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This paper discusses the quantum mechanics of closed timelike curves (CTCs) and of other potential methods for time travel. We analyze a specific proposal for such quantum time travel, the quantum description of CTCs based on post-selected teleportation (P-CTCs). We compare the theory of P-CTCs to previously proposed quantum theories of time travel: the theory is inequivalent to Deutsch's theory of CTCs, but it is consistent with path-integral approaches (which are the best suited for analyzing quantum field theory in curved spacetime). We derive the dynamical equations that a chronology-respecting system interacting with a CTC will experience. We discuss the possibility of time travel in the absence of general relativistic closed timelike curves, and investigate the implications of P-CTCs for enhancing the power of computation.

Einstein's theory of general relativity allows the existence of closed timelike curves, paths through spacetime that, if followed, allow a time traveler – whether human being or elementary particle – to interact with her former self. The possibility of such closed timelike curves (CTCs) was pointed out by Kurt Gödel [1], and a variety of spacetimes containing closed timelike curves have been proposed [2, 3]. As in all versions of time travel, closed timelike curves embody apparent paradoxes, such as the grandfather paradox, in which the time traveller inadvertently or on purpose performs an action that causes her future self not to exist. Einstein (a good friend of Gödel) was himself seriously disturbed by the discovery of CTCs [4].

Reconciling closed timelike curves with quantum mechanics is a difficult problem that has been addressed repeatedly, for example, using path integral techniques [5–10]. This paper explores a particular version of closed timelike curves based on combining quantum teleportation with post-selection. The resulting post-selected closed timelike curves (P-CTCs) provide a self-consistent picture of the quantum mechanics of time-travel. P-CTCs offer a theory of closed timelike curves that is inequivalent to other Hilbert-space based theories, e.g., that of Deutsch [11]. Because the theory of P-CTCs rely on post-selection, they provide self-consistent resolutions to such paradoxes: anything that happens in a P-CTC can also happen in conventional quantum mechanics with some probability. Similarly, the post-selected nature of P-CTCs allows the predictions and retrodictions of the theory to be tested experimentally, even in the absence of an actual general-relativistic closed timelike curve.

To provide our unifying description of closed timelike curves in quantum mechanics, we start from the prescription that time travel effectively represents a communication channel from the future to the past. Quantum time travel, then, should be described by a quantum communication channel to the past. A well-known quantum communication channel is given by quantum teleportation, in which shared entanglement combined with quantum measurement and classical communication allows quantum states to be transported between sender and receiver. We show that if quantum teleportation is combined with post-selection, then the result is a quantum channel to the past. The entanglement occurs between the forward- and backward- going parts of the curve, and post-selection replaces the quantum measurement and obviates the need for classical communication, allowing time travel to take place. The resulting theory allows a possible description of the quantum mechanics of general relativistic closed timelike curves.

The two basic types of paradoxes that arise in time travel to the past are the grandfather paradox, in which the time traveller, accidentally or on purpose, kills her grandfather as young man before he as had any children. So she does not exist; so she can't go to the past and kill her grandfather, etc. Phrased in physical terms, the grandfather paradox is essentially an issue of the self-consistency of dynamics in the presence of closed timelike curves. The second type of paradox is based on the unproved theorem paradox, in which the time traveller reads an elegant proof of a theorem in a book. Going back in time, she shows the proof to a mathematician, who decides to include the proof in his book. The book, of course, is the same book in which she read the theorem in the first place. Phrased in terms of physical law, the unproved theorem paradox raises issues of indeterminacy. Spacetimes that possess closed timelike curves typically do not possess Cauchy surfaces: the behavior of particles and fields cannot be obtained simply by specifying initial

conditions and integrating the equations of motion over time. Both the grandfather paradox and the unproved theorem paradox give rise to physical issues which must be resolved in a quantum theory of CTCs.

Although Einstein's theory of general relativity implicitly allows travel to the past, it took several decades before Gödel proposed an explicit space-time geometry containing closed timelike curves (CTCs). The Gödel universe consists of a cloud of swirling dust, of sufficient gravitational power to support closed timelike curves. Later, it was realized that closed timelike curves are a generic feature of highly curved, rotating spacetimes: the Kerr solution for a rotating black hole contains closed timelike curves within the black hole horizon; and massive rapidly rotating cylinders typically are associated with closed timelike curves [2, 9, 12]. The topic of closed timelike curves in general relativity continues to inspire debate: Hawking's chronology protection postulate, for example, suggests that the conditions needed to create closed timelike curves cannot arise in any physically realizable spacetime [13]. For example, while Gott showed that cosmic string geometries can contain closed timelike curves [3], Deser *et al.* showed that physical cosmic strings cannot create CTCs from scratch [14, 15].

At bottom, the behavior of matter is governed by the laws of quantum mechanics. Considerable effort has gone into constructing quantum mechanical theories for closed timelike curves. The initial efforts to construct such theories involved path integral formulations of quantum mechanics. Hartle and Politzer pointed out that in the presence of closed timelike curves, the ordinary correspondence between the path-integral formulation of quantum mechanics and the formulation in terms of unitary evolution of states in Hilbert space breaks down [6, 8]. Morris *et al.* explored the quantum prescriptions needed to construct closed timelike curves in the presence of wormholes, bits of spacetime geometry that, like the handle of a coffee cup, 'break off' from the main body of the universe and rejoin it in the the past [5]. Meanwhile, Deutsch formulated a theory of closed timelike curves in the context of Hilbert space, by postulating self-consistency conditions for the states that enter and exit the closed timelike curve [11].

General relativistic closed timelike curves provide one potential mechanism for time travel, but they need not provide the only one. For example, even in the context of special relativity, faster-than-light-communication is known to generate temporal paradoxes and causal loops (e.g. see [16] for a recent review). Nonetheless, quantum mechanics supports a variety of counter-intuitive phenomena which might allow time travel even in the absence of a closed timelike curve in the geometry of spacetime. One of the best-known versions of non-general relativistic quantum versions of time travel comes from Wheeler, as described by Feynman in his Nobel Prize lecture [17]. As we will see, post-selected closed timelike curves make up a precise physical theory which instantiates Wheeler's whimsical idea.

