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# Next-to-Minimal Supersymmetric Model Higgs Scenarios for Partially Universal GUT Scale Boundary Conditions

John F. Gunion<sup>1</sup>, Daniel E. López-Fogliani<sup>2,3</sup>, Leszek Roszkowski<sup>3,4</sup>, Roberto Ruiz de Austri<sup>5</sup>, and Tom A. Varley<sup>3</sup>

<sup>1</sup> *Department of Physics,*

*University of California at Davis, Davis, CA, USA*

<sup>2</sup> *Laboratoire de Physique Théorique,*

*Université Paris-Sud, F-91405 Orsay, France.*

<sup>3</sup> *Department of Physics and Astronomy,*

*The University of Sheffield, Sheffield S3 7RH, England*

<sup>4</sup> *The Andrzej Soltan Institute for Nuclear Studies, Warsaw, Poland*

<sup>5</sup> *Instituto de Física Corpuscular,*

*IFIC-UV/CSIC, Valencia, Spain*

We examine the extent to which it is possible to realize the NMSSM “ideal Higgs” models espoused in several papers by Gunion *et al.* in the context of partially universal GUT scale boundary conditions. To this end we use the powerful methodology of nested sampling. We pay particular attention to whether ideal-Higgs-like points not only pass LEP constraints but are also acceptable in terms of the numerous constraints now available, including those from the Tevatron and  $B$ -factory data,  $(g-2)_\mu$  and the relic density  $\Omega h^2$ . In general for this particular methodology and range of parameters chosen, very few points corresponding to said previous studies were found, and those that were found were at best  $2\sigma$  away from the preferred relic density value. Instead, there exist a class of points, which combine a mostly singlet-like Higgs with a mostly singlino-like neutralino coannihilating with the lightest stau, that are able to effectively pass all implemented constraints in the region  $80 < m_h < 100$ . It seems that the spin-independent direct detection cross section acts as a key discriminator between ideal Higgs points and the hard to detect singlino-like points.

Keywords: Supersymmetric Effective Theories, Cosmology of Theories beyond the SM, Dark Matter

## I. INTRODUCTION

As the LHC begins its first physics runs, more scrutiny can be placed on possible regions where Beyond the Standard Model (BSM) physics can exist. One intriguing possibility is the “ideal Higgs” scenario pointed out by Gunion *et al.* [1–3]. In this Next-to-Minimal Supersymmetric Model (NMSSM) scenario, the lightest Higgs,  $h_1$ , has mass  $m_{h_1} \sim 100$  GeV and SM-like couplings to  $WW, ZZ$ , but decays in such a way that LEP limits are obeyed. In general, this is possible if the  $h_1$  decays primarily to a pair of the lightest pseudoscalar Higgs bosons,  $h_1 \rightarrow a_1 a_1$ , (resulting in a small branching ratio for  $h_1 \rightarrow b\bar{b}$ ). Because  $m_{a_1} < 2m_B$ , the  $a_1$  then subsequently decays to  $2\tau, 2g$ , or  $2c$ , the precise mixture depending on  $\tan\beta$ , with  $2\tau$  being dominant at high  $\tan\beta$  while all states are important at low  $\tan\beta$ . This allows these points to sidestep the LEP results [4] on  $C_{\text{eff}}^{2b}$  and  $C_{\text{eff}}^{4b}$ , defined as:

$$\begin{aligned} C_{\text{eff}}^{2b} &= |C_V(1)|^2 \times BR(h_1 \rightarrow b\bar{b}), \\ C_{\text{eff}}^{4b} &\equiv |C_V(1)|^2 \times BR(h_1 \rightarrow a_1 a_1) \times [BR(a_1 \rightarrow b\bar{b})]^2, \end{aligned} \quad (1)$$

where  $|C_V(1)|^2 \equiv g_{h_1 ZZ}^2 / g_{h_{SM} ZZ}^2$ , where  $h_{SM}$  denotes the Higgs boson of the Standard Model. Roughly speaking, LEP constraints are evaded if  $BR(h_1 \rightarrow a_1 a_1) \gtrsim 0.7$ .

There are three main motivations for such a scenario.

- Precision electroweak (PEW) data is most consistent with a SM-like Higgs having mass below 100 GeV.
- There is an excess in the combined LEP data for  $e^+e^- \rightarrow Zb\bar{b}$  for  $M_{b\bar{b}}$  in the region 80 – 100 GeV that is consistent with  $BR(h_1 \rightarrow b\bar{b}) \lesssim 0.3$ , as obtained for  $BR(h_1 \rightarrow a_1 a_1) \gtrsim 0.7$ .
- A mass  $m_{h_1} \lesssim 100$  GeV is consistent with a light superparticle spectrum for which GUT scale parameters need not be fine tuned in order to obtain the correct value of  $m_Z$  at low scales.

One of the primary goals of this paper is to elucidate the extent to which the emergence of ideal-Higgs-like scenarios depends on the extent to which the pattern of soft supersymmetry breaking is highly constrained/universal.

Indeed, a key uncertainty in both the MSSM and NMSSM is the pattern of soft supersymmetry breaking, as described by the scalar masses  $m_0$ , gaugino masses  $m_{1/2}$  and trilinear couplings  $A_0$ . These presumably originate from

physics at some high-energy scale, e.g., from some supergravity or superstring theory, and then evolve down to lower energy scale according to well-known renormalization-group equations. What is uncertain, however, is the extent to which universality applies to the scalar masses  $m_0$  for different squark, slepton and Higgs fields, the gaugino masses  $m_{1/2}$  for the  $SU(3)$ ,  $SU(2)$  and  $U(1)$  gauginos, and the trilinear couplings  $A_0$  corresponding to different Yukawa couplings. Certain types of universality are much better motivated than others.

The suppression of flavour-changing neutral interactions suggests that the  $m_0$  may be universal for different matter fields with the same quantum numbers, e.g., the different squark and slepton generations. However, there is no very good reason to postulate universality between, say, the spartners of left- and right-handed quarks, or between squarks and sleptons. In Grand Unified Theories (GUTs), there must be universality between fields in the same GUT multiplet, e.g.,  $u_L, d_L, u_R$  and  $e_R$  in a  $\mathbf{10}$  of  $SU(5)$ , and this would extend to all matter fields in a  $\mathbf{16}$  of  $SO(10)$ . However, there is less reason to postulate universality between these and the Higgs fields. Nevertheless, this extension of universality to the Higgs masses (UHM) is often assumed, resulting in what is commonly termed the constrained MSSM (CMSSM) or constrained NMSSM (CNMSSM). Alternatively, there may be non-universal Higgs masses (NUHM) in the more general MSSM and NMSSM. As regards the  $A$  parameters, in the MSSM or CMSSM context it is primarily the 3rd generation  $A_t$  that matters and so universality or lack thereof does not have significant phenomenological impact. However, in the NMSSM there are two new  $A$  parameters: one,  $A_\lambda$ , associated with the singlet-Higgs-Higgs interaction at the superfield level; and a second,  $A_\kappa$ , associated with the singlet-cubed superpotential terms. Currently, there is no reason for these to have the same value as  $A_t$ . Indeed, the limit in which  $A_\kappa$  and  $A_\lambda$  are zero at the GUT scale is one in which the NMSSM has an extra  $U(1)_R$  symmetry, independent of the GUT-scale value of  $A_t$  [5]. Although it can certainly be the case that the universalities assumed in the NUHM relaxation of the CMSSM and the NUHM plus  $A_\kappa, A_\lambda$  relaxation of the CNMSSM are still more restrictive than suggested by many schemes of supersymmetry breaking, comparing results incorporating such relaxations to those obtained using the most restrictive CMSSM or CNMSSM universalities provides a simplified framework for understanding what new phenomena arise when poorly-motivated restrictive boundary conditions are relaxed. In particular, we will see that in the NMSSM context relaxation of the CNMSSM boundary conditions to include NUHM and non-universality for  $A_\lambda$  and  $A_\kappa$  relative to  $A_t$  already gives rise to dramatic new physics possibilities, including the possibilities of ideal-Higgs-like scenarios and singlino-singlet dark matter scenarios. If nothing else, this shows that overly restrictive universality assumptions should be avoided in order that dramatic new physics scenarios are not “needlessly” excluded.

As referred to above, previous studies in the context of the Constrained NMSSM (CNMSSM) [6, 7], in which all  $A$  parameters and all soft-SUSY-breaking masses-squared are unified, did not find ideal-Higgs-like points. In this paper, we allow the Higgs soft masses-squared to be independent of the other (unified) soft masses-squared and we allow the soft-SUSY-breaking  $A_\kappa$  parameter associated with the singlet field to range freely, independent of the other  $A$  parameters (which are taken to be universal). Using the advanced scanning techniques from SuperBayeS coupled to MultiNest [8] we can efficiently scan for interesting regions of parameter space and see how often points of an ideal-Higgs nature are found and look at the many phenomenological aspects that these points entail. In addition, the power of this approach to scan over *all* interesting parameters instead of fixing crucial ones to some canonical value can provide us with some insights into the full structure of the parameter space. In particular, we will not be using fine-tuning as the defining criteria when searching for ideal-Higgs-like points. This said, our choices of parameters are influenced by [1–3] and the findings therein.

## II. THE IMPLEMENTATION OF NMSSM MODEL.

The NMSSM superpotential contains a new superfield  $S$  which is a singlet under the SM gauge group  $SU(3)_c \times SU(2)_L \times U(1)_Y$ . (For simplicity, we use the same notation for superfields and their respective spin-0 component fields.)

$$W = \epsilon_{ij} (Y_u H_u^j Q^i u + Y_d H_d^i Q^j d + Y_e H_d^i L^j e) - \epsilon_{ij} \lambda S H_d^i H_u^j + \frac{1}{3} \kappa S^3, \quad (2)$$

where  $H_d^T = (H_d^0, H_d^-)$ ,  $H_u^T = (H_u^+, H_u^0)$ ,  $i, j$  are  $SU(2)$  indices with  $\epsilon_{12} = 1$ , while  $\lambda$  and  $\kappa$  are dimensionless couplings. With the addition of a scalar singlet superfield field, there will be five neutralinos and the Higgs content of the NMSSM is extended to include three scalar Higgses,  $h_1, h_2$  and  $h_3$ , and two pseudoscalars,  $a_1$  and  $a_2$ . The lightest Higgs  $h_1$  plays an important role in the scenarios we consider here, and in particular the composition will be commented on. The state composition can be written as,

$$h_1 = S_u H_u + S_d H_d + S_s S. \quad (3)$$

The superpotential in Eq. (2) is scale invariant, and the EW scale will only appear through the soft-SUSY-breaking

terms in  $\mathcal{L}_{\text{soft}}$ , which in our conventions is given by

$$\begin{aligned}
-\mathcal{L}_{\text{soft}} = & m_{\tilde{Q}}^2 \tilde{Q}^* \tilde{Q} + m_{\tilde{U}}^2 \tilde{u}^* \tilde{u} + m_{\tilde{D}}^2 \tilde{d}^* \tilde{d} + m_{\tilde{L}}^2 \tilde{L}^* \tilde{L} + m_{\tilde{E}}^2 \tilde{e}^* \tilde{e} \\
& + m_{H_d}^2 H_d^* H_d + m_{H_u}^2 H_u^* H_u + m_S^2 S^* S \\
& + \epsilon_{ij} \left( A_u Y_u H_u^j \tilde{Q}^i \tilde{u} + A_d Y_d H_d^i \tilde{Q}^j \tilde{d} + A_e Y_e H_d^i \tilde{L}^j \tilde{e} + \text{H.c.} \right) \\
& + \left( -\epsilon_{ij} \lambda A_\lambda S H_d^i H_u^j + \frac{1}{3} \kappa A_\kappa S^3 + \text{H.c.} \right) \\
& - \frac{1}{2} (M_3 \lambda_3 \lambda_3 + M_2 \lambda_2 \lambda_2 + M_1 \lambda_1 \lambda_1 + \text{H.c.}) .
\end{aligned} \tag{4}$$

When the scalar component of  $S$  acquires a VEV,  $s = \langle S \rangle$ , an effective interaction  $-\epsilon_{ij} \mu H_d^i H_u^j$  is generated, with  $\mu \equiv \lambda s$ .

In addition to terms from  $\mathcal{L}_{\text{soft}}$ , the tree-level scalar Higgs potential receives the usual  $D$  and  $F$  term contributions:

$$\begin{aligned}
V_D = & \frac{g_1^2 + g_2^2}{8} (|H_d|^2 - |H_u|^2)^2 + \frac{g_2^2}{2} |H_d^\dagger H_u|^2 , \\
V_F = & |\lambda|^2 (|H_d|^2 |S|^2 + |H_u|^2 |S|^2 + |\epsilon_{ij} H_d^i H_u^j|^2) + |\kappa|^2 |S|^4 \\
& - (\epsilon_{ij} \lambda \kappa^* H_d^i H_u^j S^{*2} + \text{H.c.}) .
\end{aligned} \tag{5}$$

Using the minimization equations we can re-express the soft breaking Higgs masses in terms of  $\lambda$ ,  $\kappa$ ,  $A_\lambda$ ,  $A_\kappa$ ,  $v_d = \langle H_d^0 \rangle$ ,  $v_u = \langle H_u^0 \rangle$  (with  $\tan \beta = v_u/v_d$ ), and  $s$ :

$$m_{H_d}^2 = -\lambda^2 (s^2 + v^2 \sin^2 \beta) - \frac{1}{2} M_Z^2 \cos 2\beta + \lambda s \tan \beta (\kappa s + A_\lambda) , \tag{6}$$

$$m_{H_u}^2 = -\lambda^2 (s^2 + v^2 \cos^2 \beta) + \frac{1}{2} M_Z^2 \cos 2\beta + \lambda s \cot \beta (\kappa s + A_\lambda) , \tag{7}$$

$$m_S^2 = -\lambda^2 v^2 - 2\kappa^2 s^2 + \lambda \kappa v^2 \sin 2\beta + \frac{\lambda A_\lambda v^2}{2s} \sin 2\beta - \kappa A_\kappa s . \tag{8}$$

