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ABSTRACT: The Standard Model and its extensions predict multiple phase transitions in the early universe. In addition to the electroweak phase transition, one or several of these could occur at energies close to the weak scale. Such phase transitions can leave their imprint on the relic abundance of TeV-scale dark matter. In this paper, we enumerate several physical features of a generic phase transition and parameterize the effect of each on the relic abundance. In particular, we include among these effects the presence of the scalar field vacuum energy and the cosmological constant, which is sensitive to UV physics. Within the context of the Standard Model Higgs sector, we find that the relic abundance of generic TeV-scale dark matter is affected by the vacuum energy at the order of a fraction of a percent. For scalar field sectors with strong first order phase transitions, an order one percent apparent tuning of coupling constants may allow corrections induced by the vacuum energy to be of order unity.

1 Introduction

Phase transitions (PTs) are expected to be generic in the early universe [1]. The high temperature environment gives rise to significant corrections to the vacuum structure, and symmetries which are broken in the universe today can be restored at earlier times [2–4]. The Standard Model (SM) predicts early universe phase transitions in both the electroweak and QCD sectors. Beyond the SM, it is well-known that additional degrees of freedom can modify the dynamics of the electroweak phase transition significantly [5–23]. Furthermore, because almost all scenarios beyond the SM have extended symmetries, an even richer thermal history is expected in general (e.g., [3, 24–35]).

Most phenomenologically viable, cosmological PTs do not leave significant observable signals today. The successful and precisely measured theories of big bang nucleosynthesis, cosmic microwave background, and large scale structure formation strongly constrain late time PTs. At earlier stages of the cosmic evolution, thermal equilibrium erases most of the traces of PTs. Therefore, particle species which decouple early offer us perhaps the best probe PTs in the early universe. Among candidate particles, possibly the most obvious is the TeV-scale dark matter (DM), which is expected to freeze out of thermal equilibrium around $O(10 - 100)$ GeV. The successful prediction of its relic abundance, sometimes referred to as the WIMP miracle, is considered to be one the most important hints of new physics at the TeV-scale. Such a scenario is expected to be thoroughly probed at the LHC.

In this paper, we assess the sensitivity with which DM may probe the physics of PTs by exploring how PTs occurring nearly coincident with freeze out can modify the relic abundance calculation and alter the predicted relic density. In PTs for which supercooling is non-negligible, we find that several competing effects contribute to an overall shift in the DM relic abundance (as compared to the usual calculation without a PT). Two of these effects, the decoupling of non-relativistic species and the vacuum energy contribution to the Hubble expansion rate, tend to increase the relic abundance, while the entropy produced by the PT tends to decrease it. The principal result of this paper is summarized in Eq. (2.14), and the central discussion will emphasize the role of vacuum energy during the PT [36], since that is the most novel aspect of this letter as compared to previous studies. We find that a parametric tuning of order one percent can lead to an order unity dark matter abundance shift due to the presence of vacuum energy, assuming that a tuning of the cosmological constant sets the vacuum energy

today. In such situations, it may be possible to use DM as a probe of vacuum energy during the early universe by measuring the DM properties at terrestrial experiments and making mild assumptions about cosmology and UV completions of the effective field theory.

This work is related to past papers which discuss moduli dilution, such as [37] (and hundreds of inflationary papers), in that we calculate how the PT effects (including the vacuum energy) alter the relic abundance.[1] However, unlike the present work, most of these papers do not consider the case of a PT which nearly coincides with freeze out, nor do they consider the case of a low scale (e.g., electroweak scale) PT with electroweak scale vacuum expectation values. As in [41, 42], our calculation incorporates the possibility that the dark matter annihilation cross section may change after the freeze out, and as in [43], we estimate the dilution of dark matter due to a release of entropy at the PT. However, we also include additional features of the PT such as the changing vacuum energy.

This paper is organized as follows. In Sec. 2, we derive a generic parameterization with which one can discuss the effects of a PT on the DM relic abundance. In Sec. 3, we use this parameterization to estimate the correction to the relic abundance in two toy models in which a real scalar field experiences a phase transition. In Sec. 4 we summarize and briefly discuss which aspects of a generic model could be favorable for enhancing the effect of vacuum energy on the relic abundance. Being a letter, we restrict ourselves to the highlights.

Throughout the paper, we work in the FLRW spacetime $ds^2 = dt^2 - a^2(t)|d\mathbf{x}|^2$ in which $a(t)$ is a monotonically increasing function of t.

2 General framework

In this section, we discuss the various ways in which phase transitions can affect the relic density, and we provide a general parameterization which is useful for analyzing specific models.

Integrating the thermally averaged Boltzmann equation, we obtain the number density of dark matter today ($a = a_0$ and $t = t_0$)

$$n_X(t_0) = x_0^{-3} \left(\int_0^{\ln x_0} \frac{d\ln(x)}{H} \langle \sigma v \rangle \right)^{-1}, \qquad x = \frac{a}{a_f}, \quad x_0 = \frac{a_0}{a_f} \tag{2.1}$$

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[1]It is also related to papers such as [38–40] and many others which consider the change in the relic density due to a change in the equation of state during the freeze out process. Instead of listing all papers, interested readers can consider finding citations to and references within these papers.

where a_f corresponds to the scale factor at the time of the freeze out. We define the fractional deviation of the relic abundance as

$$\delta n_X(t_0) = \frac{n_X(t_0)}{n_X^{(U)}(t_0)} - 1 \tag{2.2}$$

where $n_X^{(U)}(t_0)$ is the "usual" relic density that one finds assuming that the PT does not occur. In Eq. (2.1), the quantities that will be affected by the PT are the Hubble expansion rate $H(a)$, the thermally averaged cross section $\langle \sigma v \rangle$ (also a function of a), and the dilution factor $x_0^{-3} = (a_f/a_0)^3 \ll 1$, which accounts for the expansion of the universe from freeze out until today (related to $T(a)$).

