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Phys. Rev. D **84**, 071105 — Published 14 October 2011

DOI: 10.1103/PhysRevD.84.071105

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A measurement of the top-quark mass is presented using Tevatron data from proton-antiproton collisions at center-of-mass energy $\sqrt{s} = 1.96$ TeV collected with the CDF II detector. Events are selected from a sample of candidates for production of $t\bar{t}$ pairs that decay into the lepton+jets channel. The top-quark mass is measured with an unbinned maximum likelihood method where the event probability density functions are calculated using signal and background matrix elements, as well as a set of parameterized jet-to-parton transfer functions. The likelihood function is maximized with respect to the top-quark mass, the signal fraction in the sample, and a correction to the jet energy scale (JES) calibration of the calorimeter jets. The simultaneous measurement of the JES correction ($\Delta_{\rm JES}$) amounts to an additional *in situ* jet energy calibration based on the known mass of the hadronically decaying W boson. Using the data sample of 578 lepton+jets candidate events, corresponding to 3.2 fb^{-1} of integrated luminosity, the top-quark mass is measured to be $m_t = 172.4 \pm 1.4\,({\rm stat} + \Delta_{\rm JES}) \pm 1.3\,({\rm syst})\ {\rm GeV}/c^2$.

PACS numbers: 14.65.Ha,13.85.-t,12.15.Ff

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The top-quark mass, m_t, is an intrinsic parameter of the standard model (SM) of particle physics and is of particular importance due to its strikingly large value. As a result, the top quark has a large effect on radiative corrections to electroweak processes and has a Yukawa coupling to the Higgs field of $\mathcal{O}(1)$, which may provide insight into the mechanism of electroweak symmetry breaking [1].

The Higgs boson mass, m_H, is not predicted by the SM, but constraints on its value can be derived from the calculation of radiative corrections to the W boson mass, m_W, and from the values of other precision electroweak variables [2]. These corrections depend primarily on $\ln m_H$ and m_t^2, and thus precision measurements of m_W and m_t provide important constraints on m_H.

The dominant top-quark production process is pair production via the strong interaction. At Fermilab's Tevatron this process is initiated by $p\bar{p}$ collisions at center-of-mass energy $\sqrt{s} = 1.96$ TeV. Because of its large mass, the top quark decays rapidly with lifetime $\tau_t \sim 10^{-25}$ s [3] — fast enough that it has essentially no time to interact and may be considered as a free quark. This allows a direct measurement of its mass from the daughter particles from its decay, and as a result m_t has the lowest relative uncertainty of all of the quark masses [4].

In the SM top quarks decay via the weak interaction, predominantly to W bosons and b quarks as $t\bar{t} \to W^+b\, W^-\bar{b}$. W bosons decay into lower-mass fermion-antifermion pairs: a charged lepton and a neutrino ($W^+ \to \bar{\ell}\nu_\ell$ or $W^- \to \ell\bar{\nu}_\ell$), "leptonic decay"; or an up-type quark and a down-type quark ($W^+ \to q\bar{q}'$ or $W^- \to \bar{q}q'$), "hadronic decay". The result presented here uses the lepton+jets decay channel (with $q\bar{q}'b\ell\bar{\nu}_\ell\bar{b}$ or $\bar{\ell}\nu_\ell b\bar{q}q'\bar{b}$ in the final state), where one of the two W bosons decays leptonically into an electron or a muon, and the other decays hadronically. All the quarks in the final state evolve into jets of hadrons. Events with tau leptons are not selected directly, but may contribute a few percent of the total sample via leptonic cascade decays or fake jets. The most recent m_t measurements obtained at the Tevatron using the lepton+jets topology are reported in Ref. [5], while the results of an earlier version of the present analysis using 955 pb^{-1} of integrated luminosity are reported in Ref. [6]. The distinctive feature of this analysis is the use of matrix element calculations to describe the dominant background contribution. The result presented here uses a more than three times larger data sample than the earlier version, and employs a more detailed likelihood function.