We are now looking at a Lagrangian that is identical in structure to our previous work [6] but with a few important differences in terms of unification. In contrast to [6], where we took CMSSM-like boundary conditions, in the present paper, we allow the Higgs mass parameters  $m_{H_u}$  and  $m_{H_d}$  to freely vary, in a similar manner to [9]. In addition  $A_\kappa$  is no longer taken equal to the universal value,  $A_0$ , of the other  $A$  parameters. This freedom, specifically allowing  $|A_\kappa|$  to be small, will make it possible to obtain lighter pseudoscalar and scalar Higgs masses than in [6] that are nonetheless still allowed by collider constraints. This additional freedom in Higgs mass can be seen by looking at the tree level pseudoscalar mass matrix in the basis  $(A^0, S)$  [10]:

$$M_A^2 = \begin{pmatrix} \frac{2\lambda s}{\sin 2\beta} (\kappa s + A_\lambda) & \lambda v (A_\lambda - 2\kappa s) \\ \lambda v (A_\lambda - 2\kappa s) & \lambda \left( 2\kappa + \frac{A_\lambda}{2s} \right) v^2 \sin 2\beta - 3\kappa A_\kappa s \end{pmatrix} . \tag{9}$$

After diagonalization of  $M_A^2$ , there will be two mass eigenstates. The lightest state will be a mixture of the CP-odd doublet state  $A_{MSSM}$  that is present in the Minimal Supersymmetric Model and the new CP-odd component,  $A_S$ , of the complex scalar  $S$  field. We write

$$a_1 \equiv \cos \theta_A A_{MSSM} + \sin \theta_A A_S , \tag{10}$$

where the entries in  $M_A^2$  are such that the 11 entry is the MSSM diagonal entry. From Eqs. (9) and (10) it is clear that having the freedom to vary  $A_\kappa$  is crucial if we want to control the mass of the singlet component independently of other parameters. Despite these changes, just as in [6] the minimisation equations, (6)-(8) will be used to fix  $m_S$ ,  $\kappa$ , and  $s$  giving us a model with input parameters  $m_{1/2}$ ,  $m_0$ ,  $m_{H_u}$ ,  $m_{H_d}$ ,  $A_0$ ,  $A_\kappa$ ,  $\tan \beta$  and  $\lambda$ , in addition to  $\text{sgn}(\mu)$ . In particular note that in our procedure, the value of  $\kappa$  is an output that depends on  $A_\kappa$ . It is important to emphasize that in contrast to previous studies looking at the ideal-Higgs region, where scanning was done at the EW scale and run up, here all of our parameters (excluding  $\lambda$ ) are searched over at the GUT scale and then run down. One implication is that in the studies we present here it is impossible to obtain the values of the gaugino masses in [1] for a given parameter point as we are constrained by unification considerations. That said, our choice of parameters, although constrained, leads to a scan that is practicable and is a useful starting point to perturb from in order to better satisfy, for example, fine-tuning or phenomenology.

We also present the neutralino sector since the lightest neutralino will, by assumption, play the rôle of dark matter. The mass term in the Lagrangian is given by

CNMSSM parameters $\theta$	SM (nuisance) parameters $\psi$
$10^{-2} < m_{1/2} < 4000$ GeV	$160 < M_t < 190$ GeV
$10^{-2} < m_0 < 4000$ GeV	$4 < m_b(m_b)^{\overline{MS}} < 5$ GeV
$10^{-2} < m_{H_u} < 4000$ GeV	$0.10 < \alpha_s(M_Z)^{\overline{MS}} < 0.13$
$10^{-2} < m_{H_d} < 4000$ GeV	
$ A_0  < 100$ GeV	
$ A_\kappa  < 10$ GeV	
$2 < \tan \beta < 20$	
$10^{-3} < \lambda < 0.7$	

TABLE I: Initial ranges for our basis parameters  $m = (\theta, \psi)$ .

$$\mathcal{L}_{\text{mass}}^{\chi^0} = -\frac{1}{2}(\Psi^0)^T \mathcal{M}_{\chi^0} \Psi^0 + \text{H.c.}, \quad (11)$$

with  $\mathcal{M}_{\chi^0}$  given by a  $5 \times 5$  matrix in the basis  $(\tilde{B}, \tilde{W}, \tilde{H}_u, \tilde{H}_d, \tilde{S})$ ,

$$\mathcal{M}_{\chi^0} = \begin{pmatrix} M_1 & 0 & -M_Z \sin \theta_W \cos \beta & M_Z \sin \theta_W \sin \beta & 0 \\ 0 & M_2 & M_Z \cos \theta_W \cos \beta & -M_Z \cos \theta_W \sin \beta & 0 \\ -M_Z \sin \theta_W \cos \beta & M_Z \cos \theta_W \cos \beta & 0 & -\lambda s & -\lambda v_u \\ M_Z \sin \theta_W \sin \beta & -M_Z \cos \theta_W \sin \beta & -\lambda s & 0 & -\lambda v_d \\ 0 & 0 & -\lambda v_u & -\lambda v_d & 2\kappa s \end{pmatrix}, \quad (12)$$

where  $M_1$  ( $M_2$ ) denotes the soft mass of the bino (wino) and  $\theta_W$  denotes the weak mixing angle. After diagonalization the lightest neutralino  $\chi_1$  (which we will denote as  $\chi$  from now on) can be written as:

$$\chi = N_B \tilde{B} + N_W \tilde{W} + N_u \tilde{H}_u + N_d \tilde{H}_d + N_s \tilde{S}. \quad (13)$$

Finally, we note that couplings of the Higgs bosons and  $\chi$  to SM particle states depend upon the compositions of the former. In particular, the coupling of  $h_1$  to  $WW, ZZ$  is given by  $C_V(1) \equiv g_{h_1 WW} / g_{h_{SM} WW} = S_u \sin \beta + S_d \cos \beta$  (with analogous results for  $h_2$  and  $h_3$ ) and the coupling of  $a_1$  to  $b\bar{b}$  is given by  $C_{a_1 b\bar{b}} \equiv g_{a_1 b\bar{b}} / g_{h_{SM} b\bar{b}} = \cos \theta_A \tan \beta$ .

### III. OUTLINE OF THE METHOD

Following the discussion of section II, in this constrained version of the NMSSM the free parameters are given by

$$\theta = (m_{1/2}, m_0, m_{H_u}, m_{H_d}, A_0, A_\kappa, \tan \beta, \lambda). \quad (14)$$

Without loss of generality, one can choose  $\lambda > 0$  [10]. However, we will also (as in our previous work) fix  $\text{sgn}(\mu) = +1$  and then  $\mu = \lambda s$  implies  $s > 0$ . The “nuisance” parameters are treated in the same manner as in our previous work [6], and are shown in Eq. (15):

$$\psi = (M_t, m_b(m_b)^{\overline{MS}}, \alpha_s(m_Z)^{\overline{MS}}). \quad (15)$$

Using notation consistent with previous analyses we define our eleven dimensional *basis parameter* set as

$$m = (\theta, \psi), \quad (16)$$

all of which will be simultaneously scanned over. For each choice of  $m$  a number of collider and cosmological observables are calculated. These derived variables are denoted by  $\xi = (\xi_1, \xi_2, \dots)$ , which are then compared with the relevant measured data,  $d$ .

In this study we will be using the “nested sampling” method [11] as implemented in the MultiNest [12] algorithm to efficiently explore the likelihood space. Generally speaking, for this study we will be looking at points of interest irrespective of statistical considerations. MultiNest provides an extremely efficient sampler even for likelihood functions defined over a parameter space of large dimensionality with a very complex structure. (See, *e.g.*, Refs. [9, 13].)

It should be emphasised here that although nested sampling was used to obtain the points, we will be drawing no statistical inferences from the results. What in effect we are doing here is using the useful properties of the

SM (nuisance) parameter	Mean value	Uncertainty	Ref.
	$\mu$	$\sigma$ (exper.)	
$M_t$	172.6 GeV	1.4 GeV	[21]
$m_b(m_b)^{\overline{MS}}$	4.20 GeV	0.07 GeV	[22]
$\alpha_s(M_Z)^{\overline{MS}}$	0.1176	0.002	[22]

TABLE II: Experimental mean  $\mu$  and standard deviation  $\sigma$  adopted for the likelihood function for SM (nuisance) parameters, assumed to be described by a Gaussian distribution.

technique (especially fast scans of high dimensionality parameter spaces and the ability to simultaneously scan in *all* parameters) to get a sample of representative points for the part of the parameter space we are investigating. What is presented below then, must be viewed with some observations in mind. Firstly, although data is implemented in the scans, the trading off of poor fits in one variable for good fits in another can lead to some outlandish values for key phenomenological values. In addition as the chain has to start somewhere, an unweighted scatter plot such as we will be showing below will also show these initial, poorly fitting points. To address this, for the key points we will state more precisely what the key phenomenological values are, and in general they correspond closely to experimental values. Also, regions that have clearly unacceptable experimental values will be identified where possible.

The specific region we are investigating is defined by our range of priors which are specified in table I. The above choice of priors was influenced by previous work and the preference to focus on lower values of the soft masses in order to explore ideal-Higgs-like scenarios. This also leads us to choose a log prior, defined here as being flat in  $\log m_{1/2}$ ,  $\log m_0$ ,  $\log m_{H_u}$  and  $\log m_{H_d}$  and flat in  $A_0$ ,  $A_\kappa$ ,  $\lambda$  and  $\tan \beta$ . For the nuisance parameters we use flat priors (although this is not important as they are directly constrained by measurements) and apply Gaussian likelihoods representing the experimental observations (see table II), as before [9, 14–16].

The region specified by this set of priors is by no means a fair and even-handed exploration of the parameter space, but the aim here is to try and find points in a particular regime. In particular, allowing  $A_\kappa$  to be independent of  $A_0$  (unlike in our previous paper) was very important, in addition to having non-universal soft Higgs masses ( $m_{H_u} \neq m_{H_d} \neq m_0$ ). With this additional flexibility compared to the CNMSSM type scan we were able to find points with the desired characteristics, although as displayed below these points are still in the minority.

Two alternate exploratory scans were implemented to better understand this region, the first being to allow a much more generous range in the parameters, and the second was to do a scan with similar constrained priors but with the unification conditions  $m_{H_u} = m_{H_d} = m_0$  and  $A_\kappa = A_0$  enforced, as in the so-called CNMSSM studied in [6]. The objective was to see if it was the focusing into a small region or the extra freedoms in the Higgs sector that lead to us finding points of interest. What we found is that both the freedom and the focus (especially in  $A_\kappa$ ) seem to be important.

We compute our mass spectra and observable quantities using the publicly available NMSSMTools (version 2.1.1) that includes NMSPEC with a link to Micromegas [17]; for details see Ref. [18]. We list the observables that the current version of NMSPEC is applying to points found in the scan in Table III. The relic density  $\Omega h^2$  of the lightest neutralino is computed with the help of Micromegas, which is also linked to NMSPEC. We further use the same code to compute the cross section for direct detection of dark matter via its elastic scattering with targets in underground detectors but do not include it in the likelihood due to large astrophysical uncertainties. The likelihoods for the measured observables are taken as Gaussian about their mean values,  $\mu$  as tabulated in Table III, the Gaussian widths being determined by the experimental and theoretical errors,  $\sigma$  and  $\tau$ , respectively (see the detailed explanation in Refs. [14, 15]). In the case where only an experimental limit is available, this is given, along with the theoretical error. The smearing out of bounds and combination of experimental and theoretical errors is handled in an identical manner to Refs. [14, 15], with the notable exception of the Higgs mass and LEP limits on sparticle masses, which are calculated as a step function with values of the cross section times branching ratio (in the case of the Higgs) or mass that are within two standard deviations of the experimental limit being accepted. Finally, any points that fail to provide radiative EWSB, give us tachyons or the LSP other than the neutralino are rejected.

The above discussion does not yet include the constraints from the recent analysis by ALEPH for the  $e^+e^- \rightarrow Zh$ ,  $h \rightarrow aa \rightarrow 4\tau$  channel [19] nor the constraints from BaBar data on  $\Upsilon(3S) \rightarrow \gamma a$  with  $a \rightarrow \tau^+\tau^-$  [20]. These will be considered ex-post-facto. In this way, we can see what the impact of these latter constraints is upon a less biased sample of otherwise acceptable points in parameter space.

Observable	Mean value	Uncertainties		Ref.
		$\mu$	$\sigma$ (exper.)	
$\delta(g-2)_\mu \times 10^{10}$	29.5	8.8	1	[23]
$BR(\bar{B} \rightarrow X_s \gamma) \times 10^4$	3.55	0.26	0.21	[23]
$BR(\bar{B}_u \rightarrow \tau \nu) \times 10^4$	1.32	0.49	0.38	[24]
$\Omega h^2$	0.1099	0.0062	$0.1 \Omega h^2$	[25]
	Limit (95% CL)		$\tau$ (theor.)	Ref.
$BR(\bar{B}_s \rightarrow \mu^+ \mu^-)$	$< 5.8 \times 10^{-8}$		14%	[26]
$m_h$	As implemented in NMSSMTools.			[18]
sparticle masses	As implemented in NMSSMTools.			[18]
$C_{\text{eff}}^{2b}$	As implemented in NMSSMTools.			[18]
$C_{\text{eff}}^{4b}$	As implemented in NMSSMTools.			[18]

TABLE III: Summary of the observables used in the analysis. For more details on how these are implemented, see [14].

#### IV. RESULTS

In this section we present our numerical results from global scans, mostly in the form of scatter plots for some of the most interesting combinations of observables. Most of the figures displayed are scatter plots with three distinct populations visible on them. There are a large number of grey points in the figures that follow; these so-termed *full scan* points come from all the points obtained in the scan and are thinned by a factor of one hundred for clarity. This will also eliminate many of the outlying initial points talked about earlier. This population of points can be thought of as a representation of the general structure of the parameter space, the backdrop against which we specify points of interest.

To better display the points with  $BR(a_1 \rightarrow b\bar{b}) = 0$  and  $BR(h_1 \rightarrow a_1 a_1) > 0.5$  we have marked them as triangles in the relevant figures. Such points will be termed Type I points. In addition, there are points with  $BR(h_1 \rightarrow b\bar{b}) < 0.5$  by virtue of substantial  $BR(h_1 \rightarrow a_1 a_1)$  but for which  $m_{a_1} > 2m_B$  and  $BR(a_1 \rightarrow b\bar{b}) \neq 0$ . These are shown by squares in the figures. We call these points Type II points. In fact, there are two subclasses of Type II points. Type IIA points are such that the light Higgs is mainly doublet. Type IIA points pass constraints on  $C_{\text{eff}}^{2b}$  and  $C_{\text{eff}}^{4b}$  considered individually, but may struggle to pass the overall constraint implicit in LEP data on  $hZ$  production with  $h \rightarrow b's$ , where  $b's$  represents any final state containing one or more  $b$  quarks. This overall constraint becomes important for  $m_h$  below about 110 GeV. Because of the need to consider this overall constraint, one cannot be certain of whether or not the Type IIA points with  $m_{h_1} \lesssim 110 - 112$  GeV should be eliminated without submitting them to the LEP collaborations for full analysis. Thus, we will depict them in the figures. However, in general the Type I points where  $m_{a_1} < 2m_B$  are more interesting. Type IIB points are ones with  $BR(h_1 \rightarrow b\bar{b}) < 0.5$ , large  $BR(h_1 \rightarrow a_1 a_1)$  and  $m_{a_1} > 2m_B$  but for which the light Higgs is mainly singlet. These easily pass the LEP constraints and have other interesting properties that we shall elucidate later.

