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Non-Gaussianity in Two-Field Inflation

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We derive semi-analytic formulae for the local bispectrum and trispectrum in general two-field inflation and provide a simple geometric recipe for building observationally allowed models with observable non-Gaussianity. We use the δN formalism to express the bispectrum in terms of spectral observables and the transfer functions, which encode the super-horizon evolution of modes. Similarly, we calculate the trispectrum and show that the trispectrum parameter τ_{NL} can be expressed entirely in terms of spectral observables, which provides a new consistency relation unique to two-field inflation. We show that in order to generate observably large non-Gaussianity during inflation, the sourcing of curvature modes by isocurvature modes must be extremely sensitive to a change in the initial conditions orthogonal to the inflaton trajectory and that the amount of sourcing must be non-zero. Under some minimal assumptions, we argue that the first condition is satisfied only when neighboring trajectories through the two-dimensional field space diverge during inflation. Geometrically, this means that the inflaton must roll along a ridge in the potential V for some time during inflation and that its trajectory must turn somewhat in field space. Therefore, it follows that under our assumptions, two-field scenarios with attractor solutions necessarily produce small non-Gaussianity. This explains why it has been so difficult to achieve large non-Gaussianity in two-field inflation, and why it has only been achieved in a narrow class of models where the potential and/or the initial conditions are fine-tuned. Some of our conclusions generalize at least qualitatively to multi-field inflation and to scenarios where the interplay between curvature and isocurvature modes can be represented by the transfer function formalism.

I. INTRODUCTION

Cosmological inflation [1–5] is widely thought to be responsible for producing the density perturbations that initiated the formation of large-scale structure. During such an inflationary expansion, quantum fluctuations would have been stretched outside the causal horizon and then frozen in as classical perturbations. These primordial perturbations would later be gravitationally amplified over time into the cosmological large-scale structure that we observe today [6–11].

Pinning down the specific nature of inflation or whatever physics seeded the primordial density fluctuations is one of the greatest open problems in cosmology. The simplest models of inflation are driven by a single scalar field whose fluctuations are adiabatic, nearly scale-invariant, and nearly Gaussian. But these are assumptions and need to be tested. Whether the primordial fluctuations were indeed adiabatic and near scale-invariant can be determined by measuring the power spectra of fluctuations; the upper limit on the isocurvature spectrum constrains non-adiabaticity, while the slope of the scalar (curvature) power spectrum constrains the deviation from scale-invariance. Similarly, whether the primordial fluctuations obey Gaussian statistics can be tested by measuring reduced n -point correlation functions, where $n \geq 3$. For Gaussian fluctuations, these higher-point functions all vanish, and only the two-point function (the power spectrum) is non-zero. Any deviations from adiabaticity, near scale invariance, and Gaussianity would signal some non-minimal modifications to the simplest scenarios and hence would provide exciting insight into ultra-high energy physics.

Of these observational measures, non-Gaussianity has the potential to be the most discriminating probe, given all the information contained in higher-point statistics. This is particularly valuable given how challenging it has been to discriminate among the myriad different inflationary models.

The two lowest order non-Gaussian measures are the bispectrum and the trispectrum. Just like the power spectrum $\mathcal{P}_{\mathcal{R}}$ represents the two-point function of the comoving curvature perturbation \mathcal{R} in Fourier space, the bispectrum $\mathcal{B}_{\mathcal{R}}$ represents the three-point function and the trispectrum $\mathcal{T}_{\mathcal{R}}$ represents the four-point function:

$$\langle \mathcal{R}(\mathbf{k}_1) \mathcal{R}(\mathbf{k}_2) \rangle = (2\pi)^3 \delta^3 \left(\sum_{i=1}^2 \mathbf{k}_i \right) \mathcal{P}_{\mathcal{R}}(\mathbf{k}_1, \mathbf{k}_2), \quad (1)$$

$$\langle \mathcal{R}(\mathbf{k}_1) \mathcal{R}(\mathbf{k}_2) \mathcal{R}(\mathbf{k}_3) \rangle = (2\pi)^3 \delta^3 \left(\sum_{i=1}^3 \mathbf{k}_i \right) \mathcal{B}_{\mathcal{R}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3), \quad (2)$$

$$\begin{aligned} \langle \mathcal{R}(\mathbf{k}_1) \mathcal{R}(\mathbf{k}_2) \mathcal{R}(\mathbf{k}_3) \mathcal{R}(\mathbf{k}_4) \rangle \\ = (2\pi)^3 \delta^3 \left(\sum_{i=1}^4 \mathbf{k}_i \right) \mathcal{T}_{\mathcal{R}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4). \end{aligned} \quad (3)$$

The δ -functions in equations (1)-(3) reflect the fact that the statistical properties are translationally invariant in real space, which makes the above three correlations vanish except if all \mathbf{k} -vectors add up to zero — *i.e.*, are the negative of one another for the power spectrum, form the

three sides of a triangle for the bispectrum, and form the sides of a (perhaps non-flat) quadrangle for the trispectrum. Since the statistical properties are also rotationally invariant, the power spectrum depends only on the length and not the direction of its vector argument, and the bispectrum depends only on the lengths of the three triangle sides, so they can be written simply as $\mathcal{P}_{\mathcal{R}}(k)$ and $\mathcal{B}_{\mathcal{R}}(k_1, k_2, k_3)$, respectively.

