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On strong coupling in nonrelativistic general covariant theory of gravity

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We study the strong coupling problem in the Horava-Melby-Thompson setup of the Horava-Lifshitz theory of gravity with an arbitrary coupling constant λ , generalized recently by da Silva, where λ describes the deviation of the theory in the infrared from general relativity that has $\lambda_{GR} = 1$. We find that a scalar field in the Minkowski background becomes strong coupling for processes with energy higher than $\Lambda_\omega \equiv (M_{pl}/c_1)^{3/2} M_{pl} |\lambda - 1|^{5/4}$, where generically $c_1 \ll M_{pl}$. However, this problem can be cured by introducing a new energy scale M_* , so that $M_* < \Lambda_\omega$, where M_* denotes the suppression energy of high order derivative terms of the theory.

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I. INTRODUCTION

Horava recently proposed a new theory of quantum gravity [1], based on the perspective that Lorentz symmetry may appear as an emergent symmetry at low energies, but can be fundamentally absent at high energies. His starting point is the anisotropic scalings of space and time,

$$\mathbf{x} \rightarrow b^{-1}\mathbf{x}, \quad t \rightarrow b^{-z}t. \quad (1.1)$$

In (3+1)-dimensions, in order for the theory to be power-counting renormalizable, the critical exponent z needs to be $z \geq 3$ [2]. At long distances, all the high-order curvature terms are negligible, and the linear term R becomes dominant. Then, the theory is expected to flow to the relativistic fixed point $z = 1$, whereby the general covariance is “accidentally restored.” The special role of time is realized with the Arnowitt-Deser-Misner decomposition [3],

$$ds^2 = -N^2 c^2 dt^2 + g_{ij} (dx^i + N^i dt) (dx^j + N^j dt), \quad (i, j = 1, 2, 3). \quad (1.2)$$

Under the rescaling (1.1) with $z = 3$, N , N^i and g_{ij} scale, respectively, as,

$$N \rightarrow N, \quad N^i \rightarrow b^{-2}N^i, \quad g_{ij} \rightarrow g_{ij}. \quad (1.3)$$

The gauge symmetry of the system now is broken down to the foliation-preserving diffeomorphisms $\text{Diff}(M, \mathcal{F})$,

$$\delta t = -f(t), \quad \delta x^i = -\zeta^i(t, \mathbf{x}), \quad (1.4)$$

under which, N , N^i and g_{ij} transform as,

$$\begin{aligned} \delta N &= \zeta^k \nabla_k N + \dot{N} f + N \dot{f}, \\ \delta N_i &= N_k \nabla_i \zeta^k + \zeta^k \nabla_k N_i + g_{ik} \dot{\zeta}^k + \dot{N}_i f + N_i \dot{f}, \\ \delta g_{ij} &= \nabla_i \zeta_j + \nabla_j \zeta_i + f \dot{g}_{ij}, \end{aligned} \quad (1.5)$$

where $\dot{f} \equiv df/dt$, ∇_i denotes the covariant derivative with respect to the 3-metric g_{ij} , $N_i = g_{ik} N^k$, and $\delta g_{ij} = \tilde{g}_{ij}(t, x^k) - g_{ij}(t, x^k)$, etc. Eq.(1.5) shows clearly that the lapse function N and the shift vector N^i play the role of gauge fields of the $\text{Diff}(M, \mathcal{F})$. Therefore, it is natural to assume that N and N^i inherit the same dependence on space and time as the corresponding generators,

$$N = N(t), \quad N_i = N_i(t, x), \quad (1.6)$$

while the dynamical variables g_{ij} in general depend on both time and space, $g_{ij} = g_{ij}(t, x)$. This is often referred to as the *projectability condition*.

Abandoning the general covariance, on the other hand, gives rise to a proliferation of independently coupling constants, which could potentially limit the prediction powers of the theory. Inspired by condensed matter systems [4], Horava assumed that the gravitational potential \mathcal{L}_V can be obtained from a superpotential W_g via the relations,

$$\begin{aligned} \mathcal{L}_{V,detailed} &= w^2 E_{ij} \mathcal{G}^{ijkl} E_{kl}, \\ E^{ij} &= \frac{1}{\sqrt{g}} \frac{\delta W_g}{\delta g_{ij}}, \end{aligned} \quad (1.7)$$

where w is a coupling constant, and \mathcal{G}^{ijkl} denotes the generalized De Witt metric, defined as $\mathcal{G}^{ijkl} = (g^{ik} g^{jl} + g^{il} g^{jk})/2 - \lambda g^{ij} g^{kl}$, with λ being a coupling constant. The general covariance, $\delta x^\mu = \zeta^\mu(t, x)$, ($\mu = 0, 1, \dots, 3$), requires $\lambda = 1$. The superpotential W_g is given by

$$W_g = \int_\Sigma \omega_3(\Gamma) + \frac{1}{\kappa_W^2} \int d^3x \sqrt{g} (R - 2\Lambda), \quad (1.8)$$

with $\omega_3(\Gamma)$ being the gravitational Chern-Simons term,

$$\omega_3(\Gamma) = \text{Tr} \left(\Gamma \wedge d\Gamma + \frac{2}{3} \Gamma \wedge \Gamma \wedge \Gamma \right). \quad (1.9)$$

The condition (1.7) is usually referred to as the *detailed balance condition*.

However, with this condition it was found that the Newtonian limit does not exist [5], and a scalar field in

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the UV is not stable [6]. Thus, it is generally believed that this condition should be abandoned [7]. But, it has several remarkable features [8]: it is in the same spirit of the AdS/CFT correspondence [9]; and in the nonequilibrium thermodynamics, the counterpart of the superpotential W_g plays the role of entropy, while the term E^{ij} the entropic forces [10]. This might shed light on the nature of the gravitational forces, as proposed recently by Verlinde [11]. Due to these desired properties, together with Borzou, two of the present authors recently studied this condition in detail, and found that the scalar field can be stabilized, if the detailed balance condition is allowed to be softly broken [12]. This can also solve the other problems [5, 7]. In addition, such a breaking can still reduce significantly the number of independent coupling constants. For detail, we refer readers to [12].

