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Mass Measurement in Boosted Decay Chains with Missing Energy

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We explore a novel method of mass reconstruction in events with missing energy at hadron colliders. In events with sizeable boost factors in the final steps of dual multi-stage decay chains, the missing energy particles may each be approximately collinear with a visible standard model particle, spanning a narrow “MET-cone.” We exploit this collinear approximation, when applicable, to reconstruct the masses of exotica.

Introduction — The start of the Large Hadron Collider (LHC) at CERN gives hope to the discovery of long-anticipated TeV scale new physics. In many of these scenarios, discrete symmetries are motivated by the requirement that higher dimensional operators which contribute to amplitudes affecting weak gauge boson form factors and baryon number violation must be highly suppressed. Such discrete symmetries may simultaneously be responsible for stabilizing the dark matter component of our universe. The typical collider signatures of such discrete symmetries are characterized by events with large amounts of missing energy. The undetected particles in such events, the lightest particle charged under the discrete symmetry, complicate the reconstruction of the masses of new exotica. This is particularly the case at hadron colliders, where the initial parton momenta are unknown. There has been recent substantial progress in mass measurement in such scenarios over the past few years (see [1] for a recent review, and complete citation list). Most of these methods rely on the kinematics of the events and fall into several broad categories: invariant mass endpoint methods [2–4], mass relation/polynomial methods [5–9], M_{T2} -like methods [10–12], and various combinations of them. The existence of multiple methods is crucial, providing complementary techniques for extracting information about the underlying physics model.

Here, we explore a conceptually new method of mass determination that is particularly useful when a decay chain terminates with the disintegration of a relatively boosted exotic particle to the lightest exotic plus a visible standard model (SM) particle.¹ Such events are characteristic of models in which pair produced color charged exotica are somewhat heavy, and in which there is simultaneously a small amount of phase space available for the NLSP decay. As a motivator for such scenarios, in the MSSM, the LEP II bound on the higgs mass prefers a large top squark mass, or in general TeV scale squark masses in models with minimal flavor violation. On the other hand, the neutralinos and charginos can naturally

be light, near the weak-scale and with relatively small mass splitting.

We consider production of a generic new heavy particle Q in a collider, which decays in the following way: $Q \rightarrow \cdots \chi_2 \rightarrow \cdots \chi_1 X$. Here $\chi_{1,2}$ are the LSP and NLSP respectively, while X is a SM particle. The set of dots represents a multiplicity of SM states arising from the intermediate stages of the decay chain. This rather general decay topology is shown in Fig. 1. If the mass difference between Q and χ_2 is large, $m_Q - m_{\chi_2} \gg m_{\chi_2}$, the daughter particle χ_2 will often be significantly boosted.

For a given momentum configuration of the visible X particles on each decay chain, and for given NLSP and LSP masses, there is a kinematic boundary for the total allowed missing three-momentum in such events [9]. For boosted decays, the total missing three-momentum vector is constrained to lie in a cone around the total three-momentum of both X ’s from each side of the event. The size and shape of this cone is sensitive to the NLSP and LSP masses. This observation is the central focus of this letter, and motivates the construction of a simple variable which extracts information about the mass spectrum from these MET-cones. We begin with a brief discussion of collinearity and its dependence on the mass parameters.²

FIG. 1: A schematic picture of a boosted decay chain.

Collinearity and the MET-Cone — The kinematics of two-body χ_2 decays are straightforward. We take the χ_2 particle to have relativistic boost factor γ and velocity β in the lab frame. The angle $\theta_{\chi_2 X}$ between the visible particle X and the direction of motion of the parent χ_2 is then given by

$$\tan \theta_{\chi_2 X} = \frac{\beta_0^X}{\gamma} \left(\frac{\sin \theta_0}{\beta_0^X \cos \theta_0 + \beta} \right). \quad (1)$$

where θ_0 is angle between X and χ_2 in the rest frame of

¹ From here on, we use the semantics of supersymmetry, and refer to the lightest particle charged under the discrete symmetry as the LSP, and the next lightest as the NLSP. However, our method is generic, and applies to all TeV scale physics in which the exotica carry a conserved Z_2 charge under which the SM fields are neutral.