For the bispectrum, it is standard in the literature to define a dimensionless quantity $f_{NL}(k_1, k_2, k_3)$ by dividing by appropriate powers of the power spectrum [12]¹:

$$-\frac{6}{5}f_{NL}(k_1, k_2, k_3) = \frac{B_{\mathcal{R}}(k_1, k_2, k_3)}{\left[\mathcal{P}_{\mathcal{R}}(k_1)\mathcal{P}_{\mathcal{R}}(k_2) + \text{cyclic permutations}\right]}. \quad (5)$$

Although f_{NL} can in principle depend on the triangle shape of the \mathbf{k} -vectors in very complicated ways, it has been shown that in practice, essentially all models produce an f_{NL} that is well approximated by one of merely a handful of particular functions of triangle shape, with names like “local”, “equilateral”, “orthogonal”, “warm”, and “flat” [13, 14]. For example, the local function peaks around triangles that are degenerate (with one angle close to zero, like for $k_3 \ll k_1 \approx k_2$), while the equilateral function peaks around triangles that are equilateral ($k_1 = k_2 = k_3$). Bispectra dominated by different triangle shapes correspond to different inflationary scenarios and different physics. In particular, for multi-field inflation, barring non-canonical kinetic terms or higher-order derivative terms in the Lagrangian, the dominant type of bispectra is of the local form [13, 14]. Local non-Gaussianity arises from the non-linear evolution of density perturbations once the field fluctuations are stretched beyond the causal horizon. Seven-year data from WMAP constrains non-Gaussianity of the local form to [15]

$$-10 < f_{NL}^{local} < 74 \quad (95\% \text{ C.L.}), \quad (6)$$

and a perfect CMB measurement has the potential to detect a bispectrum as low as $|f_{NL}| \approx 3$ [16].

Similarly, the dominant form of trispectra for standard multi-field inflation is also of the local form, and it can be characterized by two dimensionless non-linear parameters, τ_{NL} and g_{NL} . Five-year data from WMAP con-

strains these two parameters to [18, 19]

$$\begin{aligned} -0.6 \times 10^4 < \tau_{NL} < 3.3 \times 10^4 & \quad (95\% \text{ C.L.}), \\ -5.4 \times 10^5 < g_{NL} < 8.6 \times 10^5 & \quad (95\% \text{ C.L.}). \end{aligned} \quad (7)$$

Interestingly, for standard single-field inflation, the non-linear parameters representing the bispectrum [12, 20–27] and trispectrum [28–30] are all of order the slow-roll parameters (*i.e.*, at the percent level) and will not be accessible to CMB experiments. However, if inflation is described by some non-minimal modification, such as multiple fields or higher derivative operators in the inflationary Lagrangian, then non-Gaussianity might be observable in the near future. Indeed, there have been many attempts to calculate the level of non-Gaussianity in general multi-field models (e.g., [31–42]), as well as in two-field models (e.g., [43–51]). However, it has been very difficult to find models that produce large non-Gaussianity, though some exceptions have been found such as in the curvaton model [52–58], hybrid and multi-hybrid inflation (e.g., [59–67]), in certain modulated and tachyonic (p)re-heating scenarios (e.g., [68–74]), and in some quadratic small-field, two-field models by taking appropriate care of loop corrections [75, 76]. Moreover, it has not been wholly clear why it is so difficult to produce large non-Gaussianity in such models. Though some authors [31, 32, 41, 44, 45, 47] have found spikes in non-Gaussianity whenever the inflaton trajectory changes direction sharply, these spikes in non-Gaussianity are transient and die away before the end of inflation. That makes a comprehensive study of non-Gaussianity generation timely, to understand any circumstances under which observably large non-Gaussianity arises in such models.

In this paper, we calculate the bispectrum and trispectrum in general two-field inflation models with standard kinetic terms. We provide conditions for large non-Gaussianity, and we formulate a geometric recipe for building two-field inflationary potentials that give rise to large non-Gaussianity. In the process, we provide a unified answer to the mystery of why it has been so hard to produce large non-Gaussianity in two-field inflationary models. The rest of this paper is organized as follows. In Section II, we present the background equations of motion for the fields and discuss the field vector kinematics. Section III presents the equations of motion for the field perturbations and some necessary results for the power spectra. In Section IV, we describe the δN formalism, which we use to calculate the bispectrum, both for general two-field inflation and for specific models. Thereafter, we discuss the necessary conditions for large non-Gaussianity. Finally, we tackle the trispectrum in Section V. We summarize our conclusions in Section VI.