Suppose that a PT occurs after the time of the freeze out. This PT can affect $H(a)$ (though the energy density) in three ways: exotic energy, reheating, and decoupling. First, the PT is a change in the vacuum state and is typically accompanied by a decrease in the vacuum energy. For all (cosmological) intents and purposes, this vacuum energy behaves as a cosmological constant (CC). Speaking more generally, we can collectively refer to the vacuum energy, cosmological constant, and any other non-thermal sources of energy (e.g., quintessence) as "exotic energy." We assume that the exotic energy density can be written as $\rho(x) = \rho_{\rm ex} \kappa(x)$ with

$$\kappa(x) \approx \Theta((1+\delta) - x) + \Theta(x - (1+\delta)) \left(1 - \frac{\Delta\rho_{\rm ex}}{\rho_{\rm ex}}\right) \kappa_2(x) , \tag{2.3}$$

where $\Theta(z)$ is a step function and $\delta \equiv \frac{a_{PT}}{a_f} - 1 \lesssim 1$ quantifies the delay between freeze out and the phase transition. During the phase transition, the exotic energy decreases by $\Delta\rho_{\rm ex} > 0$, and the step function approximation corresponds to restricting ourselves to only phase transitions that occur on a time scale much shorter than $1/H$. Such short time scale phase transitions are expected to be generic for models in which the thermal bounce action has a strong temperature dependence.[2] In the case that the exotic energy is simply composed of vacuum energy plus a tuned cosmological constant,[3] we have $\Delta\rho_{\rm ex} \approx 0$ if the phase transition is of the second order or a smooth cross over and $\Delta\rho_{\rm ex} \approx \rho_{\rm ex}$ if the phase transition is first order with large supercooling. In the case $\Delta\rho_{\rm ex} \neq \rho_{\rm ex}$, the behavior of $\kappa_2(x)$ can parameterize quintessence dynamics which we assume decreases approximately as $(x\, a_f/a_{PT})^{-n_d}$ where n_d is a computable model dependent parameter. We focus on phase transitions that can be parameterized by a weakly coupled scalar field description.

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[2]For a recent discussion of situations with a longer time scale transitions, see for example [44].

[3]This has been considered as an acceptable possibility [45, 46], and it is a consequence of recently proposed string landscape scenario [47].

The remaining ways in which a PT can affect $H(a)$ are via the radiation energy density. From energy conservation, the change in exotic energy $\Delta\rho_{\text{ex}}$ must be compensated by a release of radiation energy, or equivalently, a reheating with entropy release Δs. In addition, generically particle masses may depend upon the scalar field vacuum expectation value (VEV) and may increase during the phase transition (e.g., this is the case in the SM electroweak phase transition). The heavier degrees of freedom can become non-relativistic and decouple. Consequently, the remaining relativistic species have a relatively lower energy density. We can parameterize this decoupling effect by writing the effective number of degrees of freedom for radiation energy g_E and entropy g_S as

$$g_{E/S}(x) = g_{E/S}(1) - h(x) \tag{2.4}$$

$$h(x) = \frac{7}{8}N_{PT}\,\Theta(x-(1+\delta)) + \frac{7}{8}Nf(x)\,, \tag{2.5}$$

where (N_{PT}) N represents the number of fermionic degrees of freedom which have (non-) adiabatically decoupled, and $f(x)$, which rises from 0 to 1, is given by Eq. (A.4).

Treating all of the aforementioned effects as small perturbations and using $T(a)$ from Eq. (B.8), the modification to $H(a)$ can be expressed as

$$H \approx H_R^{(U)}(x)\left[1 + \frac{\epsilon_1}{2}x^4\,\kappa(x) + \frac{2}{3}\epsilon_2\Theta(x-(1+\delta)) + \frac{\epsilon_{31}\Theta(x-(1+\delta)) + \epsilon_{32}f(x)}{6}\right] \tag{2.6}$$

where

$$H_R^{(U)}(x) \equiv \frac{T_f^2}{3\,M_p\,x^2}\sqrt{\frac{\pi^2}{10}\,g_E(T_f)} \tag{2.7}$$

is the "usual" Hubble parameter in the absence of a PT, T_f is the temperature at freeze out, and

$$\epsilon_1 \equiv \frac{\rho_{\text{ex}}}{\frac{\pi^2}{30}g_E(T_f)\,T_f^4} = \text{fractional energy of the exotic during freeze out} \tag{2.8a}$$

$$\epsilon_2 \equiv (1+3\,\delta)\frac{\Delta s}{\frac{2\pi^2}{45}g_S(T_f)\,T_f^3} = \text{fractional entropy increase during PT} \tag{2.8b}$$

$$\epsilon_{31} \equiv \frac{\frac{7}{8}N_{PT}}{g_E(T_f)} = \text{fractional decoupling degrees of freedom during PT} \tag{2.8c}$$

$$\epsilon_{32} \equiv \frac{\frac{7}{8}N}{g_E(T_f)} = \text{fractional decoupling degrees of freedom} \tag{2.8d}$$

are small, dimensionless quantities.

