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# Anomalous $\omega$-$Z$-$\gamma$ Vertex from Hidden Local Symmetry

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We formulate the general form of $\omega$-$Z$-$\gamma$ vertex in the framework based on the hidden local symmetry (HLS), which arises from the gauge invariant terms for intrinsic parity-odd (IP-odd) part of the effective action. Those terms are given as the homogeneous part of the general solution (having free parameters) to the Wess-Zumino (WZ) anomaly equation and hence are not determined by the anomaly, in sharp contrast to the Harvey-Hill-Hill (HHH) action where the relevant vertex is claimed to be uniquely determined by the anomaly. We show that, even in the framework that HHH was based on, the $\omega$-$Z$-$\gamma$ vertex is actually not determined by the anomaly but by the homogeneous (anomaly-free) part of the general solution to the WZ anomaly equation having free parameters in the same way as in the HLS formulation: The HHH action is just a particular choice of the free parameters in the general solution. We further show that the $\omega$-$Z$-$\gamma$ vertex related to the neutrino ($\nu$) - nucleon ($N$) scattering cross section $\sigma(\nu N \rightarrow \nu N(N')\gamma)$ is determined not by the anomaly but by the anomaly-free part of the general solution having free parameters. Nevertheless we find that the cross section $\sigma(\nu N \rightarrow \nu N(N')\gamma)$ is related through the Ward-Takahashi identity to $\Gamma(\omega \rightarrow \pi^0\gamma)$ which has the same parameter-dependence as that of $\sigma(\nu N \rightarrow \nu N(N')\gamma)$ and hence the ratio $\sigma(\nu N \rightarrow \nu N(N')\gamma)/\Gamma(\omega \rightarrow \pi^0\gamma)$ is fixed independently of these free parameters. Other set of the free parameters of the general solution can be fixed to make the best fit of the $\omega \rightarrow \pi^0 l^+ l^-$ process, which substantially differs from the HHH action. This gives a prediction of the cross section $\sigma(\nu N \rightarrow \nu N(N')\gamma^*(l^+l^-))$ to be tested at $\nu$-$N$ collision experiments in the future.

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## I. INTRODUCTION

The chiral Lagrangian describes the low-energy properties of QCD governed by the low-lying hadron spectrum including the pseudo Nambu-Goldstone bosons (NGBs) associated with the spontaneous chiral symmetry breaking. In this framework Ward-Takahashi identities of QCD play an essential role to determine forms of interactions between the NGB and external gauge fields arising from gauging the chiral symmetry. Among those symmetry identities, the anomalous Ward-Takahashi identity is of great importance, which is tied with non-Abelian anomaly arising from the underlying quark currents in QCD. It is reproduced in the chiral Lagrangian by (covariantized) Wess-Zumino-Witten (WZW) term $\Gamma_{\rm WZW}$ [1, 2], which satisfies the non-Abelian anomaly equation, called the Wess-Zumino (WZ) anomaly equation,

$$\delta\Gamma[\mathcal{L}, \mathcal{R}, U] = \delta\Gamma_{\rm QCD} \ = \ -\frac{N_c}{24\pi^2}\int_{M^4}{\rm tr}\left[\epsilon_L\left\{(d\mathcal{L})^2 - \frac{i}{2}d\mathcal{L}^3\right\} - (\mathcal{L} \leftrightarrow \mathcal{R})\right] \equiv \mathbf{A}\,, \tag{1}$$

where $M^4$ denotes four-dimensional Minkowski manifold, and $U$, $\mathcal{L}$ and $\mathcal{R}$ denote the chiral field parameterizing the NGBs ($\pi$) as $U = e^{2i\pi/F_\pi}$ and external gauge fields, respectively. Hence the low-energy interactions among the NGBs and external gauge fields such as $\pi^0 \rightarrow \gamma\gamma$ and $\gamma^* \rightarrow \pi^0\pi^+\pi^-$ are completely determined by the anomaly (Low Energy Theorem). How about inclusion of the vector mesons ($\omega$, $\rho$, etc.) in the intrinsic parity odd (IP-odd) processes such as $\omega$-$Z$-$\gamma$, $\omega$-$\pi^0$-$\gamma$, $\omega$-$\pi^0$-$\pi^+$-$\pi^-$, etc. without affecting the Low Energy Theorem? This problem was solved long time ago [3] in the framework of the Hidden Local Symmetry (HLS) [4–6] which incorporates the vector mesons as the (composite) gauge bosons of HLS in the nonlinear sigma model. The resulting expression [3] implicitly contained the $\omega$-$Z$-$\gamma$ vertex which was not analyzed so far.

The method was based on the fact that since the WZ anomaly equation (1) is an inhomogeneous linear differential equation, its general solution is given by the linear combination of a special solution to Eq.(1), $\Gamma_{\text{WZW}}$, and the general solution to the homogeneous part of Eq.(1) ($\delta\Gamma = 0$). While the special solution $\Gamma_{\text{WZW}}$ is completely fixed by the anomaly, the general solution to the homogeneous part $\delta\Gamma = 0$ cannot be fixed by the anomaly. The general solution $\Gamma_{\text{HLS}}^{\text{inv}}$ to the homogeneous WZ equation must be gauge-invariant (anomaly-free) and has actually been found [3]:

$$\Gamma_{\text{HLS}}^{\text{inv}} = \frac{N_c}{16\pi^2} \int_{M^4} \sum_{i=1}^{4} c_i \mathcal{L}_i \,, \quad \delta\Gamma_{\text{HLS}}^{\text{inv}} = 0 \,, \tag{2}$$

where the explicit expression of $\mathcal{L}_i$ will be given later. Then the general solution $\Gamma_{\text{HLS}}^{\text{full}}$ to the WZ anomaly equation Eq.(1) is given by the sum of the special solution $\Gamma_{\text{WZW}}$ and the gauge invariant terms in Eq.(2), $\Gamma_{\text{HLS}}^{\text{inv}} = \Gamma_{\text{HLS}}^{\text{inv}}[V, \mathcal{L}, \mathcal{R}, \xi_L^\dagger \xi_R]$, having odd-property under intrinsic parity (IP-odd):

$$\Gamma_{\text{HLS}}^{\text{full}}[V, \mathcal{L}, \mathcal{R}, \xi_L^\dagger \xi_R] = \Gamma_{\text{WZW}}[\mathcal{L}, \mathcal{R}, U] + \Gamma_{\text{HLS}}^{\text{inv}}[V, \mathcal{L}, \mathcal{R}, \xi_L^\dagger \xi_R] \,, \tag{3}$$

where $U$ was decomposed as $U = \xi_L^\dagger \xi_R$ [4–6] and the vector mesons ($V$) as the gauge bosons of the HLS were involved. This gauge invariant part $\Gamma_{\text{HLS}}^{\text{inv}}$ fully describes the $\omega$-$Z$-$\gamma$ vertex as well as the other IP-odd hadronic processes such as $\omega$-$\pi^0$-$\gamma$ and $\omega$-$\pi^0$-$\pi^+$-$\pi^-$ which were intensively studied in Refs. [3, 4, 6]. It is remarkable that these processes are *not* determined by the anomaly, but by the *invariant* part $\Gamma_{\text{HLS}}^{\text{inv}}$ which contains free parameters $c_1$-$c_4$.

On the other hand, it has recently been advocated by Harvey-Hill-Hill (HHH) [7, 8] that the interactions such as $\omega$-$Z$-$\gamma$ are uniquely determined by the non-Abelian anomaly in the presence of "background fields" $B_L$ and $B_R$ for vector and axialvector mesons in addition to the external gauge fields $\mathcal{L}$ and $\mathcal{R}$. Based on the resultant action ("HHH action"), they further suggested that the $\omega$-$Z$-$\gamma$ vertex can explain an excess of electron-like events in a low-energy region of neutrino-nucleon collision processes observed at the Fermilab Booster Neutrino Experiment MiniBooNE [9–11].

In this paper, we shall first present explicit form of the $\omega$-$Z$-$\gamma$ vertex arising from the gauge-invariant (anomaly-free) term in the HLS formalism. We then clarify the claim of HHH that the $\omega$-$Z$-$\gamma$ vertex is determined by the anomaly, which is in obvious contradiction with the HLS formalism.

The HHH incorporated the "background fields" $B_L$ and $B_R$ transforming homogeneously under the chiral (gauge) symmetry into the WZW term, $\Gamma_{\text{WZW}}[\mathcal{L}, \mathcal{R}, U]$, in such a way that $\mathcal{L}$ and $\mathcal{R}$ are simply replaced as $\mathcal{L} \to \mathcal{L} + B_L$ and $\mathcal{R} \to \mathcal{R} + B_R$:

$$\Gamma_{\text{HHH}}[\mathcal{L}, \mathcal{R}, B_L, B_R, U] = \Gamma_{\text{WZW}}[\mathcal{L} + B_L, \mathcal{R} + B_R, U] \,. \tag{4}$$

They included the counterterm $\Gamma_c[\mathcal{L}, \mathcal{R}, B_L.B_R]$, what they call "generalized Bardeen's counterterm", so that the full action under the presence of $B_L$ and $B_R$,

$$\Gamma_{\text{HHH}}^{\text{full}}[\mathcal{L}, \mathcal{R}, B_L, B_R, U] \equiv \Gamma_{\text{HHH}}[\mathcal{L}, \mathcal{R}, B_L, B_R, U] + \Gamma_c[\mathcal{L}, \mathcal{R}, B_L, B_R] \,, \tag{5}$$

satisfies the WZ anomaly equation Eq.(1): $\delta\Gamma_{\text{HHH}}^{\text{full}} = \mathbf{A}$. They claimed that counter term $\Gamma_c$ should be uniquely determined by the anomaly through Eq.(1). This would imply that the difference of them

$$\Delta\Gamma_{\text{HHH}}[\mathcal{L}, \mathcal{R}, B_L, B_R, U] \equiv \Gamma_{\text{HHH}}^{\text{full}}[\mathcal{L}, \mathcal{R}, B_L, B_R, U] - \Gamma_{\text{WZW}}[\mathcal{L}, \mathcal{R}, U] \tag{6}$$

should also be determined by the anomaly.