¹ The factor of $-\frac{6}{5}$ arises from the fact that the non-linear parameter f_{NL} was originally introduced to represent the degree of non-Gaussianity in the metric perturbation [16, 17],

$$\Phi = \Phi_G + f_{NL}\Phi_G^2, \quad (4)$$

where Φ_G is Gaussian and Φ is not. Here, Φ is the metric perturbation in the Newtonian gauge, which equals the gauge-invariant Bardeen variable. After inflation ends and during the matter-dominated era, $2\Phi = -\frac{6}{5}\mathcal{R}$.

II. BACKGROUND FIELD EQUATION & KINEMATICS

In this section, we review the background equations of motion and discuss the kinematics of the background fields. This discussion will help us calculate the primordial bispectrum and trispectrum in two-field inflation and understand what features are necessary for non-Gaussianity to be observably large.

We consider general two-field inflation where the non-gravitational part of the action is of the form

$$S = \int \left[-\frac{1}{2} g^{\mu\nu} \delta_{ij} \frac{\partial \phi^i}{\partial x^\mu} \frac{\partial \phi^j}{\partial x^\nu} - V(\phi_1, \phi_2) \right] \sqrt{-g} d^4x, \quad (8)$$

where $V(\phi_1, \phi_2)$ is a completely arbitrary potential of the two fields, $g_{\mu\nu}$ is the spacetime metric, and δ_{ij} reflects the fact that we assume the kinetic terms are canonical.

In this paper, we adopt similar notation to [77]: bold-face for vectors and standard vector product notation. We use the symbol T to denote the transpose of a vector, and we use this symbol to convert a naturally covariant vector into a contravariant vector and vice versa. Gradients ($\nabla \equiv \{\frac{\partial}{\partial \phi^i}\}$) and partial derivatives ($\partial_i \equiv \frac{\partial}{\partial \phi^i}$) represent derivatives with respect to the fields, except where explicitly indicated otherwise. We set the reduced Planck mass, $\bar{m} \equiv \frac{1}{\sqrt{8\pi G}}$, equal to unity, so that all fields are measured in units of the reduced Planck mass. To simplify the equations of motion and connect them more directly with observables, we use the number of e -folds, N , as our time variable. N is defined through the relation

$$dN = H dt, \quad (9)$$

where t is the comoving time and H is the Hubble parameter. We denote derivatives with respect to N using the notation

$$' = \frac{d}{dN}. \quad (10)$$

Using N as the time variable, we showed in [77] that the background equation of motion for the fields can be written as

$$\frac{\eta}{(3-\epsilon)} + \phi' = -\nabla^T \ln V. \quad (11)$$

The parameter ϵ is defined as

$$\epsilon \equiv -(\ln H)' = \frac{1}{2} \phi' \cdot \phi', \quad (12)$$

and η is the field acceleration, defined as

$$\eta \equiv \phi''. \quad (13)$$

In [77], we also explained how the two quantities ϕ' and η represent the kinematics of the background fields. If we view the fields as coordinates on the field manifold, then ϕ' represents the field velocity, and

$$v \equiv |\phi'| \quad (14)$$

represents the field speed. Similarly, η is the field acceleration.

The velocity vector, ϕ' , is also useful because it can be used to define a *kinematical basis* [78–80]. In this basis, the basis vector e_{\parallel} points along the field trajectory, while the basis vector e_{\perp} points perpendicularly to the field trajectory, in the direction that makes the scalar product $e_{\perp} \cdot \eta$ positive. To denote the components of a vector and a matrix in this basis, we use the short-hand notation

$$X_{\parallel} \equiv \mathbf{e}_{\parallel} \cdot \mathbf{X}, \quad X_{\perp} \equiv \mathbf{e}_{\perp} \cdot \mathbf{X}, \quad (15)$$

and

$$M_{\parallel\perp} \equiv \mathbf{e}_{\parallel}^T \mathbf{M} \mathbf{e}_{\perp}, \quad \text{etc.} \quad (16)$$

The kinematical basis is useful for several reasons. First, the field perturbations naturally decompose into components parallel and perpendicular to the field trajectory, and the former represent *bona fide* density perturbations, while the latter do not. This decomposition of the field perturbations is helpful in finding expressions for the power spectra. Second, it allows us to consider separate aspects of the background field kinematics, which in [77], we encapsulated in a set of three quantities. The first quantity is the field speed, v . The second and third quantities arise from decomposing the field acceleration into components parallel and perpendicular to the field velocity. In particular, the quantity $\frac{\eta_{\parallel}}{v}$ represents the logarithmic rate of change in the field speed (the *speed-up rate*), while the quantity $\frac{\eta_{\perp}}{v}$ represents the rate at which the field trajectory changes direction (the *turn rate*) [77].

This distinction between the speed-up rate and the turn rate is important for two reasons. First, the turn rate represents uniquely multi-field behavior (as the background trajectory cannot turn in single-field inflation), whereas the speed-up rate represents single-field-like behavior. Second, the speed-up and turn rates have very different effects on the evolution of the field perturbations and hence on the power spectra. Indeed, the features in the power spectra depend not only on the absolute sizes of the two rates but also on their relative sizes to each other; in particular, the ratio of the turn rate to the speed-up rate is an indicator of the relative impact of multi-field effects. So disentangling the two quantities allows for a better understanding of the power spectra and all the ways that the spectra can be made consistent with observations.

