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Abstract

We present a Lagrangian describing a massive charged spin-2 field and a scalar in a constant electromagnetic background, and we provide a consistent description of the system. The Lagrangian, derived from string field theory through a suitable dimensional reduction, propagates the correct number of degrees of freedom within the light cone in any space-time dimension less than 26. We briefly discuss the higher-spin generalization of this construction, that cures the pathologies of a massive charged particle of arbitrary integer spin by introducing only finitely many new massive degrees of freedom.

1 INTRODUCTION

Electromagnetic (EM) or gravitational interactions generally produce severe inconsistencies in local actions that attempt to describe massive fields with spin larger than one, not only when EM and gravity are dynamical, but even when these interactions are treated as external classical backgrounds [1, 2, 3]. The interacting Lagrangian may not propagate the correct number of degrees of freedom (DoF), or may allow the physical modes to move faster than light. Such pathologies indeed originate from the presence of high-spin gauge modes [4].

Even the very simple interaction setup of a constant external EM background is fraught with difficulties. A well-known example is that of a massive charged spin-2 field, for which, although there exists a unique minimally coupled Lagrangian that preserves the DoF count [5], the resulting modes suffer from a "Velo-Zwanziger acausality" [1], i.e. they cease to propagate within the light cone. This problem persists for a wide class of non-minimal models [3] making it quite challenging, field theoretically, to construct consistent EM interactions for a charged massive spin-2 particle (and generically for any higher-spin field).

However, it is not impossible to restore causal propagation. In fact, there are a number of explicit examples where such difficulties are circumvented either through the addition of new dynamical DoFs or through appropriate non-minimal terms. For instance, the $\mathcal{N} = 2$ gauged supergravity [6], with supersymmetry broken without a cosmological constant [7], contains a massive spin-3/2 gravitino that can be charged under the graviphoton and still propagate consistently. Here, it is the presence of dynamical gravity that rescues causality for the charged gravitino [3]. On the other hand, ref. [8] presented a consistent non-minimal model containing a *single* massive spin-3/2 field propagating in a constant EM background.

String theory also bypasses the Velo-Zwanziger problem (at least in a constant EM background) for an arbitrary integer spin s, in that any field belonging to the first Regge trajectory of the open bosonic string interacts consistently and causally with a constant EM background [9, 10], thanks to the highly non-minimal (kinetic) terms that the theory itself spells out. While the explicit string-theoretic Lagrangian for spin 2 was given in [9], and for any higher spin the corresponding non-minimal Lagrangian is guaranteed to exist by the results of ref. [10], these Lagrangians do not hold good in arbitrary space-time dimensions. Unlike the free-string case, a unitary-gauge Lagrangian in the presence of the background is consistent only in the critical dimension $D = 26$. The same is true for the spin-2 Argyres-Nappi Lagrangian [9], as was shown in [10].

Of course, one can perform a dimensional reduction of the 26-dimensional Lagrangian provided by String Theory for an arbitrary integer spin s, to a space-time dimension $d < 26$.

The d-dimensional model obtained thereby would be a system of all integer spins from s down to 0, coupled to a constant EM background, with each field propagating causally. The spin-s acausality in d dimensions would then be cured by the presence of additional dynamical fields with spins $s-1$, $s-2$, ..., 0. The main point presented in this paper, however, is that one can consistently throw away the non-singlets of the $SO(26-d)$ symmetry rotating the internal coordinates. By this reduction, one ends up with a d-dimensional model that contains just the spins $s, s-2, s-4, ..., \frac{1}{2}\left[1+(-)^{s+1}\right]$, all of which propagate within the light cone with the correct DoFs. In particular, such a reduction of the Argyres-Nappi Lagrangian [9] gives rise to a coupled system of a spin 2 and a scalar that is consistent in any space-time dimension less than 26. Thus one *can* restore causality for a massive charged spin 2, in $d = 4$ for example, just by adding a dynamical scalar along with non-minimal terms.

The organization of this paper is as follows. In Section 2 we construct the aforementioned model for a massive charged spin-2 field and show its consistency: in particular, Section 2.1 presents the Argyres-Nappi spin-2 Lagrangian with some necessary details, while Section 2.2 carries out the dimensional reduction for obtaining the actual model involving a spin 2 and a scalar. Section 2.3 proves the consistency of the model for arbitrary dimensions less than 26, whereas Section 2.4 proves a crucial property of the dimensional reduction procedure, i.e. that it is free of negative-norm states. We discuss similar constructions for higher integer spins in Section 3, focusing on the equations of motion (EoM) for simplicity. Finally, we make some concluding remarks in Section 4.