² There is a recent study [13] which also uses boosted decays for mass measurement, but in a rather different way.

χ_2 , and β_0^X is the velocity of X in the rest frame of χ_2 :

$$(\beta_0^X)^2 = \frac{(m_{\chi_2}^2 - (m_{\chi_1} + m_X)^2)(m_{\chi_2}^2 - (m_{\chi_1} - m_X)^2)}{(m_{\chi_2}^2 + (m_X^2 - m_{\chi_1}^2))^2} \quad (2)$$

The angle takes on values in the range (for $\beta_0^X < \beta$):

$$0 \leq \tan \theta_{\chi_2 X} \leq \frac{\beta_0^X}{\gamma \beta} \frac{1}{\sqrt{1 - (\beta_0^X/\beta)^2}} \xrightarrow{\gamma \gg 1} \frac{\beta_0^X \gamma_0^X}{\gamma}. \quad (3)$$

The velocity β_0^X is a function of the masses of the three particles involved, and it characterizes the allowed phase space of the χ_2 decay. The angle $\theta_{\chi_2 \chi_1}$ can be obtained by exchanging m_X with m_{χ_1} in the above equations. An approximately collinear configuration is achieved either with large $\gamma \gg \gamma_0^{X, \chi_1}$, or with narrow phase space for the χ_2 decay.

The boost factors of X and χ_1 in the lab frame are given by

$$\begin{aligned} \gamma^{\chi_1} &= \gamma \gamma_0^{\chi_1} (1 + \beta \beta_0^{\chi_1} \cos \theta_0) \\ \gamma^X &= \gamma \gamma_0^X (1 - \beta \beta_0^X \cos \theta_0). \end{aligned} \quad (4)$$

The magnitudes of the 3-momenta of X and χ_1 in the lab frame can be written as $p^{\chi_1} = \gamma^{\chi_1} \beta^{\chi_1} m_{\chi_1}$ and $p^X = \gamma^X \beta^X m_X$.

The χ_2 boost factor γ is determined by several variables. As a simple example, we consider a heavy exotic, with mass m_Q which decays to a massless SM particle (e.g. a jet) and the NLSP, with mass m_{χ_2} . For $m_Q \sim 2$ TeV and $m_{\chi_2} \sim 200$ GeV, a boost factor of $\gamma = 5$ is achieved in the rest frame of Q . However, at a hadron collider the Q particle will be produced with some transverse as well as longitudinal momentum, providing a distribution of boost factors. In addition, in multistage decay chains, the typical boost factors will depend on the mass spectrum of particles participating in the cascade.

Using Eq.'s (3,4), for a full dual-chain event we can parametrize the total three-momentum of both χ_1 particles as a function of m_{χ_1} , m_{χ_2} , the momenta of each X particle $\vec{p}_{1,2}^X$, and 4 angles that characterize the orientation of each X in each χ_2 rest frame. Varying those invisible angles with given m_{χ_1} , m_{χ_2} and $\vec{p}_{1,2}^X$ defines the kinematically allowed region for the total missing 3-momentum. We have found the boundaries by performing a numerical scan of the NLSP rest frame angles (with flat prior) for different NLSP-LSP mass pairings. We illustrate this in Figure 2, where we display the region allowed for the MET vector in each event for given NLSP and LSP masses. We assume an event topology in which each of the two decay chains terminates with the NLSP decaying to a Z -boson plus the LSP. We also restrict to a particular configuration of Z momenta. The y -axis reflects the component of the MET vector parallel to the total transverse momentum of Z -bosons while the x -axis displays the remaining MET vector component. As expected, the MET vector is correlated with the $\vec{p}_{Z,T}^{\text{tot}}$ (the vector sum of the transverse momenta of the two Z

bosons) with kinematic boundaries. More importantly, the MET-cone boundary is sensitive to the overall mass scale of the underlying theory, not simply the mass splitting, as shown in Figure 2.

