Scaling dimensions in hidden Kerr/CFT correspondence

David A. Lowe, Ilies Messamah, and Antun Skanata

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David A. Lowe, Ilies Messamah and Antun Skanata*

Department of Physics, Box 1843, Brown University, Providence, RI 02912, USA

Abstract

It has been proposed that a hidden conformal field theory (CFT) governs the dynamics of low frequency scattering in a general Kerr black hole background. We further investigate this correspondence by mapping higher order corrections to the massless wave equations in a Kerr background to an expansion within the CFT in terms of higher dimension operators. This implies the presence of infinite towers of CFT primary operators with positive conformal dimensions compatible with unitarity. The exact Kerr background softly breaks the conformal symmetry and the scaling dimensions of these operators run with frequency. The scale-invariant fixed point is dual to a degenerate case of flat spacetime.

* lowe, ilies_messamah, antun_skanata@brown.edu

I. INTRODUCTION

Several years ago an intriguing conjecture was made that quantum gravity around a general Kerr black hole background is dual to a conformal field theory [1]. Most of the evidence for this conjecture has been established for the case of extremal Kerr. For example, the properties of near-super radiant modes in extremal Kerr could be explained via a dual two-dimensional conformal field theory [2].

The conjecture is surprising because the general Kerr black hole does not have any obvious geometric symmetry near its horizon that might explain the conformal structure. More recently, it was observed that the limit of low-frequency scattering in the near-region of a black hole does possess such a hidden conformal structure [3]. This was observed by studying the massless scalar wave equation in the general Kerr background.

If this hidden CFT viewpoint can be put on a sound footing, these techniques would lead to a radically new way to treat the quantum physics of the entire class of Kerr black holes, including the Schwarzschild limit. In addition to accounting for the quantum entropy of the black hole, it would provide an efficient mechanism for computing of scattering (at least in the near-region) in a small frequency expansion. Moreover if the central charge can be computed in a reliable way from the gravity side, this proposal would yield dramatic new insight into the physics of the black hole microstates that account for the Bekenstein-Hawking entropy.

In the present work we study this hidden conformal symmetry in more detail, and further develop the correspondence between CFT primaries and bulk fields. We find that the dual CFT must contain infinite towers of quasi-primary operators with positive conformal weights, compatible with unitarity. However the full Kerr geometry softly breaks the conformal symmetry, and induces a nontrivial running of the scaling dimensions of these operators. The fixed point where the hidden conformal symmetry becomes exact is flat spacetime. This indicates that if there is an exact CFT underlying the dynamics of Kerr, there is not a smooth geometrical limit connecting the low frequency limit of general Kerr, with the promising studies of dynamics of extremal Kerr.

II. REVIEW OF HIDDEN KERR/CFT

The Kerr metric in Boyer-Lindquist coordinates (with $c = G = 1$) is

$$ds^2 = (1 - 2Mr/\Sigma)dt^2 + \left(4Mar\sin^2(\theta)/\Sigma\right)dtd\phi - (\Sigma/\Delta)\,dr^2 - \Sigma d\theta^2 - \\ \sin^2(\theta)\left(r^2 + a^2 + 2Ma^2 r\sin^2(\theta)/\Sigma\right)d\phi^2$$

where M is the black hole mass, $J = aM$ is the angular momentum, $\Sigma = r^2 + a^2\cos^2\theta$ and $\Delta = r^2 - 2Mr + a^2$. The outer and inner horizons sit at $r_\pm = M \pm \sqrt{M^2 - a^2}$. It will be convenient later to define left/right temperatures

$$T_L = \frac{M^2}{2\pi J}\,, \quad T_R = \frac{\sqrt{M^4 - J^2}}{2\pi J}\,.$$

Let us consider the wave equations for massless fields in this background. Teukolsky [4] found that it was possible to separate the wave equation in this background for general spin massless fields. Decomposed into spin-weighted spheroidal harmonics, with spin-weight s (see [4] for details) the solutions can be written

$$\psi_s = e^{-i\omega t}e^{im\phi}S(\theta)R(r) \tag{1}$$

The angular equation takes the form

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dS}{d\theta}\right)+\left(a^2\omega^2\cos^2\theta - \frac{m^2}{\sin^2\theta} - 2a\omega s\cos\theta - \frac{2ms\cos\theta}{\sin^2\theta} - s^2\cot^2\theta + E - s^2\right)S = 0 \tag{2}$$

