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Gravitational Radiation in Hořava Gravity

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Abstract

We study the radiation of gravitational waves by self-gravitating binary systems in the low-energy limit of Hořava gravity. We find that the predictions for the energy-loss formula of General Relativity are modified already for Newtonian sources: the quadrupole contribution is altered, in part due to the different speed of propagation of the tensor modes; furthermore, there is a monopole contribution stemming from an extra scalar degree of freedom. A dipole contribution only appears at higher post-Newtonian order. We use these findings to constrain the low-energy action of Hořava gravity by comparing them with the radiation damping observed for binary pulsars. Even if this comparison is not completely appropriate – since compact objects cannot be described within the post-Newtonian approximation – it represents an order of magnitude estimate. In the limit where the post-Newtonian metric coincides with that of General Relativity, our energy-loss formula provides the strongest constraints for Hořava gravity at low-energies.

1 Introduction

General Relativity (GR) continues to stubbornly agree with every observation related to gravity [1]. This would be extremely desirable if the theory could be merged with quantum mechanics in a straightforward way. Unfortunately, the current situation is far from this: the search for a consistent theory of quantum gravity remains elusive and there is no experimental guidance to shed light on it. Furthermore, the cosmological constant problem aside[1], the success of GR as a low-energy effective field theory (EFT) points towards the Planck mass $M_P \approx 10^{19}$ GeV as the physical frontier where one expects to learn anything about quantum gravity. If the preceding arguments are realized in Nature, experimental information about quantum gravity will indeed be sparse in the foreseeable future.

More interesting for phenomenology are the proposals for ultraviolet (UV) completions of GR where the previous logic fails. These include models of gravitation with a low-energy cutoff beyond which GR ceases to be valid [2, 3]. If this cutoff scale is as low as the TeV, these proposals may have interesting phenomenology and may even be relevant for the resolution of the hierarchy problem. Another recent proposal in this category is Hořava gravity [4, 5]. This framework yields a concrete UV completion of GR, with effects that may permeate basically any gravitational experiment. It is on the implications of Hořava gravity for gravitational radiation that we pursue in this paper.

Essentially, Hořava's proposal consists of considering the existence of a preferred time-foliation of spacetime. Assuming the presence of this absolute structure, the GR Lagrangian can be supplemented by operators which render it power-counting renormalizable without destroying the unitarity of the theory [4]. The result is a non-relativistic theory of quantum gravity [5] (in the sense that it is Lorentz-violating). The preferred foliation is in principle detectable at any energy scale, and it is not surprising that this approach (which is designed to cure the unsatisfactory behaviour of GR at distances of the order $M_P^{-1} \approx 10^{-33}\ cm$) generically also modifies the theory at large distances[2] [5]. Among the different implementations of Hořava's idea, we consider the so-called "healthy extension" [7]. This version possesses a stable Minkowski background where the issues about strong coupling appearing in other approaches are absent. Furthermore, variants of Hořava's original proposal can be retrieved

[1] One may argue that the cosmological constant problem is a hint towards the actual theory of quantum gravity, and that a successful framework of quantum gravity should provide a mechanism to explain this phenomenological observation. We do not address this particular issue here.

[2] A counterexample to this argument can be found in [6]. However, it is not clear how GR is recovered at large distances in this proposal.

for a particular limit of this (generic and stable) case [7].

The low-energy (large-distance) sector of the theory is encoded into a scalar field φ, called[3] the "khronon", that characterizes the foliation structure and interacts with a metric field. We refer to this low-energy scalar-tensor theory as "khronometric" theory [5]. The extra scalar field φ turns out to be massless, and its presence modifies most of the predictions of GR, including the parametrized-post-Newtonian (PPN) parameters [5, 7, 9] and cosmological phenomena [10, 11]. These modifications differ from those of standard scalar-tensor theories [1, 12]. They are close, however, to the predictions of Einstein-aether theory (or æ-theory for short) [13]. This is not surprising since both theories incorporate a field whose expectation value violates Lorentz invariance (a unit timelike vector in the case of Einstein-aether, and φ in our case), and are otherwise generic. It can be shown that the khronon φ corresponds to the hypersurface-orthogonal mode of æ-theory, and many of the predictions of both theories are indeed identical [5, 8]. The PPN parameters derived from æ-theory and khronometric theory restrict the parameter space of those theories but are otherwise in agreement with current observations. Thus, both (low-energy) models represent interesting alternatives to GR, which, furthermore, have a high energy cutoff . The further advantage of khronometric theory is that beyond this energy cutoff there is a known UV completion, in the form of Hořava gravity.

The aim of this paper is to further constrain khronometric theory based on the loss of energy due to the emission of gravitational waves (GWs) from a binary self-gravitating system. This is a relevant test for gravitational theories given its sensitivity to the way gravity propagates (e.g. the degrees of freedom and corresponding properties), and also to the strong-field regime since known astrophysical sources of GWs tend to have strong gravitational self-energies [12, 14, 15]. The confirmation of GR's famous quadrupole formula in the damping of a binary pulsar's orbit is indeed one of its triumphs [16, 17]. Radiation tests have also been used in the past to constrain possible modifications of GR [1, 18, 19]. A priori for both æ-theory and khronometric theory, one expects this radiation formula to be modified due to a different speed of propagation of the tensor modes and the presence of new propagating fields. These modifications imply new ways to constrain the parameter space of the theory, independently of PPN and cosmological considerations. While the above expectations have been verified for æ-theory in the weak-field regime in [20], the constraints obtained are not final since the astrophysical systems for which radiation damping has been observed are not in the weak-field regime [1]. The incorporation of strong-field effects in the

[3] From Greek $\chi\rho o\nu o\varsigma$ – time. The khronon is also known as the "T-field" [8].

Einstein-aether began in [21].

We focus on the radiation formula in the post-Newtonian (weak field, slow-motion and weakly stressed [14]) regime of khronometric theory. This restriction is interesting for two reasons. First, we find deviations from GR's quadrupole formula already at leading order. (This is similar to what happens in æ-theory, as computed in [20].) Second, and from a purely pragmatic point of view, many of the formulae we present in this paper are useful for the phenomenologically relevant situation of compact sources. First results relevant for the study of gravitational radiation from these systems include the black hole solutions derived in [22, 23], and those for neutron stars in æ-theory [24]. The use of binary pulsar observations to constraint Hořava gravity was suggested in [25].

To extract information about the damping of the orbit of a binary self-gravitating system from the emission of GWs, we take advantage of the fact that khronometric theory is semi-conservative (in the language of [12]). Then, for the bound system there exists a conserved energy that decreases due to the emission of gravitational radiation. By computing the energy flux at infinity, we can derive the flux of energy lost by the binary. Under the assumption that this energy is extracted entirely from the orbital motion of the binary, the subsequent damping of the orbits can be computed using Kepler's third law. This assumption has been tested to lowest order in GR [15], and is plausible for khronometric theory.

This work is structured as follows. In Sec. 2, we define the action for khronometric theory and the equations of motion relevant for low-energy phenomenology. Sec. 3 is devoted to the linearized equations for the fields far away from the source (far-zone). In Sec. 4, we study the conserved properties of the source relevant for the post-Newtonian (PN) calculation. We derive the explicit expressions for the the different waveforms, up to and including the first PN order corrections in Sec. 5. In Sec. 6, we determine the formula for the average power loss in GWs. This formula is evaluated for a Newtonian system of two point-masses in Sec. 7, where the Peters-Mathews parameters for khronometric theory are derived. We summarize our results and conclude in Sec. 8. Appendix A contains a derivation of the PPN parameters for our model (whose full expressions appear here for the first time). Appendix B compares the monopole contribution, or lack thereof, in both khronometric theory and æ-theory for a particular choice of parameters. Finally, Appendix C provides a summary of the notion of energy relevant for our study.

Conventions

We use the $(+---)$ signature. For an arbitrary expression X, the overbar $\bar{X}$ denotes the part of X linear in perturbations. The superscript X^{NL} is the non-linear part of X, i.e. $X^{NL} \equiv X - \bar{X}$. The dot $\dot{X}$ denotes the derivative of X with respect to time. Greek indices refer to spacetime, whereas Latin indices refer to space only. Repeated Latin indices are to be summed, e.g. $X_{ii} \equiv \delta^{ij} X_{ij}$. We define the symmetrization of indexes as $T_{(ij)} \equiv \frac{1}{2}\left(T_{ij} + T_{ji}\right)$. We choose units where $c = \hbar = 1$.

2 Action for khronometric theory at low energies

As outlined in the introduction, Hořava gravity is based on the existence of an absolute time foliation of spacetime. This allows for the GR Lagrangian to be supplemented with higher dimensional operators that render the theory power-counting renormalizable [4]. These operators are suppressed by a scale M_* whose magnitude is constrained by various phenomenological tests. The most stringent of these tests comes from absence of deviations from Newton's law at short distances [5] which implies that $M_* \gtrsim (10\ \mu m)^{-1} \sim 10^{14}$ Hz [1, 5]. Thus, these higher dimensional operators are irrelevant for the binary systems of interest[4] and we neglect them in the following. The presence of a preferred foliation also has consequences at energy scales below M_*. Indeed, at low-energies new operators appear (compared to GR) that are compatible with the group of gauge invariance preserving the preferred foliation, i.e. the foliation-preserving diffeomorphism [4, 5]. Renormalization group arguments imply that these relevant operators should be added to the GR action, which has been done in the Stückelberg (or covariant) formulation of the theory in [5, 26]. In this formulation, the preferred-time foliation corresponds to the expectation value of a scalar field φ called the "khronon". This field is such that the normal to the surfaces of constant field is timelike,

$$\partial_\mu \varphi\, \partial^\mu \varphi > 0. \tag{1}$$

The action of the theory is invariant under diffeomorphisms, and Lorentz invariance is broken by condition (1) in a spontaneous way. Also, the action must be endowed with invariance under field reparametrizations

$$\varphi \mapsto f(\varphi), \tag{2}$$

[4] As an example, the famous PSR 1913+16 binary pulsar has a characteristic frequency of 10^2 Hz [1]. We assume that the speed of propagation of all the modes is similar to the speed of light. We comment on this assumption when we derive the energy-loss formula in Sec. 6.

which follows from our requirement of a preferred time-foliation as opposed to a preferred time. It corresponds to the time reparametrization invariance of the theory in the original formulation of [4]. The invariance under the transformations (2) is readily implemented by making the action depend on φ through the combination

$$u_\mu \equiv \frac{\partial_\mu \varphi}{\sqrt{\partial_\rho \varphi \partial^\rho \varphi}}. \tag{3}$$

Clearly, u_μ is non-singular whenever condition (1) is satisfied. Notice also that u_μ is a unit timelike vector.