The leptons and jets resulting from the top-antitop quark pair ($t\bar{t}$) decay are detected in the CDF II general-purpose particle detector that is described in detail elsewhere [7]. Azimuthally and forward-backward symmetric about the beam-line, the detector contains a high precision particle tracking system immersed in a 1.4 T magnetic field and surrounded by calorimetry, with muon detectors on the outside. A right-handed spherical coordinate system is employed, with the polar angle θ measured from the proton beam direction, the azimuthal angle ϕ in the plane perpendicular to the beam-line, and the distance r from the center of the detector. Transverse energy and momentum are defined as $E_T \equiv E \sin\theta$ and $p_T \equiv p \sin\theta$, where E and p denote energy and momentum. Pseudorapidity is defined as $\eta \equiv -\ln\tan(\theta/2)$.

Electron	$E_T > 20$ GeV	$\lvert\eta\rvert < 1.1$
or Muon	$p_T > 20$ GeV/c	$\lvert\eta\rvert < 1.0$
$\not\!\!E_T$	$\not\!\!E_T > 20$ GeV	$\lvert\eta\rvert < 3.6$
Jets	$E_T > 20$ GeV	$\lvert\eta\rvert < 2.0$
Four jets; at least one from a b quark		

TABLE I: Event selection criteria.

This measurement makes use of CDF II data collected between February 2002 and August 2008, representing approximately 3.2 fb^{-1} of integrated luminosity. The event selection criteria (Table I) are tuned to select the lepton+jets final-state particles, requiring that each event must have exactly one high-E_T electron or high-p_T muon, exactly four high-E_T jets, and a significant amount of missing E_T, $\not\!\!E_T$ [8], characteristic of the undetected neutrino. Jets are reconstructed using a cone algorithm [9], with the cone radius $\Delta R \equiv \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} = 0.4$. At least one of the four jets must be identified as originating from a b quark via the SECVTX algorithm [10], which detects displaced secondary vertices characteristic of the decay of long-lived b hadrons. A total of 578 events are selected, of which 76% are expected to be $t\bar{t}$ events (Table II). Of the 24% of events expected to be background, it is predicted that 69% arise from the production of a W boson in conjunction with 4 jets (W+jets), 19% come from multi-jet QCD production (non-W), while the remaining 12% are from sources such as diboson and single-top-quark production. These fractions are estimated using theoretical cross-sections, Monte Carlo (MC) simulated events, and data. The $t\bar{t}$ events are generated using the Lund Monte Carlo program PYTHIA [11], with a top-quark mass of 175 GeV/c^2 and a $t\bar{t}$ production cross section of 6.7 ± 0.8 pb [12]. The W+jets and Z+jets events are generated using the ALPGEN generator [13] while the single-top-quark events are generated using the MADEVENT package [14], in both cases also using PYTHIA to perform the parton showering and hadronization. Diboson events are also generated using PYTHIA. In addition, data are used for non-W events [15].

This analysis employs an unbinned maximum likelihood method [6, 16, 17]. The m_t-dependent probability density function (p.d.f.) is calculated for each event in the data sample:

$$\mathcal{P}(k) = \nu_{\rm sig} P_s(k) + (1 - \nu_{\rm sig}) P_b(k), \qquad (1)$$

where $k \equiv (E_i, \vec{p}_i)$ represents the measured kinematic quantities of the event, P_s and P_b are respectively the

sample	# of events	% of total	% of bkg
$t\bar{t}$ signal	425.0 ± 58.9	76.0%	-
W+jets	92.6 ± 15.9	16.6%	69.0%
non-W	25.0 ± 12.5	4.5%	18.7%
single top quark	6.6 ± 0.4	1.2%	4.9%
diboson	6.0 ± 0.6	1.1%	4.5%
Z+jets	3.9 ± 0.5	0.7%	2.9%
total	559.2 ± 67.0	100%	-
Observed	578		