As described in previous work [18], the notion that entanglement and projection can give rise to closed timelike curves to has arisen independently in a variety of contexts. This combination lies at the heart of the Horowitz-Maldacena model for information escape from black holes [19–22], and Gottesman and Preskill note in passing that this mechanism might be used for time travel [21]. Pegg explored the use of a related mechanism for 'probabilistic time machines' [23]. Bennett and Schumacher have explored similar notions in unpublished work [24]. Laforest, Baugh, and Laflamme analyzed their proposal, its consistency with the tensor product structure, and a proof-of-principle experiment that tests the symmetry of information flow and of apparent causality breaking [25]. Coecke [26] studies the symmetry of information flow using entanglement as mediation, and interpreting entanglement as sending information back in time in such a way that the information sent back in time is altered depending on the outcome of a (future) Bell measurement. Ralph suggests using teleportation for time traveling, although in a different setting, namely, displacing the entangled resource in time [27]. Svetlichny describes experimental techniques for investigating quantum travel based on entanglement and projection [28]. Chiribella *et al.* consider this mechanism while analyzing extensions to the quantum computational model [29]. Brukner *et al.* have analyzed probabilistic teleportation (where only the cases in which the Bell measurement yields the desired result are retained) as a computational resource in [30]. Greenberger and Svozil [31] show how the grandfather paradox can be solved using quantum interference when feedback backward-in-time is allowed using unitary couplings similar to beam-splitters: one can set up the quantum interference in this interferometer-analogue such that self-contradictory events cannot happen. In [32] it is shown that the existence of time travel paradoxes would lead to violations in the probability rules in a simple finite-state model.

The outline of the paper follows. In Sec. I we describe P-CTCs and Deutsch's mechanism in detail, emphasizing the differences between the two approaches. Then, in Sec. II we relate P-CTCs to the path-integral formulation of quantum mechanics. This formulation is particularly suited for the description of quantum field theory in curved spacetime [33], and has been used before to provide quantum descriptions of closed timelike curves [6–8, 10, 34–37]. Our proposal is consistent with these path-integral approaches. In particular, the path-integral description of fermions using Grassmann fields given by Politzer [6] yields a dynamical description which coincides with ours for systems of quantum bits. Other descriptions, such as Hartle's [8], are more difficult to compare as they do not provide an explicit prescription to calculate the details of the dynamics of the interaction with systems inside closed timelike curves. In any case, their general framework is consistent with our derivations. By contrast, Deutsch's CTCs are not compatible with the Politzer path-integral approach, and are ana-

lyzed by him on a different footing [6]. Indeed, suppose that the path integral is performed over classical paths which agree both at the entrance to– and at the exit from– the CTC, so that x-in, p-in are the same as x-out, p-out. Similarly, in the Grassmann case, suppose that spin-up along the z axis at the entrance emerges as spin-up along the z axis at the exit. Then, the quantum version of the CTC must exhibit the same perfect correlation between input and output. But, as the grandfather paradox experiment [18] shows, Deutsch's CTCs need not exhibit such correlations: spin-up in is mapped to spin-down out (although the overall quantum state remains the same). By contrast, P-CTCs exhibit perfect correlation between in- and out- versions of all variables. Note that a quantum-field theoretical justification of Deutsch's solution is proposed in [38, 39] and is based on introducing additional Hilbert subspaces for particles and fields along the geodesic: observables at different points along the geodesic commute because they act on different Hilbert spaces.

The path-integral formulation also shows that using P-CTCs it is impossible to assign a well defined state to the system in the CTC. This is a natural requirement (or, at least, a desirable property), given the cyclicity of time there. In contrast, Deutsch's consistency condition (2) is explicitly built to provide a prescription for a definite quantum state ρ_{CTC} of the system in the CTC.

In Sec. III we go beyond the path-integral formulation and provide the dynamical evolution formulas in the context of generic quantum mechanics (the Hilbert-space formulation). Namely, we treat the CTC as a generic quantum transformation, where the transformed system emerges at a previous time "after" eventually interacting with some chronology-respecting systems that are external to the CTC. In this framework we obtain the explicit prescription of how to calculate the nonlinear evolution of the state of the system in the chronology-respecting part of the spacetime. This nonlinearity is exactly of the form that previous investigations (e.g. Hartle's [8]) have predicted.

In Sec. IV we consider time travel situations that are independent from general-relativistic CTCs. We then conclude in Sec. V with considerations on the computational power of the different models of CTCs.

I. P-CTCS AND DEUTSCH'S CTCS

Any quantum theory of gravity will have to propose a prescription to deal with the unavoidable [8] nonlinearities that plague CTCs. This requires some sort of modification of the dynamical equations of motions of quantum mechanics that are always linear. Deutsch in his seminal paper [11] proposed one such prescription, based on a self-consistency condition referred to the state of the systems inside the CTC. Deutsch's theory has recently been critiqued by several authors as exhibiting self-contradictory features [38–41]. By contrast, although any quantum theory of time travel quantum mechanics is likely to yield strange and counter-intuitive results, P-CTCs appear to be less pathological [18]. They are based on a different self-consistent condition that states that self-contradictory events do not happen (Novikov principle [34]). Pegg points out that this can arise because of destructive interference of self-contradictory histories [23]. Here we further compare Deutsch's and post-selected closed timelike curves, and give an in-depth analysis of the latter, showing how they can be naturally obtained in the path-integral formulation of quantum theory and deriving the equations of motions that describe the interactions with CTCs. As noted, in addition to general-relativistic CTCs, our proposed theory can also be seen as a theoretical elaboration of Wheeler's assertion to Feynman that 'an electron is a positron moving backward in time' [17]. In particular, any quantum theory which allows the nonlinear process of post-selection supports time travel even in the absence of general-relativistic closed timelike curves.