There is a third class of points, Type III, that are worth singling out. These have  $BR(h_1 \rightarrow b\bar{b}) > 0.5$  but pass our primary selection criteria. As for Type II points, there are two subclasses: Type IIIA for which the  $h_1$  is mainly doublet and  $S_s^2$  is small; and Type IIIB for which  $h_1$  is mainly singlet,  $S_s^2 \sim 1$ . The latter easily pass LEP limits on  $C_{\text{eff}}^{2b}$  and  $C_{\text{eff}}^{4b}$ , by virtue of  $h_1$  being mainly singlet in composition, implying very small  $ZZh_1$  coupling. In common with Type IIB, the Higgs with SM-like coupling to  $WW, ZZ$  is instead the  $h_2$  which must have mass and/or decays that allow it to obey all of the bounds imposed by LEP. Type IIIA points pass LEP limits by virtue of  $m_{h_1} \gtrsim 114$  GeV, and are therefore not “special” in any way. Type IIIB points have  $m_{h_1} \in [80 - 100]$  GeV and are particularly interesting in that they have a high probability of providing the correct value of  $\Omega h^2$ . In fact, in what follows *we will include in the definition of Type III points the requirement that they yield the correct value of  $\Omega h^2$  within  $2\sigma$* . This will mean that Type III points will have the lowest overall  $\chi^2$  of all the points in our scans. However, they cannot explain the LEP excess in this range since, as stated before, the  $h_1$  is for the most part decoupled and it is the  $h_2$  which plays the role of the SM-like Higgs, and it has mass above 110 GeV. Finally, we also include in our final definition of Type III points the requirement that they satisfy the “ex-post-facto” constraints from ALEPH (on the  $h_1 \rightarrow a_1 a_1 \rightarrow 4\tau$  channel) and from BaBar (from  $\Upsilon(3S) \rightarrow \gamma a_1$  decays) mentioned earlier. In short, our final definition of Type III points is such that they are very consistent with *all* available experimental constraints. We explicitly state our selection criteria for the various points in Table IV.

A later table will provide a fuller list of the properties of the different classes of points.

Given the motivation for searching in those regions of parameter space that might yield ideal-Higgs scenarios, we

Point Type	$BR(h_1 \rightarrow b\bar{b})$	$BR(a_1 \rightarrow b\bar{b})$	$S_s^2$
Type I	$< 0.5$	$= 0$	$\sim 0$
Type IIA	$< 0.5$	$\neq 0$	$\sim 0$
Type IIB	$< 0.5$	$\neq 0$	$\sim 1$
Type IIIA	$> 0.5$	$\neq 0$	$\sim 0$
Type IIIB	$> 0.5$	$\neq 0$	$\sim 1$

TABLE IV: Definition of various points shown in figures. In the case of Type III points, we have also required that  $\Omega h^2$  be within  $\pm 2\sigma$  of the observed value and that they obey the ALEPH constraints on  $h_1 \rightarrow a_1 a_1 \rightarrow 4\tau$  and the BaBar constraints on  $\Upsilon(3S) \rightarrow \gamma a_1$ .

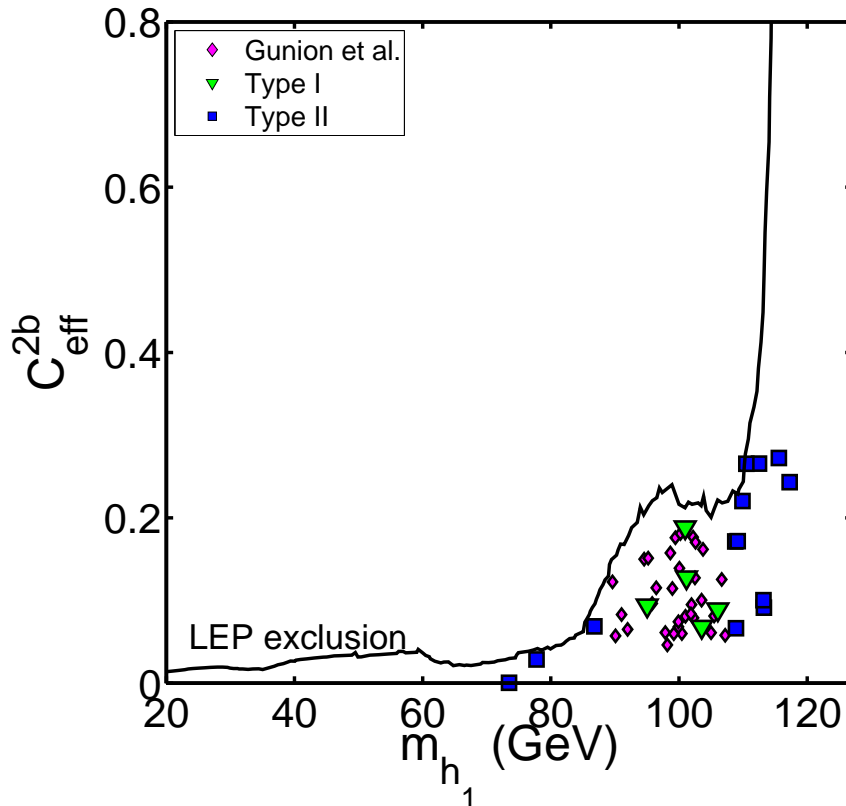


FIG. 1: A plot of the LEP limit from [4] in the  $C_{\text{eff}}^{2b}$  vs  $m_{h_1}$  plane, superimposed on points obtained by Gunion *et al.* in [1] and so called “Type I” points ( $m_{a_1} < 2m_B$ ) and “Type II” points ( $m_{a_1} > 2m_B$ ) from this study.

first look at the  $h_1$  branching ratios and  $C_{\text{eff}}^{2b}$ . The first result is shown in Fig. 1, where we plot  $C_{\text{eff}}^{2b}$  as a function of  $m_{h_1}$  for our study, points from previous work by Gunion *et al.* and the experimental limits from LEP. This plot already shows that the *only* points in our scans that reproduce exactly the desired qualities ( $BR(h_1 \rightarrow b\bar{b}) \rightarrow 0$  with  $BR(a_1 \rightarrow b\bar{b}) = 0$ ) while passing all LEP constraints are precisely those with a Higgs mass in the range  $80 \text{ GeV} \lesssim m_{h_1} \lesssim 100 \text{ GeV}$ .

In Fig. 1, one also clearly sees the Type IIA points with  $m_{h_1} \gtrsim 108 \text{ GeV}$  and the Type IIB points with  $m_{h_1} \lesssim 90 \text{ GeV}$ . As stated above, those of the Type IIA points that have  $m_{h_1} \lesssim 114 \text{ GeV}$ , especially those with  $m_{h_1} \lesssim 110 \text{ GeV}$ , might be ruled out by a combined  $Z + b$ 's LEP analysis, even if not ruled out by the  $C_{\text{eff}}^{2b}$  and  $C_{\text{eff}}^{4b}$  separate limits.

We display in Fig. 2 an expanded look at this region. In the top left plot we put into context the correlation between  $m_{h_1}$  and  $C_{\text{eff}}^{2b}$  for Type I and Type II points by showing “background” points from the full scan. There, we see that there are many background points with  $C_{\text{eff}}^{2b}$  of order 0.7 to 0.8 but with large enough  $m_{h_1}$  to escape LEP limits, as characteristic of Type IIIA points, as well as background points with very low  $m_{h_1}$  and  $C_{\text{eff}}^{2b}$ , which include Type IIIB points. Other plots in this figure show that we obtain the points with small  $C_{\text{eff}}^{2b}$  in two different ways. The



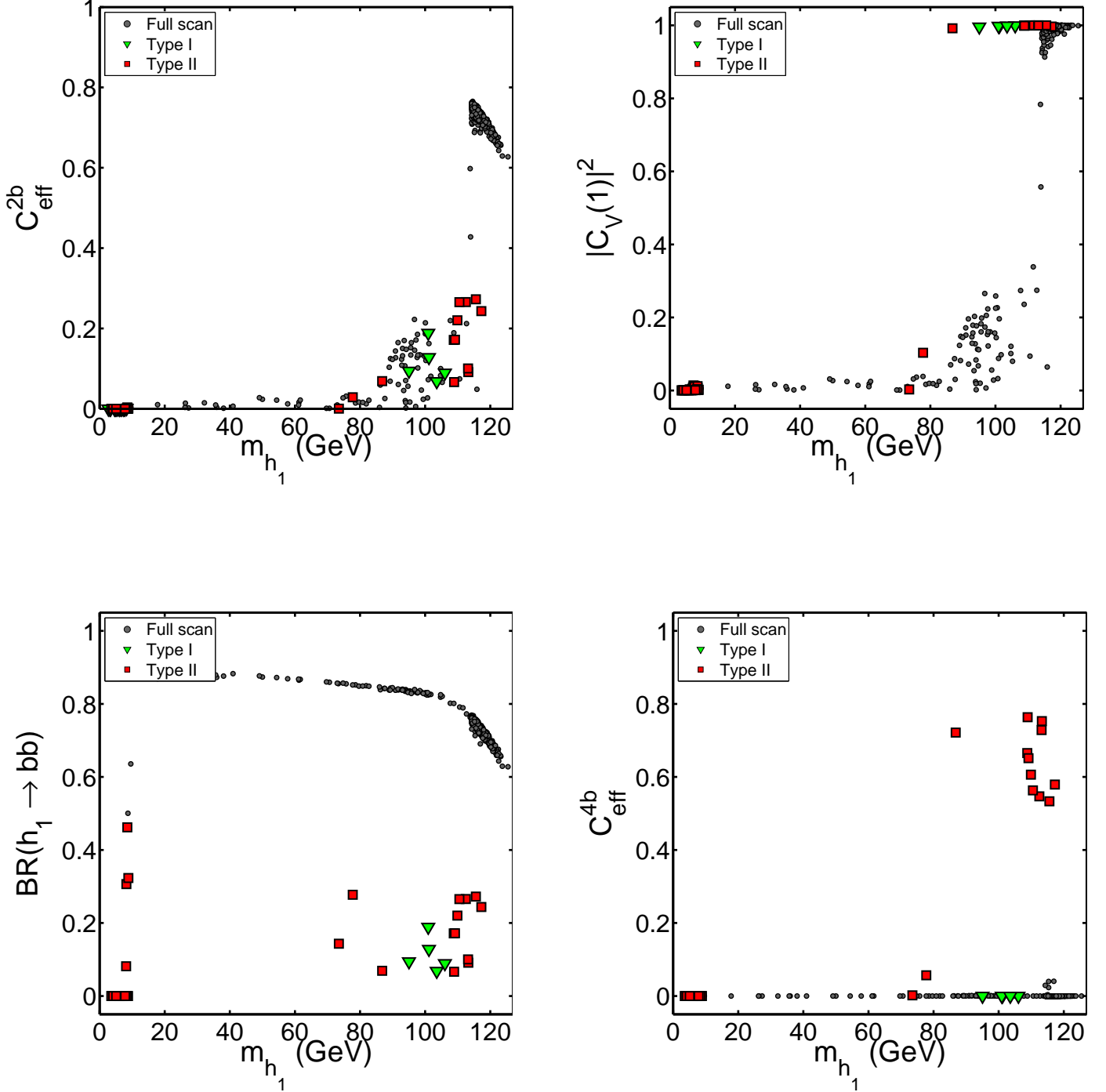


FIG. 2: Plots of various phenomenological quantities — see Eq. (1) — as a function of  $m_{h_1}$ . Note that all plots and all figures to come use the color red for the Type II points, as opposed to the coloring in Fig. 1, where the color blue was used for Type II points.

first means is to suppress the branching ratio,  $BR(h_1 \rightarrow b\bar{b})$ , as is the case with most Type I and Type IIA points. The second is to suppress the squared-coupling  $|C_V(1)|^2$ , as happens if the  $h_1$  is sufficiently singlet in composition, which is the case in particular for Type IIB points. Indeed there can be a lot of points in a similar  $m_{h_1}$  region to that identified by Gunion *et al.* (*i.e.*  $80 \text{ GeV} \lesssim m_{h_1} \lesssim 100 \text{ GeV}$ ) that escape LEP limits via suppression of  $|C_V(1)|^2$  rather than via suppression of  $BR(h_1 \rightarrow b\bar{b})$ . Small  $|C_V(1)|^2$  implies that the  $h_1$  cannot act as a “ideal” Higgs defined as

having SM-like  $WW, ZZ$  coupling but mass  $\lesssim 105$  GeV.

The bottom right figure in Fig. 2 shows  $C_{\text{eff}}^{4b}$  against  $m_{h_1}$ . This can be compared with a similar figure in [1]. In general it is clear that the Type I points all have  $C_{\text{eff}}^{4b} = 0$  (since  $m_{a_1} < 2m_B$ ) while for Type IIA points  $C_{\text{eff}}^{4b}$  is quite significant and, for those points with  $m_{h_1} < 114$  GeV, is not far below the LEP limit. As noted earlier, since the limits on  $C_{\text{eff}}^{2b}$  and  $C_{\text{eff}}^{4b}$  are being applied individually and not in combination it could well be that the Type IIA points with  $m_{h_1} < 114$  GeV, especially those with  $m_{h_1} \lesssim 110$  GeV, are in fact in contradiction with LEP. But, without a full LEP analysis, it can be instructive to leave them in with this caveat in mind. One thing to notice in general is that the ideal-Higgs-like Type I and the Type IIA points with  $m_{h_1} \lesssim 110$  GeV and fairly large  $|C_V(1)| \sim 1$  are very rare, even in the context of a scan looking for these regions; this could be an artifact of our scanning technique or it could be that these points are truly hard to find given the criteria and high-scale boundary conditions we have used.