To fully take advantage of this distinction between the speed-up and turn rates, we redefined the standard slow-roll approximation in [77], splitting it into two different approximations that can be invoked either separately or together. As background, the standard slow-roll approximation is typically expressed as

$$\epsilon \approx \frac{1}{2} |\nabla \ln V|^2 \ll 1, \quad (17)$$

and

$$\left| \frac{\partial_i \partial_j V}{V} \right| \ll 1. \quad (18)$$

However, as argued in [77], the latter condition lumps together and simultaneously forces the speed-up rate, the turn rate, and a quantity called the entropy mass to be small. So instead, we redefined the slow-roll approximation to mean that the field speed is small,

$$\epsilon = \frac{1}{2}v^2 \ll 1, \quad (19)$$

and is slowly changing,

$$\left| \frac{\eta_{\parallel}}{v} \right| \ll 1. \quad (20)$$

In other words, the above slow-roll approximation represents the minimum conditions necessary to guarantee quasi-exponential inflationary expansion, and it corresponds to limits on single-field-like behavior. As for the turn rate, we endowed it with its own separate approximation, the *slow-turn approximation*, which applies when the turn rate satisfies

$$\frac{\eta_{\perp}}{v} \ll 1. \quad (21)$$

The slow-turn limit corresponds to limits on multi-field behavior. Finally, this alternative framework does not restrict the value of the lowest order entropy mass. (See [77] for further discussion of these points.)

When the background field vector is both slowly rolling and slowly turning, we call the combined slow-roll and slow-turn limits (which is equivalent to the conventional slow-roll limit minus the constraint on the entropy mass) the SRST limit for brevity. In the combined limit, the evolution equation for the fields can be approximated by

$$\phi' \approx -\nabla^T \ln V. \quad (22)$$

Also in this combined limit, the speed-up rate and the turn rate can be approximated by

$$\frac{\eta_{\parallel}}{v} \approx -M_{\parallel\parallel}, \quad \frac{\eta_{\perp}}{v} \approx -M_{\parallel\perp}, \quad (23)$$

respectively, where we define the *mass matrix*, \mathbf{M} , as the Hessian of $\ln V$, *i.e.*,

$$\mathbf{M} \equiv \nabla^T \nabla \ln V. \quad (24)$$

Being a symmetric 2×2 matrix, \mathbf{M} is characterized by three independent coefficients. In the kinematical basis, these three coefficients are $M_{\parallel\parallel}$, $M_{\parallel\perp}$, and $M_{\perp\perp}$, the third of which is the lowest order *entropy mass*.² In other words, in the kinematical basis and under the SRST limit, we can interpret the mass matrix as follows:

$$\mathbf{M} = \begin{pmatrix} M_{\parallel\parallel} & M_{\parallel\perp} \\ M_{\parallel\perp} & M_{\perp\perp} \end{pmatrix} = \begin{pmatrix} \text{-speed up rate} & \text{-turn rate} \\ \text{-turn rate} & \text{entropy mass} \end{pmatrix}, \quad (25)$$

² We refer to $M_{\perp\perp}$ as the entropy *mass*, even though we constructed it to be dimensionless.

where the speed-up rate and turn rate alone determine the background kinematics. However, all three quantities — the speed-up rate, the turn rate, and the entropy mass — affect how the perturbations evolve, as described in the next section.

III. PERTURBATIONS, TRANSFER FUNCTIONS, AND POWER SPECTRA

In this section, we summarize the general results for the evolution of perturbations and for the power spectra. These expressions will enable us to calculate the bispectrum and trispectrum in two-field inflation and to express the results in terms of spectral observables.

Because we know the power spectra at horizon exit ($k = aH$), to find the spectra at the end of inflation, we need to know how the modes evolve after they exit the horizon. In [77], we derived the following general equation of motion for the field perturbations in Fourier space:

$$\begin{aligned} \frac{1}{(3-\epsilon)}\delta\phi'' + \delta\phi' + \left(\frac{k^2}{a^2V}\right)\delta\phi \\ = -\left[\mathbf{M} + \frac{\eta\eta^T}{(3-\epsilon)^2}\right]\delta\phi, \end{aligned} \quad (26)$$

where $\delta\phi$ represents the field perturbation in the flat gauge, which coincides with the gauge-invariant Mukhanov-Sasaki variable [81, 82]. (In comparison to [77], we drop the subscript f on $\delta\phi_f$ since we are working exclusively in the flat gauge.) When the field perturbations are well outside the horizon ($k \ll aH$), we can drop the subhorizon term $\left(\frac{k^2}{a^2V}\right)\delta\phi$ in equation (26). If additionally the background fields are in SRST limit, the acceleration of both the unperturbed and perturbed fields can be neglected. Therefore, in the combined SRST and super-horizon limits, equation (26) reduces to [77]

$$\delta\phi' \approx -\mathbf{M}\delta\phi. \quad (27)$$

In the first half of this section, we use the symbol \approx to indicate relationships that hold in the SRST limit, whereas for relationships that hold in the super-horizon limit, we use $=$ and note the domain of validity.

