Comment on “Atomic gravitational wave interferometric sensor”

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The use of laser interferometers for detecting and studying gravitational wave signals from many types of astronomical sources is being pursued actively by a number of groups in different countries. However, it has been suggested recently that cooled atom clouds in atom interferometers could be used to replace the test masses in space-based gravitational wave detectors and the end mirrors in ground-based detectors [1]. Some new error sources that apparently have not been included in proposals of atom interferometer gravitational wave detectors will be discussed in this comment. They are based on additional effects of aberrations in the laser wave fronts that interact with the atom clouds.

I. INTRODUCTION

Several groups have suggested the use of atom interferometers in antennas for detecting gravitational waves. The most extensive proposal for detectors of this kind was made in 2008 by Dimopoulos et al. [1]. Many error sources are considered in [1], including known systematic errors due to static wave-front curvature effects, which have been discussed previously by Weiss, Young, and Chu (1994) [2] and by a number of others. However, some additional limitations due to aberrations in the laser beams used and the effect of these aberrations on the interactions with the cooled atom clouds in the atom interferometers were not included. These limitations will be discussed here.

The proposed detectors are called Atom Interferometer Gravitational-wave Sensors (AGISs), and include both ground-based and space-based instruments. Attention will be focused here on the most ambitious space-based proposal, AGIS-Satellite 3. Under this proposal, measurements would be made between two spacecraft, S1 and S2, separated by L = 10,000 km. Atom interferometers 100 m long would be operated at each end of the path. (See Fig. 11 in [1].)

In each interferometer, a cloud of cooled atoms would be launched once per second along the line between the spacecraft with a speed of roughly 0.5 m/s. Each cloud would contain roughly 10^{10} atoms, with cesium as one suitable choice, and be cooled to an internal temperature of about 100 pK. Dimensions of roughly 100 mm are suggested for the clouds. This will be interpreted here as a tentative value of b = 50mm for the radius of the clouds.

The main scenario discussed involves the use of stimulated two-photon Raman transitions to operate the two interferometers. A sequence of three short Raman pulses would be applied to the atoms at times t – T, t, and t + T, where T is about 100 s. Each Raman pulse would be fed by two lasers at opposite ends of the path, and would connect the two ground state sublevels of interest via a virtual level close to a real excited level. One laser, called the passive laser, would be on continuously. The other, called the control laser, would be pulsed on briefly to control the length and timing of the Raman pulses.

The three Raman pulses would be a $\pi/2$, a π, and a $\pi/2$ pulse. The first $\pi/2$ pulse would take the atoms from one ground state sublevel to a 50-50 superposition of the two levels, with one part of the wave function having considerably different momentum than the other. The spatial position for the two parts of the wave function thus would separate during the first 100 s period. Then the π pulse would reverse the momentum difference, and cause the wave functions to overlap at the time of the second $\pi/2$ pulse, which would bring the momentum difference back to zero. But any net acceleration of the atoms over the 200 s period would affect the final population difference between the two levels. From the difference in the accelerations of the atoms in the two interferometers, the effect of gravitational waves can be detected, in principle.

Under the above scenario, it is assumed that the control laser and the passive laser would be tightly offset phase locked with respect to each other in order to minimize the effects of laser phase noise. The offset frequency between the two lasers will be called the Raman frequency, and will be very close to the frequency difference between the ground state sublevels. The phase corresponding to the Raman frequency minus the ground state splitting frequency will be called the Raman phase. It can be shown that any variation in this phase between the Raman pulses will affect the population difference measured after the third Raman pulse [3].

With the suggested AGIS approach, strong use is made of the fact that the travel time for light between the two interferometers is 30 microseconds or less, and thus the effects of time variations in the Raman phase would be almost the same for the two interferometers for many causes of the time variations. However, other effects such as those associated with laser wave-front aberrations can cause the Raman phase to be averaged differently from second to second over the atoms in the clouds in the two interferometers. Such effects apparently represent an important additional source of possible noise in the proposed gravitational wave measurements, as discussed below.