It should be noted that, even the detailed balance condition is allowed to be broken softly, the theory is still plagued with several other problems, including the instability, ghost, and strong coupling [13–17]. To overcome those problems, recently Horava and Melby-Thompson (HMT) [18] extended the foliation-preserving-diffeomorphisms $\text{Diff}(M, \mathcal{F})$ to include a local $U(1)$ symmetry,

$$U(1) \ltimes \text{Diff}(M, \mathcal{F}). \quad (1.10)$$

Such an extended symmetry is realized by introducing a $U(1)$ gauge field A and a Newtonian prepotential φ . Under $\text{Diff}(M, \mathcal{F})$, these fields transform as [18–20],

$$\begin{aligned} \delta A &= \zeta^i \partial_i A + \dot{f} A + f \dot{A}, \\ \delta \varphi &= f \dot{\varphi} + \zeta^i \partial_i \varphi, \end{aligned} \quad (1.11)$$

while under $U(1)$, characterized by the generator α , they, together with N , N^i and g_{ij} , transform as

$$\begin{aligned} \delta_\alpha A &= \dot{\alpha} - N^i \nabla_i \alpha, \quad \delta_\alpha \varphi = -\alpha, \\ \delta_\alpha N_i &= N \nabla_i \alpha, \quad \delta_\alpha g_{ij} = 0 = \delta_\alpha N. \end{aligned} \quad (1.12)$$

HMT showed that, similar to GR, the spin-0 graviton is eliminated [18]. This was further confirmed in [19]. Then, the instability of the spin-0 gravity is out of question. In addition, in the linearized theory Horava noticed that the $U(1)$ symmetry only pertains to the case $\lambda = 1$ [1], and it was believed that this was also the case when the Newtonian prepotential is introduced [18]. If this were true, the ghost and strong coupling problems would be also resolved, as both of them are related to the very fact that $\lambda \neq 1$ [20].

However, it has been soon challenged by da Silva [21], who argued that the introduction of the Newtonian prepotential is so strong that actions with $\lambda \neq 1$ also has the extended symmetry, Eq.(1.10). Although the spin-0 graviton is also eliminated even with any λ as shown explicitly in [20–22], the ghost and strong coupling problems rise again, because now λ can be different from one. Indeed, it was shown [20] that to avoid the ghost problem, λ must satisfy the same constraints,

$$\lambda \geq 1, \quad \text{or} \quad \lambda < \frac{1}{3}, \quad (1.13)$$

as found previously [1, 23–25].

In addition, the strong coupling problem also arise [20]. In this paper, we shall address this important issue. In particular, in Sec. II we briefly review the HMT setup with any coupling constant λ with detailed balance condition softly breaking, a version presented in [12] in detail, while in Sec. III we study the strong coupling problem when a scalar field is present. We find that a scalar field in the Minkowski background becomes strong coupling for processes with energy higher than $\Lambda_\omega \equiv (M_{pl}/c_1)^{3/2} M_{pl} |\lambda - 1|^{5/4}$. For $c_1 \simeq M_{pl}$, this gives precisely the strong coupling strength found in [20]. However, this problem can be resolved by introducing a new energy scale M_* [26], so that $M_* \lesssim \Lambda_\omega$, where M_* defines the energy scale that suppresses the sixth-order derivative terms of the theory. In Sec. IV, we present our main conclusions.

It should be noted that the strong coupling problem in other versions of the Horava-Lifshitz (HL) theory has been studied extensively by using both effective field theory [20, 27–30] and Stückelberg formalism [26, 31, 32]. In this paper, we shall follow the approach of the effective field theory [33], although the final conclusions are independent of the methods to be used. In addition, strong coupling can happen not only due to gravitational/matter self-interactions, but also to the interactions among gravitational and matter fields. The latter was studied in [20, 30], while strong coupling due to the self-gravitational interactions were studied in [26–29, 32]. In this paper, we shall study it due to the interaction between gravitational and scalar fields.

II. GENERAL COVARIANT THEORY WITH DETAILED BALANCE CONDITION SOFTLY BREAKING

As mentioned above, HMT considered only the case $\lambda = 1$. Later, da Silva generalized it to the cases with any λ [21], in which the total action can be written in the form [20, 21],

$$S = \zeta^2 \int dt d^3 x N \sqrt{g} \left(\mathcal{L}_K - \mathcal{L}_V + \mathcal{L}_\varphi + \mathcal{L}_A + \mathcal{L}_\lambda + \zeta^{-2} \mathcal{L}_M \right), \quad (2.1)$$

where $g = \det g_{ij}$,

$$\begin{aligned} \mathcal{L}_K &= K_{ij} K^{ij} - \lambda K^2, \\ \mathcal{L}_\varphi &= \varphi \mathcal{G}^{ij} \left(2K_{ij} + \nabla_i \nabla_j \varphi \right), \\ \mathcal{L}_A &= \frac{A}{N} \left(2\Lambda_g - R \right), \\ \mathcal{L}_\lambda &= (1 - \lambda) \left[(\Delta \varphi)^2 + 2K \Delta \varphi \right], \end{aligned} \quad (2.2)$$