Note, however, that both $\Gamma_{\text{HHH}}^{\text{full}}$ and $\Gamma_{\text{WZW}}(= \Gamma_{\text{HHH}}[B_L = B_R = 0])$ in Eq.(6) satisfy the same anomaly equation (1), i.e., $\delta\Gamma_{\text{HHH}}^{\text{full}} = \delta\Gamma_{\text{WZW}} = \mathbf{A}$, and hence $\Delta\Gamma_{\text{HHH}}$ should be invariant (anomaly-free) under the gauge transformation:

$$\delta\left(\Delta\Gamma_{\text{HHH}}[\mathcal{L}, \mathcal{R}, B_L, B_R, U]\right) = 0 \,, \tag{7}$$

in contradiction with the HHH claim. Actually, there are a lot of solutions which satisfy Eq.(7). We shall call the general solution to Eq.(7) $\Gamma_{\text{G-HHH}}^{\text{inv}}$ (G-HHH action),

$$\delta\Gamma_{\text{G-HHH}}^{\text{inv}} = 0 \,, \tag{8}$$

which is not determined by the anomaly, precisely the same situation as $\Gamma_{\text{HLS}}^{\text{inv}}$ in the HLS formulation.

In order to make the above our argument more explicit, we shall present the general solution to Eq.(8), the G-HHH action $\Gamma_{\text{G-HHH}}^{\text{inv}}$, in the framework that HHH was based on. (We call it "HHH formulation"). It turns out that $\Gamma_{\text{G-HHH}}^{\text{inv}}$ is given by a linear combination of fourteen chiral (gauge) invariant IP-odd terms, which clarifies that the definite form of the HHH action $\Delta\Gamma_{\text{HHH}}$ is actually a *particular* choice of the general solution $\Gamma_{\text{G-HHH}}^{\text{inv}}$.

To be concrete, we next discuss $\omega$-$Z$-$\gamma$ vertex. We formulate the general form of $\omega$-$Z$-$\gamma$ vertex arising from the gauge invariant HLS action $\Gamma_{\text{HLS}}^{\text{inv}}$ in Eq.(2) as well as the G-HHH action $\Gamma_{\text{G-HHH}}^{\text{inv}}$ having free parameters. In spite of the free parameters, certain combinations of the physical quantities can be fixed independently of these free parameters by taking the ratio having the same parameter-dependence. We find that the $\omega$-$Z$-$\gamma$ vertex is related to the $\omega$-$\pi^0$-$\gamma$ vertex through the Ward-Takahashi identity. We evaluate contributions from the $\omega$-$Z$-$\gamma$ vertex to a neutrino ($\nu$) - nucleon ($N$) cross section $\sigma(\nu N \rightarrow \nu N^{(')}\gamma)$ using the experimental input for the $\omega \rightarrow \pi^0\gamma$ decay. Furthermore, existence of other free parameters in the general solution enables us to make the best fit of the $\omega$-$\pi^0$-$\gamma^*$ process, which is substantially different from the HHH action. Based on the best fit parameter choice, we give a prediction of the cross section $\sigma(\nu N \rightarrow \nu N\gamma^*)$ to be tested at $\nu$-$N$ collision experiments in the future.

This paper is organized as follows: In Sec. II we derive the explicit expression of $\omega$-$Z$-$\gamma$ vertex arising from the gauge invariant IP-odd terms in the HLS formulation. In Sec. III to make a comparison of the HLS result with the HHH one, we show that the $\omega$-$Z$-$\gamma$ vertex is not determined by the anomaly but by the general solution to Eq.(8), the G-HHH action $\Gamma_{\text{G-HHH}}^{\text{inv}}$, which also has free parameters, based on the same framework that HHH was based on. We then demonstrate that the HHH action $\Delta\Gamma_{\text{HHH}}$ is nothing but a particular choice of $\Gamma_{\text{G-HHH}}^{\text{inv}}$. In Sec. IV we discuss phenomenological applications associated with the $\omega$-$Z$-$\gamma$ vertex: $\sigma(\nu N \rightarrow \nu N\gamma)$ and $\sigma(\nu N \rightarrow \nu N\gamma^*(l^-l^+))$. Summary is devoted to Sec. V. In Appendix A we show the explicit relation between the HLS and HHH action by integrating out the axialvector mesons of the HHH action. In Appendix B we will also show the relation between the general solution $\Gamma_{\text{G-HHH}}^{\text{inv}}$ and the IP-odd gauge invariant terms [12] formulated in the generalized HLS (GHLS) [4, 13, 14].

## II. THE $\omega$-$Z$-$\gamma$ VERTEX IN THE HLS FORMALISM

We begin by briefly reviewing the HLS formalism [4–6] to introduce the explicit form of the gauge invariant IP-odd terms in $\Gamma_{\text{HLS}}^{\text{inv}}$ of Eq.(2) [3]. The basic dynamical variables are nonlinear base $\xi_{L,R}(x)$ embedded into the chiral field $U(x)$ as $U(x) = \exp(2i\pi(x)/F_\pi) = \xi_L^\dagger(x)\xi_R(x)$ associated with the spontaneously breaking of global chiral symmetry $G_{\text{global}} = U(N_f)_L \times U(N_f)_R$ down to $H_{\text{global}} = U(N_f)_{V=L+R}$, where $\pi(x)$ denotes the NGB fields having the decay constant $F_\pi$ and $N_f$ the number of massless quark flavors (quark mass is disregarded throughout this paper). The chiral field $U$ transforms as $U \rightarrow g_L U g_R^\dagger$ with $g_{L,R} \in G_{\text{global}}$. There is an arbitrariness or a gauge degree of freedom (HLS) in dividing $U(x)$ into a product of $\xi_L^\dagger$ and $\xi_R$ in such a way that they transform as $\xi_{L,R} \rightarrow h(x)\xi_{L,R}g_{L,R}^\dagger$ and do the HLS gauge fields as $V_\mu(x) \rightarrow h(x)V_\mu(x)h^\dagger(x) + ih(x)\partial_\mu h^\dagger(x)$ with $h(x) \in H_{\text{local}}$. After gauge fixing of HLS, $H_{\text{local}}$, the direct sum of the $H_{\text{local}}$ and the subgroup $H_{\text{global}}(\in G_{\text{global}})$ becomes $H$ of the usual nonlinear sigma model manifold $G/H$. As done in the nonlinear sigma model, we may freely gauge $G_{\text{global}}$ by introducing the external gauge fields $\mathcal{L}_\mu(x)$ and $\mathcal{R}_\mu(x)$ including the standard model gauge bosons $W, Z, \gamma$ through the standard promotion $g_{L,R} \Rightarrow g_{L,R}(x)$.

The HLS action $\Gamma_{\text{HLS}}^{\text{inv}}$ is thus constructed from invariant terms under parity ($P$) and charge conjugation ($C$) with IP-odd property [#1] as well as the gauge transformation:

$$\delta_{\text{HLS}}\Gamma_{\text{HLS}}^{\text{inv}} = 0\,,$$
$$\delta_{\text{HLS}}: \quad \xi_{L,R} \rightarrow h(x)\xi_{L,R}g_{L,R}^\dagger(x)\,,$$
$$\mathcal{L} \rightarrow g_L(x)\mathcal{L}g_L^\dagger(x) + ig_L(x)dg_L^\dagger(x)\,,$$
$$\mathcal{R} \rightarrow g_R(x)\mathcal{R}g_R^\dagger(x) + ig_R(x)dg_R^\dagger(x)\,,$$
$$V \rightarrow h(x)Vh^\dagger(x) + ih(x)dh^\dagger(x)\,, \tag{9}$$

where the differential form has been introduced: $d = (\partial_\mu)dx^\mu$, $V = V_\mu dx^\mu$, and so on. The explicit expression of $\Gamma_{\text{HLS}}^{\text{inv}}$

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[#1] The intrinsic parity of a particle is assigned to be even, if its parity equals $(-1)^{\text{spin}}$, and odd otherwise.
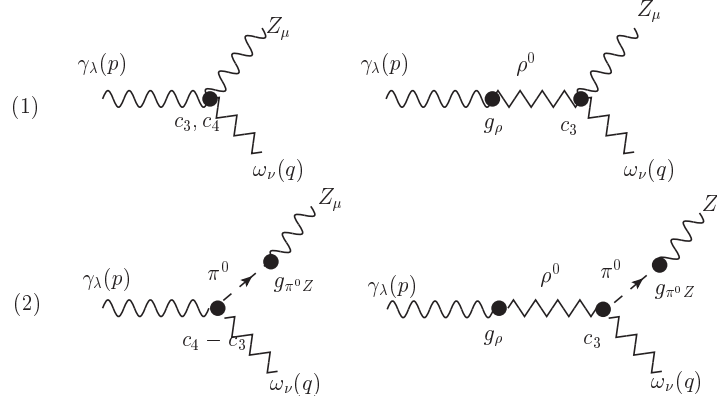
FIG. 1: The diagrams relevant to the $\omega$-$Z$-$\gamma$ vertex (18).