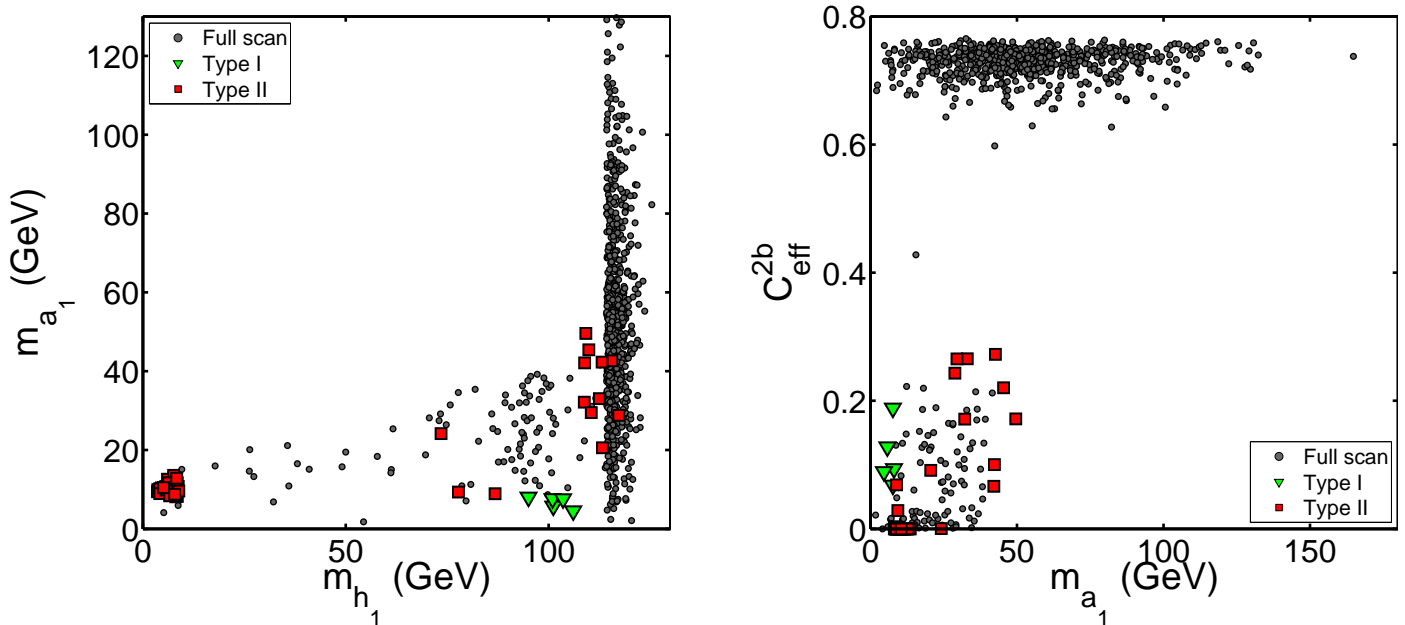


FIG. 3: Plots of  $m_{a_1}$  as a function of  $m_{h_1}$  and of  $C_{\text{eff}}^{2b}$  as a function of  $m_{a_1}$ .

Since  $m_{a_1}$  is so crucial to whether or not a given point is ruled out by LEP data, it is useful to understand how  $m_{h_1}$  and  $C_{\text{eff}}^{2b}$  correlate with  $m_{a_1}$ . In the left plot of Fig. 3 we display all the different types of points in the  $m_{h_1}$ - $m_{a_1}$  plane. Note again the Type IIA points with  $m_{a_1} > 2m_B$  and  $109 \text{ GeV} \lesssim m_{h_1} < 114 \text{ GeV}$  that escape LEP limits on  $C_{\text{eff}}^{4b}$  despite having large  $BR(h_1 \rightarrow a_1 a_1)$  and large  $BR(a_1 \rightarrow b\bar{b})$ . These also appeared in Fig. 2.

To better demonstrate the interplay between the various branching ratios needed to evade LEP constraints, some of the crucial ones are depicted in the same convention in Fig. 4. One can see that the green Type I points are clearly isolated, with the key discriminator from Type II points being  $BR(a_1 \rightarrow b\bar{b})$ . In Fig. 4, some key differences between Type IIA and Type IIB points are apparent, the most notable being the very small  $BR(h_1 \rightarrow a_1 a_1)$  for Type IIB (singlet  $h_1$ ) points. Note also that *all* Type III points have very small  $BR(h_1 \rightarrow a_1 a_1)$ .

We will shortly discuss whether or not the Type I points escape the latest ALEPH limits on  $h_1 \rightarrow a_1 a_1$  with  $a_1 \rightarrow \tau^+ \tau^-$ . Such escape is possible when  $\tan \beta$  is small, since at small  $\tan \beta$  one predicts that  $BR(a_1 \rightarrow \tau^+ \tau^-)$  is significantly suppressed due to substantial branching ratios for  $a_1$  to  $c\bar{c}$ ,  $s\bar{s}$  and  $g\bar{g}$  and the resulting final states in  $h_1 \rightarrow a_1 a_1$  are less strongly constrained than the  $h_1 \rightarrow a_1 a_1 \rightarrow 4\tau$  final state. This was discussed in [27].

In Fig. 5 we show the square of the singlet component of the  $h_1$ , and the square of the singlino component of the lightest neutralino, the  $\chi$ , as functions of  $m_{h_1}$ . These figures illustrate a number of things. First note the large number of points with  $N_s^2 \sim 1$  and  $S_s^2 \sim 1$ , the latter implying that  $|C_V(1)|^2$  is greatly suppressed. Included in this set of points are the Type IIB points with  $m_{h_1} < 80$  GeV as well as the Type IIIB points with low  $m_{h_1}$ , large  $BR(h_1 \rightarrow a_1 a_1)$  and non-zero  $BR(a_1 \rightarrow b\bar{b})$  — all these points escape LEP limits since the  $h_1$  is very singlet-like. Second, we observe that the  $S_s^2$  plot is closely related to the top right panel in Fig. 2. As noted in the discussion of the latter figure, it is the Type I points and Type IIA points with  $m_{h_1} \gtrsim 80$  GeV for which the  $h_1$  is highly doublet-like whereas the Type IIB points with  $m_{h_1} \lesssim 80$  GeV have a singlet-like  $h_1$ . And, finally, there is the large collection of points with  $m_{h_1} > 114$  GeV that are unconstrained by LEP data and typically are very doublet-like.

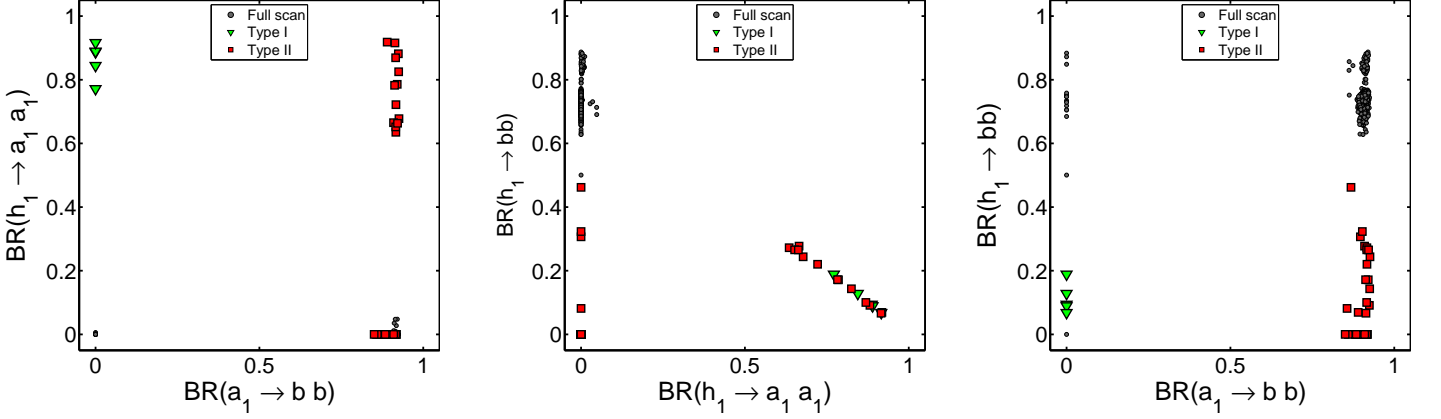


FIG. 4: Plots of:  $BR(h_1 \rightarrow a_1 a_1)$  vs.  $BR(a_1 \rightarrow b\bar{b})$ ;  $BR(h_1 \rightarrow b\bar{b})$  vs.  $BR(h_1 \rightarrow a_1 a_1)$ ; and  $BR(h_1 \rightarrow b\bar{b})$  vs.  $BR(a_1 \rightarrow b\bar{b})$ .

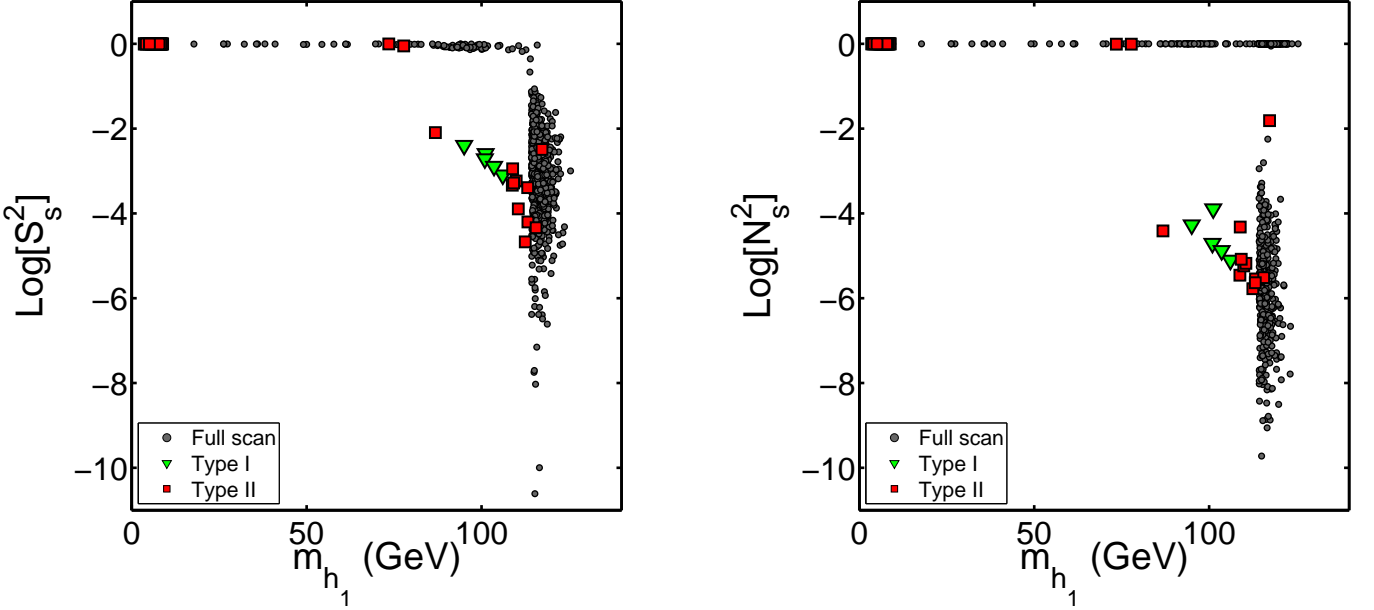


FIG. 5: We plot the square of the singlet component of the  $h_1$  and the square of the singlino component of  $\chi$  as functions of  $m_{h_1}$ .

The right hand plot of Fig. 5 will be of more use below when we consider dark matter, but it does show an interesting correlation, namely that the Type I points and the Type IIA points with  $m_{h_1} \gtrsim 80$  GeV stand out by having a  $\chi$  that is bino-like instead of singlino-like, the latter being more typical of the majority of points found in our scans, including Type IIB points. We further note that the Type IIB points that have  $m_{h_1} \lesssim 110$  GeV and  $S_s^2 \sim 1$  also have  $N_s^2 \sim 1$ . In contrast, the Type IIIA points which have  $m_{h_1} > 114$  GeV and small  $S_s^2$  (with  $S_u^2$  being large instead) can have either large  $N_s^2$  or large  $N_B^2$  (*i.e.* singlet-like  $\chi$  or bino-like  $\chi$ ).