FIG. 2: In this figure we display the MET-cone boundaries for different NLSP and LSP masses (keeping the mass splitting constant). The MET vector must lie within the boundary for particular choices of exotica masses. The Z bosons are in a configuration where each has boost factor $\gamma^Z = 5$, both lie in the transverse plane, and are separated by a 90 degree angle.

The $m_{\chi_1}^{\text{cone}}$ variable — The MET-cone boundary found in the previous section depends on the unknown masses of the exotic particles χ_1 and χ_2 , and for an incorrect spectrum hypothesis, some events will have a MET vector which lies outside the cone. This can potentially be used as a method of mass determination [14]. However, the computation of the boundary is complicated by the dependence on the specific momentum configuration of X 's in the two decay chains, and the cone must be computed event-by-event. To avoid this event-by-event analysis, we consider a variable $m_{\chi_1}^{\text{cone}}$ which inherits enough information from the 2-dimensional MET-cone to determine the mass spectrum:

$$m_{\chi_1}^{\text{cone}} = \not{p}_{T,y} \frac{m_X}{|\vec{p}_{X,T}^{\text{tot}}|}. \quad (5)$$

This is simply the MET-vector in a given event, rescaled by the total 3-momentum of the 2 X particles.

This variable is motivated to approximate the mass of χ_1 through the introduction of test momentum of χ_1 of each decay chain (distinguished by the notation $i = a, b$):

$$\vec{p}_{\chi_1}^{i, \text{test}} \equiv \vec{p}_X^i m_{\chi_1}^{\text{cone}} / m_X, \quad (6)$$

with $m_{\chi_1}^{\text{cone}}$ defined for each event by minimization of the following quantity

$$\Delta \mathcal{E}_T(m_{\chi_1}^{\text{cone}}) \equiv \left| \sum_{i=a,b} \left(\vec{p}_{\chi_1,T}^{i, \text{test}} \right) - \vec{p}_T^{\text{exp}} \right|. \quad (7)$$

This is an analytic procedure, as this formula is quadratic in $m_{\chi_1}^{\text{cone}}$. The minimization of $\Delta \mathcal{E}_T(m_{\chi_1}^{\text{cone}})$ gives Eq. (5)

and $(\Delta E_T)^{\min}/E_T = |\vec{p}_{T,x}/E_T|$. It is easy to see that these variables $m_{\chi_1}^{\text{cone}}$ and $(\Delta E_T)^{\min}/E_T$ rescale respectively the y and x components of the $\vec{E}_T$ vector event-by-event.

In Figure 3, we show rescaled MET-cones in the $m_{\chi_1}^{\text{cone}}$ vs $\vec{p}_{T,x}/E_T$ plane for both a scenario where the mass splitting is very small, and another where an $\mathcal{O}(10\%)$ splitting is assumed. From these figures, one observes that $m_{\chi_1}^{\text{cone}}$ has two approximate endpoints in the small $\vec{p}_{T,x}/E_T$ region, where the cones with randomly chosen X momentum configurations all intersect the $m_{\chi_1}^{\text{cone}}$ axis. These endpoints are a manifestation of the MET-cone boundaries when projected onto the transverse plane, as illustrated in Figure 4. Below, we show analytically that, in the relativistic limit, these endpoints are relatively insensitive to the X particle kinematics.

FIG. 3: In these two plots, we compare the rescaled MET-cones of two scenarios with decay $\chi_2 \rightarrow \chi_1 Z$, with identical χ_2 mass (200 GeV). In the top (bottom) plot, we take $\Delta m = m_{\chi_2} - (m_{\chi_1} + m_Z) = 1$ GeV (10 GeV). Different contours correspond to randomly chosen momentum configurations of two Z bosons from Monte Carlo events. Note that $m_{\chi_1}^{\text{test}}$ has two endpoints in the small $|\vec{p}_{T,x}/E_T|$ region.

