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F. Karsch [1,2] and K. Redlich [3,4]

[1] *Fakultät für Physik, Universität Bielefeld, D-33615 Bielefeld, Germany*
[2] *Physics Department Brookhaven National Laboratory, Upton, NY 11973*
[3] *Institute of Theoretical Physics University of Wroclaw, PL-50204 Wroclaw, Poland*
[4] *ExtreMe Matter Institute EMMI, GSI, D-64291 Darmstadt, Germany*

Abstract

We discuss the role of cumulants of net baryon number fluctuations in the analysis of critical behavior in QCD and the study of freeze-out conditions in heavy ion experiments. Through the comparison of the current set of measurements of higher order cumulants of net baryon number fluctuations with lattice QCD calculations and results from hadron resonance gas model we can learn to what extent freeze-out as, determined by such cumulants, occurs close to the QCD transition temperature and thus can probe critical behavior at small values of the baryon chemical potential. Understanding how the relation between freeze-out conditions and the QCD crossover transition is reflected in properties of the experimentally determined cumulants is an important prerequisite to search for the QCD critical point. We point out that even if perfect continuum extrapolated lattice QCD results would be available, it would be inappropriate to use these observables to extract the value of the QCD transition temperature at vanishing baryon chemical potential from experimental data. We furthermore provide indications that a recently performed comparison of lattice QCD results on cumulants with data from heavy ion experiments suffer from systematic as well as statistical uncertainties in the lattice QCD calculations. This makes such comparison of lattice QCD calculations with experimental data at present not useful.

Keywords: Quark Gluon Plasma, Lattice QCD, Heavy Ion Collisions

1 Introduction

The outstanding goal of the on-going RHIC low energy run is to search for the elusive critical point in the QCD phase diagram. Promising observables used in this search are higher order cumulants of net baryon number fluctuations, which have been advocated to be sensitive to critical behavior in the vicinity of the chiral phase transition of QCD at vanishing baryon chemical potential (μ_B) [1] as well as in the vicinity of the QCD critical point at $\mu_B > 0$ [2].

A first comparison of experimental results on higher order cumulants of baryon number fluctuations [3] with theoretical calculations performed in the framework of lattice regularized QCD [4] as well as hadron resonance gas [5] calculations led to quite good agreement. Recently the analysis of [4] has been extended in Ref. [6] by treating the QCD transition temperature as a free parameter that might be constrained through the experimental data. The striking statement made in this publication is that through comparison of the lattice QCD calculations with experimental findings for certain cumulants of net baryon number fluctuations the QCD transition temperature has been determined. In the following we will discuss if such an observation can indeed be substantiated.

2 Lattice Cut-off effects and the continuum limit of QCD

Cumulants of net baryon number fluctuations play an important role when analyzing properties of QCD in the vicinity of the chiral phase transition temperature, $T_c(\mu_B)$. They are defined as derivatives of the free energy density or pressure (P) of a thermodynamic system at temperature (T) with respect to the baryon chemical potential (μ_B). The n-th order cumulant is given by

$$\chi_n^B = \frac{\partial^n P(T,\mu_B)/T^4}{\partial(\mu_B/T)^n} \ . \tag{1}$$

Higher order cumulants are increasingly sensitive to critical behavior. They diverge in the chiral limit at $T_c(\mu_B)$ as well as at a possible critical point at $T_c(\mu_B^c)$ for non-zero quark masses.

Properties of cumulants at finite temperature and their dependence on μ_B have been studied in lattice QCD calculations using different lattice discretization schemes. The lattice calculations [4, 7], on which the analysis of cumulants presented in [6] is based, have been performed within a specific lattice regularization scheme for 2-flavor QCD. Thus, any influence of the strange quark on the thermodynamics has been ignored. The specific lattice regularization scheme, the standard staggered fermions used in [7], is known to be subject to large lattice discretization errors. This is even more true on the rather coarse lattice that have been used for the thermodynamics studies in [6, 7] and is well known since a long time. It becomes, for instance, apparent in the high temperature limit of QCD where discretization errors in the standard staggered fermion scheme on lattices with temporal extent $N_\tau = 4$ and 6 lead to deviations of bulk thermodynamic observables, like energy density and pressure, from their known values in the continuum limit by more than 80% [8]. It is also known, that calculations with this action performed on such coarse lattices lead to severe and different discretization errors in various hadronic observables. Such errors can to some extent be reduced by forming appropriate ratios as it has, for example, been done in the analysis performed in [4] and also previously when lattice results on ratios of cumulants have been compared to HRG calculations [1]. However, discretization errors can not be neglected when one wants to determine absolute scales in a lattice calculation.