takes the form [3, 4, 6]:

$$\Gamma^{\mathrm{inv}}_{\mathrm{HLS}}[V, \mathcal{L}, \mathcal{R}, \xi_L^\dagger \xi_R] = \frac{N_c}{16\pi^2} \int_{M^4} \sum_{i=1}^{4} c_i \mathcal{L}_i \,, \tag{10}$$

$$\mathcal{L}_1 = i\mathrm{tr}[\hat{\alpha}_L^3 \hat{\alpha}_R - \hat{\alpha}_R^3 \hat{\alpha}_L] \,, \tag{11}$$

$$\mathcal{L}_2 = i\mathrm{tr}[\hat{\alpha}_L \hat{\alpha}_R \hat{\alpha}_L \hat{\alpha}_R] \,, \tag{12}$$

$$\mathcal{L}_3 = \mathrm{tr}[F_V (\hat{\alpha}_L \hat{\alpha}_R - \hat{\alpha}_R \hat{\alpha}_L)] \,, \tag{13}$$

$$\mathcal{L}_4 = \frac{1}{2}\mathrm{tr}[\hat{\mathcal{F}}_L (\hat{\alpha}_L \hat{\alpha}_R - \hat{\alpha}_R \hat{\alpha}_L) - \hat{\mathcal{F}}_R (\hat{\alpha}_R \hat{\alpha}_L - \hat{\alpha}_L \hat{\alpha}_R)] \,, \tag{14}$$

where the normalization of $c_1$-$c_4$ terms followed Ref. [6] and

$$\hat{\alpha}_L = \frac{1}{i}d\xi_L\xi_L^\dagger - V + \xi_L \mathcal{L}\xi_L^\dagger \,, \qquad \hat{\alpha}_R = \frac{1}{i}d\xi_R\xi_R^\dagger - V + \xi_R \mathcal{R}\xi_R^\dagger \,,$$

$$\hat{\mathcal{F}}_L = \xi_L \mathcal{F}_L \xi_L^\dagger \,, \qquad \hat{\mathcal{F}}_R = \xi_R \mathcal{F}_R \xi_R^\dagger \,,$$

$$\mathcal{F}_L = d\mathcal{L} - i\mathcal{L}^2 \,, \qquad \mathcal{F}_R = d\mathcal{R} - i\mathcal{R}^2 \,,$$

$$F_V = dV - iV^2 \,. \tag{15}$$

It is evident that $\Gamma^{\mathrm{inv}}_{\mathrm{HLS}}$ is invariant under the gauge transformation Eq.(9).

Note that the $\omega$-$Z$-$\gamma$ vertex is not contained in the anomalous term $\Gamma_{\mathrm{WZW}}[\mathcal{L}, \mathcal{R}, U]$ in Eq.(3), but only exists in the gauge invariant terms $\Gamma^{\mathrm{inv}}_{\mathrm{HLS}}[V, \mathcal{L}, \mathcal{R}, \xi_L^\dagger \xi_R]$ given by Eq.(10) which is *not determined by the anomaly*. We now demonstrate that the $\omega$-$Z$-$\gamma$ vertex indeed arises from $\Gamma^{\mathrm{inv}}_{\mathrm{HLS}}$.

We employ the case with $N_f = 2$ in which the two lightest quarks $(u, d)$ externally couple to the gauge fields $\mathcal{L}$ and $\mathcal{R}$. The $\mathcal{L}$ and $\mathcal{R}$ are then parametrized as

$$\mathcal{L}|_{\mathrm{neutral}} = eQA + \frac{e}{sc}(T^3 - s^2 Q)Z \,, \qquad \mathcal{R}|_{\mathrm{neutral}} = eQ\left(A - \frac{s}{c}Z\right) \,, \tag{16}$$

where we have focused only on neutral gauge boson fields ($Z$ boson and photon ($A$) fields) which are relevant to the present study; $s \equiv \sin\theta_W$ is the weak mixing angle ($c^2 = 1 - s^2$); $e$ the electromagnetic coupling constant; $Q$ the electric charge matrix $Q = \mathrm{diag}(2/3, -1/3)$ and $T^3$ the isospin matrix for $(u, d)$ $T^3 = \mathrm{diag}(1/2, -1/2)$. The vector meson fields $(\rho^{\pm,0}, \omega)$ are embedded in the HLS gauge field $V$ with the gauge couplings $g$ for $\rho^{\pm,0}$ and $g'$ for $\omega$, in such a way that

$$V = \frac{g}{2}\begin{pmatrix} \rho^0 & \sqrt{2}\rho^+ \\ \sqrt{2}\rho^- & -\rho^0 \end{pmatrix} + \frac{g'}{2}\begin{pmatrix} \omega & 0 \\ 0 & \omega \end{pmatrix} \,. \tag{17}$$

Note that $g' = g$ for $U(3)_L \times U(3)_R$ case.

The $\omega$-$Z$-$\gamma$ vertex function is thus constructed from diagrams depicted in Fig. 1 as

$$\Gamma^{\mu\nu\lambda}[q, p+q, p]\bigg|_{\omega Z\gamma} = \frac{N_c}{32\pi^2}\frac{e^2}{sc}g'\left(\Gamma^{\mu\nu\lambda}_{(1)}[q, p+q, p] + \Gamma^{\mu\nu\lambda}_{(2)}[q, p+q, p]\right),\tag{18}$$

$$\Gamma^{\mu\nu\lambda}_{(1)}[q, p+q, p] = (c_3 + c_4)\,\epsilon^{\mu\nu\lambda\rho}p_\rho + c_3\,\epsilon^{\mu\nu\rho\lambda'}(q-p)_\rho D_\rho(p^2)\left(\frac{p^2\delta^\lambda_{\lambda'} - p_{\lambda'}p^\lambda}{m_\rho^2}\right),\tag{19}$$

$$\Gamma^{\mu\nu\lambda}_{(2)}[q, p+q, p] = \frac{(p+q)^\mu}{(p+q)^2}\epsilon^{\nu\lambda\alpha\beta}p_\alpha q_\beta\left[(c_4 - c_3) + 2c_3 D_\rho(p^2)\right],\tag{20}$$

where we have read off the $\rho$ meson propagator $D_\rho(p^2) = m_\rho^2/(m_\rho^2 - p^2)$, the $\rho^0$-$\gamma$ and the $\pi^0$-$Z$ mixing strengths, $g_\rho = m_\rho^2/g$ and $g_{\pi^0 Z} = e/(2sc)F_\pi$, from the IP-even sector [6]. We thus conclude that the $\omega$-$Z$-$\gamma$ vertex includes free parameters $c_3$ and $c_4$ which are not determined by the anomaly in contrast to the claim of HHH [7, 8]. In the next section we show that the $\omega$-$Z$-$\gamma$ vertex in the HHH formulation [7] also arises from the anomaly free (gauge invariant) terms having free parameters which are not determined by the anomaly.

Although the $\omega$-$Z$-$\gamma$ vertex generically includes the undetermined parameters $c_3$ and $c_4$, it turns out that the form can be fixed by using phenomenological inputs associated with $\omega$-$\pi^0$-$\gamma$ vertex: This is possible because of the fact that the $\omega$-$Z$-$\gamma$ vertex is related to the $\omega$-$\pi^0$-$\gamma$ vertex by the chiral symmetry through the Ward-Takahashi identity,

$$k_\nu\,\Gamma^{\mu\nu\lambda}[p+k, k, p]\bigg|_{\omega Z\gamma} = k_\nu\left(\Gamma^{\mu\nu\lambda}_{(1)}[p+k, k, p] + \Gamma^{\mu\nu\lambda}_{(2)}[p+k, k, p]\right) = 0,\tag{21}$$

which reads

$$k_\nu\,\Gamma^{\mu\nu\lambda}_{(1)}[p+k, k, p]\bigg|_{\omega Z\gamma} = -k_\nu\,\Gamma^{\mu\nu\lambda}_{(2)}[p+k, k, p]\bigg|_{\omega Z\gamma} = \frac{e}{2sc}F_\pi\,\Gamma^{\mu\lambda}[p+k, k, p]\bigg|_{\omega\pi^0\gamma}.\tag{22}$$

The amplitudes concerning the $\omega$-$Z$-$\gamma$ vertex can thus be free from $c_3$ and $c_4$ to be fixed by using experimental values associated with the $\omega$-$\pi^0$-$\gamma$ process, as we will see more explicitly later.

## III. THE GENERAL SOLUTION IN THE HHH FORMULATION

We have shown that the $\omega$-$Z$-$\gamma$ vertex comes from the anomaly-free term in the HLS formulation. As we discussed in the Introduction, the same should be the case also in the framework that HHH was based on. In this section, in order to make the argument more concrete, we present the explicit form of the general solution, $\Gamma^{\rm inv}_{\rm G-HHH}[\mathcal{L}, \mathcal{R}, B_L, B_R, U]$, to the homogeneous part of the WZ anomaly equation, Eq.(8), including "background fields" $B_L$ and $B_R$ introduced in Refs. [7, 8]. It is shown that the HHH action $\Delta\Gamma_{\rm HHH}$ defined in Eq.(6) is a particular choice of the general solution, so is the expression of the $\omega$-$Z$-$\gamma$ vertex given by HHH [7, 8]:

$$S_{\omega Z\gamma}\bigg|_{\rm HHH} = \frac{N_c}{16\pi^2}\frac{e^2}{sc}g'\int_{M^4}\omega Z dA.\tag{23}$$

The explicit comparison of the HHH action with the HLS action $\Gamma^{\rm inv}_{\rm HLS}$ in Eq.(10) will be given in Appendix A.