2   A Consistent Model for Charged Massive Spin 2

In this Section, we dimensionally reduce the spin-2 Argyres-Nappi Lagrangian [9] to construct a model containing a spin 2 and a scalar, and show that it indeed propagates both fields causally with the correct number of DoFs in any $d < 26$.

2.1 The Spin-2 Argyres-Nappi Lagrangian

The Argyres-Nappi Lagrangian [9], which describes a massive charged spin-2 field in a constant EM background, is given by

$$
\begin{aligned}
L_{\text{AN}} = {} & \mathcal{H}^*_{\mu\nu}\left(\mathfrak{D}^2 - 2 - \tfrac{1}{2}\text{Tr}G^2\right)h^{\mu\nu} - 2i\mathcal{H}^*_{\mu\nu}(G\cdot h - h\cdot G)^{\mu\nu} - \mathcal{H}^*\left(\mathfrak{D}^2 - 2 - \tfrac{1}{2}\text{Tr}G^2\right)\mathcal{H} \\
& - \mathcal{H}^*_{\mu\nu}\left\{\mathfrak{D}^\mu\mathfrak{D}^\rho\left[(1+iG)\cdot h\right]_\rho^{\ \nu} - \tfrac{1}{2}\mathfrak{D}^\mu\mathfrak{D}^\nu\mathcal{H} + (\mu \leftrightarrow \nu)\right\} + \mathcal{H}^*\mathfrak{D}^\mu\mathfrak{D}^\nu\mathcal{H}_{\mu\nu},
\end{aligned} \quad (2.1)
$$

where the mass-squared[1] of the spin-2 field has been set equal to 2, and $\mathfrak{D}^\mu$ is related to the covariant derivative $\mathcal{D}^\mu$ as

$$
\mathfrak{D}^\mu = \left(\sqrt{G/eF}\right)^{\mu\nu}\mathcal{D}_\nu, \qquad [\mathfrak{D}^\mu, \mathfrak{D}^\nu] = -iG^{\mu\nu}, \qquad [\mathcal{D}^\mu, \mathcal{D}^\nu] = -ieF^{\mu\nu}. \quad (2.2)
$$

The charge of the spin-2 field is e, and $F^{\mu\nu}$ is the EM field strength. Moreover,

$$
\mathcal{H}_{\mu\nu} \equiv (1+iG)_\mu^{\ \alpha}(1+iG)_\nu^{\ \beta}\,h_{\alpha\beta}, \qquad \mathcal{H} \equiv \mathcal{H}^\mu_{\ \mu}. \quad (2.3)
$$

The antisymmetric rank-2 Lorentz tensor $G^{\mu\nu}$ is given by the expression

$$
G = \frac{1}{\pi}\left[\tanh^{-1}(\pi e_0 F) + \tanh^{-1}(\pi e_\pi F)\right], \quad (2.4)
$$

which reflects the fact that the Lagrangian (2.1) has been derived from the theory of open bosonic strings, as it contains in it the string endpoint charges e_0 and e_π, with $e \equiv e_0 + e_\pi$ being the total string charge.

The Lagrangian (2.1) is Hermitian, and gives rise to the spin-2 Fierz-Pauli system [9, 10]

$$
\left(\mathfrak{D}^2 - 2 - \tfrac{1}{2}\text{Tr}G^2\right)\mathcal{H}_{\mu\nu} - 2i(G\cdot\mathcal{H} - \mathcal{H}\cdot G)_{\mu\nu} = 0, \qquad \mathfrak{D}^\mu\mathcal{H}_{\mu\nu} = 0, \qquad \mathcal{H} = 0, \quad (2.5)
$$

which preserves the right number of DoFs, namely $\tfrac{1}{2}(D+1)(D-2)$ in D space-time dimensions, and admits only causal propagation [9].

However, as already mentioned in the Introduction, the Fierz-Pauli system (2.5) follows from the Argyres-Nappi Lagrangian (2.1) *only* when $D = 26$ [9, 10]. Away from the critical dimension, the trace of $\mathcal{H}_{\mu\nu}$ becomes dynamical [10], so that one has more propagating DoFs than those of a massive spin-2 field. Therefore, the Lagrangian (2.1) does *not* cure the original Velo-Zwanziger problem for arbitrary space-time dimensions.