Full details regarding Type I points appear in Table V. The upper table shows the input parameter values for each of the Type I points. The corresponding “light- $a_1$ ” fine-tuning measure,  $G$ , defined by:

$$G \equiv \text{Min} \{ [\text{Max}(|F_{A_\lambda}|, |F_{A_\kappa}|)], |F_{A_\lambda} + F_{A_\kappa}| \}, \quad (17)$$

Point	$m_0$ (GeV)	$m_{1/2}$ (GeV)	$m_{H_u}$ (GeV)	$m_{H_d}$ (GeV)	$A_\kappa$ (GeV)	$A_0$ (GeV)	$\lambda$	$\tan \beta$	$G$
1	452	223	3.54	543	5.69	33.7	0.481	2.54	30.4
2	10.9	287	710	180	7.89	51.7	0.436	3.61	30.9
3	7.57	467	2.23	655	-4.65	28.1	0.408	2.15	17.6
4	0.717	393	0.724	622	2.43	46.6	0.328	2.52	15.2
5	0.804	387	42.0	526	7.17	46.3	0.399	2.05	18.8

Point	$\Omega h^2$	$\delta(g-2)_\mu$	$BR(\overline{B} \rightarrow X_s \gamma)$	$BR(\overline{B}_s \rightarrow \mu^+ \mu^-)$	$m_{h_1}$ (GeV)	$m_{a_1}$ (GeV)	$\chi^2$
1	2.35	$1.78 \times 10^{-10}$	$3.11 \times 10^{-4}$	$2.54 \times 10^{-8}$	95.0	8.04	70.3
2	0.276	$10.3 \times 10^{-10}$	$2.951 \times 10^{-4}$	$0.127 \times 10^{-8}$	101	5.77	39.6
3	0.344	$2.45 \times 10^{-10}$	$3.211 \times 10^{-4}$	$4.30 \times 10^{-8}$	106	4.58	30.1
4	0.341	$3.90 \times 10^{-10}$	$3.191 \times 10^{-4}$	$3.30 \times 10^{-8}$	101	7.66	29.9
5	0.245	$3.42 \times 10^{-10}$	$3.391 \times 10^{-4}$	$3.22 \times 10^{-8}$	104	7.63	25.7

Point	$\cos\theta_A^{Max}$	$\cos\theta_A$	$C_{a_1 b \bar{b}}^{Max}$	$C_{a_1 b \bar{b}}$	$BR(h_1 \rightarrow a_1 a_1)$	$BR(h_1 \rightarrow b \bar{b})$	$BR(a_1 \rightarrow \tau^+ \tau^-)$	$BR(a_1 \rightarrow \mu^+ \mu^-)$	$(\xi^2)_{ALEPH}^{Max}$	$\xi^2$
1	0.341	0.0186	0.867	0.0472	0.887	0.0943	0.833	0.0034	.2902	0.615 !
2	0.199	0.0197	0.719	0.0711	0.844	0.128	0.881	0.0042	.49809	0.655 !
3	0.309	0.00631	0.664	0.0136	0.890	0.0893	0.771	0.0047	.75916	0.529
4	0.336	0.00527	0.849	0.0133	0.772	0.189	0.837	0.0035	.52524	0.541 !
5	0.443	0.00716	0.906	0.0146	0.916	0.0682	0.786	0.0034	.67593	0.566

TABLE V: Displayed are some values of interest for the Type I points found in our scans. In the upper table we show the base parameters that give us our population of interesting (Type I) points. The final column, denotes  $G$ , defined in Eq. (18), a measure of the fine-tuning needed to obtain the (low) value of  $m_{a_1}$ . In the middle table are some of the phenomenological values for the points of interest. Notice the likelihood (to be precise the  $-2\log(\text{likelihood}) = \chi^2$ ) in general is large reflecting a poor fit, and this is largely being driven by poor fits to  $\Omega h^2$  [13]. The bottom table shows some of the key branching ratios of interest for Type I points, and compares the ALEPH limits on  $\xi^2 \equiv |C_V(1)|^2 \times BR(h_1 \rightarrow a_1 a_1) \times [BR(a_1 \rightarrow \tau^+ \tau^-)]^2$  with the predicted values. Points appended with an exclamation mark are excluded by the ALEPH analysis. However, as discussed later, by adjusting  $A_\kappa$  by a very small amount they can be brought into agreement with the ALEPH limits without affecting any other phenomenology.

where,

$$F_{A_\lambda} \equiv \frac{A_\lambda}{m_{a_1}^2} \frac{dm_{a_1}^2}{dA_\lambda} \qquad F_{A_\kappa} \equiv \frac{A_\kappa}{m_{a_1}^2} \frac{dm_{a_1}^2}{dA_\kappa} \qquad (18)$$

is also shown. One can see some common threads for all of the Type I points. Perhaps the most intriguing is the need for  $\lambda$  to be quite large and away from the decoupling limit. This suggests that these points are in some sense specific to the NMSSM and are unlikely to be found in similar parametrizations of the MSSM. It is also nice to see that despite the light- $a_1$  fine-tuning measure  $G$  not being used in the scans, the resultant values for Type I points are not wholly unreasonable. Finally, we note that the values of  $\tan \beta$  for which Type I points were found are relatively low. In the less constrained scans of parameter space performed in [1–3] Type I (ideal-Higgs) points were found at large  $\tan \beta$  as well.

In Fig. 6, we show the values of  $m_0$  and  $m_{1/2}$  (at the GUT scale) for the various different types of points. We see that many of the points of Type II and Type III have quite small values of  $m_0$  and that most Type I points have quite modest  $m_0$ . As regards  $m_{1/2}$ , it is typically of order 250 – 300 GeV for Type I points but ranges from  $\sim 250$  GeV up to 2 TeV for Type II and Type III points. Regardless, the resulting gluino and (non-stop) squark masses are always at least as large as 650 GeV and often significantly larger. Such values are above the limits currently being set by LHC data, which limits are typically of order 500 – 600 GeV (assuming universality for the gaugino masses and for the non-Higgs scalar masses at the GUT scale). Of course, the LHC will probe gluino and squark masses of order 1 TeV after another year or two of running. In common with other models employing universality at the GUT scale,

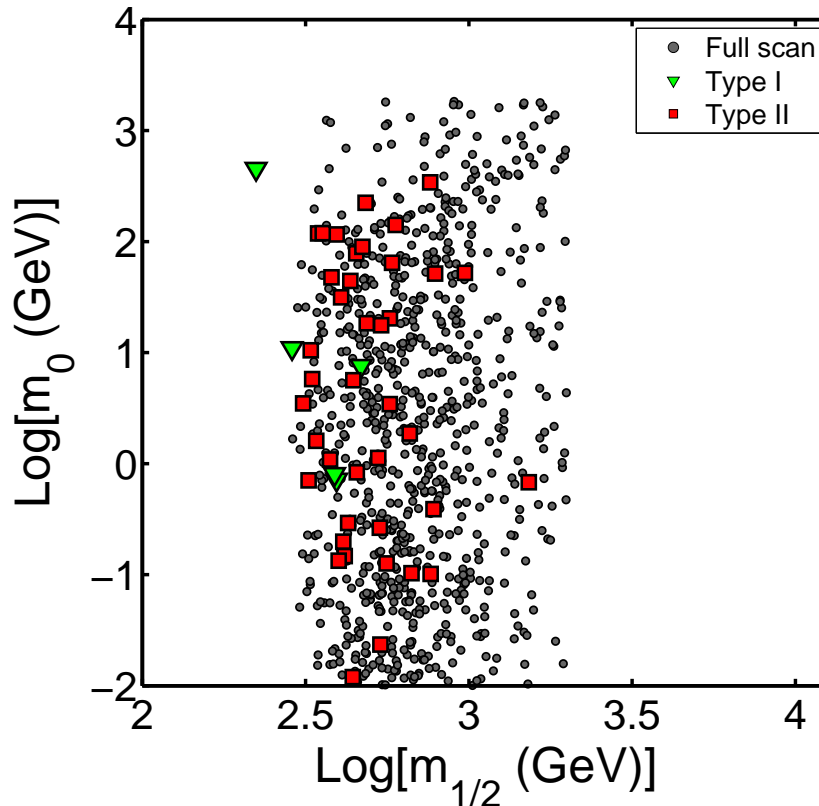


FIG. 6: A plot of our points in the  $m_0 - m_{1/2}$  plane.

the parameter points typical of our study will then start to be ruled out.

The middle table in Table V gives some corresponding experimental values for the Type I points. It is interesting to see that the points provide phenomenologically viable results for  $BR(\bar{B} \rightarrow X_s \gamma)$  and  $BR(\bar{B}_s \rightarrow \mu^+ \mu^-)$ . However, the lower  $2\sigma$  boundary for the observed  $\delta(g-2)_\mu$  is  $\sim 10 \times 10^{-10}$  and only Type I point 2 barely predicts this high a value, the other Type I points predicting values in the range  $(1.78 - 3.90) \times 10^{-10}$ . The relic density is equally problematical, with the best value barely getting to within  $2\sigma$  of the WMAP value. The likelihood is dominated by this contribution as in general the relic density is the strongest constraint on the parameter space [13]. Hence, the points with best likelihood correspond to  $\Omega h^2$  closest to its experimental value. As discussed shortly, the Type IIIB points (*i.e.* points  $m_{h_1} \lesssim 104$  GeV that escape LEP limits by virtue of the  $h_1$  being mainly singlet) quite readily achieve an  $\Omega h^2$  near the WMAP value; as a result, the  $\chi^2$  for Type IIIB points ranges from a low of  $\sim 1.9$  to a high of  $\sim 6$ , vs. the best value of  $\sim 26$  found for Type I points in our scans. Type IIIA points have  $\chi^2$  values only slightly larger than Type IIIB points and definitely below 26, as consistent with our requirement that Type III points be consistent with the observed  $\Omega h^2$  within  $\pm 2\sigma$ . Note that small  $\chi^2$  can be achieved within the other defining characteristics for Type IIIA and Type IIIB points because small  $\Omega h^2$  is possible despite the singlino or bino nature of the  $\chi$  by virtue of near mass degeneracy of the  $\chi$  and  $\tilde{\tau}_1$ .

The bottom table of Table V gives the values of the coupling  $C_{a_1 b \bar{b}} \equiv \tan \beta \cos \theta_A$  in comparison to the maximum absolute value allowed by BaBar data in the  $\Upsilon_{3S} \rightarrow \gamma \tau + \tau^-$  channel. Here,  $\cos \theta_A$  is the doublet component of  $a_1$  as defined by  $a_1 = \cos \theta_A a_{MSSM} + \sin \theta_A a_S$ . This bottom table also gives the value of  $\xi^2 \equiv |C_V(1)|^2 \times BR(h_1 \rightarrow a_1 a_1) \times [BR(a_1 \rightarrow \tau^+ \tau^-)]^2$  in comparison to the upper limit for each point from the recent ALEPH analysis. We observe that the Type I points have no problem obeying the limits from BaBar but that the ALEPH limits are very problematical for three out of five of the Type I points. However, we show below that a very small change in  $A_\kappa$  will bring these points into agreement with the ALEPH limits without affecting any other phenomenology. We also wish to note that the ALEPH constraints are much stronger than what was expected on the basis of Monte Carlo and so, in our opinion, some relaxation of the ALEPH bounds could be considered. If  $\sim 1\sigma$  relaxation is allowed, then all our Type I points survive “as is”. We also wish to note that  $C_{a_1 b \bar{b}}^2 \times BR(a_1 \rightarrow \mu^+ \mu^-)$  roughly determines the ability to detect  $gg \rightarrow a_1 \rightarrow \mu^+ \mu^-$  at hadron colliders [28]. Very roughly, in the  $m_{a_1} < 2m_B$  region where  $BR(a_1 \rightarrow \mu^+ \mu^-) \sim (0.003 - 0.005)$  detection

will only be “easy” if  $|C_{a_1 b \bar{b}}| \geq 1$ . Unfortunately, for our Type I points,  $|C_{a_1 b \bar{b}}|$  is always small, with point 2 providing the largest value of  $\sim 0.07$ .

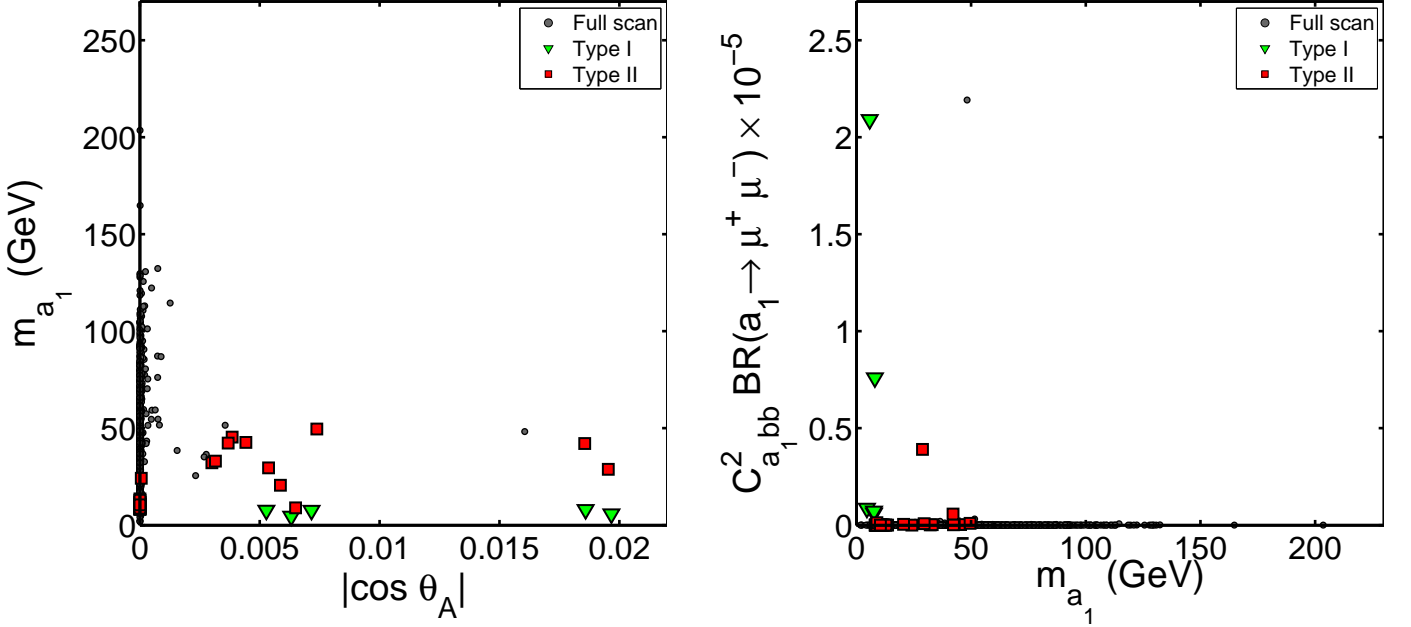


FIG. 7: We plot  $m_{a_1}$  as a function of its singlet component,  $|\cos \theta_A|$  and  $C_{a_1 b \bar{b}}^2 BR(a_1 \rightarrow \mu^+ \mu^-)$  as a function of  $m_{a_1}$ .

A more global picture of  $|\cos \theta_A|$  and  $C_{a_1 b \bar{b}}^2 BR(a_1 \rightarrow \mu^+ \mu^-)$  is provided by Fig. 7. In the left plot, we give  $m_{a_1}$  as a function of the magnitude of its singlet component,  $|\cos \theta_A|$ , for all points in order to show more generally how singlet the  $a_1$  is for the different classes of points. We observe that the  $a_1$  is extremely singlet for the vast bulk of points, including the Type IIB and Type IIIB points. This correlates with the fact that the  $h_1$  is mainly singlet for these same two types of points.

In the right plot of Fig. 7 we show  $C_{a_1 b \bar{b}}^2 BR(a_1 \rightarrow \mu^+ \mu^-)$  as a function of  $m_{a_1}$  to indicate which points have a reasonable probability that the production/decay channel  $gg \rightarrow a_1 \rightarrow \mu^+ \mu^-$  could be detected. Points for which this product is  $\gtrsim 0.001$  would have a viable signal at the LHC for accumulated luminosities of order  $10 \text{ fb}^{-1}$  [28] (more being required if  $m_{a_1}$  is in the region of the  $\Upsilon$  resonances). We see that none of our points are even close to allowing such detection.