$\Delta \equiv \nabla^2 = g^{ij} \nabla_i \nabla_j$, and Λ_g is a coupling constant. The Ricci and Riemann terms all refer to the 3-metric g_{ij} ,

and

$$\begin{aligned} K_{ij} &= \frac{1}{2N} (-\dot{g}_{ij} + \nabla_i N_j + \nabla_j N_i), \\ \mathcal{G}_{ij} &= R_{ij} - \frac{1}{2} g_{ij} R + \Lambda_g g_{ij}. \end{aligned} \quad (2.3)$$

\mathcal{L}_V is an arbitrary $\text{Diff}(\Sigma)$ -invariant local scalar functional built out of the spatial metric, its Riemann tensor and spatial covariant derivatives. With the detailed balance condition softly breaking, it takes the form [12, 34],

$$\begin{aligned} \mathcal{L}_V &= \zeta^2 \gamma_0 + \gamma_1 R + \frac{1}{\zeta^2} (\gamma_2 R^2 + \gamma_3 R_{ij} R^{ij}) \\ &+ \frac{\gamma_4}{\zeta^3} \epsilon^{ijk} R_{il} \nabla_j R_k^l + \frac{\gamma_5}{\zeta^4} C_{ij} C^{ij}, \end{aligned} \quad (2.4)$$

where the coupling constants γ_s ($s = 0, 1, 2, \dots, 5$) are all dimensionless, and $\gamma_5 \equiv w^2 \zeta^4$. The relativistic limit in the IR requires

$$\lambda = 1, \quad \gamma_1 = -1, \quad \zeta^2 = \frac{1}{16\pi G}. \quad (2.5)$$

The existence of the γ_4 term explicitly breaks the parity, which could have important observational consequences on primordial gravitational waves [35].

\mathcal{L}_M is the matter Lagrangian density. For a scalar field χ with detailed balance conditions softly breaking, it is given by [12],

$$\mathcal{L}_M = \mathcal{L}_\chi^{(A, \varphi)} + \mathcal{L}_\chi^{(0)}, \quad (2.6)$$

where

$$\begin{aligned} \mathcal{L}_\chi^{(A, \varphi)} &= \frac{A - \mathcal{A}}{N} \left[c_1 \Delta \chi + c_2 (\nabla \chi)^2 \right] \\ &- \frac{f}{N} \left(\dot{\chi} - N^i \nabla_i \chi \right) (\nabla^k \varphi) (\nabla_k \chi) \\ &+ \frac{f}{2} \left[(\nabla^k \varphi) (\nabla_k \chi) \right]^2, \\ \mathcal{L}_\chi^{(0)} &= \frac{f}{2N^2} \left(\dot{\chi} - N^i \nabla_i \chi \right)^2 - \mathcal{V}, \end{aligned} \quad (2.7)$$

and

$$\begin{aligned} \mathcal{V} &= V(\chi) + \left(\frac{1}{2} + V_1(\chi) \right) (\nabla \chi)^2 + V_2(\chi) \mathcal{P}_1^2 \\ &+ V_3(\chi) \mathcal{P}_1^3 + V_4(\chi) \mathcal{P}_2 + V_5(\chi) (\nabla \chi)^2 \mathcal{P}_2 \\ &+ V_6 \mathcal{P}_1 \mathcal{P}_2, \end{aligned} \quad (2.8)$$

with $V(\chi)$ and $V_n(\chi)$ being arbitrary functions of χ , and

$$\mathcal{P}_n \equiv \Delta^n \chi, \quad V_6 \equiv -\sigma_3^2. \quad (2.9)$$

The corresponding field equations are given in Appendix A and by Eqs.(3.13) and (3.14) in [12].

III. STRONG COUPLING

To study the strong coupling problem, it is found sufficient to consider perturbations of the Minkowski space-time,

$$\begin{aligned} \bar{N} &= 1, \quad \bar{N}_i = 0, \quad \bar{g}_{ij} = \delta_{ij}, \quad \bar{A} = \bar{\varphi} = 0, \\ \bar{\chi} &= \bar{\chi}_0, \quad V(\bar{\chi}_0) = V'(\bar{\chi}_0) = 0 = \Lambda = \Lambda_g, \end{aligned} \quad (3.1)$$

where χ_0 is a constant. Without loss of generality, we can set $\chi_0 = 0$, a condition that will be assumed in the rest of the paper. The perturbed fields with the (generalized) quasilongitudinal gauge are given by [19, 20, 25],

$$\begin{aligned} N &= 1, \quad N_i = B_{,i}, \quad g_{ij} = (1 - 2\psi)\delta_{ij}, \\ A &= \delta A, \quad \varphi = 0, \quad \chi = \delta \chi. \end{aligned} \quad (3.2)$$

Then, we find

$$\begin{aligned} N^i &= (1 + 2\psi + 4\psi^2) B^{,i} + \mathcal{O}(\epsilon^4), \\ g^{ij} &= (1 + 2\psi + 4\psi^2 + 8\psi^3) \delta^{ij} + \mathcal{O}(\epsilon^4), \\ \sqrt{g} &= 1 - 3\psi + \frac{3}{2}\psi^2 + \frac{1}{2}\psi^3 + \mathcal{O}(\epsilon^4), \end{aligned} \quad (3.3)$$

where $B^{,i} \equiv \delta^{ij} B_{,j}$, etc. After simple but tedious calculations, to third order we find that $\mathcal{L}_\lambda = \mathcal{L}_\varphi = 0$, and