It can be shown that the transverse momenta of the individual χ_1 particles must lie within an ellipse in the transverse plane [9]. If we parametrize the boundaries of the two ellipses corresponding to the two χ_1 's in the event with angles $t^{a,b}$, then the total missing momentum

lies on the boundary of the region spanned by these two angles. We can then calculate that, for a given mass spectrum and X particle kinematics, $m_{\chi_1}^{\text{cone}}$ lies within the following region:

$$m_{\chi_1}^{\text{cone}} = \frac{m_X}{2} \left[r + \frac{(\vec{p}_X)_y^a \cos t^a + (\vec{p}_X)_y^b \cos t^b}{|p_X^T|} \lambda + \mathcal{O}\left(\frac{1}{\gamma_X^{a,b}}\right) \right], \quad (8)$$

where corrections due to deviations from the relativistic limit are given by:

$$\sum_{i=a,b} \frac{m_X}{2} \lambda \frac{(\vec{p}_X)_y^i}{|p_X^T|} \left[\left(\frac{1 - \beta_X^i}{\beta_X^i} \right) \cos t^i + \frac{1}{\gamma_X^i \beta_X^i} \tan \theta_X^i \sin t^i \right]. \quad (9)$$

We have made the following definitions:

$$r \equiv \frac{m_{\chi_2}^2 - (m_{\chi_1}^2 + m_X^2)}{m_X^2}; \quad \lambda \equiv \sqrt{r^2 - 4 \frac{m_{\chi_1}^2}{m_X^2}}, \quad (10)$$

θ_X^i are the angles between the individual X particle momenta and the y axis in the transverse plane, $\gamma_X^{a,b}$ are the transverse boost factors for the two X particles, and $\beta_X^{a,b}$ are their transverse velocities.

At zeroth order in the $1/\gamma_X^{a,b}$ expansion, the endpoints of $m_{\chi_1}^{\text{cone}}$ are given by:

$$m_{\text{lower}}^{\text{cone}} \approx m_X \left(\frac{1}{2} r - \lambda \right) = m_{\chi_1} \frac{\gamma_0^{\chi_1}}{\gamma_0^X} \frac{1 - \beta_0^{\chi_1}}{1 + \beta_0^X} \\ m_{\text{upper}}^{\text{cone}} \approx m_X \left(\frac{1}{2} r + \lambda \right) = m_{\chi_1} \frac{\gamma_0^{\chi_1}}{\gamma_0^X} \frac{1 + \beta_0^{\chi_1}}{1 - \beta_0^X} \quad (11)$$

The upper and lower values are achieved when $\theta_0^a = 0$ and π respectively, i.e. χ_1 moving forward or backward along the χ_2 boost direction in its rest frame. The range of $m_{\chi_1}^{\text{cone}}$ becomes smaller as the phase space for the χ_2 decay shrinks, and the distribution centers on the LSP mass, as can be seen in the second equalities in Eq. (11). In the relativistic limit, $\beta \sim 1$, the endpoint positions determine the unknown masses m_{χ_1} and m_{χ_2} . The non-collinear corrections to $m_{\chi_1}^{\text{cone}}$ will give rise to small corrections to the endpoint positions as seen from Eq. (9). To make sure the expansion coefficients not too large, we should cut on event configurations where either of the $\tan \theta_X^i$ become large relative to the boost factor.

In summary, if the endpoint positions of the $m_{\chi_1}^{\text{cone}}$ distribution are measured from the data, we can find solutions for the masses of the LSP and NLSP using the relation in Eq. (11).