Furthermore, if the dark matter is coupled to the scalar sector directly, a PT in the scalar sector may alter the annihilation cross section $\langle \sigma v \rangle$. This effect can be parameterized by

$$\langle \sigma v \rangle = \langle \sigma v \rangle^{(U)} \left(1 - \epsilon_4 \, \Theta(x - (1 + \delta)) \right) \qquad \text{with} \qquad \epsilon_4 \equiv -\frac{\Delta_\sigma}{\langle \sigma v \rangle^{(U)}} \, . \tag{2.9}$$

Since the derivation of Eq. (2.1) assumes that the dark matter is decoupled after T_f, we will assume that $\epsilon_4 \gtrsim 0$ in order to prevent re-thermalization due to an increase $\langle \sigma v \rangle$.

Finally, we turn our attention to the dilution factor x_0^{-3}. Phase transitions occurring close to the freeze out can change the Hubble expansion rate, which in turn can cause dark matter to freeze out earlier or later. We can parameterize this effect by approximating the freeze out temperature as

$$T_f \approx \frac{m_X}{\ln A} \left[1 + \frac{\epsilon_1}{2} \left(\frac{1}{\ln A} + O\left((\ln A)^{-2} \right) \right) \right] \tag{2.10}$$

where

$$A \equiv \frac{N_{DM} 3\sqrt{5} M_p \sqrt{m_X T_f} \langle \sigma v \rangle}{4\pi^{5/2} \sqrt{g_E(T_f)}} \sim \exp[20] \, , \tag{2.11}$$

m_X is the dark matter mass, and N_{DM} counts the real dynamical degrees of freedom of the dark matter. By also taking into account the effect of a late time entropy release associated with the PT, we obtain

$$x_0 = \left. \frac{a_0}{a_f} \right|_{\text{usual}} \times \left[1 + \frac{\epsilon_1}{2} \frac{1}{\ln A} + \frac{\epsilon_2}{3} \right] \tag{2.12}$$

where

$$\left. \frac{a_0}{a_f} \right|_{\text{usual}} \equiv \left(\frac{g_S(T_f)}{g_S(T_0)} \right)^{1/3} \frac{m_X}{T_0} \frac{1}{\ln A} \tag{2.13}$$

is the "usual" dilution factor in the absence of a PT, and T_0 is the temperature today. In this paper we assume that the PT described by Eq. (2.3) is the last PT that generates appreciable entropy, but one can easily generalize Eq. (2.13) to accommodate later PTs that generate more entropy.[4]

Putting everything together, we obtain a general parameterization of the changes to the dark matter relic abundance which are induced by a PT:

$$\delta n_X(t_0) = c_1 \, \epsilon_1 + c_2 \, \epsilon_2 + c_{31} \, \epsilon_{31} + c_{32} \, \epsilon_{32} + c_4 \, \epsilon_4 \tag{2.14}$$

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[4]In particular, we assume that QCD phase transition is not a significant source of entropy [48].

where the coefficients

$$c_1 \equiv \frac{1}{2}\left(\delta + \frac{(1+3\delta)}{n_d - 3}\left(1 - \frac{\Delta\rho_{\text{ex}}}{\rho_{\text{ex}}}\right)\right) - \frac{3}{2}\frac{1}{\ln A} \tag{2.15a}$$

$$c_2 \equiv -\frac{1}{3}(1+2\delta) \tag{2.15b}$$

$$c_{31} \equiv \frac{1}{6}(1-\delta) \tag{2.15c}$$

$$c_{32} \equiv \frac{1}{6}\int_1^{a_0/a_f|_{\text{usual}}} \frac{dx}{x^2} f(x) \tag{2.15d}$$

$$c_4 \equiv 1 - \delta \tag{2.15e}$$

are order one numbers that account for the delay between freeze out and the PT (recall $\delta = a_{PT}/a_f - 1 \gtrsim 0$). Note that c_{32} receives most of contributions from near the freeze out temperature. The usefulness of this parameterization is that it is general enough to classify most phase transitions that can affect the DM relic abundance. This is one of the main results of this paper.

3 Phase transition effects as a function of Lagrangian parameters

In the section, we discuss how the parameters of a scalar field theory map to the freeze out modifying effects discussed in the previous section. In particular, we focus on a generic real scalar field for which the one-loop thermal effective potential is well-approximated by

$$V_{\text{eff}}(\phi, T) \approx \rho_{\text{ex}} + \frac{1}{2}M^2\phi^2 - \mathcal{E}\,\phi^3 + \frac{\lambda}{4}\phi^4 + c\,T^2\phi^2 \tag{3.1}$$

where $M^2, \mathcal{E}, \lambda$, and c are free parameters.[5] This can be viewed as the effective description of the dynamics of a large class of PTs with a tuned cosmological constant. This simple description contains all the information that is necessary to discuss the vacuum energy contribution ($c_1\epsilon_1$) and reheating contribution ($c_2\epsilon_2$) to $\delta n_X(t_0)$. The contributions from the decoupling ($c_{31}\epsilon_{31}$ and $c_{32}\epsilon_{32}$) depend on additional details of the model

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[5] In order to treat c as a free parameter, we must suppose that the ϕ-sector is coupled to another sector, call it sector X, which is not strongly constrained phenomenologically. The interaction between ϕ and sector X can then be considered a nearly a free parameter and generates the thermal mass cT^2. For instance, suppose a Yukawa coupling $\mathcal{L} \ni y\phi\bar{\psi}\psi$ where ψ is a spin-1/2 X-sector field with N dynamical degrees of freedom, and then $c \approx Ny^2/48$. E.g., to obtain $c \approx 0.1$ one needs $y \approx 1.1$ if $N = 4$ (Dirac fermion) and $y \approx 0.6$ if $N = 12$. Moreover, the X-sector particles must be lighter than the PT temperature. Otherwise, Boltzmann suppression drives $c \to 0$.

and, as we will see, they have the dominant effect on the DM abundance. Therefore, as far as we are concerned with the mapping of Lagrangian parameters to $c_i\epsilon_i$, we will focus our discussion on just $c_1\epsilon_1$ and $c_2\epsilon_2$. Here, we also follow the traditional abuse of language in classifying the cosmological phase transitions as first order or second order dependent on whether or not (transient) bubbles are involved during changes in the vacuum determining the 1-particle state.