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[1]Here we are talking about the mass that appears in the free theory, i.e. in the absence of the EM background. As it is well known, the constant background causes a shift in the mass [9, 10].

As was first noted in [9], the consistency of the Argyres-Nappi Lagrangian (2.1) is not affected by the substitution:

$$G^{\mu\nu} \rightarrow eF^{\mu\nu}, \qquad \mathfrak{D}^{\mu} \rightarrow \mathcal{D}^{\mu}. \tag{2.6}$$

In what follows we will use interchangeably $\mathfrak{D}^{\mu}$ with $\mathcal{D}^{\mu}$, and $G^{\mu\nu}$ with $eF^{\mu\nu}$. Thus, all our subsequent expressions will contain a single charge e, as it is suitable for a point particle.

2.2 DIMENSIONAL REDUCTION OF THE ARGYRES-NAPPI LAGRANGIAN

We would like to reduce the Argyres-Nappi Lagrangian (2.1) consistently from $D = 26$ space-time dimensions to a d-dimensional model, where, obviously, $d < D$. With this end in view, we split the D space-time coordinates, labeled by Greek indices, into two subsets:

$$\mu = \begin{cases} m = 0,\, 1,\, 2,\, ...,\, d-1, \\ M = d,\, d+1,\, ...,\, D-1. \end{cases} \tag{2.7}$$

That is, the d-dimensional space-time coordinates are labeled by lower-case Roman indices, while the remaining $(D-d)$ spatial ones, which we will consider as internal coordinates, are labeled by upper-case Roman indices.

The dimensional reduction consists in keeping only those fields that are singlets of the internal coordinates. Thus, the D-dimensional fields $h_{\mu\nu}$ and $G_{\mu\nu}$ split as

$$h_{\mu\nu} = \left(\begin{array}{c|c} h_{mn} & 0 \\ \hline 0 & \frac{1}{D-d}\,\delta_{MN}\,\phi \end{array} \right), \qquad G_{\mu\nu} = \left(\begin{array}{c|c} G_{mn} & 0 \\ \hline 0 & 0 \end{array} \right), \tag{2.8}$$

so that, in view of the first equation in (2.3), $\mathcal{H}_{\mu\nu}$ has the reduction

$$\mathcal{H}_{\mu\nu} = \left(\begin{array}{c|c} [1+iG]_{ma}[1+iG]_{nb}\,h^{ab} & 0 \\ \hline 0 & \frac{1}{D-d}\,\delta_{MN}\,\phi \end{array} \right) \equiv \left(\begin{array}{c|c} \mathfrak{h}_{mn} & 0 \\ \hline 0 & \frac{1}{D-d}\delta_{MN}\phi \end{array} \right). \tag{2.9}$$

All the fields obtained thereby are assumed to be functions of the d space-time coordinates only, and not of the internal coordinates; moreover, the EM background is nonzero only in d dimensions. This means that the covariant derivative splits as

$$\mathcal{D}^{\mu} = \left(\begin{array}{c} \mathcal{D}^m \\ \hline 0 \end{array} \right). \tag{2.10}$$

5

The Lagrangian (2.1) then reduces to

$$
\begin{aligned}
L_d \;=\;& \mathfrak{h}^*_{mn}\left(\mathcal{D}^2 - 2 - \tfrac{1}{2}\mathrm{Tr}G^2\right)h^{mn} - 2i\mathfrak{h}^*_{mn}(G\cdot h - h\cdot G)^{mn} - \mathfrak{h}^*\left(\mathcal{D}^2 - 2 - \tfrac{1}{2}\mathrm{Tr}G^2\right)\mathfrak{h} \\
& - \mathfrak{h}^*_{mn}\left\{\mathcal{D}^m\mathcal{D}^p\left[(1+iG)\cdot h\right]_p{}^n - \tfrac{1}{2}\mathcal{D}^m\mathcal{D}^n\mathfrak{h} + (m\leftrightarrow n)\right\} + \mathfrak{h}^*\mathcal{D}^m\mathcal{D}^n\mathfrak{h}_{mn} \\
& + \left[\mathfrak{h}^*_{mn}\mathcal{D}^m\mathcal{D}^n\phi - \left\{\mathfrak{h}^* + \tfrac{1}{2}\left(\tfrac{D-d-1}{D-d}\right)\phi^*\right\}\left(\mathcal{D}^2 - 2 - \tfrac{1}{2}\mathrm{Tr}G^2\right)\phi + \text{h.c.}\right],
\end{aligned}
\tag{2.11}
$$

where d is the space-time dimensionality, $D = 26$ by construction, and $\mathfrak{h}^m{}_m$ has been denoted by $\mathfrak{h}$.