The mechanism of P-CTCs [18] can be summarized by saying that they behave exactly as if the initial state of the system in the P-CTC were in a maximal entangled state (entangled with an external purification space) and the final state were post-selected to be in the same entangled state. When the probability amplitude for the transition between these two states is null, we postulate that the related event does not happen (so that the Novikov principle [34] is enforced). As we will show in the following, this is equivalent to requiring that the time evolution of the system external to the CTC is given by

$$\mathcal{N}[\rho] \propto C_A \, \rho \, C_A^\dagger \, , \qquad (1)$$

where $C_A = Tr_E[U_{AE}]$ is the partial trace, over the Hilbert space E of the system in the CTC, of the unitary evolution U_{AE} that couples it to the external system. To enforce the Novikov principle, we also have to suppose that the evolution described in (1) will not take place if the right hand side is null [63]. A formulation equivalent to Eq. (1) consists in requesting that a pure state $|\psi\rangle$ evolves to $|\psi'\rangle \propto C_A|\psi\rangle$ (when this in not a null vector). This will be derived below also using the path integral formulation of quantum theory, by showing that the transition amplitude from an initial state $|\psi\rangle$ to an arbitrary final state $|F\rangle$ is proportional to $\langle F|\mathrm{Tr}_E[U_{AE}]|\psi\rangle$.

By contrast, Deutsch's CTCs are based on imposing the consistency condition

$$\rho_{CTC} = \mathrm{Tr}_A[U(\rho_A \otimes \rho_{CTC})U^\dagger], \qquad (2)$$

where ρ_{CTC} is the state of the system inside the closed timelike curve, ρ_A is the state of the system outside (i.e. of the chronology-respecting part of spacetime), U is the unitary transformation that is responsible for eventual interactions among the two systems, and where the trace is performed over the chronology-respecting system. The existence of a state ρ that satisfies (2) is ensured by the fact that any completely-positive map of the form

$\mathcal{L}[\rho] = \mathrm{Tr}_A[U(\rho_A \otimes \rho)U^\dagger]$ always has at least one fixed point ρ (or, equivalently, one eigenvector ρ with eigenvalue one). If more than one state ρ_{CTC} satisfies the consistency condition (2), Deutsch separately postulates a "maximum entropy rule", requesting that the maximum entropy one must be chosen. Note that Deutsch's formulation assumes that the state exiting the CTC in the past is factorized with the chronology-preserving variables (the properties pertaining to systems that are external to all CTCs) at that time: the time-traveler's 'memories' of events in the future are no longer valid.
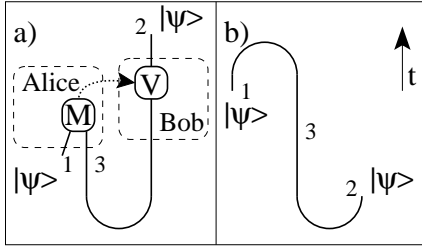

FIG. 1: Description of closed timelike curves through teleportation. a) Conventional teleportation: Alice and Bob start from a maximally entangled state shared among them represented by "⋃". Alice performs a Bell measurement M on her half of the shared state and on the unknown state $|\psi\rangle$ she wants to transmit. This measurement tells her which entangled state the two systems are in. She then communicates (dotted line) the measurement result to Bob who performs a unitary V on his half of the entangled state, obtaining the initial unknown state $|\psi\rangle$. The numbers refer to the systems as indicated by the subscripts in Eq. (3). b) Post-selected teleportation: the system in state $|\psi\rangle$ and half of the Bell state "⋃" are projected onto the same Bell state "⋂". This means that the other half of the Bell state is projected into the initial state of the system $|\psi\rangle$ even *before* this state is available.

The primary conceptual difference between Deutsch's CTCs and P-CTCs lies in the self-consistency condition imposed. Consider a measurement that can be made either on the state of the system as it enters the CTC, or on the state as it emerges from the CTC. Deutsch demands that these two measurements yield the same statistics for the CTC state alone: that is, the density matrix of the system as it enters the CTC is the same as the density matrix of the system as it exits the CTC. By contrast, we demand that these two measurements yield the same statistics for the CTC state *together with its correlations with any chronology preserving variables*. It is this demand that closed timelike curves respect both statistics for the time-traveling state together with its correlations with other variables that distinguishes P-CTCs from Deutsch's CTCs. The fact that P-CTCs respect correlations effectively enforces the Novikov principle [34], and, as will be seen below, makes P-CTCs consistent with path-integral approaches to CTCs.

The connection between P-CTCs and teleportation [42] is illustrated (see Fig. 1) with the following simple example that employs qubits (extensions to higher

dimensional systems are straightforward). Suppose that the initial Bell state is $|\Psi^{(-)}\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$ (but any maximally entangled Bell state will equivalently work), and suppose that the initial state of the system entering the CTC is $|\psi\rangle$. Then the joint state of the three systems (system 1 entering the CTC, system 2 emerging from the CTC, and system 3, its purification) is given by $|\psi\rangle_1 |\Psi^{(-)}\rangle_{23}$. These three systems are denoted by the three vertical lines of Fig. 1b. It is immediate to see that this state can be also written as

$$(-|\Psi^{(-)}\rangle_{13}|\psi\rangle_2 - |\Psi^{(+)}\rangle_{13}\sigma_z|\psi\rangle_2 + $$
$$|\Phi^{(-)}\rangle_{13}\sigma_x|\psi\rangle_2 + i|\Phi^{(+)}\rangle_{13}\sigma_y|\psi\rangle_2)/2 , \qquad (3)$$

where $|\Psi^{(\pm)}\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$ and $|\Phi^{(\pm)}\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$ are the four states in a Bell basis for qubit systems and σ_αs are the three Pauli matrices. Eq. (3) is equivalent to Eq. (5) of Ref. [42], where the extension to higher dimensional systems is presented (the extension to infinite dimensional systems is presented in [43]). It is immediate to see that, if the system 1 entering the CTC together with the purification system 3 are post-selected to be in the same Bell state $|\Psi^{(-)}\rangle_{13}$ as the initial one, then only the first term of Eq. (3) survives. Apart from an inconsequential minus sign, this implies that the system 2 emerging from the CTC is in the state $|\psi\rangle_2$, which is exactly the same state of the system that has entered (rather, will enter) the CTC.