TABLE II: Number of expected signal and background events, corresponding to the total integrated luminosity of 3.2 fb^{-1}. The percentages are used when generating Monte Carlo simulated experiments.

normalized p.d.f.s for signal and background events, and ν_{sig} is the signal fraction parameter (constrained $0 \leq \nu_{\text{sig}} \leq 1$). Signal events are defined as events consistent with $q\bar{q} \rightarrow t\bar{t}$ production and $t\bar{t}$ decay into the lepton+jets channel, as described by the leading-order (LO) matrix element evaluated by Mahlon and Parke [18]. Background events are assumed to be described by a matrix element for W+jets production, which is calculated using a sum of 1286 W+4-partons amplitudes for 592 subprocesses encoded in the VECBOS MC event generator [19]. This approximation does mean that there are some events that, in principle, are not described by either P_s or P_b, including non-W, single top, diboson, Z+jets, and $W + bb$+2-partons events, as well as W+jets events from W+0-, 1-, 2- and 3-partons processes. However, studies with MC simulated events show that the ratio P_b/P_s calculated for all of these event types is similar to that for W+4-partons events, and that, in practice, such events mostly contribute to the likelihood function via the P_b term and do not add any more bias than the W+4-partons events or than the poorly reconstructed $t\bar{t}$ events themselves [20]. Any residual bias in the measured top-quark mass is removed at the end, as described later in the paper.

The signal and background p.d.f.s, P_s and P_b, are constructed in analogous fashions, starting with the appropriately normalized parton-level differential cross-section [4], $d\hat{\sigma}_s$ or $d\hat{\sigma}_b$, which is then convolved with parton distribution functions (PDFs) and a jet-to-parton transfer function $W(k, \varkappa)$. P_s is thus given by

$$P_s(k; m_t, \Delta_{\text{JES}}) = \frac{1}{n_{jp}} \sum_{\text{jet perm.}}^{n_{jp}} \frac{1}{\hat{\sigma}_s(m_t)} \frac{1}{A_s(m_t, \Delta_{\text{JES}})} \times \tag{2}$$

$$\int d\hat{\sigma}_s(\varkappa; m_t)\, dx^1_{\text{Bj}} dx^2_{\text{Bj}}\; W(k, \varkappa; \Delta_{\text{JES}})\; f(x^1_{\text{Bj}})(x^2_{\text{Bj}}),$$

where $\varkappa \equiv (\varepsilon_i, \vec{\pi}_i)$ represents the actual event parton-level kinematic quantities corresponding to the measured quantities k, and parameter Δ_{JES} is defined in the next paragraph. The PDFs $f(x_{\text{Bj}})$ define the probability density for a colliding parton to carry a longitudinal momentum fraction x_{Bj} and are given by CTEQ5L [21]. A_s is the mean acceptance function for signal events, a normalization term that is the consequence of the constriction of the phase-space of the integral by the event selection cuts and by the detector acceptance. The average over the jet permutations, n_{jp}, is due to ambiguity in assigning final state jets to partons. The fact that the two light quarks in the final state are indistinguishable allows the reduction from the original 24 permutations to 12 in the expression for P_s, and the b-tagging information allows a further reduction to 6 assignments for events with one identified b-jet and 2 for events with both b-jets identified. In the similar expression for P_b, all 24 permutations are averaged.