Eq. (2.11) is our desired Lagrangian. We claim that, up to local field redefinitions, it consistently describes a charged massive spin-2 field coupled to a charged massive scalar in a constant external EM background in any space-time dimension $d < 26$.

It is not difficult to see that the Lagrangian (2.11) gives a decoupled system of a spin 2 plus a scalar in the absence of a background field. Indeed, the field redefinition

$$
\mathfrak{h}_{mn} \to \mathfrak{h}_{mn} - \left(\tfrac{1}{d-1}\right)\left[\eta_{mn}\phi - \tfrac{1}{4}\mathcal{D}_{(m}\mathcal{D}_{n)}\phi\right]
\tag{2.12}
$$

reduces the Lagrangian to

$$
\begin{aligned}
L_d \;\to\;& \mathfrak{h}^*_{mn}\left(\mathcal{D}^2 - 2\right)\mathfrak{h}^{mn} - \mathfrak{h}^*\left(\mathcal{D}^2 - 2\right)\mathfrak{h} - \mathfrak{h}^*_{mn}\mathcal{D}^{(m}\left[\mathcal{D}\cdot\mathfrak{h}\right]^{n)} + \mathfrak{h}^*_{mn}\mathcal{D}^m\mathcal{D}^n\mathfrak{h} + \mathfrak{h}^*\mathcal{D}^m\mathcal{D}^n\mathfrak{h}_{mn} \\
& + \left[\tfrac{D-1}{(d-1)(D-d)}\right]\phi^*\left(\mathcal{D}^2 - 2\right)\phi + \mathcal{O}(G),
\end{aligned}
\tag{2.13}
$$

so that in the limit $G \to 0$, one obtains the Fierz-Pauli Lagrangian for the massive spin-2 field $\mathfrak{h}_{mn}$, decoupled from the massive scalar ϕ. At $\mathcal{O}(G)$ there appear (kinetic) mixings of the two fields, and these are precisely what make the model consistent[2], as we will see below.

2.3 PROOF OF CONSISTENCY

The variation of the Lagrangian (2.11) gives rise to the following EoMs.

$$
\begin{aligned}
\mathcal{R}_{mn} \;\equiv\;& \left(\mathcal{D}^2 - 2 - \tfrac{1}{2}\mathrm{Tr}G^2\right)\left[\mathfrak{h}_{mn} - (1+G^2)_{mn}(\mathfrak{h}+\phi)\right] - 2i(G\cdot\mathfrak{h} - \mathfrak{h}\cdot G)_{mn} \\
& + \tfrac{1}{2}\left\{[(1+iG)\cdot\mathcal{D}]_m[(1+iG)\cdot\mathcal{D}]_n + [(1+iG)\cdot\mathcal{D}]_n[(1+iG)\cdot\mathcal{D}]_m\right\}(\mathfrak{h}+\phi) \\
& - \left\{[(1+iG)\cdot\mathcal{D}]_m\mathcal{D}^p\mathfrak{h}_{pn} + [(1+iG)\cdot\mathcal{D}]_n\mathcal{D}^p\mathfrak{h}_{pm}\right\} + (1+G^2)_{mn}\mathcal{D}^a\mathcal{D}^b\mathfrak{h}_{ab} \\
\;=\;& 0,
\end{aligned}
\tag{2.14}
$$

---

[2]We postpone for later the possibility that $\mathcal{O}(G)$ corrections to the redefinition (2.12) may eliminate these mixings, so as to give a description where either of the fields propagates consistently in isolation.

and

$$r \equiv \left(\mathcal{D}^a\mathcal{D}^b\mathfrak{h}_{ab} - \mathcal{D}^2\mathfrak{h}\right) + \left(2 + \tfrac{1}{2}\mathrm{Tr}G^2\right)\mathfrak{h} - \left(\tfrac{D-d-1}{D-d}\right)\left(\mathcal{D}^2 - 2 - \tfrac{1}{2}\mathrm{Tr}G^2\right)\phi = 0. \qquad (2.15)$$