$$\begin{aligned} \sqrt{g} \mathcal{L}_K &= (1 + \psi) \left[(1 - 3\lambda) (3\dot{\psi}^2 + 2\dot{\psi} \partial^2 B) + B_{,ij} B^{,ij} \right. \\ &\quad \left. - \lambda (\partial^2 B)^2 \right] + 2 \left[2B_{,ij} B^{,i} \psi^{,j} \right. \\ &\quad \left. - (1 - 3\lambda) \dot{\psi} B_{,i} \psi^{,i} - (1 - \lambda) B_{,i} \psi^{,i} \partial^2 B \right], \\ \sqrt{g} \mathcal{L}_A &= -2A \left[2(1 + \psi) \partial^2 \psi + 3\psi_{,i} \psi^{,i} \right], \\ \sqrt{g} \mathcal{L}_V &= 4\partial^2 \psi + {}^{(2)}(\sqrt{g} \mathcal{L}_V) + {}^{(3)}(\sqrt{g} \mathcal{L}_V), \\ \sqrt{g} \mathcal{L}_\chi &= {}^{(2)}(\sqrt{g} \mathcal{L}_\chi) + {}^{(3)}(\sqrt{g} \mathcal{L}_\chi), \end{aligned} \quad (3.4)$$

where the first order term $\partial^2 \psi$ in the expression of $(\sqrt{g} \mathcal{L}_V)$ becomes a boundary term after integration, and therefore can be discarded, and

$$\begin{aligned} {}^{(2)}(\sqrt{g} \mathcal{L}_V) &= 2\psi \partial^2 \psi + \frac{\zeta^2}{2} (8\gamma_2 + 3\gamma_3) \psi \partial^4 \psi, \\ {}^{(2)}(\sqrt{g} \mathcal{L}_\chi) &= \frac{f}{2} \dot{\chi}^2 - \frac{1}{2} V'' \chi^2 + \frac{1}{2} (1 + 2V_1) \chi \partial^2 \chi \\ &\quad - (V_2 + V_4') \chi \partial^4 \chi + \sigma_3^2 \chi \partial^6 \chi \\ &\quad + c_1 \chi \partial^2 A, \\ {}^{(3)}(\sqrt{g} \mathcal{L}_V) &= -6(\psi^2 \partial^2 \psi + 3\psi \psi_{,i} \psi^{,i}) \\ &\quad + A_1 (\gamma_2, \gamma_3) \psi^2 \partial^4 \psi + \mathcal{O}(\gamma_2, \gamma_3, \psi, \partial^4) \\ &\quad + A_2 (\gamma_5) \psi^2 \partial^6 \psi + \mathcal{O}(\gamma_5, \psi, \partial^6), \\ {}^{(3)}(\sqrt{g} \mathcal{L}_\chi) &= -\frac{3}{2} f \psi \dot{\chi}^2 - f \dot{\chi} B_{,i} \chi^{,i} - c_1 \psi A \partial^2 \chi \\ &\quad - c_1 A \psi^{,i} \chi_{,i} + c_2 A \chi^{,i} \chi_{,i} + \frac{3}{2} V'' \psi \chi^2 \end{aligned} \quad (3.5)$$

$$\begin{aligned}
& -\frac{1}{6}V''' \chi^3 - V'_1 \chi \chi_{,i} \chi^{,i} \\
& + \frac{1}{2}(1 + 2V_1) \psi \chi_{,i} \chi^{,i} \\
& + B_1(V_2, V_4) \chi^2 \partial^4 \chi \\
& \quad + \mathcal{O}(V_2, V_4, \chi, \partial^4) \\
& + B_2(V_3, V_5, V_6) \chi^2 \partial^6 \chi \\
& \quad + \mathcal{O}(V_3, V_5, V_6, \chi, \partial^6), \tag{3.6}
\end{aligned}$$

where the terms of A_i (B_i) are representatives of the fourth and sixth order derivative terms of ψ (χ), and the specific dependences of them on their arguments are not important to the analysis of the strong coupling problem, as shown below. So, for the sake of simplicity, we shall not give them explicitly here. Hence, to second order we obtain

$$\begin{aligned}
S^{(2)} = & \zeta^2 \int dt d^3x \left\{ 3(1 - 3\lambda) \dot{\psi}^2 + 2(1 - 3\lambda) \dot{\psi} \partial^2 B \right. \\
& + (1 - \lambda) (\partial^2 B)^2 - 4\psi \partial^2 A - 2\psi \partial^2 \psi \\
& - \frac{2}{\zeta^2} (8\gamma_2 + 3\gamma_3) \psi \partial^4 \psi \\
& + \frac{1}{\zeta^2} \left[\frac{1}{2} f \chi^2 - \frac{1}{2} V'' \chi^2 + \frac{1}{2} (1 + 2V_1) \chi \partial^2 \chi \right. \\
& \quad - (V_2 + V'_4) \chi \partial^4 \chi + \sigma_3^2 \chi \partial^6 \chi \\
& \quad \left. \left. + c_1 \chi \partial^2 A \right] \right\}. \tag{3.7}
\end{aligned}$$

Variations of $S^{(2)}$, respectively, with respect to ψ , B , A and χ yield,

$$\begin{aligned}
\ddot{\psi} + \frac{1}{3} \partial^2 \dot{B} = & \frac{2}{3(3\lambda - 1)} \partial^2 \left(A + \psi \right. \\
& \left. + \frac{8\gamma_2 + 3\gamma_3}{\zeta^2} \partial^2 \psi \right), \tag{3.8}
\end{aligned}$$

$$(1 - 3\lambda) \dot{\psi} + (1 - \lambda) \partial^2 B = 0, \tag{3.9}$$

$$\psi = 4\pi G c_1 \chi, \tag{3.10}$$

and

$$\begin{aligned}
f \ddot{\chi} + V'' \chi - (1 + 2V_1) \partial^2 \chi + 2(V_2 + V'_4) \partial^4 \chi \\
- 2\sigma_3^2 \partial^6 \chi = c_1 \partial^2 A. \tag{3.11}
\end{aligned}$$