Numerical Results — We now explore the effectiveness of the $m_{\chi_1}^{\text{cone}}$ method using Monte Carlo simulation. It is convenient to use the familiar MSSM language and consider squark pair production $\tilde{q}_L \tilde{q}_L$ followed by the decays $\tilde{q}_L \rightarrow q \tilde{\chi}_2 \rightarrow q \tilde{\chi}_1 Z$ in SUSY models, although the method applies more generally to all equivalent final state

FIG. 4: Illustration of the MET cone and its relation to the $m_{\chi_1}^{\text{cone}}$ endpoints.

topologies. We study the four different spectra shown in Table I, in which we vary both overall mass scale and mass splitting for the LSP and NLSP. For each of these

Model	m_{χ_1}	m_{χ_2}	$m_{\tilde{q}_L}$	$(m_{\text{lower}}^{\text{cone}})^{\text{theo}}$	$(m_{\text{upper}}^{\text{cone}})^{\text{theo}}$
1	100	200	1000	54.6	183.2
2	100	250	1250	21.6	463.0
3	200	300	1000	117.9	339.2
4	200	350	1250	52.6	761.0

TABLE I: The relevant masses in four SUSY models and the expected endpoints $(m_{\text{lower}}^{\text{cone}})^{\text{theo}}$ and $(m_{\text{upper}}^{\text{cone}})^{\text{theo}}$. Masses are given in units of GeV.

models, we simulate 20K squark events in pp collisions at $\sqrt{s} = 14$ TeV in MadGraph [15] using the $2 \rightarrow 6$ matrix element to model decays. As a test of principle, we first do a parton-level analysis, assuming a perfect measurement of particle four-momentum including those of Z bosons. For the parton level analysis, we do not include SM backgrounds.

Events are selected according to the following parton-level cuts:

- at least two Z bosons with $p_T > 50$ GeV and $|\eta| < 3$
- missing energy $\cancel{E}_T > 200$ GeV
- η of total Z three-momentum $|\eta^{Z,\text{tot}}| < 1$
- opening angle of two Z bosons $60^\circ < \theta_{ab}^Z < 120^\circ$
- $|\cancel{p}_{T,x}/\cancel{E}_T| < 0.15$

The θ_{ab}^Z cut is to ensure the coefficients in the $\theta^{a,b}$ expansion in Eq. (9) are not too large, which would otherwise obscure the endpoints. The cut on $|\cancel{p}_{T,x}/\cancel{E}_T|$ is to select those events where the MET vector is closely aligned with the total transverse momentum of two Z -bosons. For Models 2 and 4, we have slightly loosened the selection cuts in order to get better statistics near the tail of the distribution: **a)** $|\cancel{p}_{T,x}/\cancel{E}_T| < 0.2$ for Model 2 and 4. **b)** $|\eta^{Z,\text{tot}}| < 3$ for Model 4.

FIG. 5: Top: The distributions of $m_{\chi_1}^{\text{cone}}$ for Model 1 and 2. For Model 2, we have normalized the distribution by a factor of two. The fitting region for model 1 is 40–60 GeV and 120–200 GeV; for model 2, it is 10–30 GeV and 250–400 GeV. Bottom: The distributions of $m_{\chi_1}^{\text{cone}}$ for Model 3 and 4.

In Fig. 5, we show the $m_{\chi_1}^{\text{cone}}$ distribution for Models 1 and 2 in the top panel and for Models 3 and 4 in the bottom panel. For Model 1, we can see that the distribution is approximately a triangle with two endpoints at around 50 and 200 GeV. The shape of the distribution near each endpoint is slightly smeared due to deviations from collinearity. These endpoints are stable under the change of the bin size. Using naive linear fits, we can estimate the lower endpoint in the range 45–60 GeV and upper one 200–210 GeV depending on the range of the fit. This translates into a measurement of the LSP and NLSP masses: $m_{\chi_1} \in 95–112$ GeV and $m_{\chi_2} \in 199–213$ GeV. For Model 2, as seen in Fig. 5, the lower endpoint is still as sharp, around 20–30 GeV, but the upper endpoint is difficult to see. Estimation of its position is subject to a relatively large systematic uncertainty depending on the binning and the choice of the fitting method. Fortunately, the calculated masses are not very sensitive to the upper endpoint position. For a conservative estimate of the $m_{\text{upper}}^{\text{test}}$ in the range 300–500 GeV, the calculated masses $m_{\chi_1}^{\text{meas}}$ and $m_{\chi_2}^{\text{meas}}$ vary relatively mildly in the ranges 78–122 GeV and 209–268 GeV respectively. Therefore, even in this relatively less collinear case where

the upper endpoint is not well measured, we still obtain a useful estimate of the masses. The situation for Model 3 and 4 is similar, and we have summarized the obtained estimation of masses in Table II.