The jet-to-parton transfer function $W(k, \varkappa)$ is a p.d.f. describing the probability density for an event with outgoing partons and charged lepton with $\varkappa$ to be measured as reconstructed k. The charged lepton is assumed to be well-measured, allowing the use of a Dirac δ-function to represent the mapping between its parton-level momentum, $\vec{\pi}_\ell$, and its reconstructed momentum, $\vec{p}_\ell$. For the four jets, the function is obtained by parameterizing the jet-to-parton mapping observed in fully simulated PYTHIA $t\bar{t}$ events. These events contain all of the information about the original partons as well as the measured jets. The simulation includes physical effects, such as radiation and hadronization, as well as the effects of measurement resolution and of the jet reconstruction algorithm. The parameterization is made in two parts that are assumed to be independent: the energy transfer function W_E, describing the jet energies E, and the angular transfer function W_A, describing the mapping for the jet angles. The jet-to-parton transfer function is thus given by

$$W(k, \varkappa; \Delta_{\text{JES}}) = \delta^3(\vec{p}_\ell - \vec{\pi}_\ell)\, W_A \times \prod_{i=1}^{4} \left(\frac{1}{E_i p_i} W_E^i(E_i, \varepsilon_i; \Delta_{\text{JES}}) \right). \tag{3}$$

The reconstructed jet energies, E_i, used in the function W_E are not just the raw calorimeter energy deposits, but are first calibrated so that they represent the combined energies released in the calorimeter by the many particles constituting each jet. This is achieved using the CDF jet energy scale (JES) calibration [22], which is subject to a significant systematic uncertainty. The uncertainties of individual jet energy measurements, $\sigma(E_i)$, are therefore correlated, and their fractional JES uncertainty, $\sigma(E_i)/E_i$, is typically $\sim 3\%$. If this were included as a systematic uncertainty on the measured m_t it would reduce the measurement precision drastically; in fact, each 1% of fractional JES uncertainty would add about 1 GeV/c^2 uncertainty to the measured m_t [23]. However, such a treatment overestimates the uncertainty because the energies of the two daughter jets of the hadronically decaying W boson can be constrained based on the

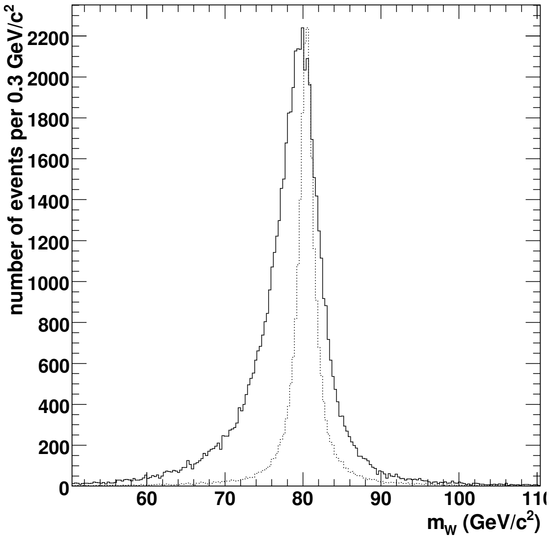

FIG. 1: The reconstructed 2-jet invariant mass of the hadronically decaying W boson, m_W, for measured jet angles (solid line) and for parton-level angles (dotted line), obtained after assuming the primary parton energy as jet energy. For ease of comparison, the parton-level distribution is normalized so that the maxima of the two distributions are the same.

known W boson mass. Applying this constraint to all events in the data sample while allowing the jet energies to be shifted results in the *in situ* measurement of the JES correction, Δ_{JES}, defined as the number of $\sigma(E_i)$ values by which the energy of each jet is shifted in the likelihood fit. This effectively re-calibrates the measured jet energies based on the known W boson mass and replaces a large component of the JES systematic uncertainty with a much smaller statistical uncertainty on the Δ_{JES}. The Δ_{JES} dependence of the jet energies is included in the parameterization of the function W_E. This parameterization is made in eight bins in pseudorapidity $|\eta|$, separately for light and b-jets, using a sum of two Gaussians as a function of the difference between the parton energies and the corrected jet energies as measured in a sample of PYTHIA $t\bar{t}$ events that pass the same selection criteria as the data.