The general solution to Eq.(8) is constructed from the chiral (gauge) invariant terms having $P$- and $C$-even but IP-odd properties. The building blocks are classified into two pieces: variables transforming homogeneously with respect to either $g_L(x)$ or $g_R(x)$. The possible set of the covariant variables is as follows:

$$\begin{aligned}\mathcal{O}_L &= \{B_L, UB_RU^\dagger, \mathcal{D}B_L, \mathcal{F}_L, \mathcal{D}UU^\dagger\} & \mathcal{O}_L &\to g_L(x)\mathcal{O}_L g_L^\dagger(x),\\\mathcal{O}_R &= \{B_R, U^\dagger B_L U, \mathcal{D}B_R, \mathcal{F}_R, \mathcal{D}U^\dagger U\} & \mathcal{O}_R &\to g_R(x)\mathcal{O}_R g_R^\dagger(x),\end{aligned}\tag{24}$$

where

$$\mathcal{D}U = dU - i\mathcal{L}U + iU\mathcal{R},\tag{25}$$

$$\mathcal{D}B_L = dB_L - i(\mathcal{L}B_L + B_L\mathcal{L}),\tag{26}$$

$$\mathcal{D}B_R = dB_R - i(\mathcal{R}B_R + B_R\mathcal{R}).\tag{27}$$

With these at hand, we can find the general solution to Eq.(8),

$$\Gamma^{\text{inv}}_{\text{G−HHH}}[\mathcal{L}, \mathcal{R}, B_L, B_R, U] \; = \; \frac{N_c}{16\pi^2} \int_{M^4} \sum_{i=1}^{14} a_i \mathcal{O}_i \,, \tag{28}$$

which consists of a linear combination of the following fourteen terms [#2]:

$$\begin{aligned}
\mathcal{O}_1 &= i \operatorname{tr}[B_L^3 U B_R U^\dagger - B_R^3 U^\dagger B_L U]\,, \\
\mathcal{O}_2 &= i \operatorname{tr}[B_L U B_R U^\dagger B_L U B_R U^\dagger]\,, \\
\mathcal{O}_3 &= \operatorname{tr}[(\mathcal{D}B_L B_L + B_L \mathcal{D}B_L) U B_R U^\dagger - (\mathcal{D}B_R B_R + B_R \mathcal{D}B_R) U^\dagger B_L U]\,, \\
\mathcal{O}_4 &= \operatorname{tr}[(\mathcal{F}_L B_L + B_L \mathcal{F}_L) U B_R U^\dagger - (\mathcal{F}_R B_R + B_R \mathcal{F}_R) U^\dagger B_L U]\,, \\
\mathcal{O}_5 &= \operatorname{tr}[B_L^3 (\mathcal{D}U U^\dagger) - B_R^3 (\mathcal{D}U^\dagger U)]\,, \\
\mathcal{O}_6 &= \operatorname{tr}[(B_L^2 U B_R U^\dagger + U B_R U^\dagger B_L^2) \mathcal{D}U U^\dagger - (B_R^2 U^\dagger B_L U + U^\dagger B_L U B_R^2) \mathcal{D}U^\dagger U]\,, \\
\mathcal{O}_7 &= \operatorname{tr}[B_L U B_R U^\dagger B_L (\mathcal{D}U U^\dagger) - B_R U^\dagger B_L U B_R (\mathcal{D}U^\dagger U)]\,, \\
\mathcal{O}_8 &= i \operatorname{tr}[(\mathcal{D}B_L B_L + B_L \mathcal{D}B_L) \mathcal{D}U U^\dagger - (\mathcal{D}B_R B_R + B_R \mathcal{D}B_R) \mathcal{D}U^\dagger U]\,, \\
\mathcal{O}_9 &= i \operatorname{tr}[(\mathcal{D}B_L U B_R U^\dagger + U B_R U^\dagger \mathcal{D}B_L) \mathcal{D}U U^\dagger - (\mathcal{D}B_R U^\dagger B_L U + U^\dagger B_L U \mathcal{D}B_R) \mathcal{D}U^\dagger U]\,, \\
\mathcal{O}_{10} &= i \operatorname{tr}[(\mathcal{F}_L B_L + B_L \mathcal{F}_L) \mathcal{D}U U^\dagger - (\mathcal{F}_R B_R + B_R \mathcal{F}_R) \mathcal{D}U^\dagger U]\,, \\
\mathcal{O}_{11} &= i \operatorname{tr}[(\mathcal{F}_L U B_R U^\dagger + U B_R U^\dagger \mathcal{F}_L) \mathcal{D}U U^\dagger - (\mathcal{F}_R U^\dagger B_L U + U^\dagger B_L U \mathcal{F}_R) \mathcal{D}U^\dagger U]\,, \\
\mathcal{O}_{12} &= i \operatorname{tr}[B_L U B_R U^\dagger (\mathcal{D}U U^\dagger)^2 - B_R U^\dagger B_L U (\mathcal{D}U^\dagger U)^2]\,, \\
\mathcal{O}_{13} &= i \operatorname{tr}[(B_L \mathcal{D}U U^\dagger)^2 - (B_R \mathcal{D}U^\dagger U)^2]\,, \\
\mathcal{O}_{14} &= \operatorname{tr}[B_L (\mathcal{D}U U^\dagger)^3 - B_R (\mathcal{D}U^\dagger U)^3]\,.
\end{aligned} \tag{29}$$

Because the terms in Eq.(28) independently satisfy the homogeneous part of the WZ anomaly equation, Eq.(8), the coefficients $a_1, a_2, \cdots, a_{14}$ cannot be fixed by the anomaly and hence should be treated as free parameters. This is in sharp contrast to the claim given by HHH [8] where the definite expression of the HHH action is given without any free parameters. It turns out that the HHH action actually corresponds to a particular choice for the free parameters $a_1, a_2, \cdots, a_{14}$, which is read off from Ref. [8] as

$$\begin{aligned}
a_1 &= \frac{1}{3}\,, & a_2 &= \frac{1}{6}\,, & a_3 &= -\frac{1}{3}\,, & a_4 &= -\frac{1}{3}\,, \\
a_5 &= \frac{1}{3}\,, & a_6 &= 0\,, & a_7 &= \frac{1}{3}\,, & a_8 &= \frac{1}{3}\,, \\
a_9 &= \frac{1}{6}\,, & a_{10} &= \frac{2}{3}\,, & a_{11} &= \frac{1}{3}\,, & a_{12} &= \frac{1}{3}\,, \\
a_{13} &= -\frac{1}{6}\,, & a_{14} &= \frac{1}{3}\,.
\end{aligned} \tag{30}$$

We may therefore express the HHH action $\Delta\Gamma_{\text{HHH}}$ defined in Eq.(6) as

$$\Delta\Gamma_{\text{HHH}} = \Gamma^{\text{inv}}_{\text{G−HHH}} \quad \text{with} \quad a_{1-14} \quad \text{in} \quad \text{Eq.(30)}\,. \tag{31}$$

By putting the $\omega$ meson field into $B_L$ and $B_R$ as done in Ref. [8]

$$B_L + B_R = g' \begin{pmatrix} \omega & 0 \\ 0 & \omega \end{pmatrix} + \cdots \,, \tag{32}$$

the $\omega$-$Z$-$\gamma$ vertex is read off from the general solution Eq.(28):

$$S_{\omega Z\gamma} = \frac{N_c}{16\pi^2} \frac{e^2}{sc} g'(a_{10} + a_{11}) \int_{M^4} \omega Z dA \,, \tag{33}$$

---

[#2] Without $a_1$ meson field, the number of the independent operators $\mathcal{O}_i$ is reduced to four as in the HLS formalism (See Appendix A). Similar four terms were introduced in Ref. [15] based on the massive Yang-Mills approach for vector meson field. On the other hand, if the external fields are turned off in Eq.(28) keeping the degree of freedom of $a_1$ meson, one would be left with eleven terms. In Ref. [16] similar eleven terms were discussed based on the massive Yang-Mills approach.

which is reduced to the HHH result [8] in Eq.(23) for $a_{10} + a_{11} = 1$ (See Eq.(30)).

The expression of $\Gamma^{\text{inv}}_{\text{G}-\text{HHH}}$ in Eq.(28) actually becomes identical to that obtained in the framework of the GHLS [4, 13, 14] which includes fourteen invariant terms as well [12]. This can be seen just by converting the gauge fields $L$ and $R$ associated with the GHLS $G_{\text{local}} = [U(N_f)_L \times U(N_f)_R]_{\text{local}}$ into the background fields $B_L$ and $B_R$ through a certain operation as shown in Appendix B.

## IV. APPLICATION TO NEUTRINO-NUCLEON COLLISION PROCESSES

In this section we shall address the phenomenological application of the $\omega$-$Z$-$\gamma$ vertex to neutrino ($\nu$) - nucleon ($N$) collision processes obtained from the HLS formulation as well as the HHH formulation.

We first discuss contributions to a $\nu N \to \nu N \gamma$ process coming from the $\omega$-$Z$-$\gamma$ vertex as depicted in Fig. 2. We consider the heavy nucleon limit so that the nucleon does not move to be completely stationary. In this limit the pion exchange contributions corresponding to terms in $\Gamma^{\mu\nu\lambda}_{(2)}$ of Eq.(18) vanish in the amplitude. Integrating out the $Z$ boson and replacing it with the neutrino current $J_\mu = \bar{\nu}_L \gamma_\mu \nu_L$, we can then write the effective action relevant to this process as

$$S_{\text{eff}} = \kappa \int d^4x d^4y \epsilon^{0ijk} \delta^{(3)}(\vec{x}) \, \mathcal{F}_{\omega^* Z^* \gamma^*}(x - y) \, J_i(y) F_{jk}(y) \,, \tag{34}$$

where $F_{jk}$ stands for the photon field strength. The $\omega^*$-$Z^*$-$\gamma^*$ form factor, the Fourier transformation of $\mathcal{F}_{\omega^* Z^* \gamma^*}(x-y)$, can be read off from the diagram shown in Fig. 2:

$$\mathcal{F}_{\omega^* Z^* \gamma^*}(q^2) = \frac{1 - \bar{c}}{2} + \frac{1 + \bar{c}}{2} D_\rho(q^2) \,, \tag{35}$$

where

$$\bar{c}\big|_{\text{HLS}} = \frac{c_3 - c_4}{c_3 + c_4} \,, \tag{36}$$

for the HLS formulation (See Eq.(18)) and likewise

$$\bar{c}\big|_{\text{G}-\text{HHH}} = \frac{2(a_8 + a_9)}{a_{10} + a_{11}} - 1 \,, \tag{37}$$

for the HHH formulation. The overall coupling $\kappa$ is given by

$$\kappa\big|_{\text{HLS}} = \frac{N_c}{8\sqrt{2}\pi^2} \frac{e g_\omega G_F}{m_\omega^2} \frac{g'(c_3 + c_4)}{2} \,, \tag{38}$$

$$\kappa\big|_{\text{G}-\text{HHH}} = \frac{N_c}{8\sqrt{2}\pi^2} \frac{e g_\omega G_F}{m_\omega^2} g'(a_{10} + a_{11}) \,. \tag{39}$$