The above equations can be obtained from Eqs.(4.13) - (4.17) and Eq.(4.20) of [12], by setting $\bar{\chi} = \bar{\chi}' = 0$ and $a = 1$, as it is expected. Using Eqs.(3.8)-(3.9), we can integrate out ψ , B and A , so $S^{(2)}$ finally takes the form,

$$\begin{aligned}
S^{(2)} = & \beta^2 \int dt d^3x \left[\dot{\chi}^2 - \alpha (\partial \chi)^2 - m_\chi^2 \chi^2 \right. \\
& \left. - \frac{1}{M_A^2} \chi \partial^4 \chi + \frac{1}{M_B^4} \chi \partial^6 \chi \right], \tag{3.12}
\end{aligned}$$

where

$$\begin{aligned}
\beta^2 & \equiv \frac{2\pi G c_1^2}{|c_\psi|^2} + \frac{f}{2}, \\
\alpha & \equiv \frac{1}{2\beta^2} (1 + 2V_1 - 4\pi G c_1^2), \\
M_A^2 & \equiv \beta^2 \left(2\pi G c_1^2 \frac{8\gamma_2 + 3\gamma_3}{\zeta^2} + V_2 + V'_4 \right)^{-1}, \\
M_B^4 & \equiv \frac{\beta^2}{\sigma_3^2}, \quad m_\chi^2 \equiv \frac{1}{2\beta^2} V'', \tag{3.13}
\end{aligned}$$

and $c_\psi^2 = (1 - \lambda)/(3\lambda - 1)$. As a consistency check, one can show that the variation of the action (3.12) with respect to χ yields the master equation (4.21) given in [12]. In addition, when λ satisfies the condition (1.13), the above expression shows clearly that the scalar field is ghost free for $f > 0$, as first noticed in [12]. The scalar field is stable in all energy scales by properly choosing the potential terms V_i , including the UV and IR. For detail, we refer readers to [12].

Therefore, in the following we focus only on the strong coupling problem. To this end, let us first note that the cubic action is given by ¹,

$$\begin{aligned}
S^{(3)} = & \int dt d^3x \left\{ \lambda_1 \left(\frac{1}{\partial^2} \ddot{\chi} \right) \chi \partial^2 \chi + \lambda_2 \left(\frac{1}{\partial^2} \ddot{\chi} \right) \chi_{,i} \chi^{,i} \right. \\
& + \lambda_3 \chi \dot{\chi}^2 + \lambda_4 \chi_{,i} \left(\frac{\partial_j}{\partial^2} \dot{\chi} \right) \left(\frac{\partial^i \partial^j}{\partial^2} \dot{\chi} \right) \\
& + \lambda_5 \dot{\chi} \chi_{,i} \left(\frac{\partial_i}{\partial^2} \dot{\chi} \right) + \lambda_6 \chi^3 + \lambda_7 \chi^2 \partial^2 \chi \\
& + \lambda_8 \chi^2 \partial^4 \chi + \lambda_9 \chi^2 \partial^6 \chi \\
& \left. + \dots \right\}, \tag{3.14}
\end{aligned}$$

where “...” represents the fourth- and sixth-order derivative terms of \mathcal{O} 's given in Eq.(3.6), which are irrelevant to the strong coupling problem, as mentioned above, and

$$\begin{aligned}
\lambda_1 & = \frac{c_1^3}{8\zeta^4 |c_\psi|^2}, \quad \lambda_2 = \frac{1}{|c_\psi|^2} \left(\frac{5c_1^3}{32\zeta^4} - \frac{c_1 c_2}{4\zeta^2} \right), \\
\lambda_3 & = \frac{c_1^3}{32\zeta^4 |c_\psi|^2} - \frac{3f c_1}{8\zeta^2}, \quad \lambda_4 = \frac{3c_1^3}{64\zeta^4 |c_\psi|^4}, \\
\lambda_5 & = \frac{c_1^3}{64\zeta^4 |c_\psi|^4} + \frac{c_1 f}{4\zeta^2 |c_\psi|^2}, \quad \lambda_6 = \frac{3c_1}{8\zeta^2} \ddot{V} - \frac{\ddot{V}}{6},
\end{aligned}$$

¹ Note the difference between the perturbations considered in [19, 20, 28, 29] and the ones studied here. In particular, in [19, 20, 28, 29] the perturbations of the form, $N = 1$, $N_i = \partial_i \beta$, $g_{ij} = e^{2\zeta} \delta_{ij}$, were studied, while in this paper we use the expansions of Eq.(3.3) to calculate the third order action. Although such obtained $S^{(3)}$ is different from the one obtained in [19, 20, 28, 29], it is not difficult to argue that the final conclusions for the strong coupling problem will be the same.

$$\begin{aligned}
\lambda_7 &= \frac{\dot{V}_1}{2} + \frac{c_1^2 c_2}{8\zeta^2} - \frac{c_1}{16\zeta^2} - \frac{c_1 V_1}{8\zeta^2}, \\
\lambda_8 &= \tilde{A}_1(\gamma_2, \gamma_3, c_1) + B_1(V_2, V_4), \\
\lambda_9 &= \tilde{A}_2(\gamma_5, c_1) + B_2(V_3, V_5, V_6),
\end{aligned} \tag{3.15}$$

where $\tilde{A}_i \equiv (4\pi G c_1)^3 A_i$. Depending on the energy scales, each term in Eq.(3.14) will have different scalings. Thus, in the following we consider them separately.

$$\mathbf{A}. \quad |\nabla| \ll M_*$$

When $|\nabla| \ll M_*$, where $M_* = \text{Min.}(M_A, M_B)$, we find that the high order derivative terms in Eq.(3.12) can be neglected, and

$$S^{(2)} \simeq \beta^2 \int dt d^3x \left[\dot{\chi}^2 - \alpha (\partial\chi)^2 \right]. \tag{3.16}$$