The low-energy action for the healthy extension of Hořava gravity corresponds to the most general action describing the coupling of φ with a metric field $g_{\mu\nu}$ at low-energies and compatible with the aforementioned invariances [5]. It is given by

$$S = -\frac{{M_b}^2}{2} \int \mathrm{d}^4 x \sqrt{-g} \Big[R + K^{\mu\nu}{}_{\sigma\rho} \nabla_\mu u^\sigma \nabla_\nu u^\rho \Big] + S_m, \tag{4}$$

where M_b is an arbitrary mass parameter to be related to the Planck mass,

$$K^{\mu\nu}{}_{\sigma\rho} = \beta\, \delta^\mu_\rho \delta^\nu_\sigma + \lambda\, \delta^\mu_\sigma \delta^\nu_\rho + \alpha\, u^\mu u^\nu g_{\sigma\rho},$$

and α, β and λ are free dimensionless constants[5]. We also introduce a term S_m in Eq. (4) representing the matter component of the theory. We assume that matter is universally coupled to the metric $g_{\mu\nu}$, which enforces the weak equivalence principle [1]. This action defines what we call "khronometric theory". For later convenience, we introduce

$$S_\chi \equiv -\frac{{M_b}^2}{2} \int \mathrm{d}^4 x \sqrt{-g} K^{\mu\nu}{}_{\sigma\rho} \nabla_\nu u^\rho \nabla_\mu u^\sigma = -\frac{{M_b}^2}{2} \int \mathrm{d}^4 x \sqrt{-g} K^\mu{}_\sigma \nabla_\mu u^\sigma,$$

where

$$K^\mu{}_\sigma \equiv K^{\mu\nu}{}_{\sigma\rho} \nabla_\nu u^\rho = K^{\nu\mu}{}_{\rho\sigma} \nabla_\nu u^\rho.$$

is used to compactify notation.

Khronometric theory can be considered on its own as an alternative to GR with an extra scalar field, independently of quantum gravity motivations. This approach is similar to the way Einstein-æther theories are constructed. The only difference is that the vector u_μ is taken to be a generic timelike vector in æ-theory [13], meaning that it has more degrees of freedom than in the khronometric case. It also implies an extra term in the generic action

[5]Note that the parameter λ corresponds to λ' in the notations of [5]. It also differs from the λ defined in [4].

with respect to Eq. (4). This extra term can be absorbed by the ones in action (4) for hypersurface orthogonal vectors, i.e. whenever u_μ satisfies Eq. (3). Khronometric theory and æ-theory share the nice feature of having a high energy cutoff. The advantage of the former is that a UV completion in the form of Hořava gravity is known.

Let the khronon and matter energy-momentum tensors be, respectively,

$$T^\chi_{\mu\nu} \equiv \frac{2}{\sqrt{-g}}\frac{\delta S_\chi}{\delta g^{\mu\nu}}, \quad T^m_{\mu\nu} \equiv \frac{2}{\sqrt{-g}}\frac{\delta S_m}{\delta g^{\mu\nu}}.$$

The explicit expression for $T^\chi_{\mu\nu}$ reads

$$\begin{aligned} M_b^{-2} T^\chi_{\mu\nu} = &-\nabla_\rho \left(K_{(\mu\nu)} u^\rho + K^\rho{}_{(\mu} u_{\nu)} - K_{(\mu}{}^\rho u_{\nu)} \right) + \frac{1}{2} g_{\mu\nu} K^\rho{}_\sigma \nabla_\rho u^\sigma \\ &+ \alpha\, a_\mu a_\nu + 2\nabla_\rho K^\rho{}_{(\mu} u_{\nu)} - u_\mu u_\nu u^\sigma \nabla_\rho K^\rho{}_\sigma - 2\alpha\, a_\sigma u_{(\mu} \nabla_{\nu)} u^\sigma + \alpha\, a^\rho a_\rho u_\mu u_\nu, \end{aligned}$$

where we have introduced the notation

$$a_\mu \equiv u^\rho \nabla_\rho u_\mu.$$

The equations of motion derived from varying the action with respect to the metric are

$$\mathcal{Q}_{\mu\nu} \equiv G_{\mu\nu} - M_b^{-2} \left(T^\chi_{\mu\nu} + T^m_{\mu\nu} \right) = 0. \tag{5}$$

The equation of motion for the khronon field is

$$\mathcal{Q}_\chi \equiv \nabla_\mu J^\mu \equiv \nabla_\mu \left(\frac{1}{\sqrt{X}} \mathcal{P}^{\mu\nu} \left[\nabla_\sigma K^\sigma{}_\nu - \alpha\, a_\sigma \nabla_\nu u^\sigma \right] \right) = 0, \tag{6}$$

where

$$\mathcal{P}^{\mu\nu} \equiv \left(g^{\mu\nu} - u^\mu u^\nu \right). \tag{7}$$

As usual, this equation follows from the covariant conservation of the khronon energy-momentum tensor. That it can be represented as the conservation of a current is a consequence of the invariance of the theory under reparametrizations of the khronon given by Eq. (2) [26].

3 Equations of motion in the far-zone

The physical system of interest for radiation damping consists of an isolated self-gravitating astrophysical source. By this we mean that there is a region of spacetime surrounding the source where the fields acquire their background values plus small perturbations. Thus, there exists a coordinate frame where the metric in this region satisfies[6],

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},$$

with $|h_{\mu\nu}| \ll 1$. For the khronon field, we fix the parametrization invariance (2) by working with a time coordinate corresponding to the background of the field. Thus, we expand it as

$$\varphi = t + \chi,$$

where $|\chi| \ll t$. It is easy to verify that the background fields are indeed solutions of the equations of motion (5) in the absence of matter. To derive the flux of energy lost by this system, it is enough to understand the behavior of the fields produced by the isolated source in this region where they are weak. This is so because the energy carried by GWs is radiated away and eventually permeates the "weak-field" zone. We can extract the power radiated by integrating the flux of energy over a sphere surrounding the source at a particular time after the emission. This calculation is further simplified in the region far away from the source due to the applicability of the both the "weak-field" and "far-zone" approximations (see below).

To derive the equations governing the perturbations $h_{\mu\nu}$ and χ, we split Eq. (5) and Eq. (6) into linear and non-linear parts as follows

$$\bar{G}_{\mu\nu} - M_b^{-2}\bar{T}^{\chi}_{\mu\nu} = M_b^{-2}\tau_{\mu\nu}, \quad \bar{\mathcal{Q}}_\chi = -\mathcal{Q}^{NL}_\chi. \tag{8}$$

The expression for $\tau_{\mu\nu}$ reads

$$\tau_{\mu\nu} = T^m_{\mu\nu} + \left(T^{\chi}_{\mu\nu}\right)^{NL} - M_b^2 G^{NL}_{\mu\nu}. \tag{9}$$

This separation into linear and non-linear parts allows us to solve for $h_{\mu\nu}$ and χ perturbatively in M_b^{-2}. The terms $\tau_{\mu\nu}$ and $\mathcal{Q}^{NL}_\chi$ can be interpreted to be source terms for the linear equations at different orders in M_b^{-2}. They include contributions from both matter and non-linear gravitational fields of lower order. For this paper, we are interested in matter sources that are weakly self-gravitating, slowly moving[7] and weakly stressed. These are known as post-Newtonian (PN) sources [14]. For these systems, one has

$$v \sim |h_{00}^{1/2}| \sim \left|\frac{T^m_{0i}}{T^m_{00}}\right| \sim \left|\frac{T^m_{ij}}{T^m_{00}}\right|^{1/2} \ll 1, \tag{10}$$

[6] In this section, Greek indices are manipulated with the Minkowski metric.

[7] For theories with modes propagating at different speeds, this means that the typical velocity v of the source is small with respect to all of them.

where v is the typical velocity of the source. Thus, we can introduce v as a new parameter of expansion and consider the predictions of the theory at different orders in v, also known as PN orders. We content ourselves with the first PN corrections, which amounts to considering Eqs. (8) where the source terms also include PN corrections. In particular, the metric should be substituted by its first PN expression (Appendix A) whenever it appears in the non-linear source terms.

This straightforward analysis is only suited for the first PN corrections. Beyond that, the analysis becomes more complicated due to the presence of tails and retardation effects. The correct treatment of the problem in general involves the separation into a near-zone and a wave-zone. In the near-zone, one can find the metric to any PN order including non-linearities and minimizing retardation effects. This corresponds to an expansion in the small parameter to desired order in v. In the wave zone, one can solve the equations of motion perturbatively in the fields and match the solution to the one found in the near-zone in a region where both approximations are valid [14, 15, 27, 28]. For the first PN corrections considered in this paper, this analysis reduces to the one outlined in the previous paragraph. For higher order corrections the matching is much less trivial [14, 15, 27, 29].