Here $g_\omega$ is a coupling strength between the nucleon current $J^N_\mu = \bar{N}\gamma_\mu N$ and the $\omega$ meson defined by $\mathcal{L}_{\omega NN} = g_\omega \omega^\mu J^N_\mu$. From the effective action (34), we compute the total cross section $\sigma(\nu N \to \nu N \gamma)$ as a function of the incident neutrino energy $E_\nu$ evaluated at the rest frame of nucleon, with nucleon recoil being neglected: For the HLS formulation, we obtain

$$\sigma(\nu N \to \nu N \gamma)\bigg|_{\text{HLS}} = \frac{3\alpha g_\omega^2 G_F^2}{640\pi^6 m_\omega^4} \left(\frac{g'(c_3 + c_4)}{2}\right)^2 E_\nu^6 \,, \tag{40}$$

and similarly for the HHH formulation:

$$\sigma(\nu N \to \nu N \gamma)\bigg|_{\text{G}-\text{HHH}} = \frac{3\alpha g_\omega^2 G_F^2}{640\pi^6 m_\omega^4} g'^2 (a_{10} + a_{11})^2 E_\nu^6 \,. \tag{41}$$

The cross section $\sigma(\nu N \to \nu N \gamma)$ is thus determined by the free parameters $c_3$ and $c_4$ ($a_{10}$ and $a_{11}$) not fixed by the anomaly, in contrast to the claim of HHH [7]: The HHH result [7] is nothing but a particular choice taking $a_{10} + a_{11} = 1$ (See Eq.(30)):

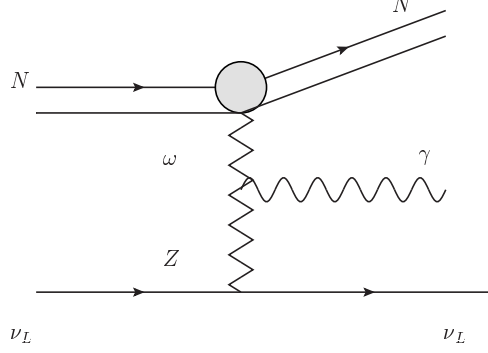$$\sigma(\nu N \to \nu N \gamma)\bigg|_{\text{HHH}} = \frac{3\alpha g_\omega^2 G_F^2}{640\pi^6 m_\omega^4} g'^2 E_\nu^6 \,. \tag{42}$$

FIG. 2: The $\nu N \to \nu N \gamma$ process arising from the $\omega$-$Z$-$\gamma$ vertex.

Although the cross section $\sigma(\nu N \to \nu N \gamma)$ includes the free parameters, it turns out that it can be fixed by using phenomenological inputs associated with $\omega$-$\pi^0$-$\gamma$ vertex. This is possible due to the Ward-Takahashi identity (21) associated with the chiral symmetry regarding the $Z$-boson current: Setting the $\omega$ momentum to zero $p + k = 0$ in Eq.(21), which corresponds to our process, we have

$$k_\nu \, \Gamma^{\mu\nu\lambda}_{(1)}[0, k, -k]\bigg|_{\omega Z \gamma} = \frac{e}{2sc} F_\pi \, \Gamma^{\mu\lambda}[0, k, -k]\bigg|_{\omega\pi^0\gamma} \,. \tag{43}$$

This implies that the cross section $\sigma(\nu N \to \nu N \gamma)$ is expressed by using the $\omega \to \pi^0 \gamma$ decay width $\Gamma(\omega \to \pi^0 \gamma)$: In the HLS formulation we have [6]

$$\Gamma(\omega \to \pi^0 \gamma)\bigg|_{\text{HLS}} = \frac{3\alpha}{64\pi^4 F_\pi^2} \left( \frac{m_\omega^2 - m_{\pi^0}^2}{2m_\omega} \right)^3 \left( \frac{g'(c_3 + c_4)}{2} \right)^2 \,. \tag{44}$$

Likewise in the HHH formulation, we have

$$\Gamma(\omega \to \pi^0 \gamma)\bigg|_{\text{G-HHH}} = \frac{3\alpha}{64\pi^4 F_\pi^2} \left( \frac{m_\omega^2 - m_{\pi^0}^2}{2m_\omega} \right)^3 g'^2(a_{10} + a_{11})^2 \,. \tag{45}$$

Although $\Gamma(\omega \to \pi^0 \gamma)|_{\text{HLS}}$ and $\Gamma(\omega \to \pi^0 \gamma)|_{\text{G-HHH}}$ have free parameters ($c_3$, $c_4$) in the HLS formulation and ($a_{10}$, $a_{11}$) in the HHH formulation, respectively, the ratio

$$\frac{\sigma(\nu N \to \nu N \gamma)}{\Gamma(\omega \to \pi^0 \gamma)} = \frac{g_\omega^2}{10\pi^2} \left( \frac{2m_\omega}{m_\omega^2 - m_{\pi^0}^2} \right)^3 \frac{G_F^2 F_\pi^2 E_\nu^6}{m_\omega^4} \tag{46}$$

is free from the parameters and hence is fixed (up to $g_\omega$) once the experimental value of $\Gamma(\omega \to \pi^0 \gamma)$ is used as input. We thus evaluate the cross section to get

$$\sigma(\nu N \to \nu N \gamma) = 3.0 \times 10^{-41} \, \text{cm}^2 \left( \frac{\Gamma(\omega \to \pi^0 \gamma)}{0.70 \, \text{MeV}} \right) \left( \frac{g_\omega}{13.4} \right)^2 \left( \frac{E_\nu}{\text{GeV}} \right)^6 \,, \tag{47}$$

where use has been made of the reference values $g_\omega \simeq 13.4$ [17] and $\Gamma(\omega \to \pi^0 \gamma) = 0.70 \pm 0.03$ MeV [18]. As was indicated in Ref. [7], this cross section may explain the excess of electron-like events in a low-energy range of the the quasi-elastic (QE) $\nu$-$N$ collision process, $200 \, \text{MeV} \lesssim E_\nu^{\text{QE}} \lesssim 475 \, \text{MeV}$, which has recently been observed at the Fermilab Booster Neutrino Experiment MiniBooNE [10, 11], where electrons could be mimicked by a hard photon $\gamma$ at the detector.

We shall next present a prediction to $\nu + N \to \nu + N + \gamma^*$ process where $\gamma^*$ can be a charged lepton pair $l^\pm$. From the effective action (34), we straightforwardly evaluate the cross section at the rest-frame of the nucleon with the incident neutrino energy $E_\nu$ and nucleon recoil being neglected, to get

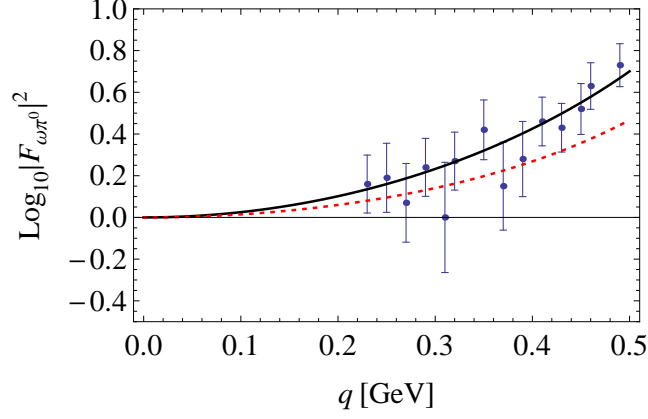$$\sigma(\nu N \to \nu N \gamma^*(q^2)) = \sigma(\nu N \to \nu N \gamma) \cdot f(q^2) \,, \tag{48}$$

FIG. 3: The best fit curve of the $\omega$-$\pi^0$ transition form factor $F_{\omega\pi^0}(q^2)$ yielding $\chi^2/\text{d.o.f} = 4.3/13 = 0.33$ with $\bar{c} = 0.74$ (black solid line) compared with the case of the $\rho$ meson dominance with $\bar{c} = 0$ (red dotted line: $\chi^2/\text{d.o.f} = 23/14 = 1.6$) together with data from the NA60 experiment [19].

where $\sigma(\nu N \to \nu N \gamma)$ is given in Eq.(40) and

$$
\begin{aligned}
f(q^2) \;=\; & |\mathcal{F}_{\omega^* Z^* \gamma^*}(q^2)|^2 \cdot \left[ \left\{ 1 + \frac{133}{16}\frac{q^2}{E_\nu^2} - \frac{643}{32}\left(\frac{q^2}{E_\nu^2}\right)^2 \right\} \sqrt{1 - \frac{q^2}{E_\nu^2}} \right. \\
& \left. + \frac{15}{4}\left\{ 6 + \frac{q^2}{E_\nu^2} \right\} \left(\frac{q^2}{E_\nu^2}\right)^2 \ln\left( \frac{E_\nu + \sqrt{E_\nu^2 - q^2}}{\sqrt{q^2}} \right) \right],
\end{aligned}
\tag{49}
$$

and $\mathcal{F}_{\omega^* Z^* \gamma^*}(q^2)$ is given in Eq.(35). In order to evaluate Eq.(48) explicitly, the free parameter $\bar{c}$ in Eq.(35) needs to be fixed. It turns out that the $\bar{c}$ can be determined by fitting the $\omega$-$\pi^0$ transition form factor to the $\omega \to \pi^0 l^+ l^-$ decay data [19]: Consider the $\omega \to \pi^0 l^+ l^-$ decay width

$$
\begin{aligned}
\Gamma(\omega \to \pi^0 l^+ l^-) \;=\; & \int_{4m_l^2}^{(m_\omega - m_{\pi^0})^2} dq^2 \, \frac{\alpha}{3\pi} \frac{\Gamma(\omega \to \pi^0 \gamma)}{q^2} \left( 1 + \frac{2m_l^2}{q^2} \right) \sqrt{\frac{q^2 - 4m_l^2}{q^2}} \\
& \times \left[ \left( 1 + \frac{q^2}{m_\omega^2 - m_{\pi^0}^2} \right)^2 - \frac{4m_\omega^2 q^2}{(m_\omega^2 - m_{\pi^0}^2)^2} \right]^{3/2} \cdot |F_{\omega\pi^0}(q^2)|^2,
\end{aligned}
\tag{50}
$$

where $F_{\omega\pi^0}$ denotes the $\omega$-$\pi^0$ transition form factor. For $0 \le q^2 \lesssim 1\,\text{GeV}^2$ the general form of $F_{\omega\pi^0}(q^2)$ normalized as $F_{\omega\pi^0}(0) = 1$ is given by

$$
F_{\omega\pi^0}(q^2) = D_\rho(q^2) + \bar{c} \cdot [D_\rho(q^2) - 1].
\tag{51}
$$