It seems that, based on what is currently known on these two approaches, we cannot conclusively choose P-CTCs over Deutsch's, or *vice versa*. Both arise from reasonable physical assumptions and both are consistent with different approaches to reconciling quantum mechanics with closed timelike curves in general relativity. A final decision on which of the two is "actually the case" may have to be postponed to when a full quantum theory of gravity is derived (which would allow to calculate from first principles what happens in a CTC) or when a CTC is discovered that can be tested experimentally. However, because of the huge recent interest on CTCs in physics and in computer science (e.g. see [40, 41, 44–48]), it is important to point out that there are reasonable alternatives to the leading theory in the field. We also point out that our post-selection based description of CTCs seems to be less pathological than Deutsch's: for example P-CTCs have less computational power and do not require to separately postulate a "maximum entropy rule" [18]. Therefore, they are in some sense preferable, at least from an Occam's razor perspective. Independent of such questions of aesthetic preference, as we will now show, P-CTCs are consistent with previous path integral formulations of closed timelike curves, whereas Deutsch's CTCs are not.

II.   P-CTCS AND PATH INTEGRALS

Path integrals [49, 50] allow one to calculate the transition amplitude for going from an initial state $|I\rangle$ to a final state $|F\rangle$ as an integral over paths of the action, i.e.

$$\langle F|\exp(-\tfrac{i}{\hbar}H\tau)|I\rangle = \int_{-\infty}^{+\infty} dx\, dy\, I(x)\, F^*(y) \int_x^y \mathcal{D}x(t)\exp[\tfrac{i}{\hbar}S], \text{ where } S = \int_0^\tau dt L(x,\dot{x}) \,, \tag{4}$$

and where L is the Lagrangian, H the Hamiltonian, S is the action, $I(x)$ and $F(x)$ are the position representations of $|I\rangle$ and $|F\rangle$ respectively (i.e. $|I\rangle = \int dx\, I(x)|x\rangle$), and the paths in the integration over paths— indicated by $\int \mathcal{D}x(t)$– all start in x and end in y. Of course in this form it is suited only to describing the dynamics of a particle in space (or a collection of particles). It will be extended to other systems in the next section.

In order to add a CTC, we first divide the spacetime into two parts,

$$\langle F|_C\langle F'|\exp(-\tfrac{i}{\hbar}H\tau)|I\rangle|I'\rangle_C = \int_{-\infty}^{+\infty} dx\, dx_C dy\, dy_C\, I(x)\, I'(x_C)\, F^*(y)\, F'^*(y_C) \int_{x,x_C}^{y,y_C} \mathcal{D}x(t)\exp[\tfrac{i}{\hbar}S] \,, \tag{5}$$

where the first part will represent the chronology-respecting system outside the CTC, and the second part (indicated with the subscript C) will represent the system in the CTC, once appropriate conditions are enforced. The "conventional" strategy to deal with CTCs using path integrals is to send the system C to a prior time unchanged (i.e. with the same values of $x, \dot{x}$), while the other system (the chronology-respecting one) evolves normally. This is enforced by imposing periodic boundary conditions on the CTC boundaries. Namely, the probability amplitude for the chronology-respecting system is

$$\langle F|\exp(-\tfrac{i}{\hbar}H\tau)|I\rangle \propto \int_{-\infty}^{+\infty} dx\, dx_C dy\, dy_C\, I(x)\, F^*(y)\, \delta(x_C - y_C) \int_{x,x_C}^{y,y_C} \mathcal{D}x(t)\exp[\tfrac{i}{\hbar}S] \,, \tag{6}$$

where the δ-function ensures that the initial and final boundary conditions in the CTC system are the same. Note that we have removed $I'(x_C)$ and $F'(y_C)$, but we are coherently adding all possible initial and final conditions (through the x_C and y_C integrals). This implies that it is not possible to assign a definite state to the system inside a CTC: one could consistently assign to the system any possible state that is compatible with the (periodic) boundary conditions. Note also that the boundary conditions of Eq. (6) have previously appeared in the literature (e.g. see [10] and, in the classical context, in the seminal paper [51]).

To show that Eq. (6) is the same formula that one obtains using post-selected teleportation, we have to calculate $\langle F|\langle\Psi|\exp(-\tfrac{i}{\hbar}H\tau)\otimes\mathbb{1}|I\rangle|\Psi\rangle$, where $|\Psi\rangle$ is a maximally entangled state in position and where the Hamiltonian acts only on the system and on the first of the two Hilbert spaces of $|\Psi\rangle$. As a maximally entangled state in position we use the EPR [52] state $|\Psi\rangle \propto \int dx|xx\rangle$. Since this state is non-normalizable, a rigorous treatment requires a regularization and will be given in the Appendix. Here we employ the non-normalizable EPR state $|\Psi\rangle$ just to provide the idea behind the proof. Use Eq. (5) for the system and for the first Hilbert space of $|\Psi\rangle$ to obtain

$$\langle F|\langle\Psi|\exp(-\tfrac{i}{\hbar}H\tau)\otimes\mathbb{1}|I\rangle|\Psi\rangle \propto \int_{-\infty}^{+\infty} dx\, dx'dy\, dy'dz\, dz'\, I(x)\, F^*(y)\, \delta(x' - z)\, \delta(y' - z')\langle z|\mathbb{1}|z'\rangle$$

$$\times \int_{x,x'}^{y,y'} \mathcal{D}x(t)\exp[\tfrac{i}{\hbar}S] = \int_{-\infty}^{\infty} dxdx'dydy'I(x)F^*(y)\,\delta(x' - y') \int_{x,x'}^{y,y'} \mathcal{D}x(t)\exp\left[\tfrac{i}{\hbar}S\right] \,, \tag{7}$$

where we have used the position representation

$$\langle\Psi| \propto \int dx'\, dz\, \delta(x' - z)\langle x'|\langle z| \text{ and}$$

$$|\Psi\rangle \propto \int dy'\, dz'\, \delta(y' - z')|y'\rangle|z'\rangle \,,$$

with $\langle z|z'\rangle = \delta(z - z')$. Note that this result is independent of the particular form of the EPR state $|\Psi\rangle$ as long as it is maximally entangled in position (and hence in momentum).