In an earlier version of this analysis [6], the jet-to-parton transfer functions for all jet angles were approximated by Dirac δ-functions. The introduction of the function W_A was motivated by a discrepancy noticed in simulated $t\bar{t}$ events in the 2-jet effective invariant mass of the hadronically decaying W boson, m_W. Even when the true simulated parton-level jet energies are used, instead of the corresponding reconstructed detector-level values, the use of the measured jet angles rather than their parton-level values causes a significant shift of the reconstructed m_W from its nominal value, as illustrated in Fig. 1.

There is also a negative skewness in the distribution for measured angles, and since parton-level jet energies are used, the observed effects are due to the differences between the measured angles and the parton-level angles alone. The peak of the m_W distribution, when fit by a Breit-Wigner distribution, corresponds to a W boson pole mass of 79.5 GeV/c^2, a -0.9 GeV/c^2 shift from its parton-level value of 80.4 GeV/c^2. This is found to be a result of a correlation between the measured jet directions: the measured angle, α_{12}, between the two jets is, on average, reduced so that the two jets appear closer together than their parent partons, which can be seen in Fig. 2. Since the apparent W boson mass is utilized to measure Δ_{JES} and thus calibrate the measured jet energies, a jet-to-parton transfer function describing the change in the angle α_{12} is important in making an accurate measurement of Δ_{JES} and thus the top-quark mass. The function W_A also describes a much smaller correlation effect seen in the angle α_{Wb} between the hadronic-side b-jet and the hadronically decaying W boson. The function W_A is thus parameterized using two different functions, W_A^{12} and W_A^{Wb}, describing the mappings for the angles α_{12} and α_{Wb}. The remaining angles describe resolution effects rather than the correlations and, due to computational constraints, are assumed to be well measured with their contributions to W_A approximated by Dirac delta functions.

The functions W_A^{12} and W_A^{Wb} are both fit using a sum of a skew-Cauchy distribution and two Gaussians, describing the change in the cosine of the relevant angle, $\Delta\cos(\alpha_{12})$ and $\Delta\cos(\alpha_{Wb})$, from partons to measured jets. Since the correlation effects are stronger in jets that are closer together, the functions are parameterized in bins of $\cos(\alpha_{12})$ and $\cos(\alpha_{Wb})$, respectively; one example for each function is shown in Fig. 2.

The m_W distribution after convolution with the function W_A is shown in Fig. 3. The skewness is removed and the mean value agrees well with the parton-level distribution.

The 20 integration variables (3 for each final-state particle and the x_{Bj} for each initial state parton, assuming zero transverse momentum for the $t\bar{t}$ pair) in the expression for the signal and background p.d.f.s (Eq. 2) are reduced to 16 by integrating over the 4-momentum conservation Dirac δ-function inherent in the expression for $d\hat{\sigma}_s$. The charged lepton 3-momentum integration and all but two of the jet angular integrations are made trivial by the Dirac δ-functions in the function $W(k,\varkappa)$, leaving 7 integration variables. In P_s, this is further reduced to 5 variables via a change of variables to the squared masses of the top quarks and by using the narrow-width approximation for the Breit-Wigner distributions of both top-quark decays in the $t\bar{t}$ matrix element. The integral is then evaluated using the VEGAS [24] adaptive Monte Carlo integration algorithm [6], which uses importance sampling, which means that the sample points are concentrated in the regions that make the largest contribution to the integral.

The treatment of P_b is unchanged since the previous version of this analysis [6], except for the updated energy