As a consequence of this it is impossible to arrive at a unique determination of the scale, i.e., the inverse lattice spacing (cut-off), that could be used to convert lattice results to physical units on such coarse lattice as they have been used in Ref. [7]. Thus, a direct comparison of such calculations with experiment seems to be excluded right from the beginning.

The problem with cut-off effects on coarse lattice has been already addressed in Ref. [7] which provided the numerical lattice QCD calculations on which the analysis in [6] is based. There, it was noted, that the quark mass used in the calculations was still to large to reproduce the correct pion to rho-meson, (m_π/m_ρ) and nucleon to rho-meson, $m_{nucleon}/m_\rho$ mass ratios. In fact, the hadron spectrum calculations relevant for setting the scale in 2-flavor QCD at finite temperature on lattices with four and six time-slices have been done long time ago [9]. It is known from theses calculations, that the nucleon to rho-meson mass ratio suffers from cut-off effects and attains a value of about 1.7 rather than its physical value of 1.22. As a consequence, a determination of a lattice scale from either of these observables would lead to large differences in a determination of the QCD transition temperature. Using the rho-meson mass to set the scale on such coarse lattices gives $T_c = (160 - 167)$ MeV, while the nucleon mass gives $T_c = (100 - 110)$ MeV. On the other hand, using a scale from gluonic observables like the string tension lead to a substantially larger transition temperature of about $T_c \sim 190$ MeV [10]. Another way to state this problem is that the nucleon mass would turn out to be about 1300 MeV if the physical value of the rho meson mass would have been used to set the scale. This illustrates the severeness of cutoff effects in the lattice calculations used in [6] for the comparison with experimental data. This also illustrates why major efforts are still being undertaken in lattice calculations to arrive at a reliable determination of the QCD transition temperature. This can be achieved by using improved actions which reduce the cut-off distortion and by performing calculations on lattices with smaller lattice cut-off, i.e., closer to the continuum limit [11, 12] where a unique scale can be determined and a reliable comparisons with continuum physics, e.g. experiments, does then become possible.

3 Are there free parameters in finite temperature Lattice QCD?

Let us set aside the problem of controlling cut-off effects in lattice QCD calculations and discuss what actually can be determine through a comparison of lattice QCD results on fluctuations of net baryon number with heavy ion experiments. As pointed out already in the previous section, the scale for lattice calculations and as such also the QCD transition temperature is ideally determined through a comparison of lattice calculations at zero temperature with known spectral properties of QCD. Nonetheless, one may take the point of view that one does not want to rely on experiments that determined the proton and nucleon mass to set the scale for lattice QCD calculations, but rather wants to use a heavy ion experiment that determined the freeze-out temperature. This, however, relies on the fundamental assumption that cumulants of net baryon number fluctuations indeed measure the same freeze-out temperature that has been extracted experimentally from ratios of particle abundances through a comparison with the hadron resonance gas model (HRG) [13]. These assumptions, however, still needed to be justified.

Thus, a much more natural approach is to accept that lattice QCD calculations (eventually) provide reliable results for the QCD transition temperature through comparison with spectral properties of QCD. In fact, the current best estimates lead to a transition temperature $T_c = (150 - 160)$ MeV [11, 12]. It then is much more sensible to use the comparison of lattice QCD results with the experimental data on ratios of cumulants to learn about the freeze-out conditions probed by these observables. In this way, one can verify whether freeze-out, as probed by ratios of cumulants, indeed happens close to the QCD transition temperature and therefore can be used to search for the QCD critical point. One can also learn whether ratios of cumulants are consistent with freeze-out conditions determined from particle yields.

All relevant scales needed to compare a lattice calculation with experimental data thus can reliably be fixed at zero temperature and no additional scales are then required to be determined at finite temperature.

4 What does the comparison of lattice QCD calculations with measured higher order cumulants tell us?

We argued in the previous section that even when perfect lattice calculations, free of cut-off errors, become available one should not use the comparison between lattice QCD calculations of ratios of cumulants and their experimental measurements for the determination of the QCD transition temperature. One should rather accept that the transition temperature is already provided by the (then perfect) lattice calculation. The comparison between experiment and theoretical calculation will then allow us to learn more about the freeze-out conditions probed by ratios of cumulants and, hopefully, will establish them as unique probes of QCD critical behavior in heavy ion experiments.

If we follow this approach one may ask whether the analysis performed in [6] confirms that the freeze-out temperature in heavy ion experiments as determined from higher order cumulants is close to T_c. Such a statement in its own would be a great success as it would confirm that cumulants are indeed the right observables that should be used in an experimental search for the QCD critical point. Unfortunately this conclusion can not yet be drawn on the basis of the analysis presented in [6].