To derive the necessary constraints from the system of equations (2.14)–(2.15) one needs to perform some algebraic manipulations, similar to those presented in Section 6.1 of [10]. One can compute the quantity $\mathcal{R}^m{}_m + \left[2\mathcal{D}\cdot(1+iG)^{-1}\right]^n\mathcal{D}^m\mathcal{R}_{mn}$, from Eq. (2.14), to find

$$(d - 6 + 2\mathrm{Tr}G^2)(\mathcal{D}^a\mathcal{D}^b\mathfrak{h}_{ab} - \mathcal{D}^2\mathfrak{h}) + \left[2(d-1) + \tfrac{1}{2}\mathrm{Tr}G^2(d + 4 + 2\mathrm{Tr}G^2)\right]\mathfrak{h}$$
$$-(d - 5 + 2\mathrm{Tr}G^2)\,\mathcal{D}^2\phi + \left[2d + \tfrac{1}{2}(d+5)\mathrm{Tr}G^2 + (\mathrm{Tr}G^2)^2\right]\phi = 0. \qquad (2.16)$$

Then one can eliminate $(\mathcal{D}^a\mathcal{D}^b\mathfrak{h}_{ab} - \mathcal{D}^2\mathfrak{h})$ from the above formula by using Eq. (2.15). The result is

$$\mathfrak{h} = -\phi + \left[\tfrac{D-6+2\mathrm{Tr}G^2}{(D-d)(10+\mathrm{Tr}G^2)}\right]\left(\mathcal{D}^2 - 2 - \tfrac{1}{2}\mathrm{Tr}G^2\right)\phi. \qquad (2.17)$$

Also, the quantity $\left[\mathcal{D}\cdot\{(1+iG)(2+iG)\}^{-1}\right]^n\mathcal{D}^m\mathcal{R}_{mn}$ gives from Eq. (2.14)

$$-\left[1 + \left(\tfrac{iG}{2+iG}\right)^{mn}\mathcal{D}_m\mathcal{D}_n\right]\left(\mathcal{D}^a\mathcal{D}^b\mathfrak{h}_{ab} - \mathcal{D}^2\mathfrak{h}\right) - \left[\tfrac{1}{4}\mathrm{Tr}G^2 + \tfrac{1}{2}(5 + \mathrm{Tr}G^2)\left(\tfrac{iG}{2+iG}\right)^{mn}\mathcal{D}_m\mathcal{D}_n\right]\mathfrak{h}$$
$$+ \left(\tfrac{iG}{2+iG}\right)^{mn}\mathcal{D}_m\mathcal{D}_n\left(\mathcal{D}^2 - \tfrac{5}{2} - \tfrac{1}{2}\mathrm{Tr}G^2\right)\phi + \left(\mathcal{D}^2 - \tfrac{1}{4}\mathrm{Tr}G^2\right)\phi = 0. \qquad (2.18)$$

In the above, Eq. (2.15) can be used once again to eliminate $(\mathcal{D}^\alpha\mathcal{D}^\beta\mathfrak{h}_{\alpha\beta} - \mathcal{D}^2\mathfrak{h})$, and then Eq. (2.17) removes $\mathfrak{h}$ from the resulting expression, giving thereby

$$0 = \left[2(D-1) + \tfrac{1}{4}\mathrm{Tr}G^2(D + 14 + 2\,\mathrm{Tr}G^2)\right]\left(\mathcal{D}^2 - 2 - \tfrac{1}{2}\mathrm{Tr}G^2\right)\phi$$
$$-\tfrac{1}{2}(D - 26)\left(\tfrac{iG}{2+iG}\right)^{mn}\mathcal{D}_m\mathcal{D}_n\left(\mathcal{D}^2 - 2 - \tfrac{1}{2}\mathrm{Tr}G^2\right)\phi. \qquad (2.19)$$

Because $D = 26$ by construction, this gives a second order equation for ϕ, namely

$$\left(\mathcal{D}^2 - 2 - \tfrac{1}{2}\mathrm{Tr}G^2\right)\phi = 0, \qquad (2.20)$$

which, in turn, implies from Eq. (2.17) that

$$\mathfrak{h} = -\phi. \qquad (2.21)$$

Eqs. (2.20) and (2.21), when used in the EoM (2.15), requires that $\mathcal{D}^a\mathcal{D}^b\mathfrak{h}_{ab}$ vanishes. The divergence equation, $\mathcal{D}^m\mathcal{R}_{mn} = 0$, i.e.