3.1 $\mathcal{E} = 0$, "second order" phase transition

We first restrict our analysis to the case of $\mathcal{E} = 0$. In this limit there is a $\mathbb{Z}_2$ symmetry, and the finite temperature effective potential can be written as

$$V_{\text{eff}}(\phi, T) \approx \frac{\lambda}{4}\left(\phi^2 - v_\phi^2\right)^2 + c\,T^2\phi^2, \tag{3.2}$$

where $v_\phi = \sqrt{-M^2/\lambda}$ is the VEV in the $\mathbb{Z}_2$ broken phase at $T = 0$. Because there is no cubic term, no sub-horizon bubbles are involved as the vacuum changes from $\phi = 0$ to $\phi = v_\phi$ at the PT.[6] The temperature at the beginning of the PT can be approximately mapped to the Lagrangian parameters as $(T_{PT}^-)^2 = \frac{\lambda}{2c}v_\phi^2$. By requiring that the exotic energy be zero today when $T = 0$, we find the exotic energy at the time of the phase transition to be

$$\rho_{\text{ex}} = V_{\text{eff}}(0,0) = \frac{\lambda}{4}v_\phi^4 = \frac{c^2}{\lambda}(T_{PT}^-)^4. \tag{3.3}$$

Therefore using Eqs. (2.8a) and (2.15a) the exotic energy contribution is given by (for $\delta < 1$)

$$c_1\epsilon_1 \approx \frac{\delta}{2g_E}\frac{c^2}{\lambda} \sim \frac{1}{10}\frac{1}{g_E}c^2\frac{v_\phi^2}{m_\phi^2} \tag{3.4}$$

where $m_\phi^2 = 2\lambda v_\phi^2$ is the approximate scalar mass in the $\phi = v_\phi$ vacuum, and typically $g_E \gtrsim 100$.

In the minimal scenario of the SM supplemented by a DM sector, one finds $c_{\text{SM}}^2/\lambda_{\text{SM}} \approx 0.28$ where c_{SM} is dominated by the top Yukawa and does not take into account the coupling of DM to the Higgs sector. If electroweak symmetry breaking occurs soon after the dark matter freeze out, Eq. (3.4) allows us to estimate that the DM relic abundance will experience a fractional change at the order of 10^{-3} due to each of the CC effect. Moreover, soon after the electroweak phase transition, the heavy quarks

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[6]Horizon sized domain walls do form, however [24].

decouple and $N \sim 20$ fermionic degrees of freedom are lost from the tally of relativistic species. Consequently, the ratio

$$\frac{c_1 \epsilon_1}{c_{32} \epsilon_{32}} \sim -\frac{c^2}{\lambda} \frac{1}{N} \lesssim 1 \tag{3.5}$$

is small, and we expect that the shift in the relic abundance is dominated by the decoupling of these heavy degrees of freedom.

In the SM, the exotic energy effect is subdominant, but Eq. (3.4) provides a guide to constructing models with enhanced $c_1 \epsilon_1$. This term can be made larger if $v_\phi^2 / m_\phi^2 \gg 1$, which could be realized by invoking fine tuning or some additional symmetry to generate a flat potential. Alternatively, one could contrive a model in which $\rho_{\text{ex}} \gg T_f^4 \geq (T_{PT}^-)^4$ and thereby enhance ϵ_1 directly. Such a scenario can be naturally realized if supercooling occurs, as in the case of a "first order" PT. We now turn our attention to this scenario.

3.2 $\mathcal{E} \neq 0$, supercooling and "first order" phase transition

At $T = 0$, the general potential in Eq. (3.1) has extrema at

$$\phi = 0 \quad \text{and} \quad \phi = v_\phi = \frac{3\mathcal{E}}{2\lambda} \left(1 + \sqrt{1 - \frac{8}{9}\alpha_0} \right) , \tag{3.6}$$

where we have introduced the dimensionless quantity $\alpha_0 \equiv \lambda M^2 / 2\mathcal{E}^2$, which controls the vacuum structure. For $\alpha_0 > 1$, $\phi = 0$ is the true vacuum; for $0 < \alpha_0 < 1$, $\phi = v_\phi$ is the true vacuum while $\phi = 0$ is metastable; and for $\alpha_0 < 0$, $\phi = 0$ becomes unstable. The barrier separating the metastable and true vacua has a height (for $0 < \alpha_0 < 1$)

$$V_{\text{barrier}} = \frac{4\mathcal{E}^4 \alpha_0^3}{27\lambda^3} \left(1 + O\left(\alpha_0\right) \right) \tag{3.7}$$

which vanishes rapidly as $\alpha_0 \to 0$. As in Eq. (3.3), by requiring the exotic energy to vanish today, we calculate the exotic energy prior to the PT to be

$$\rho_{\text{ex}} = \frac{\mathcal{E}^4}{8\lambda^3} \left[27 - 36\,\alpha_0 + 8\,\alpha_0^2 + 27 \left(1 - \frac{8}{9}\alpha_0 \right)^{3/2} \right] \tag{3.8}$$

and note that all of this energy is converted into radiation at the phase transition (i.e., $\Delta\rho_{\text{ex}} = \rho_{\text{ex}}$).