To substantiate our skepticism one needs to discuss the statistical significance of the lattice calculations used in [6]. One also needs to realize that the 'determination of T_c' presented in [6] is based on the significance of a χ^2 analysis. The χ^2-values in this analysis become large when the ratio between freeze-out temperature (T_f) and QCD transition temperature (T_c) becomes small. This is counter-intuitive as one would expect that experimental results do agree better with hadron resonance gas calculations when freeze-out happens further away from T_c. It is not obvious why lattice QCD calculations should generate discrepancies in such an 'uncritical' region. Why does a variation of T_f/T_c, as it is done in [6], then lead to such a dramatic change in the χ^2 of the fits presented in Fig. 3 of that paper?

First of all one needs to realize that the large χ^2 values reported in [6] arise in a region where T/T_c becomes small, i.e. $T/T_c \lesssim 0.92$. When looking into the data of Ref. [7] it is evident that the statistical significance of higher order cumulants rapidly decreases when T/T_c is decreased. This is most apparent for the $N_\tau = 4$ calculations reported in [14]. Already for this easier case ($N_\tau = 4$) even the fourth order cumulant (and also the sixth order cumulant [7]) is negative for $T/T_c \simeq 0.92$. Both quantities should stay positive in that temperature regime. On lattices of size $N_\tau = 6$ it does become even more difficult to get the statistics under control. Data for the sixth order cumulant given in [7] have a 50% error at $T/T_c \simeq 0.94$. In view of this one may doubt whether the eighth order term is of any use for the Pade analysis performed in [6].

We thus may safely assume that we should not rely on any results from eighth order cumulants in the comparison of lattice QCD results with the experimental observable under question. In [6] a particular combination of variance (σ), skewness (S) and kurtosis (κ), that is proportional to the ratio of third and fourth order cumulants of net baryon number fluctuations, has been analyzed

$$\frac{\kappa\sigma}{S} = \frac{\chi_B^{(4)}(T,\mu_B)}{\chi_B^{(3)}(T,\mu_B)} \; . \tag{2}$$

Written in terms of a next-to-leading order Taylor series expansion around $\mu_B = 0$ this quantity is given by

$$\frac{\kappa\sigma}{S} = \frac{T}{\mu_B} \frac{1 + \frac{1}{2}\frac{\chi_B^{(6)}(T,0)}{\chi_B^{(4)}(T,0)} \left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)}{1 + \frac{1}{6}\frac{\chi_B^{(6)}(T,0)}{\chi_B^{(4)}(T,0)} \left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)} \; . \tag{3}$$

Assuming that $\chi_B^{(6)}(T,0)/\chi_B^{(4)}(T,0) > 0$, which will be the case below T_c, one concludes from this next-to-leading order result that within this approximation $1 < \mu_B\kappa\sigma/ST < 3$. Here the lower limit corresponds to the HRG result, $\kappa\sigma/S = 1/\sinh(\mu_B/T) \simeq T/\mu_B$, and the upper limit is reached when the expansion is dominated completely by the next-to-leading order correction. At this point, of course, one should not trust the expansion anymore. In fact, the data shown in Fig. 3 of Ref. [6] cover this range of values. For the two data points which generate the large χ^2 in Fig. 3 of Ref. [6], i.e., the data labeled with $T_c = 180$ MeV and 190 MeV for $\sqrt{s} = 62.4$ GeV the next to leading order correction is 100% and 200% of the leading order result. Maybe a Pade resummation is smarter than a direct analysis of the Taylor series. However, in the absence of any systematic analysis of Pades of different order one may doubt that this is the case.

Cumulants up to 6^{th} order have also been calculated using an improved staggered action (p4) on lattices of size $16^3 \times 4$ [15]. A comparison of ratios of cumulants with the results from STAR [3] have been shown in Ref. [16]. We have used these data to perform the analysis done in Ref. [6] with numerical results based on calculations with the p4-action. We use the notation of Ref. [6], i.e., we talk about a shift of T_c, although we do not like it and would rather think of the analysis as probing a shift of the freeze-out temperature T_f relative to a fixed value of T_c.