where E is the separation constant. The eigenvalue E is constrained by the requirement that S be regular at $\theta = 0, \pi$. For the special case $a\omega = 0$ this may be computed exactly $E = \ell(\ell+1)$. For general ω this may be computed numerically, or as a series expansion. The radial equation takes the form

$$\Delta^{-s}\frac{d}{dr}\left(\Delta^{s+1}\frac{dR}{dr}\right) + \Bigg(\Big[\left(r^2 + a^2\right)^2\omega^2 - 4aMr\omega m + a^2m^2 + \\ 2ia(r - M)ms - 2iM(r^2 - a^2)\omega s\Big]\Delta^{-1} + 2ir\omega s - E + s(s+1) - a^2\omega^2\Bigg)R = 0\,. \tag{3}$$

The relation between solutions ψ_s and canonically normalized massless fields (components of the field strength for spin 1, and components of the Weyl tensor for spin 2) are given by (see [4] for further details)

$$\begin{aligned} &\text{scalar}\ \Phi = \psi_0 \\ &\text{vector}\ \varphi_0 = \psi_1 \qquad \varphi_2 = \chi^2\psi_{-1} \\ &\text{tensor}\ \Psi_0 = \psi_2 \qquad \Psi_4 = \chi^4\psi_{-2} \end{aligned} \tag{4}$$

where $\chi = -1/\left(r - ia\cos\theta\right)$.

Castro, Maloney and Strominger [3] considered the scalar case of the wave equation ($s = 0$) and noticed that in a low frequency expansion $\omega M \ll 1$, that the leading order term in the radial equation in the near-region, where $\omega r \ll 1$, reduces to a hypergeometric equation. They then showed that the full solution in this limit transformed as a representation of $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ (broken to $U(1) \times U(1)$ when the periodic identification of $\phi \sim \phi + 2\pi$ is taken into account). This led them to propose a hidden Kerr/CFT duality, with a scalar mode with angular momentum ℓ being identified with a CFT operator of conformal weight $(h_L, h_R) = (\ell, \ell)$. If one further speculates that the hidden $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ extends to a left-right Virasoro algebra with central charges $(c_L, c_R) = 12J$ then the Cardy formula for the CFT entropy agrees exactly with the Kerr horizon entropy $S = \text{Area}/4$.

In the following we will generalize CMS [3] by showing that the entire set of higher order frequency corrections can be organized into a CFT-like expansion. The precise statement is that the scaling dimensions run with frequency, which implies the CFT is deformed away from its exact conformal fixed point. Unfortunately we will see that the exact fixed point is dual to the $M = 0$ solution (i.e. flat spacetime) which seems to be a degenerate limit at odds with the $(c_L, c_R) = 12J$ proposal of [3].

III. SERIES SOLUTIONS TO THE TEUKOLSKY EQUATION

The strategy for finding exact solutions to the equations (2) and (3) will be to perform a small frequency (ω) expansion. Let us begin by considering the angular equation (2). As shown in [5], the solution in a small ω expansion is written in terms of an infinite series of Jacobi polynomials $P_j^{(\alpha,\beta)}(x)$

$$S = e^{a\omega x}\left(\frac{1-x}{2}\right)^{|m+s|/2}\left(\frac{1+x}{2}\right)^{|m-s|/2}\ {}_sU_{lm}(x)$$

where $x = \cos\theta$ and U is

$$U = \sum_{j=0}^{\infty} c_j P_j^{(|m+s|,|m-s|)}(x)\,. \tag{5}$$

Inserting (5) into (2) leads to a 3-term recurrence relation for the c_j. The expansion for U is well-defined (in the sense that each c_j is determined and finite) and convergent, provided the separation constant E satisfies an equation, that may be expressed as a continued fraction using the recurrence relations. This transcendental equation may then be readily solved as a power series expansion in $a\omega$, with the result

$$E = \ell(\ell+1) - \frac{2s^2 ma\omega}{\ell(\ell+1)} + \mathcal{O}((a\omega)^2)\,. \tag{6}$$