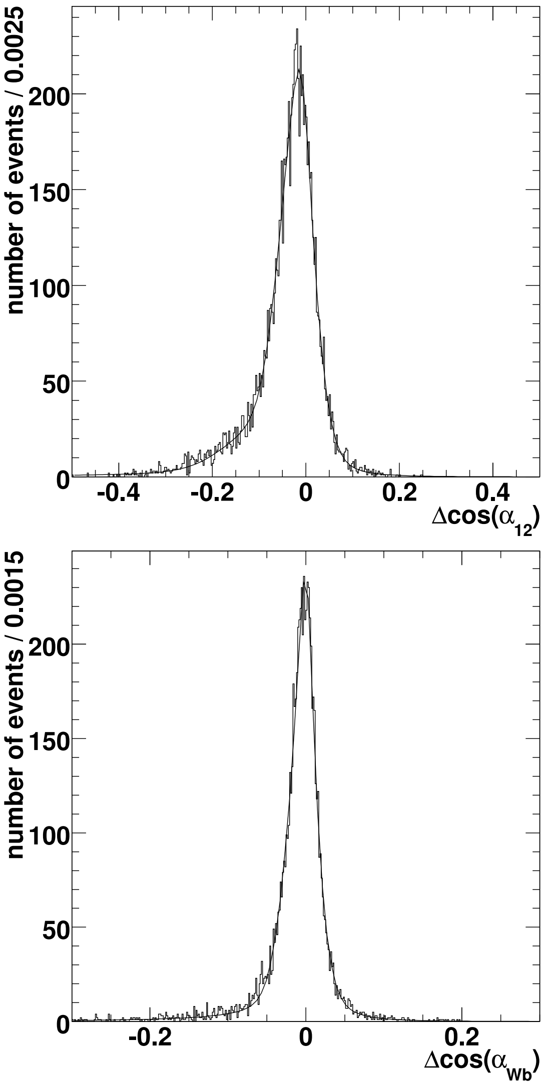

FIG. 2: Examples of parameterization of the functions W_A^{12} and W_A^{Wb} in the bins where $0.2 < \cos(\alpha_{12}) < 0.4$ and $0.2 < \cos(\alpha_{Wb}) < 0.4$. The histograms show MC simulation events and the curves represent the parameterization.

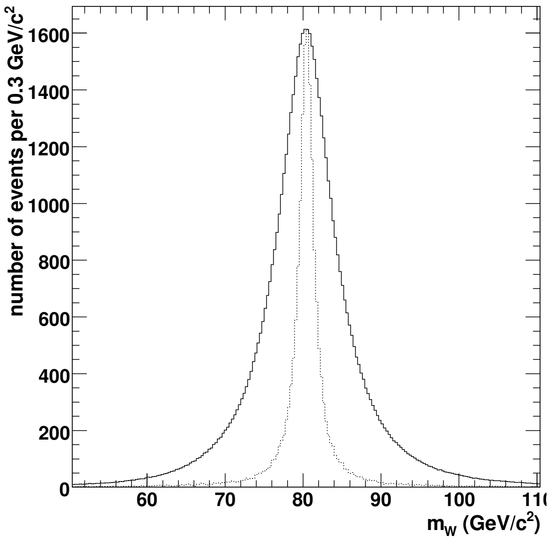

FIG. 3: The m_W distribution for measured angles from Fig. 1 is plotted (solid line) after convolution with the function W_A. For ease of comparison, the parton-level distribution (dotted line) is normalized so that the maxima of the two distributions are the same.

transfer function W_E. The integrand in the expression for P_b is much more computationally intensive than for P_s and a simplified Monte Carlo method of integration is employed, giving reasonable convergence with an execution time comparable to that of P_s. The simplifications used in this computation of P_b include setting the function W_A to a Dirac δ-function for all angles, using a narrow width approximation for the W boson decay, and neglecting the $\Delta_{\rm JES}$ dependence of the function W_E. Therefore, the value of P_b for each event does not depend on the likelihood parameters m_t and $\Delta_{\rm JES}$, while P_s is a two-dimensional function of those parameters [6]. In this approximation, the product of the background p.d.f. normalization terms (corresponding to the variables $\hat{\sigma}_b \cdot A_b$ in Eq. 2) is set to a constant, whose value is chosen to optimize the statistical sensitivity of the method, effectively providing an appropriate relative normalization with respect to P_s.