Note that $F_{\omega\pi^0}$ includes the same parameter $\bar{c}$ as in $\mathcal{F}_{\omega^* Z^* \gamma^*}$ of Eq.(35), which reflects the fact that the chiral symmetry relates the $\omega$-$\pi^0$-$\gamma$ process with the $\omega$-$Z$-$\gamma$ process through the Ward-Takahashi identity in Eq.(43). Performing the fit to the experimental data on $F_{\omega\pi^0}$ measured at the NA60 experiment [19], we find the best fit value of $\bar{c}$ (with $\chi^2/\text{d.o.f} = 4.3/13 = 0.33$),

$$
\bar{c}|_{\text{best}} = 0.74.
\tag{52}
$$

The corresponding curve of $F_{\omega\pi^0}$ is shown in Fig. 3.

In contrast, the parameter choice Eq.(30) corresponding to the HHH action [8] leads to $\bar{c}|_{\text{HHH}} = 0$ ($\rho$ meson dominance) once a canonical kinetic term of $\rho$ is assumed, and hence does not achieve the best fit form of $F_{\omega\pi^0}$.

Now, using the best fit value $\bar{c}|_{\text{best}} = 0.74$, we obtain Fig. 4 which shows the predicted curve of the total cross section $\sigma(\nu N \to \nu N \gamma^*)$ with $E_\nu = 600$ MeV fixed as a function of the virtual photon momentum $q$ which could be an invariant mass of $l^\pm$ pair production, $q = M_{l^- l^+}$. Remarkably, the cross section $\sigma(\nu N \to \nu N \gamma^*)$ has a peak around
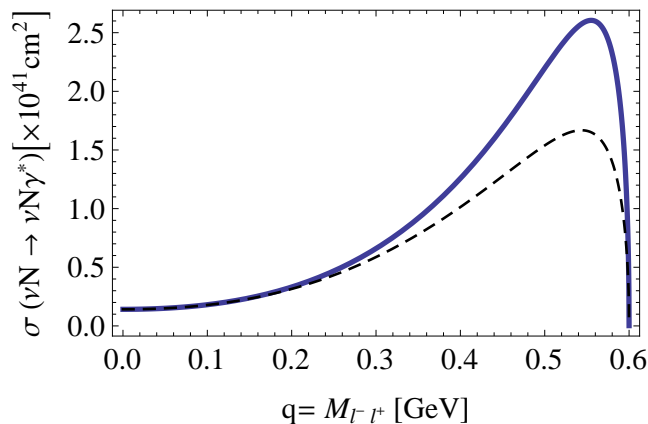
FIG. 4: The total cross section $\sigma(\nu N \to \nu N \gamma^*)$ as a function of $q = M_{l^+ l^-}$ with the incident neutrino energy $E_\nu = 600$ MeV fixed. The solid curve is drawn by using the best fit value of $\bar{c}$, $\bar{c}|_{\text{best}} = 0.74$, while the dotted curve corresponds to the $\rho$ meson dominance ($\bar{c} = 0$).

500 MeV which will yield somewhat larger number of events in this energy range [#3]. This enhancement essentially comes from the dynamical $\rho$-contribution and is independently of the value of $g_\omega$, which is to be tested at $\nu$-$N$ collision experiments like the MiniBooNE in the future.

## V. SUMMARY

In this paper we presented the general homogeneous solution to the WZ anomaly equation in the HHH formulation, which was given by a linear combination of fourteen gauge invariant terms with the coefficients not determined by the anomaly. It was clarified that the definite form of the HHH action $\Delta\Gamma_{\text{HHH}}$ is nothing but a particular expression for the general solution $\Gamma^{\text{inv}}_{\text{G−HHH}}$ in Eq.(28) (See Eq.(31)).

We formulated the general form of $\omega$-$Z$-$\gamma$ vertex arising from the gauge invariant HLS action $\Gamma^{\text{inv}}_{\text{HLS}}$ in Eq.(10) as well as the G-HHH action $\Gamma^{\text{inv}}_{\text{G−HHH}}$ in Eq.(28) having free parameters not determined by the anomaly. In spite of the free parameters, the $\omega$-$Z$-$\gamma$ vertex related to the $\nu$-$N$ collision process was determined by using the experimental input for the $\omega \to \pi^0\gamma$ decay width $\Gamma(\omega \to \pi^0\gamma)$ through the Ward-Takahashi identity. Other set of the free parameters was also used to make the best fit to the experimental data on the $\omega$-$\pi^0$-$\gamma^*$ process, which provided us with a prediction to the cross section $\sigma(\nu N \to \nu N \gamma^*)$ to be tested at $\nu$-$N$ collision experiments in the future.

---

[#3] Higher order terms of $\mathcal{O}(p^4)$ in derivative expansion such as the $z_3$ term would affect the $\rho$-$\gamma$ mixing strength $g_\rho$ by about 10% when it is evaluated at the on-shell of $\rho$ meson [6]. The definition of $\bar{c}$ in Eq.(36) will then be modified involving the $z_3$-term contribution, but the form of Eq.(35) turns out to be intact, so is the prediction about the cross section $\sigma(\nu N \to \nu N \gamma^*)$.

## Appendix A: The explicit relation between the HLS and HHH formulations

In this Appendix we explicitly compare the HHH action Eq.(31) with the HLS action Eq.(10) by eliminating axialvector mesons out of the HHH action. For that purpose, it is convenient to introduce "matter fields" $B_V$ and $B_A$:

$$B_V = \frac{\xi(\pi)B_R\xi^\dagger(\pi) + \xi^\dagger(\pi)B_L\xi(\pi)}{2}, \qquad B_A = \frac{\xi(\pi)B_R\xi^\dagger(\pi) - \xi^\dagger(\pi)B_L\xi(\pi)}{2}, \tag{A.1}$$

where $\xi(\pi)$ and $\xi^\dagger(\pi)$ denote the representatives of coset space $G/H$. Since $\xi(\pi)$ and $\xi^\dagger(\pi)$ transform under $G$ as $\{\xi(\pi), \xi^\dagger(\pi)\} \to \{h(\pi)\xi(\pi)g_L^\dagger, g_R\xi(\pi)h^\dagger(\pi)\}$ where $h(\pi) \in H$, $B_V$ and $B_A$ defined in Eq.(A.1) transform as

$$B_{V,A} \to h(\pi)B_{V,A}h^\dagger(\pi). \tag{A.2}$$

We also introduce a 1-form $\alpha_\perp(\pi)$ to replace $\mathcal{D}U$ terms in Eq.(31) by $\alpha_\perp(\pi) = \xi^\dagger(\pi)\mathcal{D}U\xi^\dagger(\pi)/(2i)$ which transforms in the same way as $B_{V,A}$:

$$\alpha_\perp(\pi) \to h(\pi)\alpha_\perp(\pi)h^\dagger(\pi). \tag{A.3}$$

We now remove axialvector mesons by putting $B_A$ in Eq.(A.1) to be zero and express all the remaining terms in Eq.(31) in terms of $B_V$ and $\alpha_\perp(\pi)$. This operation is equivalent to solving away the axialvector meson field through its equation of motion, just like a way [20] of integrating out higher Kaluza-Klein modes arising in holographic QCD, which keeps the gauge invariance manifestly. We then find that the $\mathcal{O}_1$-$\mathcal{O}_4$, $\mathcal{O}_{12}$ and $\mathcal{O}_{13}$ terms in Eq.(31) vanish and the remaining terms are reduced to only the following four terms:

$$\Delta\Gamma_{\text{HHH}}[B_V, \mathcal{L}, \mathcal{R}, \xi^2(\pi)] = \frac{N_c}{16\pi^2}\int_{M^4}\left[a_1' \, i\text{tr}[\alpha_\perp(\pi)B_V^3] + a_2' \, i\text{tr}[B_V\alpha_\perp^3(\pi)]\right.$$

$$\left. +a_3' \, \text{tr}[(\alpha_\perp(\pi)B_V - B_V\alpha_\perp(\pi))\mathcal{D}B_V] + a_4' \, \text{tr}[(\alpha_\perp(\pi)B_V - B_V\alpha_\perp(\pi))\hat{\mathcal{F}}_V(\pi)]\right], \tag{A.4}$$

with

$$a_1' = -4a_5 - 8a_6 - 4a_7 = -\frac{8}{3},$$
$$a_2' = 16a_{14} = -\frac{16}{3},$$
$$a_3' = 4a_8 + 4a_9 = 2,$$
$$a_4' = 4a_{10} + 4a_{11} = 4, \tag{A.5}$$

where

$$\mathcal{D}B_V = dB_V - i(\alpha_{||}(\pi)B_V + B_V\alpha_{||}(\pi)),$$
$$\alpha_{||}(\pi) = \frac{1}{2i}\left[d\xi(\pi)\xi^\dagger(\pi) + d\xi^\dagger(\pi)\xi(\pi) + i\xi(\pi)\mathcal{R}\xi^\dagger(\pi) + i\xi^\dagger(\pi)\mathcal{L}\xi(\pi)\right],$$
$$\hat{\mathcal{F}}_V(\pi) = \frac{1}{2}(\xi(\pi)\mathcal{F}_R\xi^\dagger(\pi) + \xi^\dagger(\pi)\mathcal{F}_L\xi(\pi)). \tag{A.6}$$