The radial equation (3) may be tackled in a similar way as studied in [6, 7]. This time, $R(r)$ is expressed as a series of hypergeometric functions. Defining a rescaled radial coordinate $\rho = \omega(r_+ - r)/\epsilon\kappa$, and the constants $\epsilon = 2M\omega$, $\kappa = \sqrt{1-(a/M)^2}$ and $\tau = (\epsilon - ma/M)/\kappa$, the radial function is factored as

$$R_s(\rho) = e^{i\epsilon\kappa\rho}(-\rho)^{-s-\frac{i}{2}(\epsilon+\tau)}(1-\rho)^{\frac{i}{2}(\epsilon-\tau)}P(\rho)\,. \tag{7}$$

The function $P(\rho)$ then admits the series expansion

$$P(\rho) = \sum_{n=-\infty}^{\infty} a_n\, {}_2F_1(n+\nu+1-i\tau, -n-\nu-i\tau; 1-s-i\epsilon-i\tau; \rho) \tag{8}$$

where the coefficients a_n satisfy a three-term linear recursion relation

$$\alpha_n a_{n+1} + \beta_n a_n + \gamma_n a_{n-1} = 0 \tag{9}$$

with

$$\begin{aligned}
\alpha_n &= \frac{i\epsilon\kappa(n+\nu+1+s+i\epsilon)(n+\nu+1+s-i\epsilon)(n+\nu+1+i\tau)}{(n+\nu+1)(2n+2\nu+3)} \\
\beta_n &= -E + 2\epsilon^2 - q^2\epsilon^2/4 + (n+\nu)(n+\nu+1) + \frac{\epsilon(\epsilon-mq)(s^2+\epsilon^2)}{(n+\nu)(n+\nu+1)} \\
\gamma_n &= -\frac{i\epsilon\kappa(n+\nu-s+i\epsilon)(n+\nu-s-i\epsilon)(n+\nu-i\tau)}{(n+\nu)(2n+2\nu-1)}\,.
\end{aligned}$$

There exist standard methods for solving such recursion relations, as discussed in [8]. The general solution can be expressed as a linear combination of two independent solutions (since, for example, one can choose arbitrary initial values for a_0 and a_1. One solution has coefficients that diverge as $|n| \to \infty$ and is called the dominant solution. The other solution, of most interest for the present work, is the minimal solution, where the a_n converge (or at least diverge less rapidly) at large $|n|$. This solution must be obtained by tuning a_1 with respect to a_0.

Furthermore, if the a_n converge, a continued fraction equation may be set up to determine the value of the eigenvalue ν. This may be arranged by solving for ν in two different ways: by setting $a_0 = 1$ and evolving the minimal solution to $n = \infty$ or by evolving the minimal solution to $n = -\infty$. To see this we define the ratios

$$R_n = \frac{a_n}{a_{n-1}}\,, \qquad L_n = \frac{a_n}{a_{n+1}}$$

so that R_n converges as $n \to \infty$ and L_n converges as $n \to -\infty$. Then, the three-term recurrence relation (9) may be rewritten

$$R_n = -\frac{\gamma_n}{\beta_n + \alpha_n R_{n+1}}\,, \qquad L_n = -\frac{\alpha_n}{\beta_n + \gamma_n L_{n-1}}$$

which may then be developed as convergent continued fractions. These continued fractions yield the equation

$$R_1 L_0 = 1$$

which generates a transcendental equation for ν. This may be solved as a low frequency expansion in ϵ together with the coefficients a_n as shown in [6]. This yields the solution to the Teukolsky equation infalling on the future outer horizon. The leading terms in the expansion for ν are

$$\nu = \ell - \frac{\epsilon^2}{2\ell+1}\left(2 + \frac{s^2}{\ell(\ell+1)} + \frac{\left(\ell^2 - s^2\right)^2}{(2\ell-1)2\ell(2\ell+1)} - \frac{\left((\ell+1)^2 - s^2\right)^2}{(2\ell+1)(2\ell+2)(2\ell+3)}\right) + \mathcal{O}(\epsilon^3)\,. \tag{10}$$

Interestingly, the expansion (8) converges for all finite r [6], so the near-region condition of CMS $\omega r \ll 1$ turns out not to be needed.