Because of the high gravitational wave sensitivity that is proposed for the AGIS-Satellite 3 detector, all additional possible sources of noise in the measurements need to be investigated carefully. The expected sensitivity shown in Fig. 14 of [1] is $3\times10^{-22}/\sqrt{\text{Hz}}$ from 0.002 to 0.5 Hz. Despite the 10^4 km path length, differential fluctuations as small as roughly 1.5×10^{-9} cycles from second to second in the Raman phase difference as averaged over the atom clouds in the two interferometers would make a noise contribution equal to the whole gravitational wave error budget, as discussed later. Such fluctuations can be caused by time variations in the laser wave-front aberrations or by uncertainties in the interactions of atom velocities with dc wave-front aberrations.

It should be noted that the Laser Interferometer Gravitational wave Observatory (LIGO) has achieved a noise level of 1×10^{-18} m/Hz$^{0.5}$ for measuring changes in the difference in length of its 4 km arms at frequencies of 40 Hz and higher. This corresponds to 1×10^{-12} cycles/Hz$^{0.5}$. However, LIGO operates with Fabry-Perot cavities in each arm that give about 100 bounces for the light and with a roughly 20 m triangular mode-cleaner cavity before the main beam-splitter in the interferometer. Thus there are enough differences in the design so that some consideration of possible additional time-dependent wave-front errors related to wave-front aberrations appears to be needed for the AGIS-Satellite 3 proposed mission. For the Laser Interferometer Space

Antenna (LISA), the requirements on the laser wave-front aberration noise are considerably less severe because of the 500 times longer paths between satellites than for AGIS-Satellite 3.

II. POSSIBLE ERROR DUE TO LASER WAVEFRONT DISTORTIONS

The passive laser beam would be sent between the two interferometers by a telescope, and the telescope needs to be large enough in diameter so that the laser beam can produce fairly rapid Raman transitions in the far interferometer. Thus it will be assumed that the telescope is substantially larger in diameter than the cooled atom clouds. Because of this, the average phase changes of the laser beams over the atoms in the near clouds won't be the same as the average phase change over the whole telescope aperture. But the atom clouds in the far interferometer will be affected more nearly by the average phase over the whole transmitting telescope aperture because of diffraction. For this reason, jitter in the aberrations of the laser wave front entering the telescope will affect the apparent phase difference between the two interferometers.

It will be assumed that the control laser at the far end of the path is offset phase locked with respect to the passive laser beam, as received there. But its time-variable laser wave-front aberrations will be different, and thus contribute also to the total instrumental noise level. If two baselines were used instead of one and fed from the same passive laser, some of the effect of the time-variable wave-front aberrations from that laser could be reduced. However, the different time-variable wave-front aberrations from the two control lasers at the far ends would still be present.

It is not clear how to estimate the amplitude for jitter in the aberrations of the laser wave fronts from second to second. Most available information for laser power levels near 1 W concerns jitter in the lateral beam position and wave-front tilt [4, 5]. But variations in the beam diameter and wave-front curvature may be more important. The most significant scales of aberrations probably will be those that, after expansion of the beam to fill the telescope aperture, are close to the size of the atom clouds or larger. Adding a filter cavity could reduce the amplitude of time variable aberrations by a large factor. However, the roughly 1.5×10^{-9} cycle level at which jitter caused by time variable wave-front aberrations would begin to be serious appears to introduce a substantial additional requirement on reducing such time variations for an AGIS-Satellite 3 type detector.

As an example, a radius a = 0.5 m will be assumed for the telescope, and $\alpha \ll 1$ wavelength for the variations from second to second in the amplitude of the primary spherical aberration. The suggested telescope radius for the AGIS-Satellite 2 proposal is 0.5 m, and the same radius is assumed here, in the absence of other information.

A roughly cos(ρ) density distribution will be assumed for the atoms in the clouds, where ρ is the distance from the center of the cloud. The distance at which the density goes to zero is taken to be $b \ll a$. The relative path delay over the telescope aperture due to primary spherical aberration is given by the corresponding Zernicke polynomial in r, where r is the distance from the telescope axis:

$$z(r) = [6(r/a)^4 - 6(r/a)^2 + 1] \; . \tag{1}$$

For $b<<a$, $z(r)\sim 1$. Thus the full amplitude of the spherical aberration time-variations will affect the atom clouds in the near atom interferometer. However, the wave-front aberration time-variations will have been reduced substantially by diffraction for the atom clouds in the far interferometer, since the distance between the spacecraft is comparable with the Rayleigh range for the telescope.