The crucial data set for the observation of a large variation in χ^2 in [6] corresponds to the RHIC low energy run at $\sqrt{s} = 62.4$ GeV. The value of the baryon chemical potential corresponding to $\sqrt{s} = 62.4$ GeV is $\mu_B \simeq 72.5$ MeV. Varying T_f/T_c and fixing μ_B/T_f we calculated $\mu_B\kappa\sigma/ST$ from Eq. 3 using $T \equiv T_f$. We never find values for $\mu_B\kappa\sigma/ST$ that become larger than 1.5 (see Fig. 1(right)), although it is apparent from Fig. 1(left) that the statistical error on the relevant input variable $\chi_B^{(6)}(T,0)/\chi_B^{(4)}(T,0)$ rapidly increase when $T/T_c \lesssim 0.92$. As $\mu_B\kappa\sigma/ST$ never gets larger than 1.5 the variation of $\mu_B\kappa\sigma/ST$ with T_f/T_c also cannot induce large

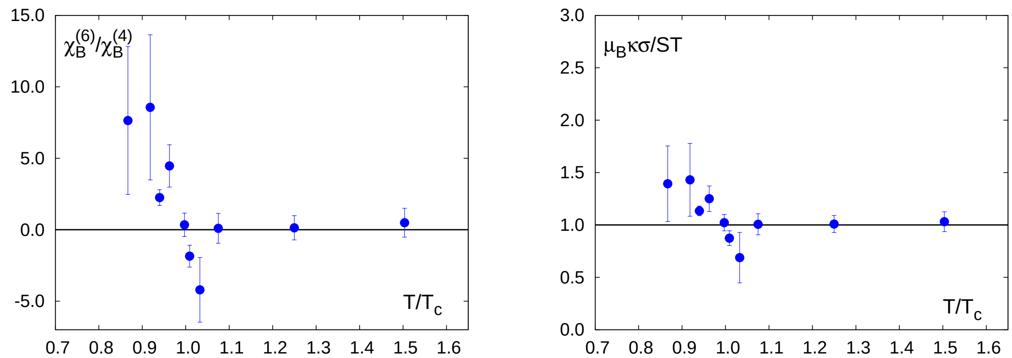

Figure 1: The ratio of sixth and fourth order cumulants, $\chi_B^{(6)}(T,0)/\chi_B^{(4)}(T,0)$, calculated at vanishing baryon chemical potential using an improved staggered fermion action (p4) [15, 16] (left). This ratio enters in next-to-leading order in the calculation of $\chi_B^{(4)}(T,\mu_B)/\chi_B^{(3)}(T,\mu_B)$ and $\mu_B\kappa\sigma/ST$ at the freeze-out temperature. The right hand figure shows this quantity for the value of the chemical potential corresponding to the RHIC low energy run at $\sqrt{s}=62.4$ GeV.

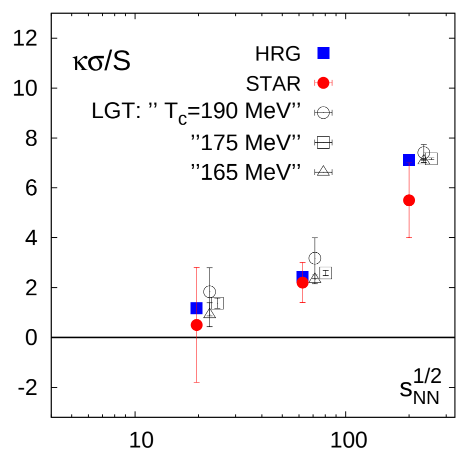

Figure 2: The ratio of fourth and third order cumulants measured by STAR in the RHIC low energy runs [3] and compared to lattice results for which the value of T_c has been shifted (see text). Also shown are results from a HRG calculation [5]. The lattice QCD data have been displaced slightly for better visibility.

variations in a χ^2 analysis as it is shown in the central Fig. 3 of [6]. We performed the analogous analysis with the p4 data. The result is shown in Fig. 2. From this figure it is evident that there is no 'best choice' for T_c. The χ^2/dof for the difference between experimental data and the lattice QCD results at the three T_f/T_c values shown in this figure vary between 0.6 and 1.3.

5 Conclusions

We noted that the analysis performed in [6], which aimed at 'Setting the scale for the QCD phase diagram' can not reach this ambitious goal because the lattice results used in these calculations suffer themselves from severe cut-off effects and do not allow for a unique determination of a scale. We have shown, that a comparison of current experimental results with another lattice discretization scheme, which at present suffers from similar discretization errors, does lead to a different conclusion. This hints at problems with the statistical significance of the analysis performed in [6] as well as with the conclusion that the critical temperature has been determined by comparing lattice QCD calculations with results from heavy ion experiments.

Putting aside the issue of actual quality of lattice results used in [6], we have argued that the procedure proposed in [6] to determine the critical temperature by comparing lattice QCD results with heavy ion data on different cumulants of net baryon number fluctuations should not have been done in the first place. The comparison of lattice QCD results with known spectral properties of QCD leads to far more accurate determinations of the scale needed to confront lattice QCD calculations with results from heavy ion experiments.

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