The solution to Teukolsky equation outgoing on the future outer horizon is obtained from the solution to (7) as

$$R_{out}(\rho) = \Delta(\rho)^{-s}(R_{-s}(\rho))^*\,.$$

IV. BULK FIELD/ CFT OPERATOR MAP

In a low frequency expansion, the exact solution (7) may be expanded as a regular series in ϵ. The leading term is

$$R_s^0(\rho) = e^{i\epsilon\kappa\rho}(-\rho)^{-s-\frac{i}{2}(\epsilon+\tau)}(1-\rho)^{\frac{i}{2}(\epsilon-\tau)}\,{}_2F_1(\nu+1-i\tau,-\nu-i\tau;1-s-i\epsilon-i\tau;\rho)\,. \quad (11)$$

We may perform a transformation $\rho \to \frac{\rho}{\rho-1}$ in the argument of the hypergeometric function to give

$$R_s^0(\rho) = e^{i\epsilon\kappa\rho}(-\rho)^{-s-\frac{i}{2}(\epsilon+\tau)}(1-\rho)^{\frac{i}{2}(\epsilon+\tau)-\nu-1}\,{}_2F_1(\nu+1-i\tau,1-s-i\epsilon+\nu;1-s-i\epsilon-i\tau;\frac{\rho}{\rho-1})$$

which agrees with eqn. (6.1) in [3] (note a typo in the argument of the hypergeometric function in the arxiv version of [3], corrected in the published version), upon replacing ν with its low frequency limit ℓ, setting $s = 0$, and dropping the first factor, as appropriate for the near-region.

The argument of [3] proceeds by noting that (11) solves the equation

$$\mathcal{H}^2\psi_0 = \bar{\mathcal{H}}^2\psi_0 = \ell(\ell+1)\psi_0$$

where $\mathcal{H}^2$ and $\bar{\mathcal{H}}^2$ are the Casimir operators of the $SL(2,\mathbb{R})\times SL(2,\mathbb{R})$ algebra generated by

$$\begin{aligned}
H_1 &= ie^{-2\pi T_R\phi}\left(\Delta^{1/2}\partial_r + \frac{1}{2\pi T_R}\frac{r-M}{\Delta^{1/2}}\partial_\phi + \frac{2T_L}{T_R}\frac{Mr-a^2}{\Delta^{1/2}}\partial_t\right)\\
H_0 &= \frac{i}{2\pi T_R}\partial_\phi + 2iM\frac{T_L}{T_R}\partial_t\\
H_{-1} &= ie^{2\pi T_R\phi}\left(-\Delta^{1/2}\partial_r + \frac{1}{2\pi T_R}\frac{r-M}{\Delta^{1/2}}\partial_\phi + \frac{2T_L}{T_R}\frac{Mr-a^2}{\Delta^{1/2}}\partial_t\right)
\end{aligned} \quad (12)$$

and

$$\begin{aligned}
\bar{H}_1 &= ie^{-2\pi T_L\phi+\frac{t}{2M}}\left(\Delta^{1/2}\partial_r - \frac{a}{\Delta^{1/2}}\partial_\phi - 2M\frac{r}{\Delta^{1/2}}\partial_t\right)\\
\bar{H}_0 &= -2iM\partial_t\\
\bar{H}_{-1} &= ie^{2\pi T_L\phi-\frac{t}{2M}}\left(-\Delta^{1/2}\partial_r - \frac{a}{\Delta^{1/2}}\partial_\phi - 2M\frac{r}{\Delta^{1/2}}\partial_t\right)
\end{aligned} \quad (13)$$

which obey

$$[H_0, H_{\pm1}] = \mp iH_{\pm1}\,, \qquad [H_{-1}, H_1] = -2iH_0 \quad (14)$$

and likewise for the others. The conjecture of [3] is that this extends to a full left-right Virasoro algebra, with central charge $c_L = c_R = 12J = 12a/M$, and that the conformal weights of the field ψ_0 are (ℓ, ℓ).