Of course, the size of  $C_{a_1 b \bar{b}}$  derives both from  $\cos \theta_A$  and  $\tan \beta$ . Thus, it is perhaps useful to display  $\tan \beta$  as a function of  $m_{a_1}$  and  $m_{h_1}$ . This is done in Fig. 8. We see that only the Type I and the Type IIA points with  $m_{h_1} \gtrsim 80 \text{ GeV}$  (for which the  $a_1$  and  $h_1$  have at least a modest doublet component) are forced into the low  $\tan \beta$  region. In contrast, Type IIB and Type IIIB points all have  $\tan \beta > 8$ . Type IIIA points can have any  $\tan \beta$  above  $\sim 3$ . In the left plot of Fig. 9 we show how all the points are distributed in the  $m_{h_2} - m_{h_1}$  plane. This plot shows very clearly two branches for all the points that are neither Type I nor Type IIA points. The vertical branch corresponds to Type IIIA points where  $m_{h_1} \gtrsim 114 \text{ GeV}$  (thereby escaping LEP limits) with  $|C_V(1)|^2 \sim 1$  (see Fig. 2). The horizontal branch encompasses the Type IIB and Type IIIB points for which the  $h_1$  is very singlet and it is instead the  $h_2$  that is SM-like with  $m_{h_2} \gtrsim 114 \text{ GeV}$  and  $|C_V(2)|^2 \sim 1$ , as displayed in the right plot of Fig. 9. Finally, we remark that  $BR(h_2 \rightarrow a_1 a_1)$  is very small for all points — this is perfectly OK since LEP limits for the  $h_2$  are evaded either because it is very singlet or because  $m_{h_2} > 114 \text{ GeV}$  — the extra  $h_2 \rightarrow a_1 a_1$  decay channel is not needed.

Let us now return to the ALEPH limits on  $\xi^2 = |C_V(1)|^2 BR(h_1 \rightarrow a_1 a_1) [BR(a_1 \rightarrow \tau^+ \tau^-)]^2$ . In Fig. 10, we show specifically the effect of varying  $A_\kappa$  on  $\xi^2$ . One can see that there exist two “discontinuities” as  $A_\kappa$  is varied. These arise because  $m_{a_1}$  decreases with increasing  $A_\kappa$ . The first abrupt change occurs at the upper range of  $A_\kappa$  plotted when  $m_{a_1}$  passes below  $2m_\tau$ , and  $\xi^2$  goes to zero because  $BR(a_1 \rightarrow \tau^+ \tau^-) = 0$ . The 2nd abrupt change arises as  $A_\kappa$  is decreased and  $m_{a_1}$  becomes larger than  $2m_B$ . In this region, the dominant decay channel for the lightest Higgs is  $h_1 \rightarrow a_1 a_1 \rightarrow 4b$  and  $BR(h_1 \rightarrow a_1 a_1 \rightarrow 4\tau)$  is very small. As  $A_\kappa$  decreases just a little bit more, the model point in question will start to exceed LEP bounds on the  $h_1 \rightarrow 4b$  final state. As seen in the left hand plot of Fig. 10, in the

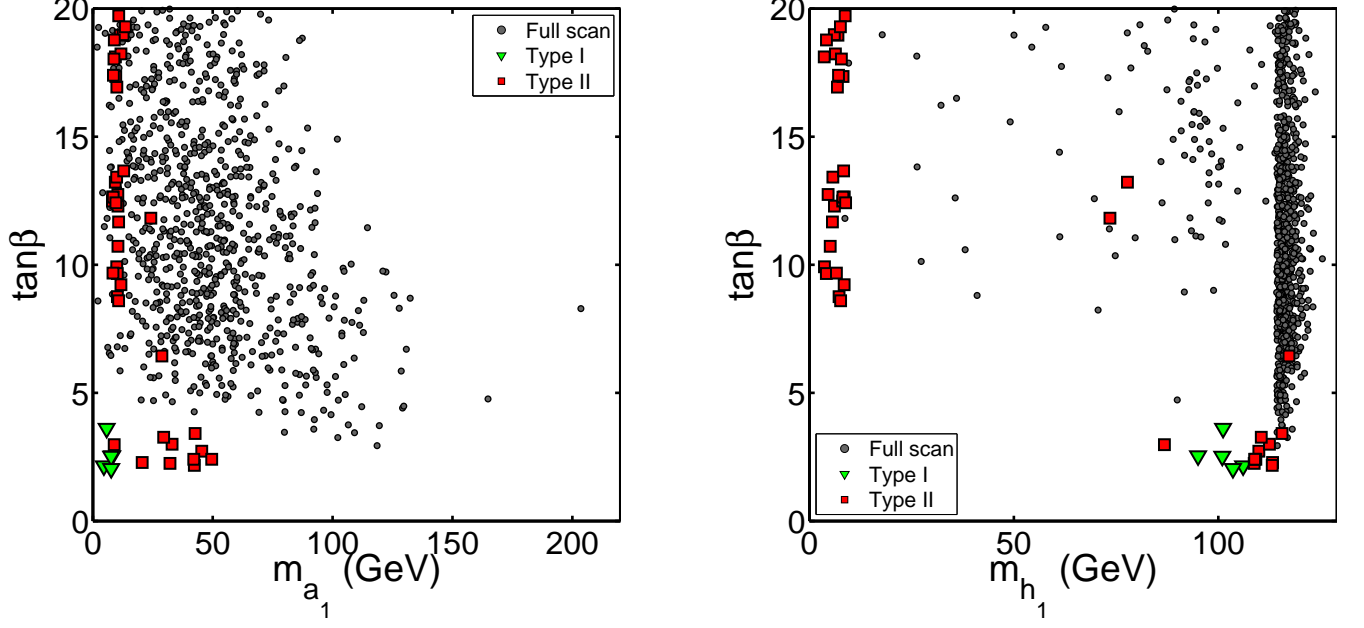


FIG. 8: We plot  $\tan\beta$  vs.  $m_{a_1}$  and vs.  $m_{h_1}$ .

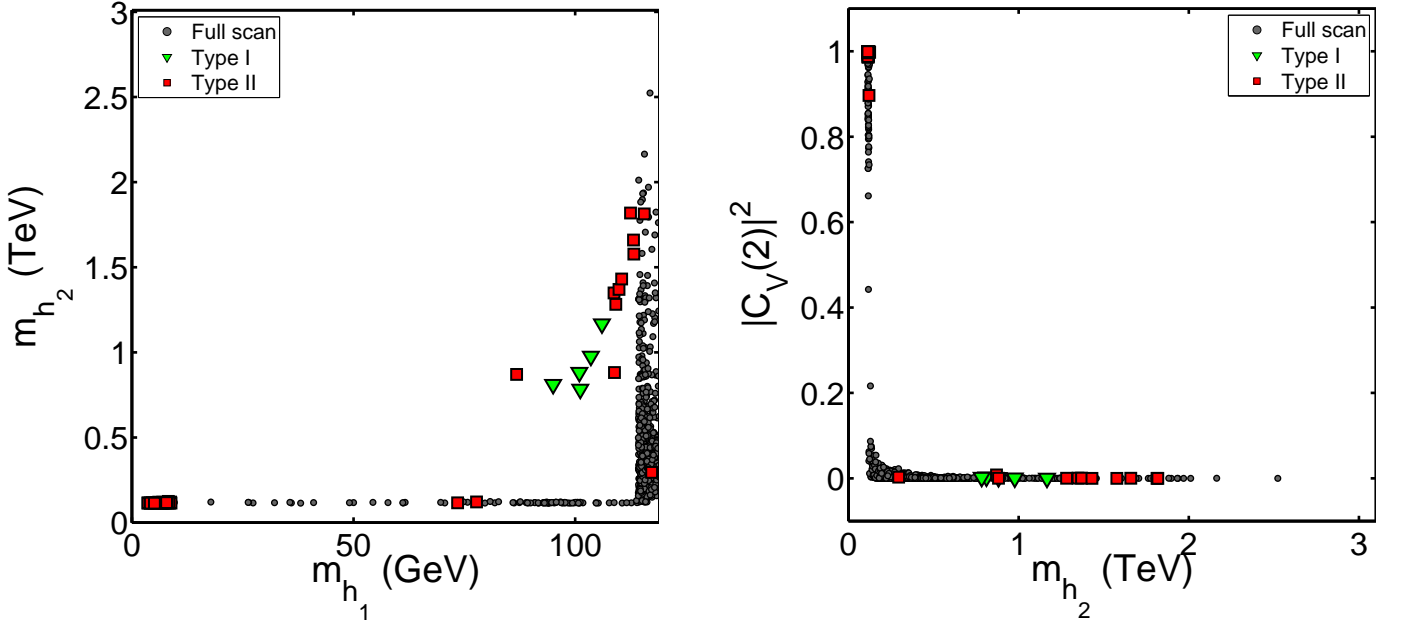


FIG. 9: We plot our points in the  $m_{h_2} - m_{h_1}$  and show  $|C_V(2)|^2$  as a function of  $m_{h_2}$ .

case of point 5 the LEP bound on  $C_{eff}^{4b}$  comes into play quite quickly as  $A_\kappa$  is decreased. In comparison, the right hand plot shows that in the case of point 1 there is a larger range of  $A_\kappa$  for which the LEP bound on  $C_{eff}^{4b}$  is satisfied and  $\xi^2 = 0$  so that the ALEPH bound is automatically satisfied. Of course, the nominal value of  $A_\kappa$  from Table V for point 5 (the left-hand plot) is such that the ALEPH bound is satisfied and no adjustment of  $A_\kappa$  is required. We presented the plot to show how sensitive the LEP phenomenology is to  $A_\kappa$ . In this case, the value of  $A_\kappa$  can be changed somewhat from its nominal Table V value without immediately encountering a problem with either the LEP bound on the  $4b$  final state or the ALEPH bound on  $\xi^2$ . In contrast, the nominal value of  $A_\kappa$  for point 1 in Table V gives a value for  $\xi^2$  that is considerably too large in comparison to the  $\xi^2 \leq 0.29$  ALEPH limit. In this case, we must lower  $A_\kappa$  in order to satisfy the ALEPH bound. Roughly, any  $A_\kappa$  above 5.58 GeV, the value at which the LEP

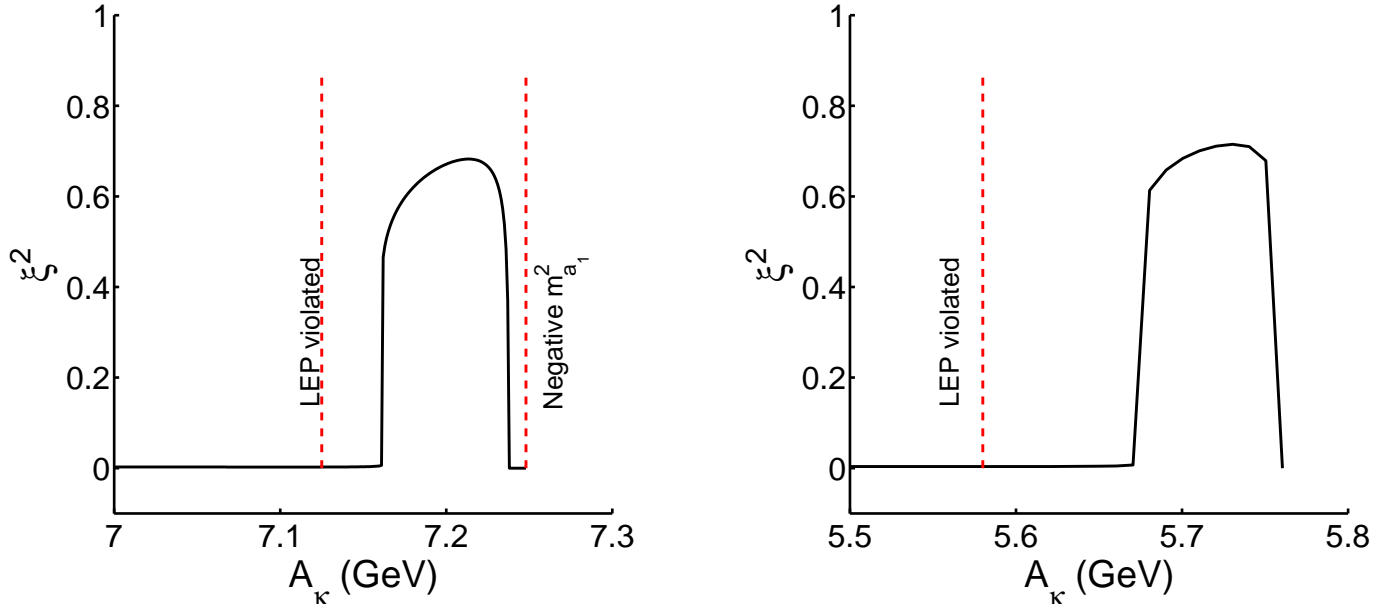


FIG. 10: The effect of varying the  $A_\kappa$  parameter of point 5 (left) and point 1 (right) in table V.

$4b$  bound (dashed line) becomes relevant, but below about 5.67 GeV (vs. the nominal value of 5.69 GeV) would be allowed.

The above discussion illustrates that acceptable  $\xi^2$  can be obtained for the type I points 1, 2 and 4 by very small shifts in  $A_\kappa$  that do not any way affect the remainder of the phenomenology of these points. Of course, it must be acknowledged that there is a certain level of fine-tuning of  $A_\kappa$  involved in getting  $m_{a_1}$  into an allowed range. This is, in fact, already reflected in the somewhat large  $G$  values of points 1 and 2 of Table V, these points being ones where the nominal  $\xi^2$  is substantially above the ALEPH limit. Point 4 has a much more modest  $G$  value and correspondingly a broader range of  $A_\kappa$  would allow it to satisfy the ALEPH limit that is only slightly below the value of  $\xi^2$  predicted by the nominal  $A_\kappa$  value tabulated in Table V.

As stated earlier,  $\Omega h^2$  plays a pivotal role in determining the likelihood of a given point in parameter space. Fig. 11 shows us that the range of values for  $\Omega h^2$  is huge, with many points having relic densities that are too large by orders of magnitude. In this context, the fact that Type I points tend to achieve the right order of magnitude seems quite remarkable. That said, despite the relic density constraint pushing our scanning quite strongly towards the WMAP value, we did not find Type I points with a relic density that is less than two sigma away from the observed value. Similar remarks apply to the Type IIA points. In contrast, Type IIB points (for which the  $\chi$  is very singlino-like) have much too large  $\Omega h^2$  as a result of too small an annihilation cross section.