All the above discussion holds for initial and final pure states. However, the extension to mixed states in the

path-integral formulation is straightforward: one only needs to employ appropriate purification spaces [53, 54]. The formulas then reduce to the previous ones.

Here we briefly comment on the two-state vector formalism of quantum mechanics [55, 56]. It is based on post-selection of the final state and on renormalizing the resulting transition amplitudes: it is a time-symmetrical formulation of quantum mechanics in which not only the initial state, but also the final state is specified. As such, it shares many properties with our post-selection based treatment of CTCs. In particular, in both theories it is impossible to assign a definite quantum state at each time: in the two-state formalism the unitary evolution forward in time from the initial state might give a different mid-time state with respect to the unitary evolution backward in time from the final state. Analogously, in a P-CTC, it is impossible to assign a definite state to the CTC system at any time, given the cyclicity of time there. This is evident, for example, from Eq. (6): in the CTC system no state is assigned, only periodic boundary conditions. Another aspect that the two-state formalism and P-CTCs share is the nonlinear renormalization of the states and probabilities. In both cases this arises because of the post-selection. In addition to the two-state formalism, our approach can also be related to weak values [55, 57], since we might be performing measurements between when the system emerges from the CTC and when it re-enters it. Considerations analogous to the ones presented above apply. It would be a mistake,

however, to think that the theory of post-selected closed timelike curves in some sense requires or even singles out the weak value theory. Although the two are compatible with each other, the theory of P-CTCs is essentially a 'free-standing' theory that does not give preference to one interpretation of quantum mechanics over another.

III. GENERAL SYSTEMS

The formula (6) was derived in the path-integral formulation of quantum mechanics, but it can be easily extended to generic quantum evolution.

We start by recalling the usual Kraus decomposition of a generic quantum evolution (that can describe the evolution of both isolated and open systems). It is given by

$$\mathcal{L}[\rho] = \text{Tr}_E[U(\rho \otimes |e\rangle\langle e|)U^\dagger] = \sum_i B_i \rho B_i^\dagger , \quad (8)$$

where $|e\rangle$ is the initial state of the environment (or, equivalently, of a putative abstract purification space), U is the unitary operator governing the interaction between system initially in the state ρ and environment, and $B_i \equiv \langle i|U|e\rangle$ is the Kraus operator ($\{|i\rangle\}$ being an arbitrary basis for the Hilbert space of the environment). In contrast, the evolution of our post-selected teleportation scheme is given by

$$\rho_A \to \text{Tr}_{EE'}\left[\left(U_{AE} \otimes \mathbb{1}_{E'}\right)\left(\rho_A \otimes |\Psi\rangle_{EE'}\langle\Psi|\right)\left(U_{AE}^\dagger \otimes \mathbb{1}_{E'}\right)\left(\mathbb{1}_A \otimes |\Psi\rangle_{EE'}\langle\Psi|\right)\right]$$
$$= \sum_{l_E, l'_E} \langle l_E|U_{AE}|l_E\rangle \rho_A \langle l'_E|U_{AE}^\dagger|l'_E\rangle = C_A \rho_A C_A^\dagger, \quad (9)$$

where $C_A \equiv \text{Tr}_E[U_{AE}]$, $\{|l_E\rangle\}$ is a set of basis states, and $|\Psi\rangle_{EE'} \propto \sum_l |l\rangle_E |l\rangle_{E'}$ (or any other maximally entangled state, which would give the same result). In Eq. (9) the subscript A refers to the Hilbert space of the external system, and E and E' to the Hilbert spaces of the forward- and backward- propagating parts of the CTC. Note that C_A is equal to one of the Kraus operators B_i of the system, as can be immediately seen by choosing a basis $\{|i\rangle\}$ of the EE' Hilbert space that contains the state $|\Psi\rangle$ as one of its elements. The evolution in (9) does not preserve the state's normalization because of the post-selection entailed by the projection onto the final state $|\Psi\rangle_{EE'}$. Then, we need to renormalize the final state, introducing a nonlinearity: according to our approach, a chronology-respecting system in a state ρ that interacts with a CTC using a unitary U will undergo the nonlinear transformation

$$\mathcal{N}[\rho] = C_A \rho C_A^\dagger / \text{Tr}[C_A \rho C_A^\dagger] , \quad (10)$$

where we suppose that this evolution is impossible whenever $\text{Tr}_A[C_A \rho C_A^\dagger] = 0$. More specifically, all evolutions that would lead to a vanishing denominator in Eq. (10) are forbidden: they cannot happen (e.g. see also [23]). An equivalent condition is to request that the evolution is possible if and only if $C_A^\dagger C_A$ is a strictly positive operator.

The comparison with (8) is instructive: there the non-unitarity comes from the inaccessibility of the environment. Analogously, in (10) the non-unitarity comes from the fact that, after the CTC is closed, for the chronology-respecting system it will be forever inaccessible. The nonlinearity of (10) is more difficult to interpret, but is connected with the periodic boundary conditions in the CTC. Note that this general evolution equation (10) is consistent with previous derivations based on path integrals. For example, it is equivalent to Eq. (4.6) of Ref. [8] by Hartle. However, in contrast to here, the actual form

of the evolution operators C is not provided there. As a further example, consider Ref. [6], where Politzer derives a path integral approach of CTCs for qubits, using Grassmann fields. His Eq. (5) is compatible with Eq. (9). He also derives a nonunitary evolution that is consistent with Eq. (10) in the case in which the initial state is pure. In particular, this implies that, also in the general qudit case, our post-selected teleportation approach gives the same result one would obtain from a specific path-integral formulation. In addition, it has been pointed out many times before (e.g. see [35, 58]) that when quantum fields inside a CTC interact with external fields, linearity and unitarity is lost. It is also worth to notice that there have been various proposals to restore unitarity by modifying the structure of quantum mechanics itself or by postulating an inaccessible purification space that is added to uphold unitarity [59, 60].