In order to compute the CC's effect on the relic abundance, we need to know the PT temperature T_{PT}^-, or equivalently the amount of supercooling, which has an interesting dependence on α_0. We require $\alpha_0 < 1$ such that there exists a temperature

$$T_c = \mathcal{E} \sqrt{\frac{1 - \alpha_0}{\lambda\,c}} \tag{3.9}$$

below which the symmetric phase $\phi = 0$ becomes metastable. The PT begins at a temperature $T_{PT}^- < T_c$ when the bubble nucleation rate per Hubble volume $\Gamma\,H^{-3} \sim T^4 e^{-S^{(3)}/T}\,H^{-3}$ is comparable to Hubble expansion rate $H \sim T^2/M_p$. Here $S^{(3)}$ is the action of the O(3) symmetric bounce. For an electroweak scale phase transition this condition is satisfied when $S^{(3)}/T$ drops below approximately 140 [49]. Provided that the potential can be expressed in the form of Eq. (3.1), then the action is well-approximated by the empirical formula [50]

$$\frac{S^{(3)}}{T} \approx 13.7 \frac{\mathcal{E}}{T} \left(\frac{\alpha}{\lambda}\right)^{3/2} f(\alpha) \tag{3.10}$$

$$f(\alpha) \equiv 1 + \frac{\alpha}{4}\left(1 + \frac{2.4}{1-\alpha} + \frac{0.26}{(1-\alpha)^2}\right) \tag{3.11}$$

where the temperature dependence is parameterized by $\alpha(T) = \alpha_0(1 - T^2/T_0^2)$, and $T_0^2 = -M^2/(2c)$ can be positive or negative.

The PT temperature is constrained by $\text{Max}\,[T_0^2, 0] < (T_{PT}^-)^2 < T_c^2$ where the lower bound depends on the sign of α_0. We will discuss the two cases separately. For $\alpha_0 > 0$ (or $T_0^2 < 0$), the vacuum $\phi = 0$ remains metastable as $T \to 0$. This suggests that the PT temperature can be arbitrarily low, and in this limit of large supercooling the CC effect may be arbitrarily large. Unfortunately, if the barrier persists as $T \to 0$, it is possible that the PT does not occur at any temperature – a obviously unphysical scenario in the case of the electroweak phase transition. This follows from the observation that for $\alpha_0 > 0$, $S^{(3)}/T$ has a minimum at $T \neq 0$: at low temperatures $S^{(3)}/T$ grows due to the explicit factor of T in the denominator, and at high temperatures $f(\alpha)$ diverges as $\alpha \to 1$. Over some of the parameter space, the inequality $S^{(3)}/T \lesssim 140$ is not satisfied at any temperature, and the PT does not occur. In particular, if α_0 is close to one, then $\alpha > \alpha_0 \approx 1$ at all temperatures, and it is very difficult for the PT to proceed. Therefore, if we require that the PT must occur via thermal bubble nucleation, we obtain an upper bound on α_0. For the case $\alpha_0 < 0$, the PT necessarily occurs at a temperature $T_{PT}^- > T_0 > 0$, since the $\phi = 0$ vacuum becomes perturbatively unstable below T_0. This case has the drawback that supercooling cannot last an arbitrarily long time, but on the other hand, one is guaranteed that the PT proceeds.

Provided that the PT does occur, we define

$$\delta_{\text{SC}} = 1 - \frac{T_{PT}^-}{T_c} \tag{3.12}$$

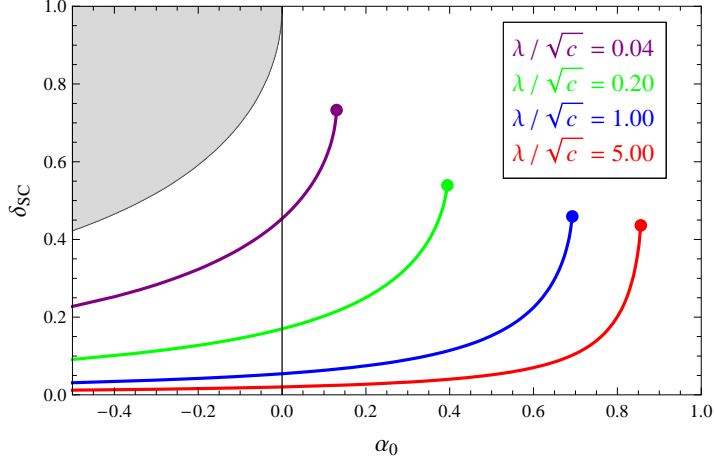which ranges from 0 to 1 and quantifies the amount of supercooling. Using δ_{SC} to

Figure 1. We have plotted the amount by which the phase transition temperature drops below the critical temperature, quantified by $\delta_{\rm SC}$, against the parameter α_0 which controls the height of the barrier. These curves only depend on the parametric combination $\lambda/\sqrt{c}$. The amount of supercooling grows as α_0 is made larger, but reaches a finite maximum $\delta_{\rm SC}^{(max)} \lesssim O\left(1\right)$ at a value of α_0 that depends on the ratio $\lambda/\sqrt{c}$.