Note that in writing the above expression, without loss of generality, we had assumed that $|\nabla| \gg m_\chi$. Setting

$$t = b_1 \hat{t}, \quad x^i = b_2 \hat{x}^i, \quad \chi = b_3 \hat{\chi}, \tag{3.17}$$

we can bring Eq.(3.16) into its “canonical” form,

$$S^{(2)} \simeq \int dt d^3\hat{x} \left[(\hat{\chi}^*)^2 - (\hat{\partial}\hat{\chi})^2 \right], \tag{3.18}$$

in which the coefficient of each term is order of one, for

$$b_2 = b_1 \sqrt{\alpha}, \quad b_3 = \frac{1}{b_1 \beta \alpha^{3/4}}, \tag{3.19}$$

where $\hat{\chi}^* \equiv d\hat{\chi}/d\hat{t}$. Note that the requirement that the coefficient of each term be order of one is important in order to obtain a correct coupling strength [20, 26, 27]. In addition, the transformations (3.17) should not be confused with the gauge choice (3.2), as it just provides a technique to obtain the correct coupling strength. In fact, when we consider physics, we will all refer to the ones obtained in the t and x coordinates, as to be shown below. Inserting Eq.(3.17) into Eq.(3.14), we obtain

$$S^{(3)} = \frac{1}{b_1 \beta^3 \alpha^{3/4}} \hat{S}^{(3)}, \tag{3.20}$$

where

$$\begin{aligned}
\hat{S}^{(3)} \equiv & \int dt d^3\hat{x} \left\{ \lambda_1 \left(\frac{1}{\hat{\partial}^2} \hat{\chi}^{**} \right) \hat{\chi} \hat{\partial}^2 \hat{\chi} \right. \\
& + \lambda_2 \left(\frac{1}{\hat{\partial}^2} \hat{\chi}^{**} \right) \hat{\partial}_i \hat{\chi} \hat{\partial}^i \hat{\chi} + \lambda_3 \hat{\chi} \hat{\chi}^{*2} \\
& + \lambda_4 \left(\hat{\partial}_i \hat{\chi} \right) \left(\frac{\hat{\partial}_j}{\hat{\partial}^2} \hat{\chi}^* \right) \left(\frac{\hat{\partial}^i \hat{\partial}^j}{\hat{\partial}^2} \hat{\chi}^* \right) \\
& \left. + \lambda_5 \hat{\chi}^* \left(\frac{\hat{\partial}_i}{\hat{\partial}^2} \hat{\chi}^* \right) \hat{\partial}_i \hat{\chi} \right\}
\end{aligned}$$

$$\begin{aligned}
& + \lambda_6 b_1^2 \hat{\chi}^3 + \frac{\lambda_7 b_1^2}{b_2^2} \hat{\chi}^2 \hat{\partial}^2 \hat{\chi} \\
& + \frac{\lambda_8 b_1^2}{b_2^4} \hat{\chi}^2 \hat{\partial}^4 \hat{\chi} + \frac{\lambda_9 b_1^2}{b_2^6} \hat{\chi}^2 \hat{\partial}^6 \hat{\chi} \\
& + \dots \left. \right\}.
\end{aligned} \tag{3.21}$$

On the other hand, from Eq.(3.18) one finds that $S^{(2)}$ is invariant under the rescaling,

$$\hat{t} \rightarrow b^{-1} \hat{t}, \quad \hat{x}^i \rightarrow b^{-1} \hat{x}^i, \quad \hat{\chi} \rightarrow b \hat{\chi}, \tag{3.22}$$

while the terms of $\lambda_{1,2,\dots,5}$ and λ_7 in $S^{(3)}$ all scale as b , and the terms of $\lambda_{6,8,9}$ scale as b^{-1} , b^3 , b^5 , respectively. Therefore, except for the λ_6 term, all the others are irrelevant and nonrenormalizable [33]. For example, considering a process with an energy E , then we find that the fourth term has the contribution,

$$\int dt d^3\hat{x} \left(\hat{\partial}_i \hat{\chi} \right) \left(\frac{\hat{\partial}_j}{\hat{\partial}^2} \hat{\chi}^* \right) \left(\frac{\hat{\partial}^i \hat{\partial}^j}{\hat{\partial}^2} \hat{\chi}^* \right) \simeq E. \tag{3.23}$$

Since the action $S^{(3)}$ is dimensionless, we must have

$$\begin{aligned}
& \frac{\lambda_4}{b_1 \beta^3 \alpha^{3/4}} \int dt d^3\hat{x} \left(\hat{\partial}_i \hat{\chi} \right) \left(\frac{\hat{\partial}_j}{\hat{\partial}^2} \hat{\chi}^* \right) \left(\frac{\hat{\partial}^i \hat{\partial}^j}{\hat{\partial}^2} \hat{\chi}^* \right) \\
& \simeq \frac{E}{\Lambda_{SC}^{(4)}},
\end{aligned} \tag{3.24}$$

where $\Lambda_{SC}^{(4)}$ has the same dimension of E , and is given by,

$$\Lambda_{SC}^{(4)} = \frac{b_1 \beta^3 \alpha^{3/4}}{\lambda_4}. \tag{3.25}$$

Similarly, one can find $\Lambda_{SC}^{(n)}$ for all the other nonrenormalizable terms. But, when $\lambda \rightarrow 1$ (or $c_\psi \rightarrow 0$), the lowest one of the $\Lambda_{SC}^{(n)}$'s is given by $\Lambda_{SC}^{(4)}$, so we have

$$\Lambda_\omega \equiv \frac{b_1 \beta^3 \alpha^{3/4}}{\lambda_4}, \tag{3.26}$$

above which the nonrenormalizable λ_4 term becomes larger than unit, and the process runs into the strong coupling regime. Back to the physical coordinates t and x , the corresponding energy and momentum scales are given, respectively, by

$$\begin{aligned}
\Lambda_\omega &= \frac{\Lambda_\omega}{b_1} \simeq \mathcal{O}(1) \left(\frac{\zeta}{c_1} \right)^{3/2} M_{pl} |c_\psi|^{5/2}, \\
\Lambda_k &= \frac{\Lambda_\omega}{b_2} \simeq \mathcal{O}(1) \left(\frac{\zeta}{c_1} \right)^{1/2} M_{pl} |c_\psi|^{3/2}.
\end{aligned} \tag{3.27}$$

In particular, for $c_1 \simeq \zeta$, we find that $\Lambda_\omega \simeq M_{pl} |c_\psi|^{5/2}$, which is precisely the result obtained in [20].