So far, our analysis is purely at parton-level. We now take into account detector smearing and check how it affects of the endpoints of $m_{\chi_1}^{\text{cone}}$ distribution. To do this, we pass the generated squark pair events to PYTHIA [16] and PGS4 [17] for parton shower and detector simulation (the default Pythia and PGS parameters in MadGraph are used). In our event sample, we force all squarks to decay through $\tilde{q} \rightarrow qZ\tilde{\chi}_1$ exclusively. We select events with the following criteria:

- 1) at least three jets with $p_T > 30$ GeV and $|\eta| < 2.5$;
- 2) the first two leading jets with $p_T > 180, 110$ GeV respectively;
- 3) $\cancel{E}_T > 200$ GeV and $M_{\text{eff}} > 800$ GeV, where the effective mass is defined as the scalar sum of $\cancel{E}_T$ and the p_T of the four leading jets
 $M_{\text{eff}} \equiv \cancel{E}_T + \sum_{i=1}^{\min(N_j, 4)} p_{T, \text{jet}}^i$;
- 4) at least two leptons with $p_T > 10$ GeV.

After these cuts, we perform Z -boson reconstruction from an opposite-sign same-flavor lepton pair and a jet pair respectively. To eliminate combinatoric backgrounds, the correct pairing is selected when the corresponding invariant mass is the closest to m_Z among all other pairings. We also require that the invariant mass should be within the window $81 - 101$ GeV in order to be considered a reconstructed Z . Once there are two reconstructed Z 's bosons, we proceed as before and calculate our $m_{\chi_1}^{\text{cone}}$ variable. Finally, we require $|\cancel{p}_{T,x}/\cancel{E}_T| < 0.6$.

The resulting $m_{\chi_1}^{\text{cone}}$ distributions for Model 1 and 2 can be seen in Figure 6 (with 50K and 80K squark pair events respectively). In the same plot, we have also included the dominant SM backgrounds to our signals, $t\bar{t} + \geq 1$ jets, which were generated using MadGraph and passed through the same analysis paths as the signal. Other backgrounds such as $W(\rightarrow l\nu)W(\rightarrow l\nu) + \geq 3$ jets, $Z(\rightarrow ll)Z(\rightarrow \nu\nu) + \geq 3$ jets are negligible after the selection cuts.

For model 1, we can see there are still two clean endpoints at around 45 and 195 GeV. This corresponds to $m_{\chi_1} \simeq 94$ GeV and $m_{\chi_2} \simeq 197$ GeV, which are in good agreement with the true masses. For Model 2, we can crudely identify the endpoint position to be at around 30 and 295 GeV. But the true upper endpoint could be larger given that the distribution is broader and flatter near the endpoint, which is indeed true for this case as the expected value for upper endpoint is 463 GeV. This leads to a conservative estimate of the mass $m_{\chi_1} \simeq 94$ GeV and $m_{\chi_2} \simeq 216$ GeV. This certainly underestimates the mass of χ_2 , although the LSP mass measurement remains robust. Better determination of the mass in this case would either require additional data and/or fitting the whole

Model	m_{χ_1}	m_{χ_2}	$m_{\text{lower}}^{\text{cone}}$	$m_{\text{upper}}^{\text{cone}}$	$m_{\chi_1}^{\text{meas}}$	$m_{\chi_2}^{\text{meas}}$
1	100	200	55	205	106	208
2	100	250	27	454	110	253
3	200	300	112	342	195	296
4	200	350	49	682	183	329