The linearized khronon energy-momentum tensor satisfies the following conservations laws

$$\partial^\mu \bar{T}^\chi_{\mu i} = 0, \quad \partial^\mu \bar{T}^\chi_{\mu 0} = M_b^2 \bar{\mathcal{Q}}_\chi. \tag{11}$$

It follows from the invariance of the linearized theory under linear diffeomorphisms. Next, from the transverse properties of the $\bar{G}_{\mu\nu}$ and when the equations of motion are imposed, one finds

$$\partial^\mu \left(T^m_{\mu\nu} + T^\chi_{\mu\nu} - M_b^2 G^{NL}_{\mu\nu} \right) = 0.$$

Together with Eq. (11), this yields the following conservation equations for the source tensor $\tau_{\mu\nu}$

$$\partial^\mu \tau_{\mu\nu} = -\partial^\mu \bar{T}^\chi_{\mu\nu} = -M_b^2 \delta^0_\nu \, \bar{\mathcal{Q}}_\chi \tag{12}$$

which is of particular importance in Sec. 4 and beyond.

3.1 Wave equations

We decompose the gravitational perturbations into irreducible representations of $SO(3)$,

$$
\begin{aligned}
&h_{00} = 2\phi, \quad h_{0i} = -\frac{\partial_i}{\sqrt{\Delta}}B + V_i, \\
&h_{ij} = t_{ij} + 2\partial_{(i}F_{j)} + 2\frac{\partial_i\partial_j}{\Delta}E + 2\left(\delta_{ij} - \frac{\partial_i\partial_j}{\Delta}\right)\psi,
\end{aligned}
\tag{13}
$$

where $t_{ii} = \partial_i t_{ij} = \partial_i V_i = \partial_j F_j = 0$. We also define the Laplacian by $\Delta \equiv \partial_i\partial_i$.

Tensors and vectors

To single out the tensorial part of the equations of motion as written in Eq. (8), we introduce the transverse-traceless projector $P_{ij,kr}$ and the transverse projector P_{ij}

$$
P_{ij,kr} \equiv P_{ik}P_{jr} - \frac{1}{2}P_{ij}P_{kr}, \quad P_{ij} \equiv \delta_{ij} - \frac{\partial_i\partial_j}{\Delta}. \tag{14}
$$

A straightforward calculation yields

$$
P_{ij,kr}\mathcal{Q}_{kr} = \frac{1}{2}P_{ij,kr}\left[\beta\,\ddot{h}_{kr} - (\partial_0^2 - \Delta)h_{kr} - 2M_b^{-2}\tau_{kr}\right],
$$

leading to the wave equation for the tensor modes

$$
(c_t^{-2}\partial_0^2 - \Delta)t_{ij} = -2M_b^{-2}P_{ij,ks}\tau_{ks}, \tag{15}
$$

with $c_t^2 = 1/(1-\beta)$ representing the speed of propagation of the tensor polarizations. This coincides with the results of æ-theory [30].

Consider now the vectorial part of the equations. Contrary to æ-theory [20, 30] this sector does not contain any propagating polarizations. Indeed, one finds

$$
\begin{aligned}
&P_{ij}\mathcal{Q}_{0j} = \frac{(1-\beta)}{2}\Delta\left(V_i - \dot{F}_i\right) - M_b^{-2}P_{ij}\tau_{j0} = 0, \\
&P_{ik}\partial_j\mathcal{Q}_{kj} = \frac{(1-\beta)}{2}\Delta\left(\dot{V}_i - \ddot{F}_i\right) - M_b^{-2}P_{ik}\partial_j\tau_{kj} = 0.
\end{aligned}
\tag{16}
$$

The first equation represents a constraint and its time derivative yields the second equation (which follows from gauge invariance). For definiteness, we choose to work in the gauge

$$
F_i = 0, \tag{17}
$$

which completely fixes the gauge freedom in the vector sector.

Scalars

The scalar sector of the theory is different from GR. In particular, it includes an extra degree of freedom. We choose to work in the gauge

$$\chi = B = 0, \tag{18}$$

which completely fixes the gauge in the scalar sector. The choices (17) and (18) are referred to as the "unitary gauge". In this gauge, the non-redundant equations of motion derived from (5) and (6) are

$$\alpha \Delta \phi = 2\Delta\psi - M_b^{-2}\tau_{00}, \tag{19a}$$

$$(\beta+\lambda)\Delta\dot{E} = -2(\lambda+1)\Delta\dot{\psi} + M_b^{-2}\partial_i\tau_{0i}, \tag{19b}$$

$$\left(c_s^{-2}\partial_0^2 - \Delta\right)\psi = \frac{\alpha M_b^{-2}}{2(\alpha-2)}\left(\frac{2}{\alpha}\tau_{00} + \tau_{ii} - \frac{(2+\beta+3\lambda)}{(\beta+\lambda)}\frac{\partial_i\partial_j}{\Delta}\tau_{ij}\right). \tag{19c}$$

The speed of propagation of the scalar perturbation is given by

$$c_s^2 = \frac{(\alpha-2)(\beta+\lambda)}{\alpha(\beta-1)(2+\beta+3\lambda)}, \tag{20}$$

which coincides with the scalar mode of æ-theory [30].

3.2 Far-zone expressions and post-Newtonian approximation

The equations of motion (15), (16) and (19) contain two types of equations that we wish to solve, Poisson and wave equations. The Poisson equation is of the form

$$\Delta\xi(t,x) = -4\pi\rho(t,x),$$

whose solution for vanishing boundary conditions at infinity is given by

$$\xi(t,x) = \int \mathrm{d}^3\tilde{x}\, \frac{\rho(t,\tilde{x})}{|x-\tilde{x}|}.$$

We assume that the isolated source of GWs can be confined within a sphere of radius R. At distances far away from the source, $r \equiv |x| \gg R$, we can perform the expansion ($\hat{r}^i = x^i/r$)

$$|x-\tilde{x}| = r - \hat{r}^i\tilde{x}^i + r\,O(R/r)^2. \tag{21}$$

We refer to this zone as the "far-zone". The leading contribution of the solution to the Poisson equation at large distances is then

$$\xi_f(t,x) = \frac{1}{r}\int \mathrm{d}^3\tilde{x}\,\rho(t,\tilde{x}). \tag{22}$$

The sourced wave equations are of the form

$$\left(c_\sigma^{-2}\partial_0^2 - \Delta\right)\sigma(t,x) = 4\pi\mu(t,x), \tag{23}$$

with speed of propagation c_σ. The solution to this equation with radiation boundary conditions is given by (see, e.g. [15])

$$\sigma(t,x) = \int \mathrm{d}^3\tilde{x}\,\frac{\mu(t-|x-\tilde{x}|/c_\sigma,\tilde{x})}{|x-\tilde{x}|}. \tag{24}$$

Besides adopting the far-zone approximation and using (21), we also assume that r is such that $r \gg \omega R^2/c_\sigma$, where ω is the largest characteristic frequency of the source. This allows us to write the leading contribution as

$$\sigma_f(t,x) = \frac{1}{r}\int \mathrm{d}^3\tilde{x}\,\mu(t-r/c_\sigma+\hat{r}^i\tilde{x}^i/c_\sigma,\tilde{x}) = \frac{1}{r}\sum_{n=0}^{\infty}\frac{1}{n!}\partial_0^n\int \mathrm{d}^3\tilde{x}\,\mu(t-r/c_\sigma,\tilde{x})\left(\hat{r}^i\tilde{x}^i/c_\sigma\right)^n, \tag{25}$$

where the last identity holds formally. This expression can be simplified further for the PN sources of interest [12, 14, 31]. As seen in the previous section, khronometric theory involves two speeds of propagation, the tensor and the scalar speeds c_t and c_s. We assume that the system is slowly moving with respect to both speeds which are considered to be of the same order, $c_t \sim c_s \sim 1$. Thus, for a typical velocity $v \sim \omega R$ of the source, the sum in Eq. (25) represents a well-defined expansion in the small parameter,

$$v \ll 1,$$

i.e. it is a PN expansion, cf. Eq. (10). In other words, every time derivative in the near zone represents an extra $O(v)$.

4 The source: conservation properties

The source terms for the equations (15), (16) and (19) are expressed in terms of the pseudotensor $\tau_{\mu\nu}$. In order to find solutions to the Poisson and wave equations in the far-zone, Eqs. (25) and (22) indicate that we need to evaluate various integrals of $\tau_{\mu\nu}$. In what follows, we present results that are relevant for simplifying those integrals (and therefore the wave forms that appear in Sec. 5) and include leading PN corrections. We refer the reader to Appendix A for more details on the first order PN approximation and the parametrized-post-Newtonian (PPN) formalism [12].

From Eq. (12), one can establish the useful integral conservation laws,

$$\int \mathrm{d}^3x\, \tau_{ij} = \frac{1}{2}\int \mathrm{d}^3x\, \ddot{\tau}_{00}x^i x^j - \frac{1}{2}\int \mathrm{d}^3x\, \partial^\mu \dot{\tau}_{\mu 0}x^i x^j. \tag{26a}$$

$$\int \mathrm{d}^3x\, \dot{\tau}_{0i}x^j = -\int \mathrm{d}^3x\, \tau_{ij}. \tag{26b}$$

$$\int \mathrm{d}^3x\, \dot{\tau}_{00}x^i = -\int \mathrm{d}^3x\, \tau_{i0} + \int \mathrm{d}^3x\, \partial^\mu \tau_{\mu 0}x^i. \tag{26c}$$