Now we switch to working in the kinematical basis, where the modes decompose into adiabatic modes, $\delta\phi_{\parallel} \equiv \mathbf{e}_{\parallel} \cdot \delta\phi$, and entropy modes, $\delta\phi_{\perp} \equiv \mathbf{e}_{\perp} \cdot \delta\phi$. We make this change of basis for two reasons: (1) it allows us to identify which modes correspond to density modes, and (2) it simplifies the super-horizon equations of motion. The adiabatic modes correspond to fluctuations forwards or backwards along the classical trajectory, and they represent density perturbations. By contrast, entropy modes are fluctuations orthogonal to the classical inflaton trajectory, and hence they represent relative perturbations among the fields that leave the overall density

unperturbed. The super-horizon equations of motion for the two mode types are

$$\begin{aligned}\delta\phi'_{\parallel} &= \left(\frac{\eta_{\parallel}}{v}\right)\delta\phi_{\parallel} + 2\left(\frac{\eta_{\perp}}{v}\right)\delta\phi_{\perp}, \\ \delta\phi'_{\perp} &\approx -M_{\perp\perp}\delta\phi_{\perp},\end{aligned}\quad (28)$$

where the first equation is valid irrespective of the behavior of the background fields, while the second super-horizon equation is valid to lowest order in the slow-turn limit. (Second-order expressions and further discussion are given in [77].) Equation (28) shows that in the SRST limit, the evolution of modes is determined by the three unique coefficients of the mass matrix. The evolution of adiabatic modes is controlled by $\frac{\eta_{\parallel}}{v} \approx -M_{\parallel\parallel}$ and by $\frac{\eta_{\perp}}{v} \approx -M_{\perp\perp}$. The third unique coefficient of the mass matrix, $M_{\perp\perp}$, alone determines the relative damping or growth of entropy modes. We call $M_{\perp\perp}$ the lowest order entropy mass (or just the entropy mass) because it approximates the effective mass in the full second-order differential equation of motion for the entropy modes [77]. In addition to being viewed as an effective mass, $M_{\perp\perp}$ can also be viewed as a measure of the curvature of the potential along the \mathbf{e}_{\perp} or entropic direction. When the curvature of the potential along the entropic direction is positive, the entropy modes decay; when the curvature is negative, the entropy modes grow.

Directly related to these two modes are the curvature and isocurvature modes, the two quantities whose power spectra are typically computed when considering the two-field power spectra. The utility of working in terms of the curvature power spectrum is that it can be directly related to the power spectrum of metric perturbations or density perturbations after inflation ends; the three are equivalent up to factors of order unity. Hence, we will also work in terms of curvature and isocurvature modes.

During inflation, the curvature and isocurvature modes are simply related to the adiabatic and entropy modes, respectively, by a factor of $\frac{1}{v}$ [83]. That is, the curvature modes are given by

$$\mathcal{R} = \frac{\delta\phi_{\parallel}}{v}, \quad (29)$$

and the isocurvature modes by

$$\mathcal{S} \equiv \frac{\delta\phi_{\perp}}{v}. \quad (30)$$

The super-horizon evolution of curvature and isocurvature modes can be determined from the equations of motion for the adiabatic and entropy modes. We parametrize the solutions through the transfer matrix formalism [83, 84]:

$$\begin{pmatrix} \mathcal{R} \\ \mathcal{S} \end{pmatrix} = \begin{pmatrix} 1 & T_{\mathcal{R}\mathcal{S}} \\ 0 & T_{\mathcal{S}\mathcal{S}} \end{pmatrix} \begin{pmatrix} \mathcal{R}_* \\ \mathcal{S}_* \end{pmatrix}, \quad (31)$$

where the transfer functions can be written as

$$\begin{aligned}T_{\mathcal{R}\mathcal{S}}(N_*, N) &\equiv \int_{N_*}^N \alpha(\tilde{N}) T_{\mathcal{S}\mathcal{S}}(N_*, \tilde{N}) d\tilde{N}, \\ T_{\mathcal{S}\mathcal{S}}(N_*, N) &\equiv e^{\int_{N_*}^N \beta(\tilde{N}) d\tilde{N}}.\end{aligned}\quad (32)$$

The script $*$ means that the quantity is to be evaluated when the corresponding modes exit the horizon. The transfer function $T_{\mathcal{S}\mathcal{S}}$ therefore represents how much the isocurvature modes have decayed (or grown) after exiting the horizon. The transfer function $T_{\mathcal{R}\mathcal{S}}$ represents the total sourcing of curvature modes by isocurvature modes; that is, it represents the importance of the multi-field effects. In [77], we found that

$$\alpha = 2\frac{\eta_{\perp}}{v} \quad (33)$$

exactly, which tells us that the curvature modes are only sourced by the isocurvature modes when the field trajectory changes direction. However, the *isocurvature mass*, β , must be approximated or computed numerically. To lowest order in the SRST limit,

$$\begin{aligned}\alpha &\approx -2M_{\perp\perp}, \\ \beta &\approx M_{\parallel\parallel} - M_{\perp\perp}.\end{aligned}\quad (34)$$

From a geometrical perspective, equation (34) for β shows that how fast the isocurvature modes evolve depends on the difference between the curvatures of the potential along the entropic and adiabatic directions. From a kinematical perspective, the isocurvature modes will grow if the entropy modes grow faster than the field vector picks up speed. This means that isocurvature modes tend to grow in two types of scenarios: when $M_{\perp\perp}$ is large and negative and when ϵ decreases quickly. Otherwise, when the entropy modes do not grow faster than the field vector picks up speed, the isocurvature modes decay. This lengthy discussion will become more important later when we consider the conditions for large non-Gaussianity.