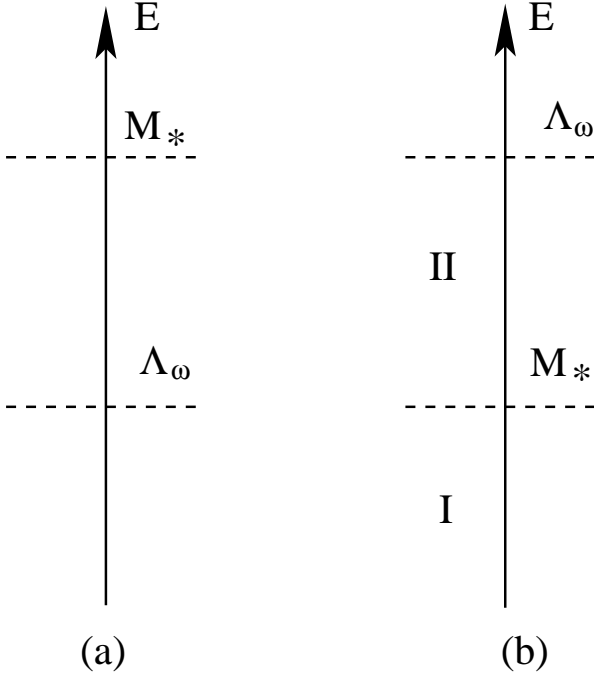


FIG. 1: The energy scales: (a) $\Lambda_\omega < M_*$; and (b) $\Lambda_\omega > M_*$.

It should be noted that the above conclusion is true only for $M_* > \Lambda_\omega$, that is,

$$M_* > \left(\frac{\zeta}{c_1}\right)^{3/2} M_{pl} |c_\psi|^{5/2}, \quad (3.28)$$

as shown by Fig. 1(a).

When $M_* < \Lambda_\omega$, the above analysis holds only for the processes with $E \ll M_*$ [Region I in Fig.1(b)]. However, when $E \gtrsim M_*$ and before the strong coupling energy scale Λ_ω reaches [cf. Fig.1(b)], the high order derivative terms of M_A and M_B in Eq.(3.12) cannot be neglected any more, and one has to take these terms into account. It is exactly because the presence of these terms that the strong coupling problem is cured [26]. In the following, we show that this is also the case here in the HMT setup [18].

B. $M_* < \Lambda_\omega$

In this case, there are two possibilities, $M_A < M_B$ and $M_A \gtrsim M_B$. In the following, let us consider them separately.

1. $M_A < M_B$

When $M_A < M_B$, we have $M_* = M_A$. For the processes with $E \gtrsim M_A$, Eq.(3.12) reduces to

$$S^{(2)} = \beta^2 \int dt d^3x \left(\dot{\chi}^2 - \frac{1}{M_A^2} \chi \partial^4 \chi \right). \quad (3.29)$$

To study the strong coupling problem, we shall follow what we did in the last case, by first writing $S^{(2)}$ in its canonical form,

$$S^{(2)} = \int dt d^3x \left(\hat{\chi}^{*2} - \hat{\chi} \hat{\partial}^4 \hat{\chi} \right), \quad (3.30)$$

through the transformations (3.17). It can be shown that now b_2 and b_3 are given by

$$b_2 = \sqrt{\frac{b_1}{M_A}}, \quad b_3 = \frac{M_A^{3/4}}{b_1^{1/4} \beta}, \quad (3.31)$$

for which the cubic action $S^{(3)}$ takes the form,

$$S^{(3)} = \frac{M_A^{3/4}}{b_1^{1/4} \beta^3} \hat{S}^{(3)}, \quad (3.32)$$

where $\hat{S}^{(3)}$ is given by Eq.(3.21). Due to the nonrelativistic nature of the action (3.30), its scaling becomes anisotropic,

$$\hat{t} \rightarrow b^{-2} \hat{t}, \quad \hat{x}^i \rightarrow b^{-1} \hat{x}^i, \quad \hat{\chi} \rightarrow b^{1/2} \hat{\chi}. \quad (3.33)$$

Then, we find that the first five terms in Eqs.(3.32) and (3.21) scale as $b^{1/2}$, while the terms of $\lambda_{6,\dots,9}$ scale, respectively, as $b^{-7/2}$, $b^{-3/2}$, $b^{1/2}$, $b^{5/2}$. Thus, except for the λ_6 and λ_7 terms, all the others are not renormalizable. It can be also shown that the processes with energy higher than $\Lambda_\omega^{(A)}$ become strong coupling, where $\Lambda_\omega^{(A)}$ is given by,

$$\Lambda_\omega^{(A)} = \frac{16}{81} \left(\frac{M_{pl}}{M_A} \right)^3 M_{pl} |c_\psi|^4, \quad (M_A < M_B). \quad (3.34)$$

Therefore, when the fourth-order derivative terms dominate, the strong coupling problem still exists. This is expected, as power-counting tells us that the theory is renormalizable only when $z \geq 3$ [cf. Eq.(1.1)]. Indeed, as to be shown below, when the sixth order derivative terms dominate, the strong coupling problem does not exist any longer.