In deriving the previous equations, we assume that all the boundary integrals cancel (the corrections to this assumption are negligible at large r). The difference with respect to the GR integral conservation laws is the presence of the terms proportional to $\partial^\mu \tau_{\mu 0}$ coming from the non-conservation of $\tau_{\mu\nu}$, Eq. (12). Remember that the current τ_{i0} is conserved for khronometric theory . Naively one expects $\partial^\mu \tau_{\mu 0}$ to contribute to order as low as $O(v^3)$. To see that this is not the case, we notice that Eqs. (26) can be simplified by writing the equation of motion (6) as an equation for a conserved current (which corresponds to the Noether current related to the invariance of the theory under reparametrizations of φ, Eq. (2)),

$$\partial_\mu(\sqrt{-g}J^\mu) = 0. \tag{27}$$

Furthermore, in the unitary gauge, $\varphi = t$ and $J^0 = 0$ (see Appendix D of [5]). Since J^i is linear in perturbations, we find that

$$\bar{\mathcal{Q}}_\chi = -\partial_i(\sqrt{-g}J^i)^{NL}. \tag{28}$$

Thus,

$$\partial^\mu \tau_{0\mu} = M_b^2 \partial_i(\sqrt{-g}J^i)^{NL} \sim O(v^5), \tag{29}$$

and the dipolar corrections turn out to be large in PN order. In particular, at order $O(v^5)$ only the Eq. (26c) is modified with respect to GR. A straightforward but tedious calculation using the PN metric displayed in Eq. (48) yields

$$\int \mathrm{d}^3x\, \partial^\mu \tau_{0\mu}x^i = \frac{1}{2}\int \mathrm{d}x\ \rho\left[(\alpha_1^{PPN} - \alpha_2^{PPN})V_i^{PPN} + \alpha_2^{PPN}W_i^{PPN}\right] + O(v^6), \tag{30}$$

where

$$\begin{aligned} \alpha_1^{PPN} &= \frac{4(\alpha - 2\beta)}{\beta - 1}, \\ \alpha_2^{PPN} &= \frac{(\alpha - 2\beta)(-\beta[3+\beta+3\lambda] - \lambda + \alpha[1+\beta+2\lambda])}{(\alpha-2)(\beta-1)(\beta+\lambda)}. \end{aligned} \tag{31}$$

These constants are the PPN parameters related to the violation of Lorentz invariance of the theory (see Appendix A). In the limit of small parameters they coincide with those found in [5]. The potentials V_i^{PPN} and W_i^{PPN} are also defined in Appendix A. Finally, the form of the Eq. (30) is identical to the one found for æ-theory [21].

The previous formulae (26) and (30) can also be derived by relating the pseudo-tensor $\tau_{\mu\nu}$ to a conserved (but asymmetric) object. Indeed, from Eqs. (12) and (6) it is evident that

$$\mathfrak{T}_{\mu\nu} \equiv \tau_{\mu\nu} + M_b^2 \delta^0_\mu \, \eta_{\nu\rho} \bar{J}^\rho$$

satisfies

$$\partial^\nu \mathfrak{T}_{\mu\nu} = 0.$$

This object has a contribution linear in the fields. To build a quadratic conserved pseudo-tensor it is enough to add the conserved current found in (27) and consider the object

$$\mathfrak{T}^q_{\mu\nu} \equiv \tau_{\mu\nu} + M_b^2 \delta^0_\mu \eta_{\nu\rho} (\bar{J}^\rho - \sqrt{-g} J^\rho) = \tau_{\mu\nu} - M_b^2 \delta^0_\mu \, \eta_{\nu\rho} (\sqrt{-g} J^\rho)^{NL}.$$

The resulting integral conservation laws for this object are then identical to the ones found in [12, 21]. The existence of this conserved quadratic current is a generic consequence of the theory being *semi-conservative* in the language of [12]. This conserved current satisfies

$$\mathfrak{T}^q_{0i} - \mathfrak{T}^q_{i0} = M_b^2 (\sqrt{-g} J^i)^{NL}.$$

Then, one can use the Eq. (4.103) in [12] to compute (29). Even if this method may save a lot of computations, it is inconvenient since the result in [12] is derived in the PPN gauge, whereas we are interested in the result in the unitary gauge (30).

5 Wave forms in the far-zone

We are now ready to compute the explicit form of the wave solutions in the far-zone, which we do consistently up to $O(v^6)$ in the PN approximation. For the tensor and vector modes, the solutions of Eqs. (15), (16), (25) and (26) are (in the gauge $F_i = 0$)

$$t_{ij}(t,x) = -\frac{1}{4\pi M_b^2 r} \hat{P}_{ij,ks} \ddot{Q}_{ks}(t - r/c_t) - \frac{1}{2\pi M_b^2 c_t \, r} \hat{P}_{ij,ks} \hat{r}^a \dot{S}_{ks,a}(t - r/c_t) + O(v^6), \quad (32a)$$

$$V_i(t,x) = -\frac{c_t^2}{2\pi M_b^2} \left(\frac{1}{r} \int \mathrm{d}^3\tilde{x} \; P_{ij} \tau_{j0}(t, \tilde{x}) \right), \quad (32b)$$

where

$$Q(t)_{ij} \equiv I(t)_{ij} - \frac{1}{3}\delta_{ij} I_{kk}(t), \quad I_{ij}(t) \equiv \int \mathrm{d}\tilde{x}\, \tau_{00}(t,\tilde{x})\tilde{x}^i\tilde{x}^j,$$
$$S_{ks,a}(t) \equiv \int \mathrm{d}^3\tilde{x}\ \tau_{ks}(t,\tilde{x})\tilde{x}^a.$$

The quantity Q_{ij} represents the quadrupole of τ_{00}. Note that in the far-zone, the longitudinal projector P_{ij} of Eq. (14) can be substituted by the longitudinal part of the algebraic projector,

$$\hat{P}_{ij} \equiv \delta_{ij} - \hat{r}^i\hat{r}^j.$$

This substitution is valid up to $O(R/r)$ terms. The object $\hat{P}_{ij,ks}$ is defined as

$$\hat{P}_{ij,kr} \equiv \hat{P}_{ik}\hat{P}_{jr} - \frac{1}{2}\hat{P}_{ij}\hat{P}_{kr}.$$

Anticipating the results of Sec. 6, we notice that the energy-loss formula depends on the time derivative of the fields. For the vector part, the previous expression yields

$$\dot{h}_{0i} = \dot{V}_i = -\frac{c_t^2}{2\pi M_b^2}\left(\frac{1}{r}\int \mathrm{d}^3\tilde{x}\ P_{ij}\dot{\tau}_{j0}(t,\tilde{x})\right). \tag{33}$$

From the conservation law (12), this term can be expressed as the integration over the boundary of the transverse component of the source, which cancels away from the source, and we can neglect the vector perturbations altogether.

Concerning the scalar field, from the wave equation (19c) one finds

$$\begin{aligned}\psi =& \frac{\alpha}{8\pi(\alpha-2)M_b^2 r}\Bigg(\frac{3}{2}\left[Z-1\right]\hat{r}^i\hat{r}^j\ddot{Q}_{ij}(t-r/c_s) + \frac{1}{2}Z\,\ddot{I}_{kk}(t-r/c_s) \\ &+ \frac{2}{c_s\alpha}\hat{r}^i\int \mathrm{d}^3\tilde{x}\ \dot{\tau}_{00}(t-r/c_s,\tilde{x})\tilde{x}^i + \frac{1}{3c_s^3\alpha}\hat{r}^i\hat{r}^j\hat{r}^k\int \mathrm{d}^3\tilde{x}\,\dddot{\tau}_{00}(t-r/c_s,\tilde{x})\tilde{x}^i\tilde{x}^j\tilde{x}^k \\ &+ \frac{1}{c_s}\hat{r}^a\dot{S}_{kk,a}(t-r/c_s) - \frac{(2+\beta+3\lambda)}{c_s(\beta+\lambda)}\hat{r}^i\hat{r}^j\hat{r}^k\dot{S}_{ij,k}(t-r/c_s)\Bigg) + O(v^6),\end{aligned} \tag{34}$$

where

$$Z \equiv \frac{(\beta-1)(\alpha_1^{PPN} - 2\alpha_2^{PPN})}{3(\alpha-2\beta)}. \tag{35}$$

Notice that the conservation law $\dot{\tau}_{00} = \partial_i(\tau_{0i}+\bar{J}_i)$ has been used to show that the first moment of τ_{00} is constant in time and therefore ignored in (34). From the results in the previous section, we see that the modification to the GR results appear at order $O(v^4)$. Notice also that it follows from (30) and the constancy of the integral of τ_{i0} that the dipolar contribution in (34) is $O(v^5)$ and suppressed by the PPN parameters. Finally, the quadrupole terms in the tensor and scalar sectors differ slightly, as they depend on different retarded times. For the remaining scalar fields ϕ and E, from Eqs. (19) one finds

$$\phi = \frac{2}{\alpha}\psi + \frac{1}{4\pi M_b^2 \alpha\, r}\int \mathrm{d}^3\tilde{x}\, \tau_{00}, \quad \dot{E} = -\frac{2(\lambda+1)}{\beta+\lambda}\dot{\psi}. \tag{36}$$

6 Energy-loss formulae for post-Newtonian systems

The definition of the energy carried by gravity waves is non-trivial (see [32] for a review on the concepts of energy and momentum in GR). For the problem at hand, we follow the procedure of [21] (see also [33]) and use the notion of energy for asymptotically flat spacetimes derived in [34]. Given an isolated source, we can compute the time variation of this notion of energy by performing an integral of the flux in the far-zone, which we idealize as being infinitely far away from the source. We associate this energy loss to the energy carried away by gravitational radiation. As shown in [34, 35], this alternative approach is equivalent to the one based on pseudo-tensors used in standard computations of energy loss due to gravitational radiation [12, 15, 31]. We give a brief review on this method in Appendix C,

In deriving the energy-loss formula, we make the following assumptions. We start by assuming that our system consists of an asymptotically Minkowski spacetime at early times, with the following fall-off properties in the unitary gauge,

$$g_{\mu\nu} = \eta_{\mu\nu} + O(1/r), \quad \partial_\alpha g_{\mu\nu} = O(1/r^2), \quad \chi = 0. \tag{37}$$

As for the matter fields, we assume that they vanish asymptotically to ensure that there are no boundary integral contributions. The previous conditions allow us to define a convenient notion of conserved energy $\mathcal{E}$, Eq. (61), as the conserved charge associated to the invariance of the asymptotic solution under asymptotic time translations[8]. To compute the flux of gravitational radiation, we consider the moment of time when the emitted GWs are already at spatial infinity, which means that the fall-off properties of the fields change to

$$h_w \sim O(1/r), \quad \dot{h}_w \sim \omega O(1/r), \quad \partial_r h_w \sim \omega/c_s O(1/r). \tag{38}$$

[8]Even if this symmetry is broken by the background for the field φ, it is still a symmetry due to the reparametrization invariance of the theory (2).