The evolution (10) coming from our approach is to be compared with Deutsch's evolution,

$$\mathcal{D}[\rho] = \mathrm{Tr}_E[U(\rho \otimes \rho_{CTC})U^\dagger], \qquad (11)$$

where

$$\rho_{CTC} = \mathrm{Tr}_A[U(\rho \otimes \rho_{CTC})U^\dagger] \qquad (12)$$

satisfies the consistency condition and where the trace in (11) refers to the degrees of freedom inside the CTC. The direct comparison of Eqs. (10) and (12) highlights the differences in the general prescription for the dynamics of CTCs of these two approaches.

Even though the results presented in this section are directly applicable only to general finite-dimensional systems, the extension to systems living in infinite-dimensional separable Hilbert spaces seems conceptually straightforward, although mathematically involved.

In his path-integral formulation of CTCs, Hartle notes that CTCs might necessitate abandoning not only unitarity and linearity, but even the familiar Hilbert space formulation of quantum mechanics [8]. Indeed, the fact that the state of a system at a given time can be written as the tensor product states of subsystems relies crucially on the fact that operators corresponding to spacelike separated regions of spacetime commute with each other. When CTCs are introduced, the notion of 'spacelike' separation becomes muddied. The formulation of closed timelike curves in terms of P-CTCs shows, however, that the Hilbert space structure of quantum mechanics can be retained.

IV. TIME TRAVEL IN THE ABSENCE OF GENERAL-RELATIVISTIC CTCS

Although the theory of P-CTCs was developed to address the question of quantum mechanics in general-relativistic closed timelike curves, it also allows us to address the possibility of time travel in other contexts. Essentially, any quantum theory that allows the nonlinear process of projection onto some particular state, such as the entangled states of P-CTCs, allows time travel even when no spacetime closed timelike curve exists. Indeed, the mechanism for such time travel closely follows Wheeler's famous telephone call above. Non-general-relativistic P-CTCs can be implemented by the creation of and projection onto entangled particle-antiparticle pairs. Such a mechanism is just what is used in our experimental tests of P-CTCs [18]: although projection is a non-linear process that cannot be implemented deterministically in ordinary quantum mechanics, it can easily be implemented in a probabilistic fashion. Consequently, the effect of P-CTCs can be tested simply by performing quantum teleportation experiments, and by post-selecting only the results that correspond to the desired entangled-state output.

If it turns out that the linearity of quantum mechanics is only approximate, and that projection onto particular states does in fact occur – for example, at the singularities of black holes [19–22] – then it might be possible to implement time travel even in the absence of a general-relativistic closed timelike curve. The formalism of P-CTCs shows that such quantum time travel can be thought of as a kind of quantum tunneling backwards in time, which can take place even in the absence of a classical path from future to past.

V. COMPUTATIONAL POWER OF CTCS

It has been long known that nonlinear quantum mechanics potentially allows the rapid solution of hard problems such as NP-complete problems [61]. The nonlinearities in the quantum mechanics of closed timelike curves is no exception [46–48]. Aaronson and Watrous have shown quantum computation with Deutsch's closed timelike curves allows the solution of any problem in PSPACE, the set of problems that can be solved using polynomial space resources [46]. Similarly, Aaronson has shown that quantum computation combined with post-selection allows the solution of any problem in the computational class PP, probabilistic polynomial time(where a probabilistic polynomial Turing machine accepts with probability $\frac{1}{2}$ if and only if the answer is "yes."). Quantum computation with post-selection explicitly allows P-CTCs, and P-CTCs in turn allow the performance of any desired post-selected quantum computation. Accordingly, quantum computation with P-CTCs can solve any problem in PP, including NP-complete problems. Since the class PP is thought to be strictly contained in PSPACE, quantum computation with P-CTCs is apparently strictly less powerful than quantum computation with Deutsch's CTCs.

In the case of quantum computing with Deutschian CTCs, Bennett et al. [40] have questioned whether the notion of programming a quantum computer even makes sense. Ref. [40] notes that in Deutsch's closed timelike curves, the nonlinearity introduces ambiguities in the definition of state preparation: as is well-known in nonlin-
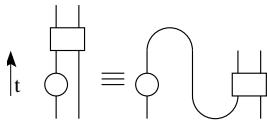
FIG. 2: Closed timelike loops can collapse the time-depth of any circuit to one, allowing to compute any problem not merely efficiently, but instantaneously.

ear quantum theories, the result of sending *either* the state $|\psi\rangle$ through a closed-timelike curve *or* the state $|\phi\rangle$ is no longer equivalent to sending the mixed state $(1/2)(|\psi\rangle\langle\psi| + |\phi\rangle\langle\phi|)$ through the curve. The problem with computation arises because, as is clear from our grandfather-paradox circuit [18], Deutsch's closed timelike curves typically break the correlation between chronology preserving variables and the components of a mixed state that enters the curve: the component that enters the CTC as $|0\rangle$ can exit the curve as $|1\rangle$, even if the overall mixed state exiting the curve is the same as the one that enters. Consequently, Bennett *et al.* argue, the programmer who is using a Deutschian closed timelike curve as part of her quantum computer typically finds the output of the curve is completely decorrelated from the problem she would like to solve: the curve emits random states.

In contrast, because P-CTCs are formulated explicitly to retain correlations with chronology preserving curves, quantum computation using P-CTCs do not suffer from state-preparation ambiguity. That is not so say that P-CTCs are computationally innocuous: their nonlinear nature typically renormalizes the probability of states in an input superposition, yielding to strange and counter-intuitive effects. For example, any CTC can be used to compress any computation to depth one, as shown in Fig. 2. Indeed, it is exactly the ability of nonlinear quantum mechanics to renormalize probabilities from their conventional values that gives rise to the amplification of small components of quantum superpositions that allows the solution of hard problems. Not least of the counter-intuitive effects of P-CTCs is that they could still solve hard computational problems with ease! The 'excessive' computational power of P-CTCs is effectively an argument for why the types of nonlinearities that give rise to P-CTCs, if they exist, should only be found under highly exceptional circumstances such as general-relativistic closed timelike curves or black-hole singularities.