2. $M_A \gtrsim M_B$

In this case, we have $M_* = M_B$, and for processes with $E \gtrsim M_B$, Eq.(3.12) reduces to

$$S^{(2)} = \beta^2 \int dt d^3x \left(\dot{\chi}^2 - \frac{1}{M_B^4} \chi \partial^6 \chi \right). \quad (3.35)$$

Then, by the transformations (3.17) with

$$b_2 = \frac{b_1^{1/3}}{M_B^{2/3}}, \quad b_3 = \frac{M_B}{\beta}, \quad (3.36)$$

we obtain,

$$S^{(2)} = \int dt d^3x \left(\hat{\chi}^{*2} - \hat{\chi} \hat{\partial}^6 \hat{\chi} \right), \quad (3.37)$$

while the cubic action $S^{(3)}$ becomes,

$$S^{(3)} = \frac{M_B}{\beta^3} \hat{S}^{(3)}. \quad (3.38)$$

Eq.(3.37) is invariant under the rescaling,

$$\hat{t} \rightarrow b^{-3}\hat{t}, \quad \hat{x}^i \rightarrow b^{-1}\hat{x}^i, \quad \hat{\chi} \rightarrow \hat{\chi}. \quad (3.39)$$

Then, it can be shown that the first five terms in Eqs.(3.38) and (3.21) are scaling-invariant, and so the last term. The terms of $\lambda_{6,7,8}$, on the other hand, scale, respectively, as b^{-6} , b^{-4} , b^{-2} . Therefore, the first five and the last terms now all become strictly renormalizable, while the λ_6 , λ_7 and λ_8 terms become super-renormalizable [33]. To have these strictly renormalizable terms to be weakly coupling, we require their coefficients be less than unit,

$$\frac{M_*}{\beta^3} \lambda_n < 1, \quad (n = 1, \dots, 5, 9). \quad (3.40)$$

For $\lambda \sim 1$ (or $|c_\psi| \sim 0$), we find that the above condition holds for

$$M_* < \frac{2}{3} M_{pl} |c_\psi|.$$

It can be shown that this condition holds identically, provided that $M_* < \Lambda_\omega$, that is,

$$M_* < \left(\frac{\zeta}{c_1} \right)^{3/2} M_{pl} |c_\psi|^{5/2}. \quad (3.41)$$

[Recall Λ_ω is given by Eq.(3.27) and $M_* = M_B$.] One can take $c_1 \simeq M_{pl}$, but now a more reasonable choice is $c_1 \simeq M_*$. Then, the condition (3.41) becomes

$$M_* < M_{pl} |c_\psi|^{1/2}, \quad (c_1 = M_*), \quad (3.42)$$

which is much less restricted than the one of $c_1 \simeq M_{pl}$. In addition, in order to have the sixth order derivative terms dominate, we must also require,

$$M_A \gtrsim M_*. \quad (3.43)$$

Therefore, it is concluded that, *provided that the conditions (3.41) and (3.43) hold, the HMT setup with any λ [18, 21] does not have the strong coupling problem.*

IV. CONCLUSIONS

In this paper, we have studied the strong coupling problem of a scalar field in the framework of the HMT setup [18], with an arbitrary coupling constant λ , generalized recently by da Silva [21]. As shown previously [20], when the energy of a process is higher than Λ_ω , it becomes strong coupling. To avoid it, one can provoke the

Blas-Pujolas-Sibiryakov (BPS) mechanism [26], in which a new energy scale M_* is introduced, so that the sixth order derivative terms become important before the strong coupling energy scale Λ_ω reaches. Once the high order derivative terms take over, the scaling behavior of the system is modified in such a way that all the nonrenormalizable terms become either strictly renormalizable or superrenormalizable, as shown explicitly in Sec. III.B.2. Whereby, the strong coupling problem is resolved.

It should be noted that, in order for the mechanism to work, λ cannot be exactly one, as one can see from Eq.(3.41). In other words, the theory cannot reduce exactly to general relativity in the IR. However, since general relativity has achieved great success in low energies, λ cannot be significantly different from one in the IR, in order for the theory to be consistent with observations. As first noticed by BPS in their model without projectability condition, the most stringent constraints come from the preferred frame effects due to Lorentz violation, which requires [26],

$$|\lambda - 1| \lesssim 4 \times 10^{-7}, \quad M_* \lesssim 10^{15} \text{ GeV}. \quad (4.1)$$

In addition, the timing of active galactic nuclei [36] and gamma ray bursts [37] requires

$$M_A \gtrsim 10^{10} \sim 10^{11} \text{ GeV}. \quad (4.2)$$

To obtain the constraint (4.1), BPS used the results from the Einstein-aether theory, as these two theories coincide in the IR [38].

In this paper, we have shown that the BPS mechanism is also applicable to the HMT setup. However, it is not clear whether the condition (4.1) is also applicable to the HMT setup, as the effects due to Lorentz violation in this setup have not been worked out, yet. On the other hand, the condition (4.2) is applicable, because this condition was obtained from the dispersion relations, which are the same in both setups.

In addition, the BPS mechanism cannot be applied to the Sotiriou-Visser-Weinfurtner generalization with projectability condition [23], because the condition $M_* < \Lambda_\omega$, together with the one that instability cannot occur within the age of the universe, requires fine-tuning,

$$|\lambda - 1| < 10^{-24}, \quad (4.3)$$

as shown explicitly in [27]. However, in the HMT setup with any λ , the Minkowski spacetime is stable [20], so such a fine-tuning does not exist.

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