The quantity $\mathcal{E}$ with these boundary conditions is in general divergent. Nevertheless, its change due to the radiation emitted during a finite interval of time is well-defined [36]. We focus on computing the time variation of $\mathcal{E}$, Eq. (64). As shown in Appendix C, $\dot{\mathcal{E}}$ is finite for conditions (38) and only has contributions that are quadratic in the fields.

We also consider the time average of the quantity $\dot{\mathcal{E}}$ over several periods of the source, $\langle\dot{\mathcal{E}}\rangle$. The final averaged energy-loss formula is a standard observable in GW experiments (including the binary system of interest, where the observed damping of the orbits occurs after several periods) and the final expression is simplified since total time derivatives vanish when integrated.

The final expression is further simplified after one takes into account the following considerations. From the form of the solution of the tensor modes, Eq. (32a), and of the field ψ, Eq. (34), then in the far-zone these fields satisfy the equation

$$c_\sigma\,\partial_i\sigma = -\hat{r}^i\dot{\sigma}, \tag{39}$$

for the corresponding speeds of propagation. Remember also that in the far-zone, the tensor modes t_{ij} are transverse with respect to the algebraic projector $\hat{r}^i t_{ij} = 0$. For the vector part, we already showed that it does not contribute to $\langle\dot{\mathcal{E}}\rangle$, as its time derivative cancels, cf. Eq. (33). Similarly, the fields E and ϕ always appear under a time or a space derivative. Thus, we notice that the dependence on the source appearing in Eq. (36) will be either higher order in R/r for the space derivatives (and therefore negligible), or of the form

$$\int \mathrm{d}^3\tilde{x}\,\dot{\tau}_{00} = -M_b^2\int \mathrm{d}^3\tilde{x}\,\bar{\mathcal{Q}}_\chi = M_b^2\int \mathrm{d}^3\tilde{x}\,\partial_i(\sqrt{-g}J^i)^{NL} = 0.$$

So, only the ψ contribution for E and ϕ is non-zero, and therefore these fields satisfy relation (39). In fact, the latter relation is also satisfied by the scalar part of h_{ij}.

The previous considerations (and some algebra presented in the Appendix C) yield the final result for the rate of energy loss of the system,

$$\langle\dot{\mathcal{E}}\rangle = -\frac{M_b^2}{4}\oint_{S^2_\infty}\mathrm{d}\Omega\,r^2\left\langle\frac{1}{c_t}\dot{t}_{ij}\dot{t}_{ij} - \frac{8(\alpha-2)}{\alpha\,c_s}\dot{\psi}\dot{\psi}\right\rangle. \tag{40}$$

Whereas the radiation emitted in the tensor modes always decreases the energy of the system, the behaviour of the emitted scalar modes depends on the parameter α. We see that the emitted energy is positive for the range $0 < \alpha < 2$, as expected since these values are also required for the stability of the Minkowski background (absence of ghosts) [7].

Up to this point, we have consistently worked to first PN order (which corresponds to and includes $O(v^5)$ in the wave-forms). Given the time derivatives in Eq. (40), substitution of the waveforms (32a) and (34) yields the energy loss of the system from gravitational radiation up to and including $O(v^{12})$, although corrections already appear at leading order, $O(v^{10})$. To simplify what follows, we therefore focus on Newtonian sources and the corrections at this order. Substituting the waveforms to lowest PN order in the previous expressions and performing the angular integrals, we find the energy-loss formula[9]

$$\langle \dot{\mathcal{E}} \rangle = -\frac{1}{8\pi M_b^2} \left\langle \frac{\mathcal{A}}{5} \dddot{Q}_{ij} \dddot{Q}_{ij} + \mathcal{B} \dddot{I} \dddot{I} \right\rangle , \tag{41}$$

where (recall Eq. (35))

$$\mathcal{A} \equiv \frac{1}{c_t} - \frac{3\alpha(Z-1)^2}{2c_s(\alpha-2)}, \quad \mathcal{B} = -\frac{\alpha Z^2}{4c_s(\alpha-2)}.$$

The final expression, Eq. (41), differs from the GR result in two ways: the coefficient corresponding to the quadrupole depends on the parameters of the model; there is a monopole contribution already at this first Newtonian order. This is similar to the formula derived for æ-theory[10] [21]. Let us note something quite remarkable in the context of the khronometric theory that we are studying. All of the Solar System tests are passed in the limit $|\alpha_1^{PPN}| \ll 1$, $|\alpha_2|^{PPN} \ll 1$, which can be achieved by the single requirement $|\alpha - 2\beta| \ll 1$, cf. (31). In this limit, $Z = 1$, the dipole term (30) cancels and the monopole contribution in Eq. (41) is still present. This last result contrasts with the æ-theory case for which there is only a modified quadrupole (in the equivalent limit). This discontinuity between the two theories is discussed in Appendix B.

7 Energy loss by a Newtonian binary system

To complete the calculation, the power-loss formula (41) must be supplemented by the equations of motion of the system to desired post-Newtonian (PN) order. We content ourselves with a 2-body Newtonian system composed of point-masses m_1 and m_2. The matter action is then given by

$$S^m = -\sum_{A=1}^{2} \int m_A \mathrm{d}s_A \,, \tag{42}$$

[9] In this final formula, we compute the quadrupole and monopole terms at a time when radiation from both the tensor and scalar modes simultaneously reaches the boundary of the isolated system. For different speeds of propagation c_t and c_s, the discrepancy in emission time is irrelevant for stationary production of GWs.

[10] For the monopole and quadrupole contributions, the definition of Z in [21] differs from Eq. (35) by a factor of two. We attribute this difference to a typo in the final formula for ψ in [21].

where ds_A represents the proper-time of the A-th particle. A priori, m_A depends on the khronon field. Since we are only interested in the Newtonian system, it is enough to Taylor expand the mass around its background value and use only the leading order contribution, hence m_A is taken to be constant. Using the preferred time as the affine parameter, the energy-momentum tensor derived from (42) is

$$T^m_{\mu\nu} = \frac{1}{\sqrt{-g}} \sum_{A=1}^{2} \frac{m_A u_{A\mu} u_{A\nu}}{\sqrt{g_{\rho\sigma} u_A^\rho u_A^\sigma}} \delta^{(3)}(x^k - x^k_A(t)),$$

where the A-th body follows the trajectory $x^k_A(t)$ with four-velocity u^μ_A. At Newtonian order this yields

$$\ddot{I}_{ij}(t - r/c_\sigma) = \partial_0^2 \sum_{A=1}^{2} m_A x^i_A(t - r/c_\sigma) x^j_A(t - r/c_\sigma).$$

We evaluate this at very late times as explained in the previous section. Next, from the geodesic equation derived from (42), we find Newton's law

$$\ddot{x}^i_1 = -G_N \frac{m_2}{r_{12}^2} \hat{r}^i_{12}, \quad \ddot{x}^i_2 = G_N \frac{m_1}{r_{12}^2} \hat{r}^i_{12},$$

where we introduce (also see Appendix A)

$$G_N \equiv \frac{1}{4\pi M_b^2 (2-\alpha)}, \quad r^i_{12} \equiv x^i_1 - x^i_2. \tag{43}$$

As usual in binary systems, it is convenient to define the problem in terms of the relative distances and the position of the center of mass $x_{CM} \equiv \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$. Finally, assuming[11] that the system is at rest with respect to the preferred frame (so that $\dot{x}_{CM} = 0$) we get

$$\dddot{I}_{ij}(t - r/c) = -\frac{2 G_N \mu M}{r_{12}^2} \left(4 \hat{r}^{(i}_{12} v^{j)} - 3 \hat{r}^i_{12} \hat{r}^j_{12} \dot{r}_{12} \right).$$

with $\mu \equiv m_1 m_2 / M$, $M \equiv m_1 + m_2$ and $v^i \equiv \dot{r}^i_{12}$ is related to the expansion parameter v. Thus, the loss of energy in gravitational radiation for a Newtonian binary system is given by

$$\langle \dot{\mathcal{E}} \rangle = -\frac{1}{\pi M_b^2} \left(\frac{G_N M \mu}{r_{12}^2} \right)^2 \left\langle \frac{1}{15} \mathcal{A} \left(12 v^2 - 11 \dot{r}_{12}^2 \right) + \frac{\mathcal{B}}{2} \dot{r}_{12}^2 \right\rangle, \tag{44}$$

from which we deduce the 'Peters-Mathews' (PM) parameters [12, 37] ($\kappa_D = 0$),

$$\kappa_1 = 12(1 - \alpha/2)\mathcal{A}, \quad \kappa_2 = (1 - \alpha/2) \left(11 \mathcal{A} - \frac{15}{2} \mathcal{B} \right).$$

[11] Corrections to this assumption are considered as higher order in the PN expansion.

Once the energy loss for the binary system is known, one can use Kepler's third law to relate it to the damping of the orbit. The expression for the change of the orbit's period for generic PM parameters in terms of other orbital parameters of the system can be found in [38].