For convenience, we identify $L_n = -iH_n$ and $\bar{L}_n = -i\bar{H}_n$ so that the L_n's satisfy the standard form of the Witt algebra

$$[L_n, L_m] = (n-m)L_{n+m}\,.$$

We begin by investigating bulk modes that satisfy a lowest weight condition, which should be dual to primary operators in the CFT. Imposing the equations $L_1\psi(r,t,\phi) = \bar{L}_1\psi(r,t,\phi) = 0$ yields the solution

$$\psi(r,t,\phi) \propto \left(rr_+ - a^2\right)^{-iam/r_+} e^{im\phi - i\omega t}$$

and the condition

$$\omega = am/\left(2Mr_+\right)\,. \tag{15}$$

The conformal weights are

$$(h_L, h_R) = (\frac{iam}{r_+}, \frac{iam}{r_+})\,.$$

So this will solve the scalar field equation of motion in Kerr if we further identify the Casimir with $\ell(\ell+1)$

$$\left(L_0^2 - \frac{1}{2}\left(L_1L_{-1} + L_{-1}L_1\right)\right)\psi(r,t,\phi) = \ell(\ell+1)\psi(r,t,\phi) \tag{16}$$

which implies $h_L(h_L - 1) = \ell(\ell+1)$ and $h_L = \ell + 1$ for the positive solution. (Here we disagree with the (ℓ, ℓ) conformal weight assignment of [3]).

This leads us to the unfamiliar situation, where the eigenvalue m must be analytically continued to imaginary values to construct a mode dual to a primary CFT operator. However this is not entirely unexpected, since the space of infalling modes is a superset containing the quasi-normal modes of Kerr, studied, for example in [9]. Likewise, the case of quasi-normal modes of the 3d black hole have been studied in [10]. These modes have complex eigenvalues for ω so it is perhaps not too surprising we also wind up with complex eigenvalues for m prior to imposing periodicity of ϕ. This phenomena is encountered in a similar context in [11].

It should also be noted that the frequency condition for a primary field (15) becomes

$$\omega = \frac{\ell + 1}{2iM} \tag{17}$$

which again takes us out of the low frequency limit $\omega M \ll 1$. We comment further on this point below.

The inner product of these primaries is rather different from the usual Klein-Gordon norm in the Kerr background. The inner product of the CFT must yield conjugation that switches $L_1 \leftrightarrow L_{-1}$ and leaves L_0 invariant. This is accomplished by Hermitian conjugation, followed by $\phi \to -\phi$ and $t \to -t$. This suggests the symmetry may be interpreted directly as acting in an analytic continuation of the Kerr geometry where $\phi \to i\phi$ and $t \to it$.

We conclude that the large r falloff $r^{-(h_L+h_R)/2}$ of a mode allows us to read off the conformal weight of the dual CFT operator $\Delta = h_L + h_R$. It is worth mentioning that in the usual AdS/CFT correspondence, the radial fall-off in Poincare coordinates is instead of the form $\tilde{r}^{-(h_L+h_R)}$. Thus if one inferred some effective AdS metric from the Kerr Laplacian in the near region, the relation between coordinates is of the form $r \sim \tilde{r}^2$ at large r.

From (11) we can generalize the above to the higher s fields, and include the higher powers of $M\omega$ on the right hand side of (16) using the expansion (6). In the near region, the large r falloff of (11) takes the form $r^{-s-\nu-1}$. To extract the behavior of the primary field we must also take into account the normalization of the component vectors used to set up the Teukolsky equation, that appear in the definition of the quantities (4) as described in [4]. Thus we extract the large r behavior of the vector potential A_μ for spin 1, and the behavior of the graviton $g_{\mu\nu}$ for spin 2, in asymptotically Minkowski coordinates. This leads us to identify the conformal weight

$$\Delta = h_L + h_R = -2s + 2\nu + 2\,.$$

Here we assumed $s \geq 0$. For $s < 0$ we must recall the canonically normalized modes are non-trivially related to the solutions of the Teukolsky equation by (4). So for $s < 0$, the large r falloff of the canonical modes takes the form $r^{s-\nu-1}$ so in general we obtain

$$\Delta = -2|s| + 2\nu + 2 \tag{18}$$

for the conformal weight of a higher spin mode. Thus, at leading order in $M\omega$, all the massless bulk fields have $\Delta = 2$ for the lowest nontrivial modes of angular momentum.