The log-likelihood function is given as a sum over the 578 events in the sample:

$$\ln \mathcal{L}(k; m_t, \Delta_{\rm JES}, \nu_{\rm sig}) = \sum_{i=1}^{578} \ln\left[\nu_{\rm sig} P_s(k_i; m_t, \Delta_{\rm JES}) + (1-\nu_{\rm sig}) P_b(k_i)\right]. \tag{4}$$

It is calculated on a two-dimensional 31 × 17 grid in m_t and $\Delta_{\rm JES}$, spanning $145 \leq m_t \leq 205$ GeV/c^2 and $-4.8 \leq \Delta_{\rm JES} \leq 4.8$, with a spacing between grid points of 2 GeV/c^2 in m_t and 0.6 in $\Delta_{\rm JES}$. To optimize computational time, the bin size is chosen to be as large as possible without appreciably affecting the fit result. The third likelihood parameter, the signal fraction parameter $\nu_{\rm sig}$, is allowed to vary continuously (within the constraint $0 \leq \nu_{\rm sig} \leq 1$), and the likelihood function is maximized with respect to $\nu_{\rm sig}$ at each point on the grid using the MINUIT program [25]. The resulting surface described on the grid is the profile log-likelihood, maximized for $\nu_{\rm sig}$. The top-quark mass, m_t, and the jet energy scale correction, $\Delta_{\rm JES}$, are measured by making a two-dimensional parabolic fit to the surface, consistent with the expectation for the likelihood function to be Gaussian near its maximum. The maximum of the parabola gives the measured m_t and $\Delta_{\rm JES}$, while the measured $\nu_{\rm sig}$ is taken from its value at the grid point of maximum likelihood. The estimated one-σ statistical uncertainty of the measurement is represented by the ellipse corresponding to a change in log-likelihood $\Delta \ln \mathcal{L} = 0.5$ from the maximum of the fitted parabola. The values of m_t and $\Delta_{\rm JES}$ are anti-correlated (Fig. 4). No correlation is observed between $\nu_{\rm sig}$ and m_t or $\Delta_{\rm JES}$.

The accuracy of the measured m_t and $\Delta_{\rm JES}$, and their uncertainties, are checked using ensembles of MC simulated experiments, using the MC samples previously mentioned with the addition of 22 $t\bar{t}$ samples generated with values of m_t between 161 and 185 GeV/c^2. The numbers of $t\bar{t}$ events and those of the various backgrounds are Poisson fluctuated around the values shown in Table II. Studies of the relationships between the known input simulation parameters and their corresponding measurements show no evidence of bias when a clean sample of MC simulated $t\bar{t}$ events is used, containing only lepton+jets events with correct jet-parton matching. However, the presence of signal events with jets which are poorly or incorrectly matched to partons and events which do not match the decay hypothesis biases the likelihood fit result and increases the pull width. The presence of background events also biases the fit, due to the backgrounds that are not well described by P_b and the approximations in P_b. The bias is removed using a set of functions obtained from a fit to the MC simulation and parameterized in terms of the measured $\Delta_{\rm JES}$ and $\nu_{\rm sig}$ [20]. This amounts to adding 1.1 GeV/c^2 to the m_t value produced by the likelihood fit and multiplying the uncertainty by 1.26 so that the pull width is consistent with unity. The systematic uncertainty due to this measurement calibration is small, as shown in Table III.

Despite the reduction from the *in situ* $\Delta_{\rm JES}$ calibration, the remaining uncertainty from JES obtained by varying the parameters in JES [22] is among the largest systematic uncertainties of the measurement (Table III). Other significant systematic uncertainties are mainly a result of assumptions made in the simulation of the events that are used in the tuning and calibration of the measurement method. In most cases, they are evaluated by varying different aspects of the MC simulation, such as signal MC generator (PYTHIA versus HERWIG [26]), color reconnection model tune (Apro versus ACRpro [27–29]), and parameters of initial and final state radiation (ISR and FSR). A detailed description of the systematic effects has been published elsewhere [30]. The systematic uncertainties for each effect are added in quadrature, resulting in a total estimated systematic uncertainty of 1.3 GeV/c^2 (Table III).

