in case of $\sin^2 2\theta_{12}$=0.87.

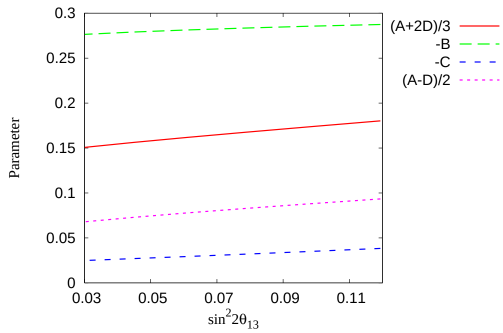

FIG. 9: T_7 parameters for inverted hierarchy in case of $\sin^2 2\theta_{12}$=0.87.