$$
\begin{aligned}
0 &= -\left\{(1+iG)(2+iG)\right\}_n{}^a \mathcal{D}^b\mathfrak{h}_{ab} - \left\{iG(1+iG)\right\}_n{}^s \mathcal{D}_s(\mathcal{D}^a\mathcal{D}^b\mathfrak{h}_{ab}) \\
&\quad + \left\{(1+iG)\left[\left(2 - \tfrac{1}{2}iG + \tfrac{3}{2}G^2\right) + iG\left(\mathcal{D}^2 - \tfrac{1}{2}\mathrm{Tr}G^2\right)\right]\right\}_n{}^a \mathcal{D}_a(\mathfrak{h} + \phi),
\end{aligned}
$$

now obviously implies the divergence constraint

$$
\mathcal{D}^m\mathfrak{h}_{mn} = 0, \tag{2.22}
$$

thanks to Eq. (2.21) and the fact that $\mathcal{D}^a\mathcal{D}^b\mathfrak{h}_{ab}$ vanishes. Finally, in view of Eqs. (2.21) and (2.22), one finds from (2.14) that

$$
\left(\mathcal{D}^2 - 2 - \tfrac{1}{2}\mathrm{Tr}G^2\right)\mathfrak{h}_{mn} - 2i(G\cdot\mathfrak{h} - \mathfrak{h}\cdot G)_{mn} = 0. \tag{2.23}
$$

Eqs. (2.20)–(2.23) are an algebraically consistent set of equations. The divergence and trace constraints ensure the correct DoF count for the spin-2 field $\mathfrak{h}_{mn}$, whose propagation, as well as that of ϕ, is causal, as is manifest from Eqs. (2.20) and (2.23). Thus Eqs. (2.20)–(2.23) consistently describe a massive charged spin-2 field plus a massive charged scalar in a constant EM background, and they follow from the Lagrangian (2.11).

If one takes the model (2.11) at face value, without knowing about its string-theoretic origin, one would discover, from the analysis presented in this Section, that it provides a consistent description of the fields $\mathfrak{h}_{mn}$ and ϕ only when the parameter D is set equal to 26. Because the scalar kinetic term appearing in (2.13) has the wrong sign for $(D - d) < 0$, the model is clearly plagued with a ghost, and therefore does not hold good, when $d > 26$. To see what happens at $d = 26$, we can of course notice that (2.11) comes from Eq. (2.1), which contains no scalar. Alternatively, we can redefine $\phi \to \sqrt{(D - d)}\,\phi$, to make (2.11) non-singular in the limit $d \to 26$. In this limit, the scalar field, which is now canonically normalized, completely decouples from the spin-2 Lagrangian.

As one expects, Eqs. (2.20)–(2.23) can also be derived directly from dimensional reduction of the Fierz-Pauli system (2.5). We will use this important fact in Section 3, where we will consider fields with arbitrary integer spin, for which we do not have any explicit Lagrangians at our disposal. Notice that the implementation of the field redefinition (2.12)

on the Eqs. (2.20)–(2.23), gives, after some algebraic manipulations, the system

$$\left(\mathcal{D}^2 - 2 - \tfrac{1}{2}\mathrm{Tr}G^2\right)\mathfrak{h}_{mn} - 2i(G\cdot\mathfrak{h} - \mathfrak{h}\cdot G)_{mn} = 0, \tag{2.24}$$

$$\mathcal{D}^m\mathfrak{h}_{mn} = -\tfrac{1}{4}\left(\tfrac{1}{d-1}\right)\left(3iG_{na} + \eta_{na}\mathrm{Tr}G^2\right)\mathcal{D}^a\phi, \tag{2.25}$$

$$\mathfrak{h} = -\tfrac{1}{4}\left(\tfrac{1}{d-1}\right)\mathrm{Tr}G^2\,\phi, \tag{2.26}$$

$$\left(\mathcal{D}^2 - 2 - \tfrac{1}{2}\mathrm{Tr}G^2\right)\phi = 0. \tag{2.27}$$

This again shows that, when $G \to 0$, the spin-2 field decouples from the scalar.

2.4   NON-EXISTENCE OF NEGATIVE-NORM STATES

It was shown in [10] that in a constant EM background the charged open-string spectrum is free of negative-norm states. One may wonder whether a dimensional reduction of the system that throws away some of the fields (non-singlets of the internal coordinates) could introduce ghosts. As we will see now, for $SO(26-d)$-singlet states obtained in the dimensional reduction of the bosonic string to d dimensions, the $D = 26$ no-ghost theorem also guarantees the absence of negative-norm states for $d < 26$.