parameterize the temperature dependence, we can rewrite Eq. (3.10) in the form

$$\left.\frac{S^{(3)}}{T}\right|_{T_{PT}^-} \approx \left(\frac{\lambda}{\sqrt{c}}\right)^{-1} \frac{1}{\sqrt{1-\alpha_0}} \left[\frac{a_{-2}}{\delta_{SC}^2} + \frac{a_{-1}}{\delta_{SC}} + a_0 + a_1\delta_{SC} + O\left(\delta_{SC}^2\right)\right], \qquad (3.13)$$

where the a_i are functions of α_0. We require $S^{(3)}/T\big|_{T_{PT}^-} = 140$ and solve for $\delta_{\rm SC}$, which we have plotted in Figure 1. The supercooling grows with increasing α_0 and decreasing $\lambda/\sqrt{c}$ as the barrier and bounce action are made larger. In the shaded region the lower bound on $T_{PT}^- > T_0$ is not satisfied. The amount of supercooling is typically of the order $\delta_{\rm SC} \lesssim 0.5$ which implies $T_{PT}^- \gtrsim T_c/2$. Above a finite value of α_0 (indicated by a dot) the barrier becomes insurmountably large, and the universe becomes trapped in the metastable vacuum. The existence of this upper bound on α_0 does not allow a phenomenologically viable, arbitrarily large supercooling, contrary to naive expectations. The largest amount of supercooling is achieved for $\lambda/\sqrt{c} \ll 1$ and $\alpha_0 \gtrsim 0$. In this parameter regime the exotic energy is large (see Eq. (3.8)), and the metastable vacuum is separated from the true vacuum by a small barrier (see Eq. (3.7)).

We have calculated the exotic energy and reheating contributions to the relic abundance shift by using Eq. (2.14), and we present the results in Figure 2. In generating these plots, we have fixed $c = 0.1$, $\mathcal{E} = 5$ GeV, and $g_{E/S} = 106.75$ (SM degrees of
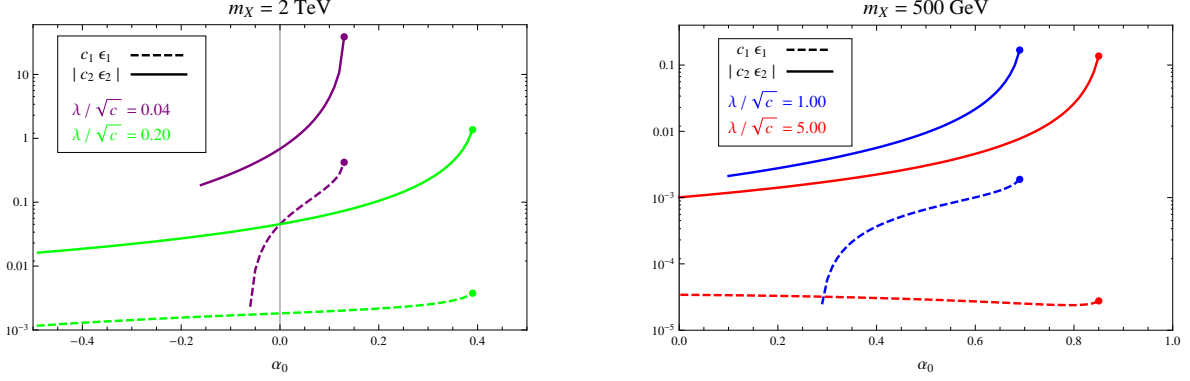
Figure 2. The fractional shift in the dark matter relic abundance due to the exotic energy (dashed, $c_1\epsilon_1$) and the reheating (solid, $c_2\epsilon_2$). Note that the two figures have different scales, and that we have plotted $|c_2\epsilon_2|$ since this quantity is negative. When $\lambda/\sqrt{c}$ is smaller than 0.04, then one may enter a regime of large supercooling for tuned values of α_0. The reheating effect dominates by an order of magnitude or more. The contours extend over a finite range of α_0 because for larger α_0 the PT does not occur, and for smaller α_0 the PT occurs before freeze out. Since our analytical approximation breaks down when $c_i\epsilon_i \sim O(1)$, the extrapolation into this region should only be treated as an indication of possible size of the effect.

freedom[7]) while allowing α_0 to vary. We select two values for the dark matter mass, which in turn fixes the freeze out temperature via Eq. (2.10). For the heavier case $m_X = 2$ TeV, the freeze out occurs quite early, and if $\lambda/\sqrt{c} = 1.00, 5.00$ (which are not shown) the PT would occur much later, in the limit where our analytic approximations break down (i.e., $\delta > 1$). Some of the curves are truncated at small α_0, because we require that the PT occur after the freeze out (i.e., $\delta > 0$), and the phase transition temperature increases with decreasing α_0 (see Figure 1). It is also for this reason that, the $\lambda/\sqrt{c} = 0.04$ and 0.2 curves are entirely absent from the $m_X = 500$ GeV plot.