As noted already, Type III points were defined by requiring not only  $BR(h_1 \rightarrow b\bar{b}) > 0.5$  and  $m_{a_1} > 2m_B$  (so that  $BR(a_1 \rightarrow b\bar{b}) \neq 0$ ) but also by demanding that  $\Omega h^2$  is within  $\pm 2\sigma$  of the observed value. Thus, it is mainly the Type III points that populate the band in the right-hand expanded plot of Fig. 11. It turns out that for all the Type III points the dominant process responsible for getting correct  $\Omega h^2$  is coannihilation of the  $\chi$  with  $\tilde{\tau}_1$  — they are quite closely degenerate in mass for all the Type IIIA and Type IIIB points. In the case of points where the  $\chi$  is very singlino-like, which comprises all Type IIIB points and a sizable fraction of Type IIIA points, the mass difference between  $\chi$  and  $\tilde{\tau}_1$  is at most about 4 GeV and the common mass is typically of order 120 GeV. The  $\chi$  has just enough gaugino and higgsino components (of order  $10^{-6}$  at the probability level) to couple effectively and coannihilate with the  $\tilde{\tau}_1$  to get the right relic density. In the case of the Type IIIA points, for which the  $\chi$  is mainly bino-like, the common mass is most often  $> 250$  GeV (but not always) and for such points the mass difference is more typically of order 10 GeV. The very smallest  $\chi^2$  values are achieved for the Type IIIB points for which both the  $h_1$  and the  $\chi$  are mainly singlet and singlino, respectively, the “singlet-singlino” (or SS) scenario. The ease with which such low  $\chi^2$  points were found in our scans suggest that the SS scenario for dark matter should be taken quite seriously as possibly being the correct paradigm for dark matter.

One question is whether small parameter changes for the Type I points could bring the predicted  $\Omega h^2$  into close agreement with observation. After all, our scans might just be slightly missing Type I points with the right relic density. To examine this, an attempt to vary the gaugino masses independently at the GUT scale was made to see if



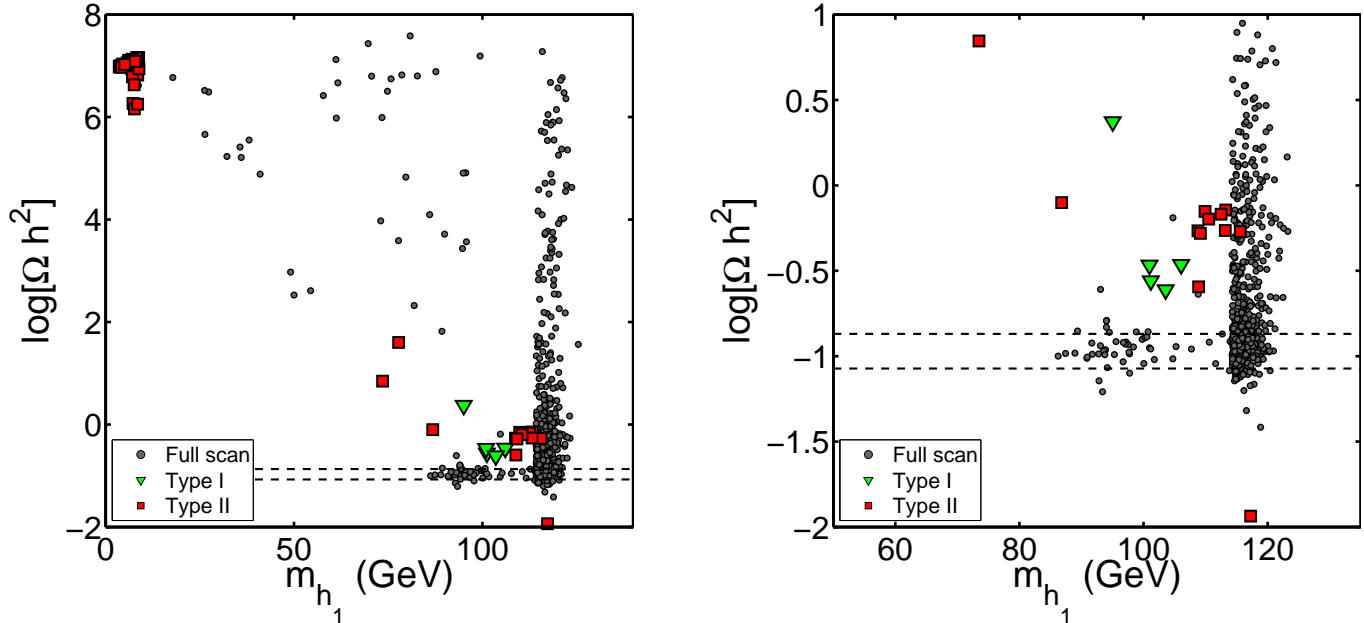


FIG. 11: The relic density,  $\log(\Omega h^2)$ , versus  $m_{h_1}$ ,

this could lead to the right amount of dark matter. In general it is not difficult to get to  $\Omega h^2 \sim 0.1$ , but in all the cases studied, it comes at the price of exceeding LEP limits on several channels, including Higgstrahlung processes such as  $Zh_1$  production with  $h_1 \rightarrow b\bar{b}$ .

In order to do this perturbation, we took the most promising Type I point (*i.e.* the one with the best relic density value, point 5 in table V) and perturbed  $M_2$  away from universality at the GUT scale. The results are shown in Figs. 12 and 13. What seems to be happening here is that as  $M_2$  changes in this region,  $m_{a_1}$  also changes dramatically, so much so that perturbing  $M_2$  by only a few GeV gives us a point that is ruled out by the  $h \rightarrow aa \rightarrow 4b$ 's limit from LEP. As  $M_2$  gets bigger, eventually we get to the crucial point where  $BR(a_1 \rightarrow b\bar{b})$  is kinematically suppressed as  $m_{a_1}$  is sufficiently light.

The curves in Fig. 12 do not extend below  $M_2 \sim 367$  GeV or so, since, as seen in Fig. 13, in this region one exceeds the LEP limits on  $C_{\text{eff}}^{2b}$ . In addition, the sensitivity of  $m_{a_1}$  means that soon after getting to a point where LEP limits are respected, (the exact point being unclear due to the resolution of the exploration done)  $m_{a_1}$  is driven tachyonic. Looking at this admittedly very specific case, one can perhaps begin to understand why we are obtaining so few points that are Type I and anywhere near the right relic density, given that  $m_{a_1}$  is very sensitive to changes and is the crucial element here. This is by no means a blanket statement about points being ruled out, merely an observation for this specific situation. For further study, a scan with all the gaugino masses disunified might be useful to shed light on this.

Let us now turn to predictions for direct detection of the neutralino via scattering on nucleons. In Fig. 14 we show the spin-independent cross section as a function of both  $m_\chi$  and  $m_{h_1}$ , and see something not entirely unexpected: the large swathe of singlino points, which comprise Type IIB, Type IIIB and a sizable fraction of Type IIIA points, will be nearly undetectable in any upcoming direct detection experiment, having cross sections of at most  $10^{-12}$  pb or so. On the other hand, about half the Type IIIA points have  $\sigma_p^{SI} > 10^{-10}$  pb, with some having  $\sigma_p^{SI} \sim 10^{-7}$  pb (and good  $\Omega h^2$ ). And, we should again note that all Type IIIA points have  $m_{h_1} > 114$  GeV. In contrast, the Type I and Type IIA points have  $m_{h_1} < 114$  GeV and often  $m_{h_1} \sim 100$  GeV, *i.e.* in the  $m_{h_1}$  region of interest for explaining the LEP excess near 100 GeV. As apparent from Fig. 14 these points for which the  $h_1$  is doublet-like seem to have a large, or at least measurable, direct detection cross section. As a result, if a largish value of  $\sigma_p^{SI}$  is eventually observed, the value of  $m_{h_1}$  could be used to distinguish between the Type IIIA and Type I/IIA regions of parameter space. Note that *a priori* there is no reason for Type I and Type IIA points to have a direct detection cross section that is both high enough to be tested by future experiments and low enough to avoid current constraints, and as such it is interesting to note that many such points do appear.

On another note, by comparing between the two figures in Fig. 14, we can again detect the correlation between singlet-like Higgses and singlino-like neutralinos that was apparent in Fig. 5 for values of the Higgs mass in the range

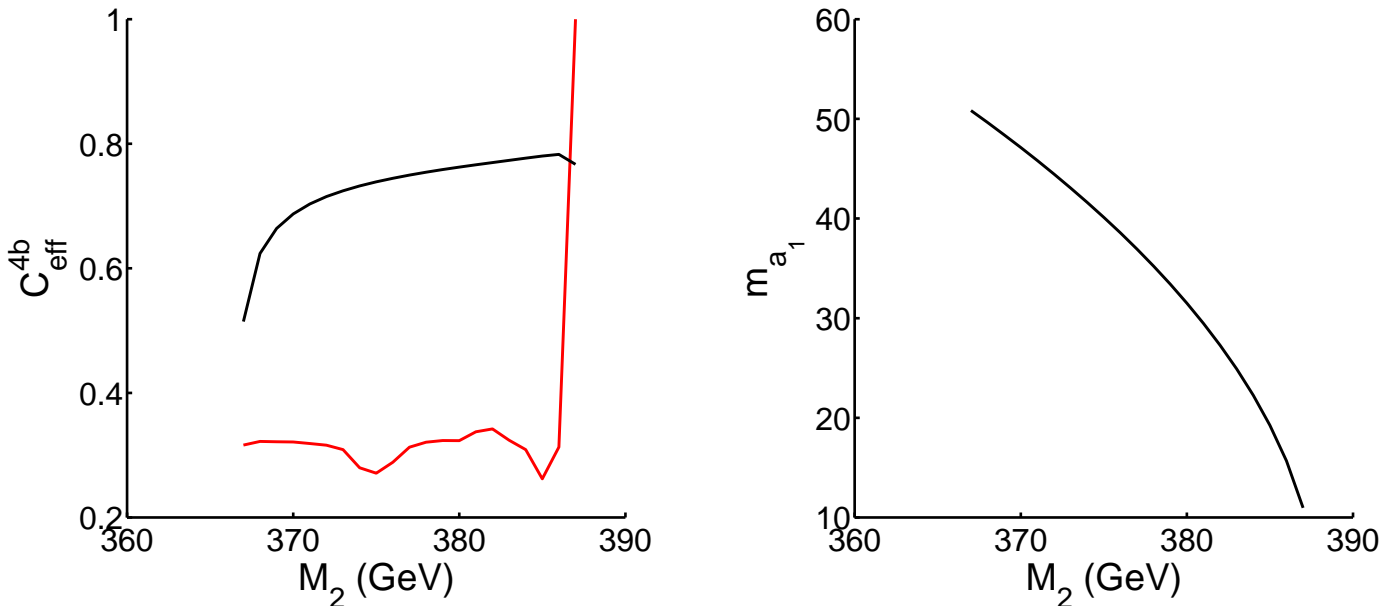


FIG. 12: The effect of varying the  $M_2$  parameter of point 5 in table V. The left-hand plot gives the values for  $C_{\text{eff}}^{4b}$  as defined in Eq.(1) with red denoting the experimental limit and black the NMSSM predicted value as a function of  $M_2$ . The black curve terminates as  $M_2$  increases when  $m_{a_1}^2$  becomes negative. The sharp rise in the experimental limit occurs as  $m_{a_1}$  approaches and then falls below  $2m_B$ , the point at which the  $a_1 \rightarrow 2b$  decay mode becomes kinematically forbidden. On the right, we show the rather dramatic change of  $m_{a_1}$  with  $M_2$ . In contrast,  $m_{h_1}$  remains roughly at 102 GeV over this range of  $M_2$ .

$m_{h_1} < 90$  GeV. As already apparent in Fig. 5, for  $114 > m_{h_1} > 90$  GeV the situation is more complicated and there is no clear correlation between the singlet nature of the  $h_1$  and the singlino nature of the  $\chi$ .

It is interesting to comment on results for  $\sigma_p^{SI}$  for points with very low  $m_\chi \lesssim 15$  GeV. We see in Fig. 14 that the Type IIB points are those that populate the very low  $m_\chi$  region. However, these points all have a singlino-like  $\chi$  and, correspondingly, the largest cross section is of order  $\sigma_p^{SI} \sim 10^{-10}$  pb, *i.e.* far below the region that is needed to explain the possible CoGeNT [30] and DAMA [31] excesses in the  $6 \text{ GeV} \lesssim m_\chi \lesssim 9 \text{ GeV}$  region for which  $\sigma_p^{SI} \gtrsim 10^{-4}$  pb is required [32]. In [33], it is shown that if one ignores GUT-scale unification then values of  $\sigma_p^{SI}$  within a factor of 10 of the above range are possible while still having a Higgs with SM-like  $WW, ZZ$  couplings that is sufficiently light to achieve “ideal-like” agreement with precision data. From the study presented in the present paper, it seems that such large values of  $\sigma_p^{SI}$  cannot be achieved in the context of the relaxed-CNMSSM boundary conditions employed here. In particular, very low  $m_\chi$  values for a bino-like  $\chi$  (which allowed for the largest  $\sigma_p^{SI}$  values in [33]) require small  $M_1$  values, a region that is quite inaccessible in the strict CNMSSM context.

Before concluding, we provide a tabular summary of the most important characteristics of the five different classes of points that we have particularly focused on: Type I, Type IIA, Type IIB, Type IIIA and Type IIIB.