TABLE II: Results of the measured $m_{\chi_1}^{\text{cone}}$ endpoints and $m_{\chi_{1,2}}$ for four SUSY spectra. As an illustration of the idea, we determine the endpoint positions by simple linear fits over the corresponding endpoint regions. For the lower endpoint positions, we take the half maximum near the lower edge to reduce the effects of smearing. For the upper endpoint, we take the x -intercept of the fit function. Note that this may not be the actual way to determine the endpoints, particularly if they are not sharp enough to be identified in the beginning. Masses are given in units of GeV.

distribution, as is done when measuring the W mass using the transverse mass variable.

Now let us estimate the cross section for $pp \rightarrow \tilde{q}\tilde{q} + X$ at the LHC. If the gluino mass is heavier than squark, this amount to the total cross section of $pp \rightarrow \tilde{q}\tilde{q}, \tilde{q}\tilde{g}, \tilde{g}\tilde{g}$, where $\tilde{q}$ being the first four light flavor squark. For simplicity, assuming $m_{\tilde{g}} = m_{\tilde{q}} + 50$ GeV, we get $\sigma_{\text{LO}} \sim 1.3$ pb for Model 1 and 0.3 pb for Model 2. The K-factors for these models is roughly 1.3 [18]. Taking all these into account, the amount of data in Figure 6 corresponds to integrated luminosity of roughly 30fb^{-1} and 200fb^{-1} for Model 1 and 2 respectively. This estimate assumes the squark and χ_2 decay with 100% branching ratio, which is typically not the case in realistic MSSM models. But it can be translated to the actual luminosity depending on the details of the model (which may not be supersymmetric). Finally, the above estimated luminosities are preliminary, and could likely be improved with optimized event selection cuts.

Summary and Outlook— In this letter, we explored a novel method of measuring the absolute mass scale of exotic particles in events with missing energy arising in cascade decays with moderate relativistic boost. This method uses the fact that in the boosted decay there is a limited variation in both the direction and magnitude of the total three-momentum of missing particles relative to the total three-momentum of the visible partners, particularly when the phase space for the NLSP decay is small. The boundary of the allowed region, or the MET-cone, is determined by the mass parameters and the configuration of the visible particles. We constructed a variable $m_{\chi_1}^{\text{cone}}$ which has endpoints which depend on the masses involved in the final step of the cascade decay. Once observed from the data, the endpoints can be used to determine the NLSP and LSP masses.

The $m_{\chi_1}^{\text{cone}}$ variable works best in the collinear limit. Given the data, the evidence of collinearity in the final step decay can be seen in various ways. First, one would see a peak at 0 in the $|\cancel{p}_{T,x}/\cancel{E}_T|$ distribution. Second, one would see well-defined endpoints in the $m_{\chi_1}^{\text{cone}}$ distribution. Once the masses of χ_1 and χ_2 are measured, one

FIG. 6: The $m_{\chi_1}^{\text{cone}}$ distributions for Model 1 (top panel) and 2 (bottom panel) using reconstructed objects after detector simulation. The signal+background distributions are shown in red solid lines. The gray straight lines indicate the linear fits of the signal in the endpoint regions. The dominant SM backgrounds from $t\bar{t} + \geq 1$ jets are shown with the black dotted lines.

can find the masses of heavier exotics upstream of the NLSP decay using more standard techniques.

We have demonstrated our method in a setup with two symmetric decay chains with a two-step cascade decay, however it applies for longer decay chains as well. An advantage of this method is that one does not require information on all of the visible SM particles. The Monte Carlo analysis performed in the paper is limited in that it studies a particular topology while in the most general model, other topologies will be present. Our purpose here is to show that the method works in principle. A more complete study is reserved for future work.

The $m_{\chi_1}^{\text{cone}}$ variable has the advantage that it is simple to calculate on an event-by-event basis. However, it does not fully utilize the information available in the MET-cones. Developing an even more effective method which takes full advantage of these kinematic boundaries is currently under investigation.

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