These figures indicate that the exotic energy effect on the relic abundance is typically on the order of 10^{-3} and is subdominant to the reheating effect by an order of magnitude. Both contributions become larger in the limit of large supercooling where $\lambda/\sqrt{c}$ is small and α_0 approaches its maximal value. For smaller values of $\lambda/\sqrt{c}$ a brief period of inflation might even be possible. The curves $\{\lambda/\sqrt{c} = 0.04, m_X = 2 \text{ TeV}\}$ and $\{\lambda/\sqrt{c} = 1.00, m_X = 500 \text{ GeV}\}$ illustrate the parametric tuning of α_0 that is required to achieve a large correction to the relic abundance. If α_0 is made too large, the PT

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[7]We choose this value as a fiducial reference. Realistically, for these parameters the PT occurs at $T_{PT}^- \approx 1 - 100$ GeV, which could be later than the electroweak phase transition. In that case, some of the SM degrees of freedom would have decoupled, $g_{E/S}$ would be smaller, and the ϵ_i would be relatively larger.

does not occur, and if α_0 is made too small, the PT occurs before freeze out. Comparing the $m_X = 2$ TeV and $m_X = 500$ GeV plots reveals the parametric tuning that must occur between the DM and scalar sectors. If the DM mass is small, for example, then the parameters of the scalar sector must conspire to generate a low scale PT, otherwise the PT occurs too early and decouples from the physics of the freeze out.

4   Conclusion

If the properties of dark matter can be measured accurately in laboratories, the information that these experiments yield can be used to probe the properties of early universe phase transitions. This is a particularly exciting prospect given that phase transition physics incorporates the energy densities of the false vacuum and the cosmological constant, and thereby it provides an empirical method to directly probe the tuning of the cosmological constant. With this in mind, we have developed a general parameterization to characterize the effects of a single field phase transition on the thermal dark matter relic abundance in a freeze out scenario.

In the context of the SM (supplemented by a DM candidate) and assuming a tuned cosmological constant, we find that the exotic energy (i.e. the Higgs field vacuum energy plus the cosmological constant energy) leads to a fractional increase in the dark matter abundance by $O(10^{-3})$. The dominant change in the dark matter abundance comes from a decoupling of relativistic degrees of freedom near the time of the freeze out, which leads to a fractional increase in the relic abundance of order 10^{-2}. Without extreme tuning, we expect that most second order PTs share the characteristics of the SM case.

In the case of a second order phase transition, models with a very flat potential (i.e., $m_\phi^2 \lesssim H_{PT}$) generally give a large dark matter abundance shift via the exotic energy contribution. In this limit, Hubble friction can enhance the supercooling as in the case of slow-roll inflation (as signaled by the enhancement attendant with large v_ϕ/m_ϕ in Eq. (3.4)). Although pseudo-Nambu-Goldstone boson models may be useful for producing such flat potentials, the required hierarchies can be somewhat unnatural during the electroweak phase transition since $H_{PT} \sim 10^{-14}$ GeV.

In first order phase transitions with supercooling, there is a somewhat surprising theoretical upper limit on the duration of supercooling which follows from the fact that the bubble nucleation rate is not a monotonically decreasing function of time. In certain parametric regimes, the phase transition never occurs. Close to this failed phase transition case, the maximum fractional increase in the relic abundance due to the exotic energy effect can become $O(0.1)$ and due to the reheating effect can become $O(1)$. However, reaching these large magnitudes requires some degree of parametric

tuning. As the parameters deviate from their tuned values, either the PT will not occur at all, or it will occur before the freeze out.

In order for dark matter freeze out to act as a probe of the phase transition, as we have considered, it must be the case that freeze out occurs soon before or concurrently with the phase transition. Since phase transitions typically occur at electroweak scale temperatures or higher and since the mass of weakly interacting dark matter is typically 20 times larger than the freeze out temperature, these DM particles must be heavy, and they may be difficult to discover at the LHC.

It is nonetheless an exciting prospect that LHC and other experiments sensitive to dark matter's non-gravitational interaction properties may unveil a new probe of dark energy. This is particularly interesting given that there is almost no other way to probe the conjecture of a tuned cosmological constant.[8]

5   Acknowledgments

A   Derivation of PT induced change in the degree of freedom

We begin with the well-known formula for the energy density of a gas of fermions at temperature T with N dynamical degrees of freedom:

$$\rho(T) = N \int \frac{d^3p}{(2\pi)^3} \frac{E_p}{1 + e^{E_p/T}} \, . \tag{A.1}$$

The gas has an effective number of degrees of freedom g_E given implicitly by $\rho(T) = \frac{\pi^2}{30} g_E(T) T^4$. We can parameterize the decrease in g_E due to the decoupling of the fermionic gas by writing

$$g_E(T) = g_E(T_f) - \frac{7}{8} N f\left(a/a_f\right) \tag{A.2}$$

where

$$f\left(x = a/a_f\right) = \left(\frac{7}{8}\frac{\pi^2}{30}\right)^{-1} \int \frac{d^3p}{(2\pi)^3} E_p \left[\frac{1}{T_f^4} \frac{1}{e^{\frac{E_p}{T_f}} + 1} - \frac{1}{T^4(a_f x)} \frac{1}{e^{\frac{E_p}{T(a_f x)}} + 1}\right] . \tag{A.3}$$

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[8]There are generic theoretical limitations on empirical reconstruction of the phase transition scenario. This study will be presented elsewhere [51].