In GR, the previous analysis suffices to predict the radiation damping of binary systems for compact (relativistic) sources, like the PSR1913+16 [12]. This is because the structure of the compact stars of the binary does not influence the orbit in GR (this is called the 'effacing principle' which is a consequence of the strong equivalence principle). This is certainly not true for most alternative theories of gravity. Thus, to yield concrete predictions about the radiation damping of systems with highly relativistic sources (sources with large self-energies), one must first understand the behaviour of the fields beyond the PN approximation. One can then use Eq. (41) to derive the energy loss resulting in a change of the orbit (at corresponding PN order). For scalar-tensor theories, the final result is a test of the strong-field regime [39, 40, 41]. For æ-theory, the first steps were performed in [21] based on the stellar solutions of [24] and using the effective field theory methods of [42, 43, 44, 45]

It is beyond the scope of our article to derive the radiation damping of these realistic systems (including relativistic self-gravitating objects) for khronometric theory. In any case, we do not expect the new corrections to cancel the ones we have already derived for the Newtonian source, and thus we find it appropriate to use Eq. (44) to set order of magnitude bounds on the free parameters of the khronometric action (4). Current data on the radiation damping of the Hulse-Taylor binary system agrees with GR up to a level slightly better than one part in one hundred [1, 17]. This means that the formula (44) should agree with GR to $O(10^{-2})$, which finally implies the bound (for the case where α, β and λ are of the same order)

$$\alpha \sim \beta \sim \lambda \lesssim 10^{-2}. \tag{45}$$

The previous bound is less stringent than the bounds coming from the PPN analysis [5, 12],

$$|\alpha_1^{PPN}| \lesssim 10^{-4}, \quad |\alpha_2^{PPN}| \lesssim 10^{-7}.$$

As can be directly seen from Eq. (31), the PPN bounds are automatically satisfied in the limit $\alpha = 2\beta$. In this limit, $Z = 1$, and our expression (45) yields the most stringent bound for the theory. Notice in particular that it constrains the propagation speeds to be close to $c = 1$. Another constraint in this limit comes from the difference between G_N as derived in (43) and the value for Newton's constant appearing in Friedmann's equation, G_c [7]. The value of G_c is constrained by nucleosynthesis and satisfies $\left|\frac{G_N}{G_c} - 1\right| \leq 0.13$ [46], which, in terms of the parameters in the action (4), implies the estimate α, β, $\lambda \lesssim 0.1$ [7]. Also, we should ensure the absence of gravitational Cherenkov radiation, which implies $c_t^2 \geq 1$ and $c_s^2 \geq 1$ (this means that a particle moving through the aether does not radiate[12]). Notice that the speeds are superluminal, which does not pose a threat to Lorentz violating theories as long as causality is maintained.

8 Discussion

Our aim has been to study the radiation loss from an isolated source in the PN approximation for khronometric theory. This theory is an interesting alternative to GR with a high energy cutoff and for which a UV completion is known in the form of Hořava gravity. It is also very similar to æ-theory, as in both cases there is a preferred time coordinate. The difference is that khronometric theory has only one extra scalar degree of freedom, the khronon, whereas æ-theory relies on a timelike unit dynamical vector leading to three extra degrees of freedom, consisting of one scalar and one vector field.

For arbitrary parameters, we have shown in Eq. (41) that the formula controlling the power loss of the system (which may be related to the change of the orbital period of a binary source) is modified with respect to GR already at lowest, Newtonian, order. In particular, the quadrupole contribution differs from GR, partly due to the different speeds of propagation of the tensor modes in both theories. Furthermore, there is also an extra monopole contribution at this order. The monopole at leading order in khronometric theory contrasts with the usual situation in other scalar-tensor theories [18]. At higher order, there are other modifications, including the dipole term (30). Quite remarkably, in the phenomenologically interesting limit where all PPN parameters coincide with GR (which corresponds to the limit $\alpha = 2\beta$ for our parameters), the monopole is still present, and its strength is proportional to the parameters appearing in the action of the theory, Eq. (4). These results for khronometric theory are similar to those of æ-theory, modulo vector propagating degrees of freedom that are absent for the khronometric case. There is a key difference, however, since æ-theory only has a modified quadrupole to lowest order in the equivalent limit.

This work has been devoted to PN sources. These types of sources do not correspond to the ones found in the binary systems of interest, which are compact and characterized by strong gravitational fields. Despite this, we have evaluated the energy-loss formula for the simplest possible system: a Newtonian binary. Doing so provides an order of magnitude estimate on the parameters of the theory (as we do not expect corrections due to strong-

[12] We thank D. Levkov and S. Sibiryakov for pointing this out to us.

fields to cancel the modifications apparent in the power loss formula). Thus, our results are relevant for constraining the case $\alpha = 2\beta$. In this case, requiring the rate of radiation damping to be close to GR sets constraints on this parameters of order $O(10^{-2})$. These constraints represent the strongest phenomenological bounds for this particular choice of parameters and are relevant for the cosmological implications of the theory, including the recently suggested model of dark-energy [47].

Sources with strong self-energies is left for future research and can be treated in our theory in the same way as scalar-tensor theories [39, 40, 41]: a phenomenon of "scalarization" modifies the orbit of these sources as compared with the post-Newtonian ones. It would also be interesting to consider our results in the parametrized post-Einsteinian framework introduced in [48] (see also [49] for the binary pulsar constraints for this framework). Finally, the consequences of alternatives theories of gravity for experiments of direct detection of GWs have been recently discussed, see e.g. [1, 50]. We hope to extend these works to khronometric theories in the future.

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A Post-Newtonian expressions

The PPN formalism is a valuable tool for comparing theories of gravitation with each other and with experiment in the weak, non-relativistic limit [12] . In this section, we briefly present the steps involved in the PPN calculation for khronometric theory (see also [5]). The final result are the parameters (all the other PPN parameters cancel)

$$\begin{aligned}
&\beta^{PPN} = \gamma^{PPN} = 1, \\
&\alpha_1^{PPN} = \frac{4(\alpha - 2\beta)}{\beta - 1}, \\
&\alpha_2^{PPN} = \frac{(\alpha - 2\beta)(-\beta(3 + \beta + 3\lambda) - \lambda + \alpha[1 + \beta + 2\lambda])}{(\alpha - 2)(\beta - 1)(\beta + \lambda)}.
\end{aligned} \tag{46}$$

Notice that the PPN parameters for khronometric theory for arbitrary values of the parameters in (4) appear here for the first time. They coincide with results in [5] in the limit of small parameters. The non-zero parameters α_1^{PPN} and α_2^{PPN} indicate that khronometric theory violates Lorentz invariance. These same two parameters are non-vanishing for æ-theory, although the dependence on the parameters α, β and λ is different. In both theories, however, the relationship between α_1^{PPN} and α_2^{PPN} is the same

$$\alpha_2^{PPN} = \frac{\alpha_1^{PPN}}{2} - \frac{(2\beta-\alpha)(3\lambda+\beta+\alpha)}{(\lambda+\beta)(2-\alpha)}.$$

To compute the previous results we closely follow [12, 51] to which we refer the reader for further details. The source is assumed to be a fluid with a covariantly conserved energy-momentum tensor

$$T^{\mu\nu} = (\rho + \rho\Pi + p)v^\mu v^\nu - p g^{\mu\nu},$$

where v^μ is the four velocity of the source, ρ the rest mass energy density, Π the internal energy density and p the isotropic pressure of the fluid. The source is assumed to satisfy (10).

In what follows, recall that the different fields have the following expansion,

$$\begin{aligned} g_{00} &= 1 + O(v^2) + O(v^4), \quad g_{0i} = O(v^3), \\ g_{ij} &= -\delta_{ij} + O(v^2), \quad \chi = O(v^2) + O(v^3). \end{aligned} \tag{47}$$

Also, we use the following potentials

$$F(x) = G_{\rm N} \int d^3y \frac{\rho(y) f}{|x-y|},$$

where $G_{\rm N}$ is defined in Eq. (43) and the correspondence $F \mapsto f$ is given by

$$U \mapsto 1, \quad \Phi_1 \mapsto v_i v_i, \quad \Phi_2 \mapsto U, \quad \Phi_3 \mapsto \Pi, \quad \Phi_4 \mapsto p/\rho,$$
$$V_i^{PPN} \mapsto v^i, \quad W_i^{PPN} \mapsto \frac{v_j(x_j-y_j)(x^i-y^i)}{|x-y|^2}.$$

The steps to take are:

1. Solve g_{00} to order $O(v^2)$. For this we use the 00 component of Eq. (5) to $O(v^2)$, which yields[13]

$$\Delta \overset{2}{h}_{00} = 8\pi G_N \rho.$$

[13]We use a number over the field to keep track of the order in v.

2. Solve g_{ij} to $O(v^2)$. Following [51], we choose the gauge conditions

$$\partial_i \overset{2}{h}_{ij} = -\frac{1}{2}\left(\partial_i \overset{2}{h}_{00} - \partial_i \overset{2}{h}_{kk}\right). \quad \partial_i \overset{3}{h}_{0i} = \Gamma \partial_0 \overset{2}{h}_{00}.$$

The arbitrary constant Γ will be chosen to write the result in the PPN gauge. Then from the ij component of Eq. (5) to $O(v^2)$, we find

$$\Delta \overset{2}{h}_{ij} = 8\pi G_N \rho\, \delta_{ij}.$$

3. Solve χ to $O(v^3)$. The khronon equation of motion (6) to leading order is given by

$$\left(\Delta \overset{3}{\chi} - \Gamma \partial_0 \overset{2}{h}_{00}\right) = -\frac{(3\lambda + \alpha + \beta)}{2(\lambda + \beta)} \partial_0 \overset{2}{h}_{00},$$

4. Solve g_{0i} to $O(v^3)$. In our gauge, the $0i$ component of Eq. (5) to $O(v^3)$ yields

$$\Delta \overset{3}{h}_{0i} = \frac{8\pi G_N \rho\, v_i(\alpha - 2) - [-2 + \alpha + \Gamma(1 - \beta)]\partial_0 \partial_i \overset{2}{h}_{00}}{\beta - 1}.$$

5. Solve g_{00} to $O(v^4)$. From the 00 component of Eq. (5) to $O(v^4)$, we find

$$\begin{aligned}\Delta \overset{4}{h}_{00} &= \partial_i \overset{2}{h}_{00} \partial_i \overset{2}{h}_{00} - \overset{2}{h}_{00} \Delta \overset{2}{h}_{00} - 4\Delta\Phi_1 + 4\Delta\Phi_2 - 2\Delta\Phi_3 - 6\Delta\Phi_4 \\ &+ \frac{(-\alpha^2 + 2\beta(3 + \beta - 2\Gamma) + 2(3 + 3\beta - 2\Gamma)\lambda + 2\alpha(\beta(\Gamma - 1) + (\Gamma - 3)\lambda))}{(\alpha - 2)(\beta + \lambda)} \Delta \partial_0^2 H,\end{aligned}$$

where $H = -G_N \int \mathrm{d}^3 y \rho |x - y|$ is known as the superpotential.