VI. CONCLUSIONS

This paper reviewed quantum mechanical theories for time travel, focusing on the theory of P-CTCs [18]. Our purpose in presenting this work is to make precise the similarities and differences between varying quantum theories of time travel. We summarize our findings here.

We have extensively argued that P-CTCs are inequivalent to Deutsch's CTCs. In Sec. II we showed that P-CTCs are compatible with the path-integral formulation of quantum mechanics. This formulation is at the basis of most of the previous analysis of quantum descriptions of closed time-like curves, since it is particularly suited to calculations of quantum mechanics in curved space time. P-CTCs are reminiscent of, and consistent with, the two-state-vector and weak-value formulation of quantum mechanics. It is important to note, however, that P-CTCs do not in any sense require such a formulation. Then, in Sec. III we extended our analysis to general systems where the path-integral formulation may not always be possible and derived a simple prescription for the calculation of the CTC dynamics, namely Eq. (10). In this way we have performed a complete characterization of P-CTC in the most commonly employed frameworks for quantum mechanics, with the exception of algebraic methods (e.g. see [62]).

In Sec. IV we have argued that, as Wheeler's picture of positrons as electrons moving backwards in time suggests, P-CTCs might also allow time travel in spacetimes without general-relativistic closed timelike curves. If nature somehow provides the nonlinear dynamics afforded by final-state projection, then it is possible for particles (and, in principle, people) to tunnel from the future to the past.

Finally, in Sec. V we have seen that P-CTCs are computationally very powerful, though less powerful than the Aaronson-Watrous theory of Deutsch's CTCs.

Our hope in elaborating the theory of P-CTCs is that this theory may prove useful in formulating a quantum theory of gravity, by providing new insight on one of the most perplexing consequences of general relativity, i.e., the possibility of time-travel.

Appendix A: Regularization of maximally entangled states in position

In this section we prove the result presented in Eq. (7) using two different normalizable states $|\Psi_a\rangle$ and $|\Psi_\gamma\rangle$ (the first allows simple calculations, whereas the second is more physically-motivated). To avoid technicalities, Eq. (7) was presented above using the un-normalizable EPR state $|\Psi\rangle \propto \int dx |xx\rangle$.

First regularization

Consider the normalized state

$$|\Psi_a\rangle \equiv \sqrt{\tfrac{2}{\pi}} \int dx\, dy\, e^{-a^2 x^2} e^{-(x-y)^2/a^2} |x\rangle|y\rangle\,. \quad \text{(A1)}$$

In the limit $a \sim 0$ this state tends to the EPR state, as

$$|\Psi_a\rangle = \sqrt{\tfrac{2}{\pi}} \int dx \, e^{-a^2 x^2} \int dy' e^{-y'^2/a^2} |x\rangle |x - y'\rangle$$

$$\simeq \sqrt{2}a \int dx \int dy' \delta(y') |x\rangle |x - y'\rangle = \sqrt{2}a \int dx |x\rangle |x\rangle \ ,$$

where we have used $y' \equiv x - y$ and the fact that $\delta(y) = \lim_{a \to 0} e^{-y^2/a^2}/(a\sqrt{\pi})$. Replacing $|\Psi\rangle$ with $|\Psi_a\rangle$ in Eq. (7), and taking $a \sim 0$, we reobtain the same result, apart from the inconsequential proportionality factor $2a^2$ that, as we will now show, can be removed by calculating the conditional probability amplitude.

The path integral is a transition amplitude between an initial state $|i\rangle$ and a final state $|f\rangle$. As such, it can

always be written as [49],

$$\langle f| \exp(-\tfrac{i}{\hbar} H \tau)|i\rangle = \langle f|U|i\rangle \ , \qquad (A2)$$

where H is the Hamiltonian and U the unitary evolution of the system. To derive (7) we consider three systems: the system A external to the CTC, which starts in the system $|I\rangle_A$ and ends in the system $|F\rangle_A$; the system E in the CTC and an ancillary system E', which are initially in a joint state $|\Psi_a\rangle_{EE'}$ and are (post-selected) in the same final state $|\Psi_a\rangle_{EE'}$. If U_{AE} is the unitary describing the interaction between the systems external and internal to the CTC, the path integral for these three systems is then

$$_A\langle F|_{EE'}\langle \Psi_a|U_{AE} \otimes \mathbb{1}_{E'}|I\rangle_A|\Psi_a\rangle_{EE'} = \tfrac{2}{\pi} \int dx \, dx' dy \ e^{-a^2(x^2+x'^2)-(x-y)^2/a^2-(x'-y)^2/a^2} {}_A\langle F|_E\langle x|U_{AE}|I\rangle_A|x'\rangle_E \ (A3)$$

$$= \sqrt{\tfrac{2}{\pi}}a \int dx \, dx' \ e^{-a^2(x^2+x'^2)-(x-x')^2/a^2} {}_A\langle F|_E\langle x|U_{AE}|I\rangle_A|x'\rangle_E \ . \ (A4)$$

The square modulus of this quantity gives the joint probability $p(|F\rangle, |\Psi_a\rangle)$ that final state of system A is $|F\rangle$ *and* that the final state of systems E and E' is $|\Psi_a\rangle$. However, our proposal is based on post-selecting the cases in which the latter event happens. To calculate the conditional probability $p(|F\rangle \mid ps)$ that the final state of A is $|F\rangle$ given that the post-selection happened, we can use $p(|F\rangle \mid ps) = p(|F\rangle, |\Psi_a\rangle)/p(ps)$, where $p(ps)$ is the probability that the post-selection succeeds independently of the final state. It can be simply calculated as

$$p(ps) = \int dF \Big| {}_A\langle F|_{EE'}\langle \Psi_a|U_{AE} \otimes \mathbb{1}_{E'}|I\rangle_A|\Psi_a\rangle_{EE'} \Big|^2 \ , \qquad (A5)$$

where the integral runs over a basis of possible final states $|F\rangle$. (Note that one could also enforce a similar condition on the initial state.)