Because the BRST charge Q commutes with $SO(26-d)$, it maps the $SO(26-d)$-singlet sector of the string Hilbert space V into itself. More generally, if one decomposes V into $SO(26-d)$ irreps as $V = \sum_I V_I$, then $Q(V_I) \subset V_I$. In other words, Q acts "diagonally" on the irreps. In particular, let us consider a Q-closed but spurious state $v = Qu$, where we decompose $v = \sum_I v_I$, $v_I \in V_I$. The diagonal action of Q means that $v_I = Qu_I$, $u_I \in V_I$. So, in particular, if a spurious v_0 is a singlet, $v_0 \in V_0$ ($I = 0$ labels the singlet subspace), then v_0 is the image of a singlet. Likewise, if v is Q-closed, then each of the component v_I's are also Q-closed: $Qv_I = 0$.

Therefore, the Q-cohomology decomposes into a direct sum of cohomologies, each of which is a positive-norm Hilbert space, with a metric that is the restriction of the metric on V to V_I (since the metric is an $SO(26)$ singlet).

3   ARBITRARY INTEGER SPIN

As we already mentioned, String Theory provides a consistent description, at least in 26 space-time dimensions, of a massive field of arbitrary integer spin s, propagating in a con-

stant EM background without the presence of other dynamical fields [10]. In this case, the generalized Fierz-Pauli conditions [10] read:

$$\left[\mathcal{D}^2 - 2(s-1) - \tfrac{1}{2}\mathrm{Tr}G^2\right]\Phi_{\mu_1\ldots\mu_s} + 2iG^{\alpha}{}_{(\mu_1}\Phi_{\mu_2\ldots\mu_s)\alpha} = 0, \tag{3.1}$$

$$\mathcal{D}^{\mu}\Phi_{\mu\mu_2\ldots\mu_s} = 0, \tag{3.2}$$

$$\Phi^{\mu}{}_{\mu\mu_3\ldots\mu_s} = 0, \tag{3.3}$$

in the units in which the mass-squared of the free theory is set equal to $2(s-1)$.

The singlets of the internal $(26-d)$ coordinates, that survive the dimensional reduction considered here, are symmetric tensors of rank $s, s-2, s-4, \ldots, \tfrac{1}{2}\left[1 + (-)^{s+1}\right]$. That is, if s is even(odd), one will end up having all fields with even(odd) spins, from s down to $0(1)$. Let us denote the field with spin $(s-2k)$ by ϕ_{s-2k}, where $0 \le k \le \tfrac{1}{4}\left[2s-1+(-)^s\right]$. The trace of such a field will be denoted with a prime. The Fierz-Pauli system (3.1)–(3.3) then reduces to

$$\left[\mathcal{D}^2 - 2(s-1) - \tfrac{1}{2}\mathrm{Tr}G^2\right]\phi_{s-2k} - 2i\left[G \cdot \phi_{s-2k}\right]_{\mathrm{symmetrized}} = 0, \tag{3.4}$$

$$\mathcal{D} \cdot \phi_{s-2l} = 0, \qquad 0 \le l \le \tfrac{1}{4}\left[2s-3+(-)^{s+1}\right], \tag{3.5}$$

$$\phi'_{s-2m} = -\phi_{s-2m-2}, \qquad 0 \le m \le \tfrac{1}{4}\left[2s-5+(-)^s\right]. \tag{3.6}$$

The system (3.4)–(3.6) is, of course, algebraically consistent. The divergence of any field with spin ≥ 1 vanishes, as seen from (3.5), whereas the constraints (3.6) render auxiliary the traces of all fields with spin ≥ 2; these ensure that each field propagates the correct number of DoFs. Causal propagation of the fields is also manifest from the dynamical equations (3.4). Therefore, the above system consistently describes a coupled set of massive fields, all having even-integer or odd-integer spins, propagating simultaneously in a constant EM background.

The d-dimensional action that would give rise to the system (3.4)–(3.6) is guaranteed to exist, and it will be free of negative-norm states, as has been shown in Section 2.4. As in the spin-2 case, by proper redefinitions of the fields, one expects to be able to show that the various fields are coupled only in the presence of a non-trivial background.

4   Concluding Remarks

In this paper, we have constructed a model that consistently describes a coupled system of a spin 2 and a scalar, propagating in a constant EM background in arbitrary dimensions less than 26. Our Lagrangian propagates the correct DoFs within the light cone, and does not contain negative-norm states. The generalization for higher spins has also been outlined.