The HLS action $\Gamma_{\text{HLS}}^{\text{inv}}$ in Eq.(10) is, on the other hand, rewritten into the following form:

$$\Gamma_{\text{HLS}}^{\text{inv}}[V, \mathcal{L}, \mathcal{R}, \xi_L^\dagger\xi_R] = \frac{N_c}{16\pi^2}\int_{M^4}\left[-4(c_1 + c_2)i\text{tr}[\hat{\alpha}_\perp\hat{\alpha}_{||}^3] + 4(c_1 - c_2)i\text{tr}[\hat{\alpha}_{||}\hat{\alpha}_\perp^3]\right.$$

$$\left. -2c_3\text{tr}[(\hat{\alpha}_\perp\hat{\alpha}_{||} - \hat{\alpha}_{||}\hat{\alpha}_\perp)F_V] - 2c_4\text{tr}[(\hat{\alpha}_\perp\hat{\alpha}_{||} - \hat{\alpha}_{||}\hat{\alpha}_\perp)\hat{\mathcal{F}}_V]\right], \tag{A.7}$$

where

$$\hat{\alpha}_{||,\perp} = \frac{1}{2}(\hat{\alpha}_R \pm \hat{\alpha}_L),$$
$$\hat{\mathcal{F}}_V = \frac{1}{2}(\hat{\mathcal{F}}_R + \hat{\mathcal{F}}_L). \tag{A.8}$$

Note that, in the unitary gauge of HLS ($\xi_L^\dagger = \xi_R = \xi(\pi)$), $\hat{\alpha}_\parallel$ transforms in the same way as the vector meson field $B_V$ in Eq.(A.4), $\hat{\alpha}_\parallel \to h(\pi)\hat{\alpha}_\parallel h^\dagger(\pi)$ [6], while $\hat{\alpha}_\perp$ becomes identical to $\alpha_\perp(\pi)$ in Eq.(A.4). We may therefore identify $\hat{\alpha}_\parallel$ with the vector meson field $B_V$ in Eq.(A.4), in such a way that $B_V = \eta \cdot \hat{\alpha}_\parallel$ with a parameter $\eta$. In the unitary gauge, Eq.(A.7) is thus expressed in terms of $B_V$ and $\alpha_\perp(\pi)$ to be

$$\Gamma_{\text{HLS: unitary}}^{\text{inv}}[B_V, \mathcal{L}, \mathcal{R}, \xi^2(\pi)]$$

$$= \frac{N_c}{16\pi^2} \int_{M^4} \left[ \left\{ \frac{-4(c_1 + c_2 - c_3)}{\eta^3} \right\} \cdot i\text{tr}[\alpha_\perp(\pi)B_V^3] + \left\{ \frac{4(c_1 - c_2 + c_3)}{\eta} \right\} \cdot i\text{tr}[B_V \alpha_\perp^3(\pi)] \right.$$

$$\left. + \left\{ \frac{2c_3}{\eta^2} \right\} \cdot \text{tr}[(\alpha_\perp(\pi)B_V - B_V\alpha_\perp(\pi))\mathcal{D}B_V] + \left\{ \frac{-2(c_3 + c_4)}{\eta} \right\} \cdot \text{tr}[(\alpha_\perp(\pi)B_V - B_V\alpha_\perp(\pi))\hat{\mathcal{F}}_V(\pi)] \right], \quad \text{(A.9)}$$

which is precisely the same form as the HHH action Eq.(A.4). We thus find the particular choice for $c_1$-$c_4$,

$$c_1 + c_2 - c_3 = \frac{2}{3}\eta^3, \tag{A.10}$$

$$c_1 - c_2 + c_3 = -\frac{4}{3}\eta, \tag{A.11}$$

$$c_3 = \eta^2, \tag{A.12}$$

$$c_3 + c_4 = -2\eta, \tag{A.13}$$

which clarifies that the HHH action corresponds to a particular expression for $\Gamma_{\text{HLS}}^{\text{inv}}$, so does the $\omega$-$Z$-$\gamma$ vertex.

The physical implication of such a choice can be seen as follows: If we apply the parameter choice (A.10)-(A.13) to the HLS formulation and impose the vector meson dominance on the $\pi^0 \to \gamma\gamma$ in accord with the experiment data on the $\pi^0$-$\gamma$ transition form factor [18], we would get $c_3 + c_4 = 2$ [3, 4, 6], which implies $\eta = -1$. Then the above relations indicate that $c_3 = c_4 = 1$, $c_1 = 1/3$, $c_2 = 0$. This is a special case of "complete vector meson dominance" $c_3 = c_4 = 1$, $c_1 - c_2 = 1/3$, which is known to be in contradiction with the experimental data on $\omega \to \pi^0\pi^+\pi^-$ (see for a recent argument Ref. [6]).

## Appendix B: The G-HHH action and the IP-odd gauge invariant terms in the GHLS formalism

In this appendix we relate the G-HHH action $\Gamma_{\text{G-HHH}}^{\text{inv}}$ in Eq.(28) with the IP-odd gauge invariant terms [12] obtained from the GHLS formalism [4, 13, 14].

The GHLS [4, 13, 14] is formulated based on the manifold $G_{\text{global}} \times G_{\text{local}}$ where $G_{\text{global}} = [U(N_f)_L \times U(N_f)_R]_{\text{global}}$ and $G_{\text{local}} = [U(N_f)_L \times U(N_f)_R]_{\text{local}}$ associated with the GHLS. The dynamics of the manifold $G_{\text{global}} \times G_{\text{local}}$ is then described by the dynamical variables $\xi_L$, $\xi_R$ and $\xi_M$ forming the chiral field $U$ as $U = \xi_L^\dagger \xi_M \xi_R$, together with the GHLS gauge fields $L_\mu$ and $R_\mu$. They transform under $G_{\text{global}} \times G_{\text{local}}$ in such a way that

$$\xi_{L,R} \to \tilde{g}_{L,R}(x)\xi_{L,R}g_{L,R}^\dagger, \tag{B.1}$$

$$\xi_M \to \tilde{g}_L(x)\xi_M\tilde{g}_R(x), \tag{B.2}$$

$$L_\mu \to \tilde{g}_L(x)L_\mu\tilde{g}_L^\dagger(x) + i\tilde{g}_L(x)\partial_\mu\tilde{g}_L(x), \tag{B.3}$$

$$R_\mu \to \tilde{g}_R(x)L_\mu\tilde{g}_R^\dagger(x) + i\tilde{g}_R(x)\partial_\mu\tilde{g}_R(x), \tag{B.4}$$

where $g_{L,R} \in G_{\text{global}}$ and $\tilde{g}_{L,R}(x) \in G_{\text{local}}$. The parity ($P$) and charge conjugation ($C$) act on $\xi_{L,R,M}$ and $L_\mu, R_\mu$ as

$$\xi_L \overset{P}{\leftrightarrow} \xi_R, \qquad \xi_M \overset{P}{\leftrightarrow} \xi_M^\dagger, \qquad L_\mu \overset{P}{\leftrightarrow} (-1)^{\mu'} R_\mu,$$

$$\xi_L \overset{C}{\leftrightarrow} \xi_R^*, \qquad \xi_M \overset{C}{\leftrightarrow} \xi_M^T, \qquad L_\mu \overset{C}{\leftrightarrow} -R_\mu^T, \tag{B.5}$$

where $(-1)^{\mu'}$ denotes 1 for $\mu = 0$ and $-1$ for $\mu \neq 0$ regarding the corresponding Lorentz vector field. The external gauge fields $\mathcal{L}$ and $\mathcal{R}$ are introduced by gauging $G_{\text{global}}$ in the usual manner ($g_{L,R} \Rightarrow g_{L,R}(x)$).

It is convenient to introduce the following variables:

$$\omega_{L\mu} = \frac{1}{i}\partial_\mu\xi_L\xi_L^\dagger - L_\mu + \xi_L\mathcal{L}_\mu\xi_L^\dagger, \tag{B.6}$$

$$\omega_{R\mu} = \frac{1}{i}\partial_\mu\xi_R\xi_R^\dagger - R_\mu + \xi_R\mathcal{R}_\mu\xi_R^\dagger, \tag{B.7}$$

$$\omega_{M\mu} = \frac{1}{i}\partial_\mu\xi_M\xi_M^\dagger - L_\mu + \xi_M R_\mu\xi_M^\dagger, \tag{B.8}$$

which transform under $[G_{\text{global}}]_{\text{gauged}} \times G_{\text{local}}$ as follows:

$$\omega_{L\mu} \rightarrow \tilde{g}_L(x)\omega_{L\mu}\tilde{g}_L^\dagger(x), \tag{B.9}$$

$$\omega_{R\mu} \rightarrow \tilde{g}_R(x)\omega_{R\mu}\tilde{g}_R^\dagger(x), \tag{B.10}$$

$$\omega_{M\mu} \rightarrow \tilde{g}_L(x)\omega_{M\mu}\tilde{g}_L^\dagger(x), \tag{B.11}$$

and under $P$- and $C$-inversions:

$$\omega_{L\mu} \overset{P}{\leftrightarrow} (-1)^{\mu'}\omega_{R\mu}, \qquad \omega_{M\mu} \overset{P}{\leftrightarrow} -(-1)^{\mu'}\xi_M^\dagger \omega_{M\mu}\xi_M, \tag{B.12}$$

$$\omega_{L\mu} \overset{C}{\leftrightarrow} -\omega_{R\mu}^T, \qquad \omega_{M\mu} \overset{C}{\leftrightarrow} \xi_M^T\omega_{M\mu}^T\xi_M^*. \tag{B.13}$$