There are now a number of puzzles we need to address. The $\bar{L}_n$ generators act on functions that may be written using the basis $R(r)e^{im\phi-i\omega t}$ but as we see from the exponential prefactors in (13), the $\bar{L}_1$ and $\bar{L}_{-1}$ generators shift the m and ω eigenvalues by imaginary amounts. The shift in ω means the low frequency approximation leading to (11) can no longer be trusted, *so that the whole $SL(2,\mathbb{R})$ associated with these generators is strongly broken down to the $U(1)$ subgroup generated by $\bar{L}_0$.*

This is not the case for the $SL(2,\mathbb{R})$ generated by the L_n because these only shift m by imaginary amounts, and no small m approximation was used. However as pointed out in (17), the primary modes with respect to L_n do take us out of the low frequency limit. We may still use the $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ representations of the leading order wave equation to organize the expansion of the higher order corrections. A priori we have no reason to expect convergence when we relate the associated CFT operators with bulk operators, but nevertheless, the mode function expansion (8) happens to converge for all finite r.

Of course, as noted in [3] $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ is explicitly broken once ϕ is periodically identified, which projects out all noninteger m modes. So in this way, we see how this can be a low frequency symmetry of the Kerr modes prior to periodic identification, without it being realized manifestly in the spectrum. For example, such $SL(2,\mathbb{R})$ towers are not observed in the numerically determined quasi-normal mode spectrum [9].

The conformal dimensions we find at lowest order in $M\omega$ are given by (18). It should be noted that the conformal dimensions encountered are all positive, indicative of an underlying unitary conformal field theory.

When the higher order $M\omega$ terms in the Teukolsky equation are included, the $SL(2,\mathbb{R})$ symmetry associated with the L_n's is softly broken. We expect the bulk scalar fields to be dual to CFT operators involving a sum of higher dimension operators. The conformal dimensions of these operators may be read off by examining the large r falloff of the expansion for the exact radial mode function (8), yielding a prediction for the dimensions of other CFT operators that must be present

$$\begin{aligned} \Delta &= -2|s| + 2n + 2\nu + 2 \qquad n > -\nu - 1 \\ &= 2|s| - 2n - 2\nu \qquad n < -\nu \end{aligned}$$

which are again all positive.

In this way, each term in (8) can be interpreted as a higher dimension correction in the mapping between the bulk mode and CFT operators. Because (8) reproduces the exact mode function for any finite r, one may deduce the exact two point function for scattering of massless modes off Kerr, including all higher $M\omega$ corrections, generalizing the lead-order matching noted in [3].

Having found a set of scaling dimensions associated with the exact solution of the Teukolsky equation, we are confronted with the problem that $\nu = \ell + \mathcal{O}(M^2\omega^2)$ as shown in (10). This means the scaling dimensions run with frequency – another sign that conformal symmetry is broken away from the $M\omega = 0$ fixed point. At first sight this seems rather disappointing: if we wish to study the conformal fixed point, we are forced to set $M = 0$. To retain a smooth geometry, this limit must be taken with $a < M$ which takes us to flat spacetime.[12] To keep the generators (12) and (13) well-defined, one must also rescale the time coordinate, keeping $\tilde{t} = t/M$ finite and the dimensionless temperatures T_L and T_R fixed. Thus the metric becomes $\mathbb{R}^3$ times a null direction $\tilde{t}$

$$ds^2 = 0d\tilde{t}^2 - dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\phi^2 \,. \tag{19}$$

Certainly we see no sign of a nontrivial central charge $c = 12J$ associated with an exact CFT dual to flat spacetime. One might have expected extremal Kerr to emerge at the fixed point, but this simply does not emerge in a small $M\omega$ limit. The generators (12) and (13) are not isometries of the metric (19). Moreover, in the extremal limit, they do not match the asymptotic symmetry generators of the NHEK geometry found in [1, 13, 14] (for further work in this direction see [15, 16]). Rather they correspond to conformal transformations of (19) that leave the massless wave equations invariant. Thus the hidden Kerr/CFT correspondence does not seem easily generalized to massive modes.

Of course our scaling dimension computations are only valid at "strong" coupling where the gravitational solution is smooth. There could still be a nontrivial CFT with $c = 12J$ with conformal dimensions that match those obtained here when its strong coupling limit is taken. Studies of the near super-radiant modes of extremal Kerr provide strong evidence for such a conformal field theory [1, 2]. While the two limits do not seem to be smoothly connected within the realm of smooth gravity solutions (for example the super-radiant modes do not satisfy the low frequency limit needed to obtain the symmetry studied in the present paper), they may well be connected within the exact microscopic CFT. Similar phenomena are observed in the duality between D1,D5-brane backgrounds and CFT.

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