Now we present expressions for the power spectra and their associated observables, using the above results. The power spectrum of a quantity \mathcal{X} is essentially the variance of its Fourier transform:

$$\mathcal{P}_{\mathcal{X}}(k_1)\delta^3(\mathbf{k}_1 + \mathbf{k}_2) = \frac{k^3}{2\pi^2}\langle\mathcal{X}(\mathbf{k}_1)\mathcal{X}^\dagger(\mathbf{k}_2)\rangle. \quad (35)$$

From here forward, we will work to first-order in the SRST limit, so we will suppress the \approx signs to avoid having to write them repeatedly. Now using the above results, the curvature, cross, and isocurvature spectra at the end of inflation can be written to lowest order as [83]

$$\begin{aligned}\mathcal{P}_{\mathcal{R}} &= \left(\frac{H_*}{2\pi}\right)^2 \frac{1}{2\epsilon_*}(1 + T_{\mathcal{R}\mathcal{S}}^2), \\ \mathcal{C}_{\mathcal{R}\mathcal{S}} &= \left(\frac{H_*}{2\pi}\right)^2 \frac{1}{2\epsilon_*}T_{\mathcal{R}\mathcal{S}}T_{\mathcal{S}\mathcal{S}}, \\ \mathcal{P}_{\mathcal{S}} &= \left(\frac{H_*}{2\pi}\right)^2 \frac{1}{2\epsilon_*}T_{\mathcal{S}\mathcal{S}}^2,\end{aligned}\quad (36)$$

where it is implied that the transfer functions are evaluated at the end of inflation, which we take to be when

$\epsilon = 1$. The associated curvature spectral index is [77]

$$n_{\mathcal{R}} = n_s - 1 \equiv \frac{d \ln P_{\mathcal{R}}}{dN} = n_T + 2\mathbf{e}_N^T \mathbf{M}_* \mathbf{e}_N, \quad (37)$$

where n_s is the standard scalar spectral index that is constrained by observations, $n_T = -2\epsilon_*$ is the tensor spectral index, and the unit vector \mathbf{e}_N is [77]

$$\mathbf{e}_N = \cos \Delta_N \mathbf{e}_{\parallel}^* + \sin \Delta_N \mathbf{e}_{\perp}^*, \quad (38)$$

where Δ_N is the correlation angle, defined by

$$\tan \Delta_N \equiv T_{\mathcal{R}\mathcal{S}}. \quad (39)$$

As we will show in the next section, the unit vector \mathbf{e}_N points in the direction of the gradient of N , which explains why we affix the subscript N to both this unit vector and the correlation angle. In turn, the correlation angle can be given in terms of the dimensionless curvature-isocurvature correlation, r_C , which we define as [77]

$$r_C \equiv \frac{C_{\mathcal{R}\mathcal{S}}}{\sqrt{P_{\mathcal{R}}P_{\mathcal{S}}}} = \sin \Delta_N, \quad (40)$$

in analogy to the tensor-to-scalar ratio,

$$r_T \equiv \frac{\mathcal{P}_T}{\mathcal{P}_{\mathcal{R}}} = 16\epsilon_* \cos^2 \Delta_N, \quad (41)$$

where \mathcal{P}_T is the tensor spectrum of gravitational waves. r_C can also be expressed in terms of other observables:

$$r_C = \sqrt{1 + \frac{r_T}{8n_T}}, \quad (42)$$

so it can be determined if either isocurvature or tensor modes are detected. Similarly, another ratio of spectra, the isocurvature fraction,

$$f_{iso} \equiv \frac{P_{\mathcal{S}}}{P_{\mathcal{R}}} = \frac{T_{\mathcal{S}\mathcal{S}}^2}{1 + T_{\mathcal{R}\mathcal{S}}^2} = \cos^2 \Delta_N T_{\mathcal{S}\mathcal{S}}^2, \quad (43)$$

constitutes another observable, and it gives the relative size of $T_{\mathcal{S}\mathcal{S}}$ to $T_{\mathcal{R}\mathcal{S}}$.

These results will enable use to calculate the bispectrum and trispectrum in general two-field inflation and will allow us to express the results in terms of spectral observables.

IV. THE BISPECTRUM

In this section, we calculate the bispectrum for general two-field inflation using the δN formalism and the transfer function formalism, which we described above. The result can be expressed entirely in terms of spectral observables and transfer functions. We then further explore the key term that determines whether the bispectrum is large, and we show how this term can be calculated for analytically solvable and similar models. We end by considering what features a general two-field inflationary potential needs in order for the bispectrum to be observably large.