The GHLS, $C$- and $P$-invariant terms having $\epsilon^{\mu\nu\rho\sigma}$-structure (IP-odd) thus take the form [12]:

$$\Gamma_{\text{GHLS}}^{\text{inv}}[L, R, \mathcal{L}, \mathcal{R}, \xi_L^\dagger\xi_M\xi_R] = \frac{N_c}{16\pi^2}\int_{M^4}\sum_{i=1}^{14} c_i\mathcal{L}_i, \tag{B.14}$$

with

$$\mathcal{L}_1 = i\,\text{tr}[\omega_L^3\xi_M\omega_R\xi_M^\dagger - \omega_R^3\xi_M^\dagger\omega_L\xi_M], \tag{B.15}$$

$$\mathcal{L}_2 = i\,\text{tr}[\omega_L\xi_M\omega_R\xi_M^\dagger\omega_L\xi_M\omega_R\xi_M^\dagger], \tag{B.16}$$

$$\mathcal{L}_3 = i\,\text{tr}[\omega_M(\omega_L^3 + \xi_M\omega_R^3\xi_M^\dagger)], \tag{B.17}$$

$$\mathcal{L}_4 = i\,\text{tr}[\omega_M(\omega_L\xi_M\omega_R\xi_M^\dagger\omega_L + \xi_M\omega_R\xi_M^\dagger\omega_L\xi_M\omega_R\xi_M^\dagger)], \tag{B.18}$$

$$\mathcal{L}_5 = i\,\text{tr}[\omega_M(\omega_L^2\xi_M\omega_R\xi_M^\dagger + \omega_L\xi_M\omega_R^2\xi_M^\dagger + \xi_M\omega_R^2\xi_M^\dagger\omega_L + \xi_M\omega_R\xi_M^\dagger\omega_L^2], \tag{B.19}$$

$$\mathcal{L}_6 = i\,\text{tr}[\omega_M^2(\omega_L\xi_M\omega_R\xi_M^\dagger - \xi_M\omega_R\xi_M^\dagger\omega_L)], \tag{B.20}$$

$$\mathcal{L}_7 = i\,\text{tr}[\omega_M(\omega_L\omega_M\omega_L - \xi_M\omega_R\xi_M^\dagger\omega_M\xi_M\omega_R\xi_M^\dagger)], \tag{B.21}$$

$$\mathcal{L}_8 = i\,\text{tr}[\omega_M^3(\omega_L + \xi_M\omega_R\xi_M^\dagger)], \tag{B.22}$$

$$\mathcal{L}_9 = \text{tr}[(F_L + \xi_M F_R\xi_M^\dagger)(\omega_L\xi_M\omega_R\xi_M^\dagger - \xi_M\omega_R\xi_M^\dagger\omega_L)], \tag{B.23}$$

$$\mathcal{L}_{10} = \text{tr}[(F_L + \xi_M F_R\xi_M^\dagger)((\omega_L + \xi_M\omega_R\xi_M^\dagger)\omega_M - \omega_M(\omega_L + \xi_M\omega_R\xi_M^\dagger))], \tag{B.24}$$

$$\mathcal{L}_{11} = \text{tr}[(F_L - \xi_M F_R\xi_M^\dagger)((\omega_L - \xi_M\omega_R\xi_M^\dagger)\omega_M - \omega_M(\omega_L - \xi_M\omega_R\xi_M^\dagger))], \tag{B.25}$$

$$\mathcal{L}_{12} = \text{tr}[(\xi_L\mathcal{F}_L\xi_L^\dagger + \xi_M\xi_R\mathcal{F}_R\xi_R^\dagger\xi_M^\dagger)(\omega_L\xi_M\omega_R\xi_M^\dagger - \xi_M\omega_R\xi_M^\dagger\omega_L)], \tag{B.26}$$

$$\mathcal{L}_{13} = \text{tr}[(\xi_L\mathcal{F}_L\xi_L^\dagger + \xi_M\xi_R\mathcal{F}_R\xi_R^\dagger\xi_M^\dagger)((\omega_L + \xi_M\omega_R\xi_M^\dagger)\omega_M - \omega_M(\omega_L + \xi_M\omega_R\xi_M^\dagger))], \tag{B.27}$$

$$\mathcal{L}_{14} = \text{tr}[(\xi_L\mathcal{F}_L\xi_L^\dagger - \xi_M\xi_R\mathcal{F}_R\xi_R^\dagger\xi_M^\dagger)((\omega_L - \xi_M\omega_R\xi_M^\dagger)\omega_M - \omega_M(\omega_L - \xi_M\omega_R\xi_M^\dagger))], \tag{B.28}$$

where the notation of coefficients for $\mathcal{L}_{1-14}$ followed Ref. [12], and

$$F_L = dL - iL^2, \qquad F_R = dR - iR^2. \tag{B.29}$$

We shall now introduce "background fields" $B_L$ and $B_R$ [8] transforming as $B_{L,R} \rightarrow g_{L,R}(x)B_{L,R}g_{L,R}^\dagger(x)$ and relate them with $\omega_L$ and $\omega_R$ as follows:

$$B_L = \xi_L^\dagger\omega_L\xi_L, \qquad B_R = \xi_R^\dagger\omega_R\xi_R. \tag{B.30}$$

The variable $\omega_M$ and the field strengths of the GHLS fields $L$ and $R$, $F_{L,R}$, are then expressed in terms of the building blocks listed in Eq.(24) as

$$\omega_M = \xi_L(B_L - UB_RU^\dagger - i\mathcal{D}UU^\dagger)\xi_L^\dagger. \tag{B.31}$$

$$F_{L(R)} = \xi_{L(R)}\left(\mathcal{F}_{L(R)} - \mathcal{D}B_{L(R)} - iB_{L(R)}^2\right)\xi_{L(R)}^\dagger. \tag{B.32}$$

Putting Eqs.(B.30)-(B.32) into Eqs.(B.15)-(B.28) we rewrite $\mathcal{L}_1$-$\mathcal{L}_{14}$ in Eq.(B.14) in terms of $\mathcal{O}_1$-$\mathcal{O}_{14}$ in Eq.(28):

$$\mathcal{L}_1 = \mathcal{O}_1 \,, \tag{B.33}$$
$$\mathcal{L}_2 = \mathcal{O}_2 \,, \tag{B.34}$$
$$\mathcal{L}_3 = \mathcal{O}_1 - \mathcal{O}_5 \,, \tag{B.35}$$
$$\mathcal{L}_4 = -\mathcal{O}_1 + 2\mathcal{O}_2 - \mathcal{O}_7 \,, \tag{B.36}$$
$$\mathcal{L}_5 = 2\mathcal{O}_1 - \mathcal{O}_6 \,, \tag{B.37}$$
$$\mathcal{L}_6 = 2\mathcal{O}_1 - 2\mathcal{O}_2 - \mathcal{O}_6 + 2\mathcal{O}_7 - \mathcal{O}_{12} \,, \tag{B.38}$$
$$\mathcal{L}_7 = 2\mathcal{O}_1 - 2\mathcal{O}_2 - 2\mathcal{O}_5 + 2\mathcal{O}_7 + \mathcal{O}_{13} \,, \tag{B.39}$$
$$\mathcal{L}_8 = 2\mathcal{O}_1 - 2\mathcal{O}_2 - \mathcal{O}_5 - \mathcal{O}_6 + 3\mathcal{O}_7 - 2\mathcal{O}_{12} + \mathcal{O}_{13} + \mathcal{O}_{14} \,, \tag{B.40}$$
$$\mathcal{L}_9 = -2\mathcal{O}_1 - \mathcal{O}_3 + \mathcal{O}_4 \,, \tag{B.41}$$
$$\mathcal{L}_{10} = 4\mathcal{O}_1 + 2\mathcal{O}_3 - 2\mathcal{O}_4 - 2\mathcal{O}_5 - \mathcal{O}_6 + \mathcal{O}_8 + \mathcal{O}_9 - \mathcal{O}_{10} - \mathcal{O}_{11} \,, \tag{B.42}$$
$$\mathcal{L}_{11} = -2\mathcal{O}_5 + \mathcal{O}_6 + \mathcal{O}_8 - \mathcal{O}_9 - \mathcal{O}_{10} + \mathcal{O}_{11} \,, \tag{B.43}$$
$$\mathcal{L}_{12} = \mathcal{O}_4 \,, \tag{B.44}$$
$$\mathcal{L}_{13} = -2\mathcal{O}_4 - \mathcal{O}_{10} - \mathcal{O}_{11} \,, \tag{B.45}$$
$$\mathcal{L}_{14} = -\mathcal{O}_{10} + \mathcal{O}_{11} \,. \tag{B.46}$$

The GHLS action (B.14) thus precisely becomes identical to the general solution in the HHH formulation Eq.(28) with the free parameters $a_1$-$a_{14}$ replaced in such a way that

$$a_1 = c_1 + c_3 - c_4 + 2c_5 + 2c_6 + 2c_7 + 2c_8 - 2c_9 + 4c_{10} \,, \tag{B.47}$$
$$a_2 = c_2 - c_4 - 2c_6 - 2c_7 - 2c_8 \,, \tag{B.48}$$
$$a_3 = -c_9 + 2c_{10} \,, \tag{B.49}$$
$$a_4 = c_9 - 2c_{10} + c_{12} - 2c_{13} \,, \tag{B.50}$$
$$a_5 = -c_3 - 2c_7 - c_8 - 2c_{10} - 2c_{11} \,, \tag{B.51}$$
$$a_6 = -2c_5 - c_6 - 2c_8 - c_{10} + c_{11} \,, \tag{B.52}$$
$$a_7 = -c_4 + 2c_6 + 2c_7 + 3c_8 \,, \tag{B.53}$$
$$a_8 = c_{10} + c_{11} \,, \tag{B.54}$$
$$a_9 = c_{10} - c_{11} \,, \tag{B.55}$$
$$a_{10} = -c_{10} - c_{11} - c_{13} + c_{14} \,, \tag{B.56}$$
$$a_{11} = -c_{10} + c_{11} - c_{13} + c_{14} \,, \tag{B.57}$$
$$a_{12} = -c_6 - 2c_8 \,, \tag{B.58}$$
$$a_{13} = c_7 + c_8 \,, \tag{B.59}$$
$$a_{14} = c_8 \,. \tag{B.60}$$

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