6. To go to the PPN gauge, we choose Γ that cancels the term depending on H in the previous equation [12].

Putting everything together, we have (to desired order)

$$\begin{aligned} g_{00}^{PPN} &= 1 - 2U + 2U^2 - 4\Phi_1 - 4\Phi_2 - 2\Phi_3 - 6\Phi_4 = 1 + \Delta H, \\ g_{ij}^{PPN} &= -(1 + 2U)\delta_{ij} = \delta_{ij}\left(-1 + \Delta H\right), \\ g_{0i}^{PPN} &= \frac{1}{2}(7 + \alpha_1^{PPN} - \alpha_2^{PPN}) V_i^{PPN} + \frac{1}{2}(1 + \alpha_2^{PPN}) W_i^{PPN}, \\ \chi^{PPN} &= \frac{(\alpha - 2\beta)(2 + \beta + 3\lambda)\dot{H}}{2(\alpha - 2)(\beta + \lambda)}, \end{aligned}$$

which, compared to the generic PPN metric (see for example, Eq. (A.11) of [51]) implies that all the PPN parameters vanish except for the ones cited in (46).

The PN metric in the unitary gauge of Eqs. (17) and (18) is easily derived from these PPN expressions. It suffices to go from the PPN gauge to the unitary gauge via a diffeomorphism $\delta x^\mu = \xi^\mu$ satisfying

$$\xi^0 = -\frac{(\alpha - 2\beta)(2+\beta+3\lambda)\dot{H}}{2(\alpha-2)(\beta+\lambda)}, \quad \xi^i = \frac{(\alpha+\beta+3\lambda)\partial_i H}{2(\beta+\lambda)}.$$

This leads to the following PN metric in the unitary gauge

$$\begin{aligned} g_{00} &= 1 + \Delta H + O(v^4), \\ g_{ij} &= \delta_{ij}\left(-1+\Delta H\right) - \frac{(\alpha+\beta+3\lambda)}{\beta+\lambda}\partial_j\partial_i H + O(v^4), \\ g_{0i} &= \frac{1}{4}(8+\alpha_1^{PPN})(V_i^{PPN} + W_i^{PPN}) + O(v^4), \\ \chi &= O(v^4). \end{aligned} \tag{48}$$

B The Einstein-aether and the monopole

In both khronometric and Einstein-aether theories, we compare the monopole contribution to the energy-loss formula in the limit for which the PPN parameters are identical to GR. The free parameters of khronometric theory are α, β and λ and those of the Einstein-aether [13] are c_i for $i = 1, \ldots, 4$. We have the correspondence[14] $c_1 = 0$, $c_2 = \lambda$, $c_3 = \beta$ and $c_4 = \alpha$. Notice that one less parameter is needed to define khronometric theory. This is because the action of a hypersurface orthogonal aether (which is equivalent to khronometric theory [5]) contains a term that can be absorbed by the others, reducing the number of independent terms from four down to three.

Comparing the results of this paper and the work presented in [20], we see that the waveforms for the spin-0 and spin-2 modes are essentially identical. The main difference comes from the expression for Z of Eq. (35). Let $\tilde{Z}$ be the equivalent expression in æ-theory,

$$\tilde{Z} \equiv \frac{(c_{13}-1)(\tilde{\alpha}_1^{PPN} - 2\tilde{\alpha}_2^{PPN})}{3(c_{14}-2c_{13})}, \tag{49}$$

where $\tilde{\alpha}_1^{PPN}$, $\tilde{\alpha}_2^{PPN}$ are the Lorentz violating PPN parameters in æ-theory and $c_{ij} = c_i + c_j$. Then the khronometric expression for Z is precisely $\tilde{Z}$, but with $c_1 = 0$.

The limit $\alpha_1^{PPN} = \alpha_2^{PPN} = 0$ in khronometric theory can be achieved by setting $\alpha = 2\beta$ and leads to $Z = 1$. By inspection of the energy-loss formula (41), we see that the monopole

[14] Recall that we are using the mostly minus signature. The mostly plus signature is used in [20] and leads to a different correspondence between the parameters.

is proportional to Z and therefore persists in this limit. In generic æ-theory, the equivalent limit that sets the PPN parameters to GR is given by

$$c_2 = \frac{-2c_1^2 - c_1 c_3 + c_3^2}{3c_1}, \quad c_4 = -\frac{c_3^2}{c_1} \tag{50}$$

and leads to $\tilde{Z} = 0$. The corresponding monopole is proportional to $\tilde{Z}$ and subsequently vanishes in this limit. Therefore, the values of Z and $\tilde{Z}$ explain the presence or absence of the monopole in the limit when the PPN parameters are identical to those of GR.

It is natural to ask if $\tilde{Z}$ can be tweaked so that æ-theory has a monopole when $\tilde{\alpha}_1^{PPN} = \tilde{\alpha}_2^{PPN} = 0$. A first possibility would be to consider the limit that ressembles khronometric theory, namely $c_1 = 0$ and $c_4 = 2c_3$. This leads to $\tilde{Z} = 1$, like in khronometric theory, indicating that a monopole may be possible. However, requiring only $c_1 = 0$ implies that $\alpha_1^{PPN} = 8$. One could try to set $c_1 = 0$ and $c_3 = 0$ to get $\alpha_1^{PPN} = 0$, but this case of æ-theory has yet to be studied [13]. Alternatively, one may try to make the denominator in (49) vanish to retrieve a finite limit. Setting $c_{14} = 2c_{13}$ yields $\tilde{Z} = 1$. However, the second condition in (50) implies $c_1 = c_3 = c_4 = 0$, which is a singular limit for æ-theory.

C Notion of energy for an asymptotically flat spacetime

To characterize the energy carried away from a system by GWs, we use a method different from the standard technique defined in terms of the Landau-Lifshitz or related pseudotensors [12, 15, 31, 32]. Here, instead of computing the energy carried by GWs, we derive the loss of energy of the isolated system during the process of gravitational radiation. This resembles the definition of energy loss by the time variation of the Bondi-Sachs mass [28, 32]. However, we will use a different notion of conserved energy that, to our knowledge, was first used in the context of GWs in [20]. This energy is well-defined for asymptotically flat spacetimes satisfying the boundary conditions (37), which we use to define isolated sources. Its conservation follows from the invariance of the asymptotic solution under time translations and it reduces to the standard notion of energy for flat spacetime [34] (see also [36, 52]). Since the method is not standard, this Appendix is devoted to presenting a succinct summary. We encourage the reader to consult the original literature to complement it.

Given a Lagrangian density $L(\Phi)$ depending on some dynamical fields Φ, we define its associated 4-form (we present the $3+1$ case) as

$$\mathbf{L}(\Phi) = L(\Phi)\mathrm{d}^4 x.$$

After integration by parts, the first variation of the previous form following from the variation $\delta\Phi$ can be expressed as,

$$\delta\mathbf{L}(\Phi) = \mathbf{E}_\Phi\delta\Phi + \mathrm{d}\mathbf{\Theta}_L(\Phi, \delta\Phi),$$

where $\mathbf{E}_\Phi = 0$ are the equations of motion of the theory. If the variation $\delta\Phi$ is a diffeomorphism generated by a vector field ξ, the previous variation should correspond to the action of this transformation over $\mathbf{L}(\Phi)$,

$$\delta_\xi\mathbf{L}(\Phi) = \mathrm{d}(i_\xi\,\mathbf{L}),$$

where $i_\xi\,\mathbf{L}$ refers to the contraction of the form $\mathbf{L}$ with the vector field ξ. Define the Noether current 3-form associated to ξ and $L(\Phi)$ as

$$\mathbf{J}_L \equiv \mathbf{\Theta}_L(\Phi, \delta_\xi\Phi) - i_\xi\,\mathbf{L}. \tag{51}$$

This form is clearly closed when the equations of motion are satisfied. In practice, to find the components of the 3-form $\mathbf{\Theta}_L$, notice that it is dual to a 1-form. In components

$$\mathbf{\Theta}_{L\mu\nu\rho} = \epsilon_{\alpha\mu\nu\rho}\Theta_L^\alpha,$$

where the index of Θ_L^α is risen with the metric $g^{\mu\nu}$ and $\epsilon_{\alpha\mu\nu\rho}$ are the components of the Levi-Civita 3-form defined for the metric $g_{\mu\nu}$. From this definition it follows that

$$\mathrm{d}\mathbf{\Theta}_L = \sqrt{-g}\nabla_\mu\Theta_L^\mu\mathrm{d}^4x = \partial_\mu(\sqrt{-g}\Theta_L^\mu)\mathrm{d}^4x, \tag{52}$$

from which one can easily identify the components of $\mathbf{\Theta}_L$.

To associate the flux generated by ξ to a Hamiltonian evolution from an initial hypersurface Σ, one must assume [34, 52] that in the boundary of the initial hypersurface, denoted by $\partial\Sigma$, it is possible to find a 3-form $\mathbf{B}_L$ such that

$$\delta\int_{\partial\Sigma} i_\xi\,\mathbf{B}_L = \int_{\partial\Sigma} i_\xi\,\mathbf{\Theta}_L.$$

If such a current exists, the flux generated by ξ corresponds to the orbits generated by of the Hamiltonian

$$H_\xi \equiv \int_\Sigma \mathbf{J}_L - \int_{\partial\Sigma} i_\xi B_L. \tag{53}$$

Finally, since $\mathbf{J}_L$ is closed when the equations of motion are satisfied, it follows that locally $\mathbf{J}_L = \mathrm{d}\mathbf{Q}_L$. Thus, when the equations of motion hold, H_ξ can be written as a pure boundary term,

$$H_\xi = \int_{\partial\Sigma} \left(\mathbf{Q}_L - i_\xi\,B_L\right). \tag{54}$$

To define a canonical notion of energy, we shall now assume that ξ is an asymptotic time translation, with components $\xi^\mu \to \delta^\mu_0$ and that the asymptotic conditions on the dynamical fields have been specified in such a way that the surface integrals appearing in Eq. (54) approach a finite limit. The Hamiltonian then corresponds to the generator of time evolution. We define the canonical energy at a hypersurface slice of constant time Σ_t to be [34]

$$\mathcal{E}_L = \int_{S^2_t} \left(\mathbf{Q}_L - i_{\delta^\mu_0} B_L \right), \tag{55}$$

where S^2_t represents the boundary sphere at the boundary of Σ_t. Whenever $\mathcal{E}_L$ is well-defined, it is a conserved quantity, and we can remove the t label in S^2_t.