A. Calculation of f_{NL} Using the δN Formalism

We consider bispectrum configurations of the local or squeezed type (*e.g.*, $k_3 \ll k_1 \approx k_2$), which is the dominant type present during standard multi-field inflation. As we showed in equation (5), the bispectrum can be expressed in terms of the dimensionless non-linear parameter f_{NL} [12]. From here on, whenever f_{NL} appears in this paper, it represents the local form, so we drop the superscript *local*. Non-Gaussianity of the local form arises from the super-horizon evolution of the modes. Since the super-horizon mode evolution is neatly encapsulated by our two transfer functions, it is natural to consider whether we can derive an expression for f_{NL} in terms of the transfer functions and the other dynamical functions we introduced earlier. This is exactly what we will do in this section.

We start by recognizing that f_{NL} can be written in terms of the δN formalism [85–87], where N represents the number of e -folds of inflation. Under the δN formalism, it can be shown that $\mathcal{R} = \nabla N \cdot \delta\phi$ [86, 87], where $\delta\phi$ is measured in the flat gauge and where it is implied that the gradient is with respect to the fields at horizon exit. (For brevity, we drop the subscript $*$ on ∇ , but restore it in later sections whenever there might be some ambiguity.) Using this result, correlators of \mathcal{R} can be written in terms of gradients of N . In particular, it has been shown [88] that the local form of f_{NL} can be written as

$$-\frac{6}{5}f_{NL} = \frac{\nabla N \nabla^T \nabla N \nabla^T N}{|\nabla N|^4}. \quad (44)$$

To use equation (44) to find an expression for f_{NL} , we first find a semi-analytic formula for ∇N in two-field inflation. By comparing equation (36) to the lowest order result for the curvature power spectrum in multi-field inflation [86],

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{H_*}{2\pi}\right)^2 |\nabla N|^2, \quad (45)$$

we obtain

$$|\nabla N| = \sqrt{\frac{1 + T_{\mathcal{R}\mathcal{S}}^2}{2\epsilon_*}}, \quad (46)$$

where again it is implied that $T_{\mathcal{R}\mathcal{S}}$ is evaluated at the end of inflation. Combining equation (46) with the fact that

$$\nabla N \cdot \phi'_* = 1, \quad (47)$$

we conclude that ∇N takes the following form in the kinematical basis:

$$\nabla N = \frac{1}{\sqrt{2\epsilon_*}} \left[(\mathbf{e}_{\parallel}^*)^T + T_{\mathcal{R}\mathcal{S}} (\mathbf{e}_{\perp}^*)^T \right]. \quad (48)$$

The above equation implies that we can also write ∇N as

$$\nabla N = \sqrt{\frac{1 + T_{\mathcal{RS}}^2}{2\epsilon_*}} \mathbf{e}_N^T = \frac{\mathbf{e}_N^T}{\sqrt{2\epsilon_*} \cos \Delta_N}, \quad (49)$$

where \mathbf{e}_N^T is the unit vector in the direction of ∇N and is given by equation (38).

Next, we re-write equation (44) for f_{NL} as

$$-\frac{6}{5} f_{NL} = \frac{\mathbf{e}_N^T \nabla^T \nabla N \mathbf{e}_N}{|\nabla N|^2}. \quad (50)$$

Since $\mathbf{e}_N \cdot \mathbf{e}_N = 1$, it follows that $\nabla \mathbf{e}_N \cdot \mathbf{e}_N = 0$, and hence

$$\nabla^T \nabla N \mathbf{e}_N = \nabla^T |\nabla N|. \quad (51)$$

Taking this result, dividing through by $|\nabla N|$, and using equations (39) and (46), we find

$$\frac{\nabla^T \nabla N \mathbf{e}_N}{|\nabla N|} = -\frac{\nabla^T \epsilon_*}{2\epsilon_*} + \sin \Delta_N \cos \Delta_N \nabla^T T_{\mathcal{RS}}. \quad (52)$$

In the SRST limit, using equations (12) and (22), it holds that

$$\nabla^T \epsilon = -M \phi'. \quad (53)$$

Substituting this result into equation (52) and dividing through by another factor of $|\nabla N|$, we find that

$$\begin{aligned} \frac{\nabla^T \nabla N \mathbf{e}_N}{|\nabla N|^2} &= \cos \Delta_N \times \\ &\left[M_* \mathbf{e}_{\parallel}^* + \sin \Delta_N \cos \Delta_N \sqrt{2\epsilon_*} \nabla^T T_{\mathcal{RS}} \right]. \end{aligned} \quad (54)$$