For the three Raman pulses occurring at times t – T, t, and t + T, the resulting second difference in the apparent spacecraft separation L will be:

$$\Delta_2 L \sim \lambda\{2\alpha(t) - \alpha(t\text{-}T) - \alpha(t + T)\}. \qquad (2)$$

Since α is assumed to be random from shot to shot:

$$(\Delta_2\ L)_{rms} \sim \sqrt{6}\ \lambda\ \alpha_{rms}. \qquad (3)$$

To see how large an effect fluctuations from second to second in α would have, the result from eq. 3 can be compared with the value of Δ_2 L that would result from a gravitational wave of amplitude h. In this case:

$$\Delta_2\ L = (hL)[\sin(\omega\ t)][1 - \cos(\omega\ T)]. \qquad (4)$$

For angular frequencies ω in the range of interest, the error in h would be:

$$\delta\ h \sim \sqrt{6}(\lambda/L)\alpha_{rms} \sim 2 \times 10^{-13}\ \alpha_{rms}. \qquad (5)$$

Thus the fluctuations from second to second in α would have to be roughly 1.5×10^{-9} or less in order to not affect the AGIS-Satellite 3 gravitational wave sensitivity.

III. ADDITIONAL UNCERTAINTY DUE TO JITTER IN THE ATOM CLOUD TEMPERATURE

Another possible source of error will be present if there are variations from second to second in the temperature of the atom clouds. If time-varying aberrations in the passive laser wave fronts feeding into the telescope are ignored, there still will be dc aberrations of amplitude ß wavelengths introduced by the telescope itself. In this case, the change in the Zernicke polynomial as r changes due to the atomic radial velocity can be important. If the rms thermal velocity of the atoms is V, then $r(t+T) = r(t)+VT/\sqrt{3}$, etc. Then the second difference in L for constant ß is:

$$\Delta_2 L = \beta\lambda\{2z(t)\text{-}z(t\text{-}T)\text{-}z(t+T)] \qquad (6)$$

$$\Delta_2 L \sim 4\beta\lambda(VT/a)^2 \qquad (7)$$

For the assumed cloud temperature of 100 pK, the rms total atom velocity is 1×10^{-4} m/s and, for the given time of T = 100 s between pulses for the AGIS-Satellite 3 sensor:

$$\Delta_2 L \sim 1.3 \times 10^{-9} \text{ß}. \tag{8}$$

Since V^2 is proportional to the temperature, θ, the change in $\Delta_2 L$ for a change Δθ in θ would be

$$\Delta_2 L \sim 1.3 \times 10^{-9} \text{ß}(\Delta\theta/\theta). \tag{9}$$

Thus the resulting offset in h would be:

$$\delta h \sim (\Delta_2 L)/L \sim 1.3 \times 10^{-16} \text{ß}(\Delta\theta/\theta). \tag{10}$$

As an example, if the dc spherical aberration amplitude ß is 0.001 and the temperature θ is 100 pK, the fluctuations in θ from second to second would need to be less than 0.2 pK in order to avoid increasing the gravitational wave noise level for the proposed AGIS-Satellite 3 mission.

IV. CONCLUSIONS

For the AGIS-Satellite 3 proposal given in [1], it appears that tight requirements on some additional error sources would be needed in order for the suggested sensitivity level for gravitational wave detection to be achieved. For the AGIS-Satellite 2 proposal, the requirements could be roughly a factor 10 less severe.

For the more ambitious of the two ground-based versions of AGIS discussed in [1], the situation would be substantially different. The much shorter baseline of 4 km proposed presumably would lead to considerably more flexibility being possible in the design of the laser optical system. However, because of the smaller product of gravitational uncertainty and baseline length, the requirements on the wave-front aberration fluctuations would be tighter.

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