These models do make sense in the regime of validity of a local effective action. For charged fields of mass m this is when the EM field invariants are smaller than $\mathcal{O}(m^4/e^2)$, so that instabilities [11] are absent. Consistency in such models is achieved because of the presence of additional dynamical fields, having the same mass as the original spin s field, together with suitable non-minimal terms. The extra DoFs can be viewed as "new physics" on top of the system of a single spin s, which show up at a scale well below $m\,e^{-1/(2s-1)}$ — the cutoff upper bound reported in [12]. All the dynamical fields present have gyromagnetic ratio equal to 2, in accordance with the conclusion in [13].

It is curious to notice that one can add a correction to the field redefinition (2.12) that decouples the spin-2 field from the scalar at all orders in G, at the level of EoM. The required field redefinition is

$$
\mathfrak{h}_{mn} \quad \rightarrow \quad \mathfrak{h}_{mn} - \frac{\left[\,(4+\mathrm{Tr}G^2)^2 + 12\,\mathrm{Tr}G^2\,\right]\eta_{mn}\phi - (4+\mathrm{Tr}G^2)\,\mathcal{D}_{(m}\mathcal{D}_{n)}\phi}{(d-1)(4+\mathrm{Tr}G^2)^2 + 12(d+1)\,\mathrm{Tr}G^2}
$$
$$
- \frac{3i\left[\,3(G\cdot\mathcal{D})_{(m}\mathcal{D}_{n)} - \mathcal{D}_{(m}(G\cdot\mathcal{D})_{n)}\,\right]\phi}{(d-1)(4+\mathrm{Tr}G^2)^2 + 12(d+1)\,\mathrm{Tr}G^2}. \tag{4.1}
$$

It reduces Eqs. (2.20)–(2.23) to

$$
\left(\mathcal{D}^2 - 2 - \tfrac{1}{2}\mathrm{Tr}G^2\right)\mathfrak{h}_{mn} - 2i(G\cdot\mathfrak{h} - \mathfrak{h}\cdot G)_{mn} = 0, \qquad \mathcal{D}^m\mathfrak{h}_{mn} = 0, \qquad \mathfrak{h} = 0, \tag{4.2}
$$

$$
\left(\mathcal{D}^2 - 2 - \tfrac{1}{2}\mathrm{Tr}G^2\right)\phi = 0. \tag{4.3}
$$

Clearly, the two fields are completely decoupled from each other.

However, the redefinition (4.1) cannot decouple them at the Lagrangian level. This is clear from the fact that (4.1) does not redefine the scalar ϕ, so that the terms quadratic in the spin-2 field (and trace thereof), obtained after the field redefinition, are identical to the first two lines of (2.11). But the latter are simply the spin-2 Argyres-Nappi Lagrangian, which holds good in no dimensions other than 26. The conclusion is that there must be cross-couplings between the fields in $d \neq 26$, because mere field redefinitions of the spin-2 field alone cannot affect the (in)consistency of a model.

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There remains the possibility, which we leave as future work, that a redefinition of the scalar field may decouple the fields at the Lagrangian level as well. One would then have a consistent model for a single charged spin 2 in any dimension less than 26, which could be useful, among others, for constructing holographic models of d-wave superconductors [14].

We conclude with some comments on how our results fit into the existing higher-spin gauge theory literature. One can use a gauge invariant formulation to construct consistent EM interactions of a massive spin 2 field, as it was done in [15]. Moreover, the $1-s-s$ cubic vertices in [16, 17] are directly relevant to the present paper, since they encode interactions of a spin-s field with a $U(1)$ gauge field. One natural question about them is whether they reproduce the cubic vertices presented here, where the $U(1)$ is treated as a background. Consider for instance the $1-s-s$ cubic vertices for massive spin-s fields written down in [17]. They were derived from string theory in 26 space-time dimensions. When the $U(1)$ field is an external source with constant field strength, one immediately finds that, up to total derivative terms, the only vertices that survive are dipole terms that contain no derivatives; such vertices cannot be field redefined away. This is in complete agreement with the finding of [10], that the cubic vertices in the 26-dimensional Argyres-Nappi Lagrangian (2.1) contain only dipole terms up to field redefinitions. A dimensional reduction, as in Eqs. (2.7)–(2.10), would thus produce cubic vertices that contain no derivatives. The full interacting theory may still contain higher derivative cubic terms, which vanish up to field redefinitions when the $U(1)$ field strength is treated as a constant background. To study unitarity in the presence of such vertices, a derivation from string theory is probably a crucial tool, as it has been in our analysis.

Acknowledgments

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