We now apply the previous formalism to our action (4). The hypersurface of constant time corresponds to a sheet of the preferred foliation. Even if not necessary, it is convenient to work with an action for which

$$\int_{S^2} i_\xi \, \mathbf{\Theta}_L = 0. \tag{56}$$

This equation is not satisfied for the Einstein-Hilbert action part of (4) (see e.g. Eq. (87) in [34]). As explained in [34], the existence of a background metric $\eta_{\mu\nu}$ makes it possible to build a covariant action (which is required to get a conserved current (51)) equivalent to Einstein-Hilbert and satisfying (56). Indeed, let us write $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and consider $h_{\mu\nu}$ and $\eta_{\mu\nu}$ as independent dynamical fields. We can then add a boundary term invariant under diffeomorphisms to the action (4) to yield

$$S' \equiv S + \frac{M_0^2}{2} \int \mathrm{d}^4x \left(\sqrt{-g} \left((\Gamma^\alpha_{\mu\nu} - \bar{\Gamma}^\alpha_{\mu\nu}) g^{\mu\nu} - (\Gamma^\mu_{\mu\nu} - \bar{\Gamma}^\mu_{\mu\nu}) g^{\nu\alpha} \right) \right)_{,\alpha} \equiv \int \mathrm{d}^4x \, L', \tag{57}$$

where $\bar{\Gamma}^\mu_{\sigma\nu}$ refers to the connection compatible with the background metric $\eta_{\mu\nu}$. The part corresponding to GR reads

$$S'_{\Gamma\Gamma} = -\frac{M_0^2}{2} \int \mathrm{d}^4x \left[\sqrt{-g} g^{\mu\rho} \left(\Gamma^\alpha_{\rho\nu} \Gamma^\nu_{\alpha\mu} - \Gamma^\nu_{\alpha\nu} \Gamma^\alpha_{\mu\rho} \right) + \left(\sqrt{-g} \left(\bar{\Gamma}^\alpha_{\mu\nu} g^{\mu\nu} - \bar{\Gamma}^\mu_{\mu\nu} g^{\nu\alpha} \right) \right)_{,\alpha} \right]. \tag{58}$$

The equations of motion derived from varying the previous action with respect to $h_{\mu\nu}$ and $\eta_{\mu\nu}$ are the same, as these fields appear only in the combination $g_{\mu\nu}$, except in the boundary term. As a consequence, $\eta_{\mu\nu}$ can be considered to be Minkowski, and we can assume that the equations of motion fix $h_{\mu\nu}$.

For the computation of $\mathbf{J}_{\Gamma\Gamma}$ corresponding to the action (58) and the vector field ∂_t, with components δ^μ_0, we first notice that ∂_t is a Killing vector of $\eta_{\mu\nu}$,

$$\delta_{\partial_t} \eta_{\mu\nu} = 2 \bar{\nabla}_{(\mu} \eta_{\nu)\alpha} \delta^\alpha_0 = 0,$$

and the boundary term in Eq. (58) does not contribute to $\mathbf{J}_{\Gamma\Gamma}$. For the first term one finds the corresponding current

$$\Theta^{\nu}_{\Gamma\Gamma} = \frac{M_0^2}{4}\Big(\Gamma^{\nu}_{\mu\alpha}(g^{\mu\alpha}g^{\rho\sigma}\delta g_{\rho\sigma} - 2g^{\mu\rho}g^{\alpha\tau}\delta g_{\tau\rho}) + g^{\nu\alpha}(2\Gamma^{\beta}_{\sigma\beta}g^{\rho\sigma}\delta g_{\rho\alpha} - \Gamma^{\beta}_{\alpha\beta}g^{\rho\sigma}\delta g_{\rho\sigma})\Big). \tag{59}$$

This term is linear in the connection and does not depend on the derivative of $\delta g_{\mu\nu}$. To construct the conserved current, we use

$$\delta_{\partial_t} g_{\mu\nu} = 2\nabla_{(\mu}g_{\nu)\alpha}\delta^{\alpha}_0 = 2g_{\alpha(\mu}\Gamma^{\alpha}_{\nu)0},$$

Thus, under the assumption that the fields fall-off at large distances as (37), the current (59) vanishes asymptotically as $O(r^{-4})$, which means that its contribution to (56) cancels. Indeed the cancellation of the contribution to (56) holds in the more general situation where one considers variations $\delta g_{\mu\nu}$ which do not change the asymptotic behaviour (37). Finally, the energy $\mathcal{E}_{\Gamma\Gamma}$ derived from Eq. (58) coincides with the ADM mass which also agrees with the energy derived from the Landau-Lifshitz pseudotensor [34, 35].

The term S_χ in the action (4) yields a current

$$\begin{aligned}\Theta^{\nu}_{\chi} = -M_b^2\Big[\,&\left(\alpha a_\sigma \nabla_\mu u^\sigma - \nabla_\rho K^{\rho}{}_{\mu}\right)\frac{\mathcal{P}^{\nu\mu}}{\sqrt{X}}\delta\chi + K^{\nu\rho}\frac{\mathcal{P}^{\alpha}_{\rho}}{\sqrt{X}}\partial_\alpha\delta\chi \\ &- \frac{1}{2}\left([K^{\nu\alpha}+K^{\alpha\nu}]\,u^\sigma - K^{\alpha\sigma}u^\nu - u^\alpha u^\sigma u_\rho K^{\nu\rho}\right)\delta g_{\alpha\sigma}\Big].\end{aligned} \tag{60}$$

Remember that the invariance under diffeomorphisms is non-linearly realized[15] on χ

$$\delta_\xi\chi = \xi^0 + \xi^\mu\partial_\mu\chi.$$

From Eq. (37) this means that $\delta_{\partial_t}\chi \sim O(1)$. Similarly $u_\alpha = \frac{\delta_{\alpha 0}}{\sqrt{g^{00}}} \sim \delta_{\alpha 0} + O(1/r)$. Thus, $\Theta^{\nu}_{\chi} \sim O(r^{-3})$, which means that the contribution of this term to (56) cancels.

Finally, we find that the conserved energy (53) for the action (57) inside a constant time hypersurface Σ_t is given by

$$\mathcal{E} = \int_{\Sigma_t} \mathrm{d}^3x\sqrt{-g}\,\mathcal{J}^0_{S'}, \tag{61}$$

with $\mathcal{J}^0_{S'}$ representing the coordinates of the 1-form dual to the corresponding 3-form, Eq. (51),

$$\mathcal{J}^{\nu}_{S'} \equiv (\Theta^{\nu}_{\Gamma\Gamma} + \Theta^{\nu}_{\chi}) - \delta^{\nu}_0 L'. \tag{62}$$

[15] We could also work with the field φ for which $\delta_\xi\varphi = \xi^\mu\partial_\mu\varphi$.

The contribution from the khronon action is simplified once one considers the equation of motion for χ. Indeed, Θ^ν_χ in Eq. (60) includes a term

$$\left(\alpha a_\sigma \nabla_\mu u^\sigma - \nabla_\rho K^\rho{}_\mu\right) \frac{\mathcal{P}^{\nu\mu}}{\sqrt{X}} = J^\nu, \tag{63}$$

where J^ν is defined in (6). In the unitary gauge, this current is purely spatial, which means that this term does not contribute to (61).

We are eventually interested in the flux of energy loss through GWs, so we want to compute the quantity,

$$\dot{\mathcal{E}} = \int_\Sigma \mathrm{d}^3x \sqrt{-g}\, \dot{\mathcal{J}}^0_{S'} = -\oint_{S^2_\infty} \mathrm{d}\Omega \sqrt{-g}\, r^2 \hat{r}^i \mathcal{J}^i_{S'}, \tag{64}$$

where we have used the fact that the current $\mathcal{J}^\mu_{S'}$ is conserved on-shell, which is a consequence of $\mathbf{J}$ being closed and (52). The final ingredient is to evaluate $\mathcal{J}^i_{S'}$. From Eq. (59),

$$\Theta^i_{\Gamma\Gamma} = \frac{M_b^2}{4} \dot{h}_{\alpha\beta} \left[\eta^{\alpha\beta}(\partial^\rho h^i_\rho - \partial^i h^\sigma_\sigma) - 2\partial^\alpha h^{\beta i} + \partial^i h^{\alpha\beta} + \eta^{\beta i}\partial^\alpha h^\sigma_\sigma\right] + O(h^3). \tag{65}$$

For the khronon terms, we find that at quadratic order in the unitary gauge

$$\Theta^i_\chi = M_b^2 \left[\bar{K}^{(\alpha i)}(\bar{\Gamma}^0_{\alpha 0} + \eta_{\alpha\rho}\bar{\Gamma}^\rho_{00}) - \bar{K}^{i0}\bar{\Gamma}^0_{00}\right]. \tag{66}$$

From this expression it is clear that the notion of energy (61) is not well defined for spacetimes with radiation at infinity satisfying conditions (38). This is an unphysical divergence, which is regularized for a flux of energy of finite duration [36]. For our purposes, it is enough to notice that the time variation (64) (and hence the flux) is well defined for these boundary conditions. Also, only the part of the integral quadratic in perturbations does not vanish, which implies that the previous expressions are enough to compute the flux of energy at infinity. The steps to go from the previous formula to the final result (40) are explained in Sec. 6.

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