## V. CONCLUSION AND SUMMARY

In the version of the NMSSM studied in this paper, we have relaxed the unification condition of  $A_\kappa$  and of the Higgs soft masses  $m_{H_u}$  and  $m_{H_d}$  (at the GUT scale) with respect to the CNMSSM in order to explore the extent to which the absence of the ideal-Higgs-like scenarios in the CNMSSM scans depended upon these particular (rather unmotivated) universality assumptions. Allowing for non-universal  $A_\kappa$  and non-universal Higgs soft masses, five parameter space points corresponding to the so called “ideal Higgs” scenario were indeed found, although these were far outnumbered by other points. The phenomenology of the ideal-Higgs points was studied, and in the context of this particular scan these points were seemingly acceptable in terms of flavour observables like  $BR(\bar{B} \rightarrow X_s \gamma)$  and  $BR(\bar{B}_s \rightarrow \mu^+ \mu^-)$  (with  $(g-2)_\mu$ , in common with every other scan we have done, struggling to fit the observed  $3\sigma$  difference relative to

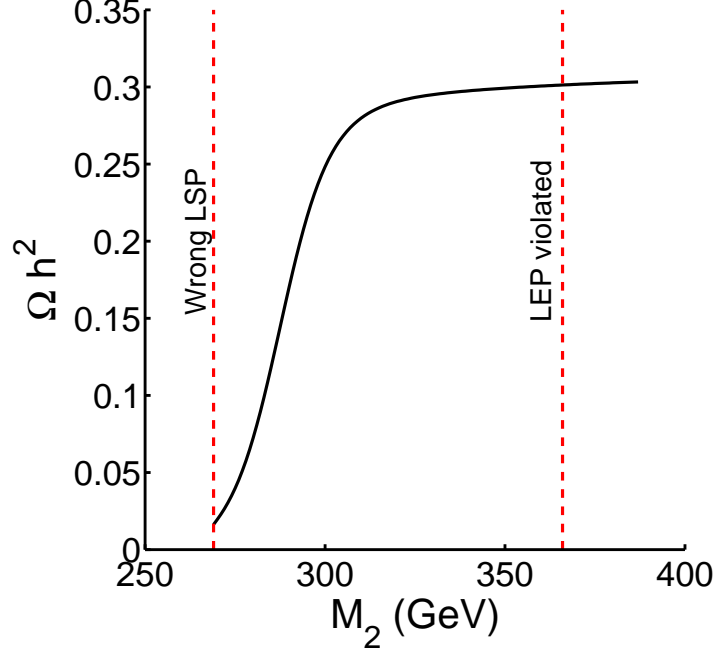


FIG. 13: The effect of varying the  $M_2$  parameter on point 5 in table V. Here we show the change in the relic density,  $\Omega h^2$ . It is possible to bring DM in line with experiment, but in doing so one violates LEP limits on several channels, below  $M_2 \sim 360$  GeV in this case, which is the reason that the curves in the previous figures don't go below this value.

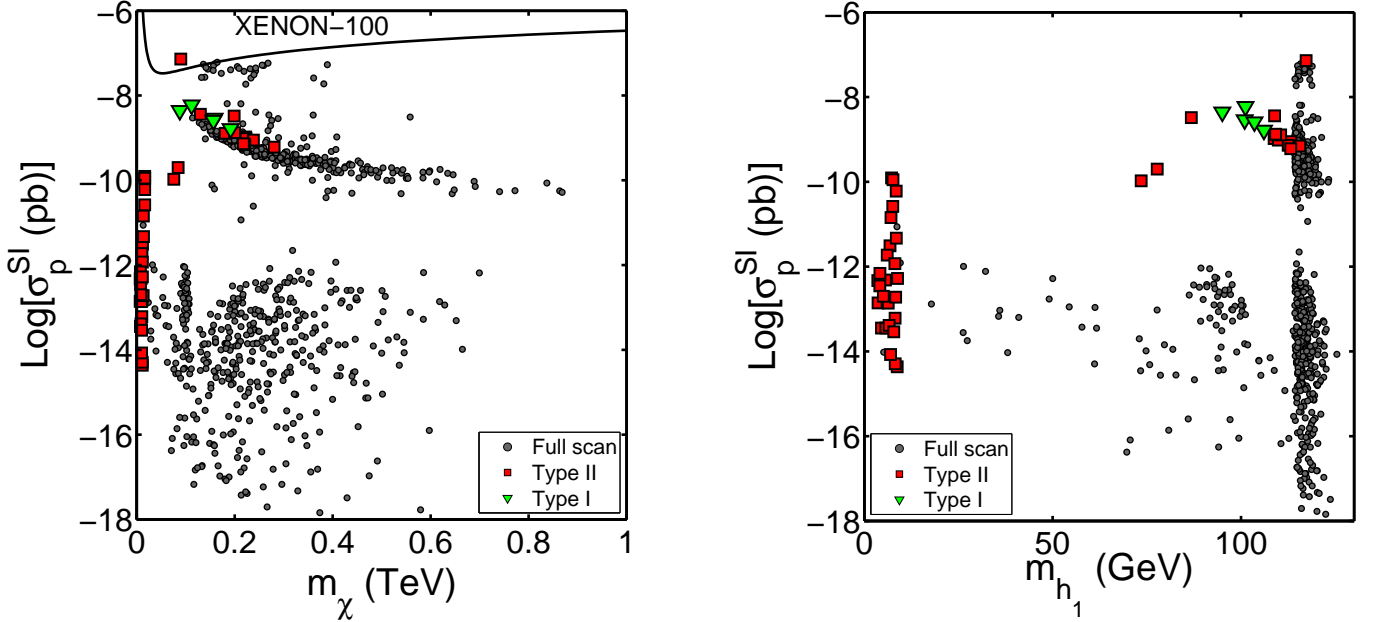


FIG. 14: The dark matter spin-independent cross section  $\sigma_P^{SI}$  as a function of, on the left,  $m_\chi$  and, on the right,  $m_{h_1}$ . Included on the left hand side is an illustrative limit from the Xenon-100 direct detection experiment [29].

the SM). However, only two of the five ideal-Higgs-like points we found are strictly consistent with the latest ALEPH limits on the Higgs to four tau mode, with a third being very close to consistency. However, we have shown that by changing  $A_\kappa$  by a very small amount compared to the nominal value found in the scan (which did not use the ALEPH limit on the four tau final state as an input to the chi-squared employed) will allow consistency with the ALEPH limit

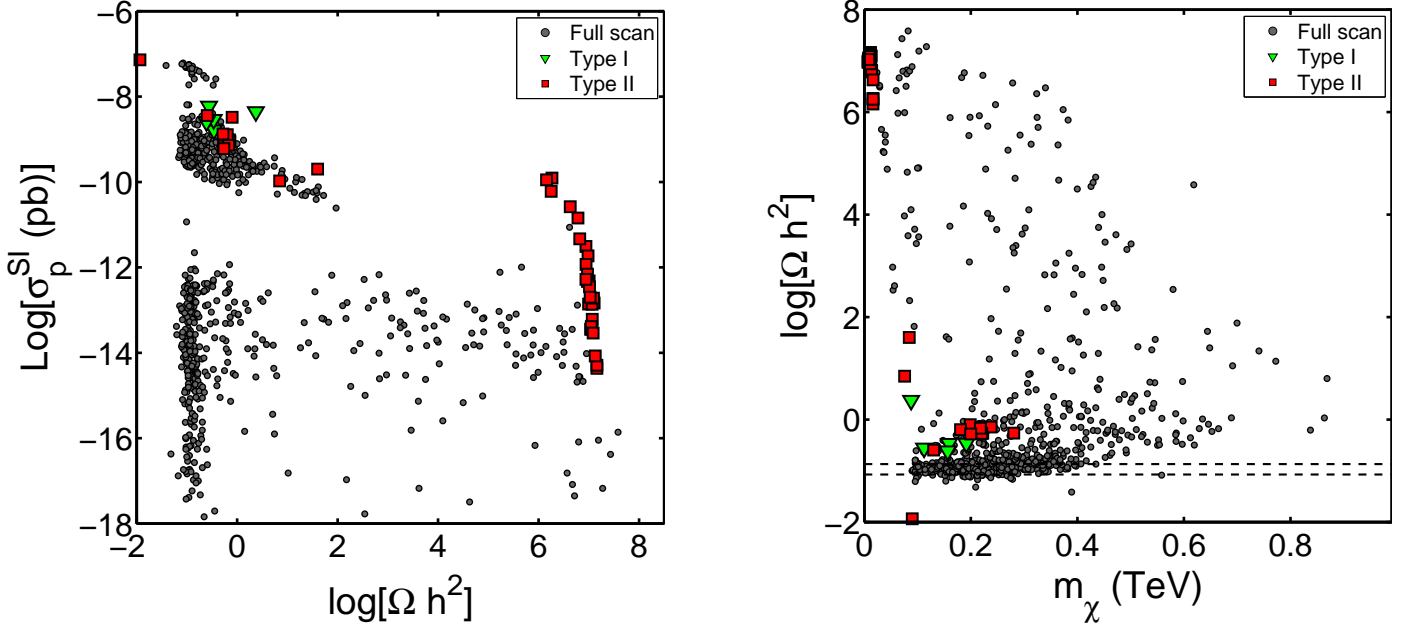


FIG. 15: Left: The spin-independent cross section,  $\log(\sigma_p^{SI})$ , vs. the relic density,  $\log(\Omega h^2)$ . Right:  $\log(\Omega h^2)$  vs.  $m_\chi$ .

TABLE VI: Important characteristics of the different classes of points. The top entries in the Table list the requirements imposed in defining the classes. The bottom entries list the resulting properties of points in the different classes. Three additional requirements are imposed for Type III points (only): (i) that  $\Omega h^2$  be within  $2\sigma$  of the observed value (roughly  $\log[\Omega h^2] \sim -1$ ); (ii) ALEPH limits on  $h_1 \rightarrow a_1 a_1 \rightarrow 4\tau$  are satisfied; and (iii) BaBar limits on  $\Upsilon(3S) \rightarrow \gamma a_1$  are satisfied.

	Type I	Type IIA	Type IIB	Type IIIA	Type IIIB
$BR(h_1 \rightarrow b\bar{b})$	$< 0.3$	$< 0.5$	$< 0.5$	$> 0.5$	$> 0.5$
$BR(a_1 \rightarrow b\bar{b})$	0	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$
$S_s^2$	$\sim 0$	$\sim 0$	$\sim 1$	$\sim 0$	$\sim 1$
$ C_V(1) $	$\sim 1$	$\sim 1$	$\ll 1$	$\sim 1$	$\ll 1$
$m_{h_1}$	$\in [95, 108]$	$> 75$	$< 90$ , mostly $< 20$	$\gtrsim 114$	$< 110$
$m_{a_1}$	$< 2m_B$	$\in [2m_B, 50]$	$\in [2m_B, 20]$	$> 2m_B$	$\in [2m_B, 40]$
$BR(h_1 \rightarrow a_1 a_1)$	$> 0.7$	$> 0.6$	$\sim 0$	$< 0.1$	$\sim 0$
$N_s^2$	$\ll 1$	$\ll 1$	$\sim 1$	$\ll 1$ or $\sim 1$	$\sim 1$
$N_B^2$	$\sim 1$	$\sim 1$	$\ll 1$	$\sim 1$ or $\ll 1$	$\ll 1$
$\tan\beta$	$< 3.5$	$< 6$	$> 8$	$\in [2, 20]$	$> 8$
$ \cos\theta_A $	$\in [0.005, 0.02]$	$\in [0.003, 0.02]$	$\sim 0$	$\in [0.002, 0.017]$	$\sim 0$
$m_\chi$	$\in [80, 200]$	$\in [70, 300]$	$< 15$	$\in [113, 400]$	$\in [91, 110]$
$\log[\Omega h^2]$	$\in [-0.65, 0.4]$	$\in [-1.9, 1.8]$	$\in [6.3, 7.3]$	$\sim -1$	$\sim -1$
$\log[\sigma_p^{SI} \text{ (pb)}]$	$\in [-8.7, -8.2]$	$\in [-10, -7]$	$\in [-14.5, -9.8]$	$\in [-10, -7]$	$< -12$
$\chi^2$	$\in [26, 70]$	$> 10$	$> 10$	$\in [2.7, 26]$	$\in [1.9, 6.0]$

without altering any other phenomenology.

As regards the relic density, the  $\Omega h^2$  values of the Type I points were not within the two sigma range of the observations, but four out of the five were within a factor of 2 or 3 of  $\Omega h^2 \sim 0.1$ . Perhaps not too much should be read into this as the relative scarcity of these points in our relaxed-CNMSSM scan could mean our understanding of these parameter points is incomplete. Correct  $\Omega h^2$  can be achieved for the Type I points by varying the most relevant gaugino mass parameter ( $M_2$ ) slightly. However, we found that for  $M_2$  such that the relic density was correct one or more of the LEP Higgs limits was not satisfied.

There was another very interesting class of points, denoted Type III, that appeared in our relaxed-CNMSSM scan. Type III points are, first of all, characterized by  $BR(h_1 \rightarrow b\bar{b}) > 0.5$  (*i.e.* the normal SM decay is dominant) and

by  $m_{a_1} > 2m_B$  (*i.e.*  $a_1 \rightarrow b\bar{b}$  is dominant). Points satisfying this criterion are, as one might expect, very numerous. Further, we found that it was very easy to find points satisfying the above criteria that gave  $\Omega h^2$  values in close agreement ( $\pm 2\sigma$ ) with the observed value (something that we included in our final definition of Type III points, along with the requirement that they obey the ALEPH limits on  $h_1 \rightarrow a_1 a_1 \rightarrow 4\tau$  decays and the BaBar limits on  $\Upsilon(3S) \rightarrow \gamma a_1$  decays). Within the Type III class, as finally defined, the very best predictions for  $\Omega h^2$  were obtained for cases in which the lightest Higgs is very singlet-like with  $85 < m_{h_1} < 110$  GeV and the lightest neutralino is very singlino-like with  $91 < m_\chi < 110$  GeV. Such scenarios are dubbed SS scenarios. For all such SS scenarios, sufficiently small  $\Omega h^2$  in agreement with experiment is achieved via  $\chi - \tilde{\tau}_1$  coannihilation. We believe that one should take these SS scenarios seriously. It will then be the second lightest Higgs boson (predicted to have mass  $m_{h_2}$  close to 114 GeV) that has SM-like couplings to WW, ZZ and its decays will also be SM-like. Unfortunately, in such scenarios the spin-independent cross section for direct dark matter detection is predicted to be very small,  $\sigma_p^{SI} < 10^{-12}$  pb. Rates for collider production of the singlet-like  $h_1$  will be very low. The  $\chi$  will appear in chain decays at the LHC and its roughly 100 GeV mass should be measurable with reasonable accuracy. However, to determine how singlet it is would require observation of a displaced vertex. Unfortunately, the predicted non-singlet content of the  $\chi$  for the SS scenarios is of order  $1 - N_s^2 \sim few \times 10^{-6} - 10^{-4}$ , sufficient to make the decays to the  $\chi$  prompt.

Overall, it is clear that even a slight extension of the strongly constrained CNMSSM to allow non-universality for the Higgs soft-masses-squared and for  $A_\kappa$  opens up the phenomenological possibilities very considerably. One finds fairly good ideal-Higgs-like scenarios. In addition, the very intriguing singlet-scenarios that are consistent with all experimental constraints and give excellent  $\Omega h^2$  become quite prominent.

## VI. ACKNOWLEDGMENTS

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