The measurement is made using the data sample of 578 events, yielding

$$m_t = 172.4 \pm 1.4\,(\text{stat+}\Delta_{\rm JES}) \pm 1.3\,(\text{sys})\ \text{GeV}/c^2$$
$$m_t = 172.4 \pm 1.9\,(\text{total})\ \text{GeV}/c^2, \qquad (5)$$

with $\Delta_{\rm JES} = 0.3 \pm 0.3$ (stat). The central value and the contour ellipses corresponding to the one-, two- and three-σ statistical confidence intervals of the measurement are illustrated in Fig. 4. The overall statistical uncertainty on the measured top-quark mass is labeled "stat+$\Delta_{\rm JES}$" because it includes the uncertainty on m_t due to the statistical uncertainty on the measured $\Delta_{\rm JES}$, i.e. the uncertainty is given by half of the full width of the one-σ contour of Fig. 4.

Systematic	(GeV/c^2)
MC Generator	0.70
Residual JES	0.65
Color Reconnection	0.56
b-jet energy	0.39
Background	0.45
ISR and FSR	0.23
Multiple Hadron Interactions	0.22
PDFs	0.13
Lepton Energy	0.12
Measurement Calibration	0.12
Total	1.31

TABLE III: Contributions to the total expected systematic uncertainty.

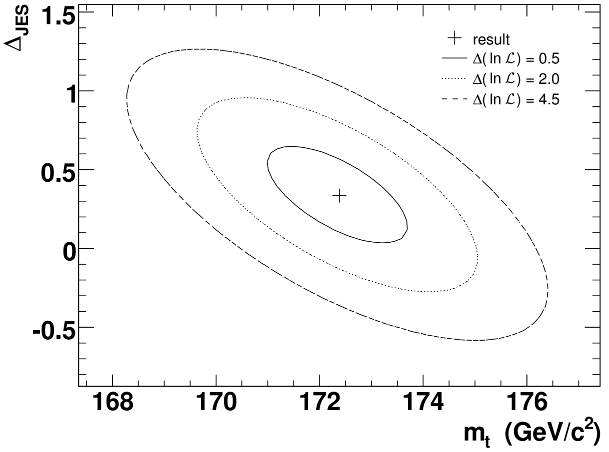

FIG. 4: The measurement result and the contour ellipses of the parabolic fit corresponding to the one-, two- and three-σ confidence intervals for the statistical uncertainty on m_t and $\Delta_{\rm JES}$.

In conclusion, a precise measurement of the top-quark mass has been presented using CDF lepton+jets candidate events corresponding to an integrated luminosity of 3.2 fb^{-1}. Using an improved matrix element method with an *in situ* jet energy calibration, the top quark mass is measured to be $m_t = 172.4 \pm 1.9$ GeV/c^2.

Acknowledgments

We thank the Fermilab staff and the technical staffs of the participating institutions for their vital contributions. This work was supported by the U.S. Department of Energy and National Science Foundation; the Italian Istituto Nazionale di Fisica Nucleare; the Ministry of Education, Culture, Sports, Science and Technology of Japan; the Natural Sciences and Engineering Research Council of Canada; the National Science Council of the Republic of China; the Swiss National Science Foundation; the A.P. Sloan Foundation; the Bundesministerium für Bildung und Forschung, Germany; the Korean World Class University Program, the National Research Foundation of Korea; the Science and Technology Facilities Council and the Royal Society, UK; the Institut National de Physique Nucleaire et Physique des Particules/CNRS; the Russian Foundation for Basic Research; the Ministerio de Ciencia e Innovación, and Programa Consolider-Ingenio 2010, Spain; the Slovak R&D Agency; the Academy of Finland; and the Australian Research Council (ARC).

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