Hence, we see that the quantity in Eq. (A3) [and also the quantities in Eq. (6) and (7)] are only *proportional* to the conditional probability amplitude we are interested in, where the proportionality constant is given by $1/\sqrt{p(ps)}$. From (A3) we see that the conditional probability amplitude is given by

$$\frac{_A\langle F|_{EE'}\langle \Psi_a|U_{AE} \otimes \mathbb{1}_{E'}|I\rangle_A|\Psi_a\rangle_{EE'}}{\sqrt{p(ps)}} = \frac{\int dx \, dx' \ e^{-a^2(x^2+x'^2)-(x-x')^2/a^2} {}_A\langle F|_E\langle x|U_{AE}|I\rangle_A|x'\rangle_E}{(\int dF| \int dx \, dx' \ e^{-a^2(x^2+x'^2)-(x-x')^2/a^2} {}_A\langle F|_E\langle x|U_{AE}|I\rangle_A|x'\rangle_E|^2)^{1/2}} \ . \ (A6)$$

We can now take the limit $a \to 0$, using again the property $\delta(y) = \lim_{a \to 0} e^{-y^2/a^2}/(a\sqrt{\pi})$. The quantity in (A6) then becomes

$$\frac{\int dx \ {}_A\langle F|_E\langle x|U_{AE}|I\rangle_A|x\rangle_E}{(\int dF|_A\langle F|_E\langle x|U_{AE}|I\rangle_A|x\rangle_E|^2)^{1/2}} \qquad (A7)$$

$$= \frac{_A\langle F|C_A|I\rangle_A}{(\mathrm{Tr}[C_A|I\rangle_A\langle I|C_A^\dagger])^{1/2}}, \qquad (A8)$$

where $C_A \equiv \mathrm{Tr}_E[U_{AE}]$. Note that the proportionality factor $2a^2$ that appears in the numerator when taking the limit $a \to 0$ is canceled by the same constant in the denominator. Recalling that the path integral can be written in the form (A2), it is simple to see that the

numerator of Eq. (A7) written in the position representation will coincide with the last term of Eq. (7) (but was here obtained from a careful regularization):

$$\int dx \ {}_A\langle F|_E\langle x|U_{AE}|I\rangle_A|x\rangle_E \qquad (A9)$$

$$= \int_{-\infty}^{\infty} dx \, dy \, dy' \ I(y)F^*(y') \int_{y,x}^{y',x} \mathcal{D}x(t) \exp\left[\tfrac{i}{\hbar}S\right] \ ,$$

where we used the position representation $|I\rangle = \int dx \, I(x)|x\rangle$ and $|F\rangle = \int dx \, F(x)|x\rangle$, and where the path integral is used to relate the initial $|x_i^A\rangle$, $|x_i^E\rangle$ and final

$|x_f^E\rangle$, $|x_f^E\rangle$ position states of the systems A and E as

$$\langle x_f^A|\langle x_f^E|U_{AE}|x_i^A\rangle|x_i^E\rangle = \int_{x_i^A,x_i^E}^{x_f^A,x_f^E} \mathcal{D}x(t)\, e^{\frac{i}{\hbar}S} \ . \quad \text{(A10)}$$

Second regularization

As a more physically-motivated alternative to the state in Eq. (A1), consider the two-mode squeezed state of two optical modes

$$|\Psi_\gamma\rangle \equiv (1-\gamma^2) \sum_{n=0}^{\infty} \gamma^n |n\rangle|n\rangle \ . \quad \text{(A11)}$$

We can write this state in the "position representation" using an eigenbasis $|x\rangle$ of the quadrature operator as

$$
\begin{aligned}
|\Psi_\gamma\rangle &= (1-\gamma^2)\sum_{n=0}^{\infty}\gamma^n \int dx\, dy\, |x\rangle\langle x|n\rangle\, |y\rangle\langle y|n\rangle \\
&= (1-\gamma^2)\int dx\, dy\, \Bigg[\sum_{n=0}^{\infty}\gamma^n \sqrt{\tfrac{2}{\pi}}\frac{1}{2^n n!} \\
&\quad \times H_n(\sqrt{2}x)H_n(\sqrt{2}y)\, e^{-x^2-y^2}\Bigg]|x\rangle|y\rangle, \quad \text{(A12)}
\end{aligned}
$$

where we have used the relation

$$\langle x|n\rangle = \left(\tfrac{2}{\pi}\right)^{1/4}\frac{1}{\sqrt{2^n n!}}H_n(\sqrt{2}x)\, e^{-x^2}\ , \quad \text{(A13)}$$

with H_n the Hermite polynomial. Use the δ-function normalization of the quadrature basis to see that $|\Psi_\gamma\rangle$ tends to the state $\int dx|x\rangle|x\rangle$ in the limit $\gamma \to 1$:

$$
\begin{aligned}
\delta(x-y) &= \langle y|x\rangle = \sum_{n=0}^{\infty}\langle y|n\rangle\langle n|x\rangle \\
&= \sum_{n=0}^{\infty}\sqrt{\tfrac{2}{\pi}}\frac{1}{2^n n!}H_n(\sqrt{2}x)H_n(\sqrt{2}y)\, e^{-x^2-y^2}, \quad \text{(A14)}
\end{aligned}
$$

whence it is clear that the term in square parentheses in Eq. (A12) tends to $\delta(x-y)$ for $\gamma \to 1$. Again, replacing $|\Psi\rangle$ with $|\Psi_\gamma\rangle$ in Eq. (7), and taking $\gamma \sim 1$, we reobtain the same result apart from an inconsequential multiplication factor $(1-\gamma^2)^2$.

Since the parameter γ is connected to the average energy of the state (A11), it is clear that the maximally entangled state $\int dx|x\rangle|x\rangle$ obtained from $|\Psi_\gamma\rangle$ with $\gamma \to 1$ requires infinite energy, and is unphysical. However, it is possible to approach it arbitrarily closely devoting sufficient energy to the task.

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