The temperature $T = T(a)$ is given by Eq. (B.8) to leading order in the perturbations ϵ_i. Since f already multiplies a small term in Eq. (2.5), we need only keep the leading factor in Eq. (B.8) which is $T = T_f\, a_f/a = T_f/x$. This lets us write Eq. (A.3) as

$$f(x) = \frac{8}{7}\left(\frac{30}{\pi^2}\right)\int \frac{d^3p}{(2\pi)^3}\frac{E_p}{T_f^4}\left[\frac{1}{e^{\frac{E_p}{T_f}}+1} - \frac{x^4}{e^{\frac{xE_p}{T_f}}+1}\right]. \tag{A.4}$$

Note that $f(x)$ increases from $f(1) = 0$ to $f(\infty) \approx 1$. Due to the exponential temperature dependence, the transition to $f \approx 1$ occurs at $T \approx m_N$ and is smoothly steplike over a time scale $\Delta t \approx 1/H$. In this discussion we have assumed $E_p = \sqrt{\mathbf{p}^2 + m_N^2}$ with m_N constant, that is, we neglect any change in the mass of the particle as a function of time. This assumption is valid sufficiently far after the PT such that the scalar VEV and field-dependent masses have approximately stopped varying.

B Derivation of T_{PT}^+, Δs, and $T(a)$

In this appendix, we calculate the temperature after the phase transition T_{PT}^+ by imposing energy conservation at the PT. This allows us to calculate Δs and ϵ_2 in terms of $\Delta\rho_{\rm ex}$. Provided that there is a negligible change in $a \approx a_{PT}$ during reheating, energy conservation can be written as

$$\frac{\pi^2}{30}\, g_E(T_{PT}^-)\left(T_{PT}^-\right)^4 + \Delta\rho_{\rm ex} = \frac{\pi^2}{30}\, g_E(T_{PT}^+)\left(T_{PT}^+\right)^4. \tag{B.1}$$

Using the perturbative expansions introduced in Section 2, Eq. (B.1) can be solved for T_{PT}^+ at leading order to obtain

$$T_{PT}^+ \approx T_{PT}^-\left[1 + \frac{1}{4}\,\epsilon_{31} + \frac{1}{4}\frac{\Delta\rho_{\rm ex}}{\frac{\pi^2}{30}\,g_E(T_f)\left(T_{PT}^-\right)^4}\right] \tag{B.2}$$

where ϵ_{31} is given by Eq. (2.8c). As expected, the amount of exotic energy released $\Delta\rho_{\rm ex} > 0$ controls the reheating from T_{PT}^- to T_{PT}^+. Additionally, the reheating is larger when more species non-adiabatically decouple (larger ϵ_{31}), because there are fewer degrees of freedom after the PT to distribute $\Delta\rho_{\rm ex}$ over, which makes them comparatively hotter.

Similarly, we can calculate the entropy density increase at the PT. Writing the entropy density as $s(T) = \frac{2\pi^2}{45}g_S(T)T^3$, we can calculate Δs as

$$\Delta s = \frac{2\pi^2}{45}\left\{g_S(T_{PT}^+)(T_{PT}^+)^3 - g_S(T_{PT}^-)(T_{PT}^-)^3\right\} \tag{B.3}$$

$$\approx \frac{2\pi^2}{45}\left\{-\frac{g_E(T_f)}{g_S(T_f)}\epsilon_{31} + \frac{3}{4}\left[\epsilon_{31} + \frac{\Delta\rho_{\rm ex}}{\frac{\pi^2}{30}g_E(T_f)(T_{PT}^-)^4}\right]\right\}g_S(T_f)\left(T_{PT}^-\right)^3 \tag{B.4}$$

where we have used Eq. (B.2) and linearized in perturbations. We can calculate ϵ_2, given by Eq. (2.8b), by noting $T_{PT}^- a_{PT} \approx T_f a_f$ and $g_S(T_f) \approx g_E(T_f)$ up to higher order terms. Doing so yields

$$\epsilon_2 \approx -\frac{1}{4}\,\epsilon_{31} + \frac{3}{4}\frac{\Delta\rho_{\mathrm{ex}}}{\frac{\pi^2}{30}g_E(T_f)(T_{PT}^-)^4}\,. \tag{B.5}$$

These expressions for Δs and ϵ_2 illustrate that the entropy increase at the PT is controlled by the amount of latent heat released and the number of particles that non-adiabatically decouple.

Lastly, we will solve the equation of entropy conservation for $T(a)$. The entropy per comoving volume $S = s\,a^3$ is conserved except for the entropy injection at reheating, which is assumed to occur rapidly at a_{PT}. Entropy conservation may be expressed as

$$g_S(T)\,T^3 a^3 = g_S(T_f)\,T_f^3 a_f^3 + \Theta(a - a_{PT})\,a_{PT}^3\left(\frac{2\pi^2}{45}\right)^{-1}\Delta s \tag{B.6}$$

and implicitly defines $T(a)$. To solve for T we use Eq. (2.4) to expand $g_S(T)$ then linearize in h and Δs to obtain

$$T(a) \approx T_f\frac{a_f}{a}\left[1 + \frac{1}{3}\frac{h(a/a_f)}{g_S(T_f)} + \Theta\left(a - a_{PT}\right)\frac{1}{3}\left(\frac{a_{PT}}{a_f}\right)^3\frac{\Delta s}{\frac{2\pi^2}{45}g_S(T_f)\,T_f^3}\right]\,. \tag{B.7}$$

Further expanding h using Eq. (2.5), approximating $g_S(T_f) \approx g_E(T_f)$, and applying Eq. (2.8b) we obtain the final expression,

$$T(a) \approx T_f\frac{a_f}{a}\left[1 + \frac{1}{3}\epsilon_{32}\,f(a/a_f) + \Theta\left(a - a_{PT}\right)\frac{1}{3}\left(\epsilon_{31} + \epsilon_2\right)\right]\,. \tag{B.8}$$

After the PT, the exotic energy component behaves approximately adiabatically.

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