To complete our calculation of f_{NL} , we need to contract equation (54) with the unit vector \mathbf{e}_N . We break this calculation into two parts, based on the fact that $\mathbf{e}_N = \cos \Delta_N \mathbf{e}_{\parallel}^* + \sin \Delta_N \mathbf{e}_{\perp}^*$. First, we contract $\cos \Delta_N (\mathbf{e}_{\parallel}^*)^T$ with equation (54). Using $\frac{d}{dN} = \phi' \cdot \nabla$ and equation (46), we can write

$$\frac{\cos \Delta_N (\mathbf{e}_{\parallel}^*)^T \nabla^T \nabla N \mathbf{e}_N}{|\nabla N|^2} = \cos^2 \Delta_N \frac{d}{dN} \ln |\nabla N|. \quad (55)$$

From equations (37) and (45), it follows that

$$\frac{\cos \Delta_N (\mathbf{e}_{\parallel}^*)^T \nabla^T \nabla N \mathbf{e}_N}{|\nabla N|^2} = \frac{1}{2} \cos^2 \Delta_N (n_{\mathcal{R}} - n_{\mathcal{T}}). \quad (56)$$

Second, we calculate $\sin \Delta_N (\mathbf{e}_{\perp}^*)^T$ contracted with equation (54), which yields

$$\begin{aligned} \frac{\sin \Delta_N (\mathbf{e}_{\perp}^*)^T \nabla^T \nabla N \mathbf{e}_N}{|\nabla N|^2} &= \sin \Delta_N \cos \Delta_N \times \\ &\left[M_{\perp}^* + \sin \Delta_N \cos \Delta_N \sqrt{2\epsilon_*} \mathbf{e}_{\perp}^* \cdot \nabla T_{\mathcal{RS}} \right]. \end{aligned} \quad (57)$$

Combining equations (56) and (57), we finally arrive at a general expression for f_{NL} :

$$\boxed{-\frac{6}{5} f_{NL} = \frac{1}{2} \cos^2 \Delta_N (n_{\mathcal{R}} - n_{\mathcal{T}}) + \sin \Delta_N \cos \Delta_N \left(M_{\perp}^* + \sin \Delta_N \cos \Delta_N \sqrt{-n_{\mathcal{T}}} \mathbf{e}_{\perp}^* \cdot \nabla T_{\mathcal{RS}} \right)}. \quad (58)$$

Equation (58) shows that f_{NL} depends on just four quantities: $n_{\mathcal{R}}$, $n_{\mathcal{T}}$, the turn rate $\frac{2a}{v} \approx -M_{\perp}$, and the transfer function $T_{\mathcal{RS}}$ (which sets the value of the trigonometric functions). We cast it in the above form to show the explicit dependence of f_{NL} on the curvature (scalar) and tensor spectral indices. In fact, f_{NL} can be written completely in terms of spectral observables, with one exception: the term $\mathbf{e}_{\perp}^* \cdot \nabla T_{\mathcal{RS}}$. To cast f_{NL} almost completely in terms of spectral observables, the terms $\sin \Delta_N$, $\cos \Delta_N$, and M_{\perp}^* can be replaced by the observables r_C , $\sqrt{1 - r_C^2}$, and $\frac{r_C}{\sqrt{1 - r_C^2}} \left(\frac{n_S - n_C}{2} \right)$, respectively, where n_C and n_S are the spectral indices for the

cross and isocurvature spectra, respectively [77].

Equation (58) has another very important benefit: although we derived it under the assumption of two-field inflation, it can be applied either directly or with minimal modifications to other scenarios where the interplay between curvature and isocurvature modes can be represented by the transfer function formalism. For example, our general expression for f_{NL} can be applied to mixed inflaton and curvaton models. To apply equation (58) to such models, we used equation (36) to tease an expression for the transfer function $T_{\mathcal{RS}}$ out of the expression for the power spectrum in [58], and then we performed the calculations for various scenarios and found good agree-

ment with the results in [58]. Also, although we have not shown it here and the calculation is more difficult, similar qualitative conditions hold for general multi-field inflation; the main difference is that for multi-field inflation, the sourcing term analogous to $T_{\mathcal{RS}}$ is a vector, rather than a scalar.

A third benefit of equation (58) is that it allows us to determine what kinds of “multi-field effects” are needed in order for f_{NL} to be large in magnitude. The standard slow-roll, single-field case produces $|f_{NL}| \ll 1$, making f_{NL} undetectably small. Indeed, taking the single-field limit of equation (58), we recover the consistency equation [26]

$$-\frac{6}{5}f_{NL} = \frac{1}{2}(n_{\mathcal{R}} - n_T). \quad (59)$$

Yet, although adding one or more fields can produce large non-Gaussianity, we know that the overwhelming majority of multi-field scenarios do not produce large non-Gaussianity. Equation (58) reveals why this is so—namely, it reveals which kinds of multi-field effects produce large non-Gaussianity. Examining equation (58), all terms must be significantly smaller than unity³ except the term $\mathbf{e}_{\perp}^* \cdot \nabla T_{\mathcal{RS}}$. Therefore, $|f_{NL}|$ cannot be greater than unity unless

$$\boxed{|\sin^2 \Delta_N \cos^2 \Delta_N (\mathbf{e}_{\perp}^* \cdot \nabla T_{\mathcal{RS}})| \gtrsim \frac{1}{\sqrt{-n_T}}}. \quad (60)$$

So we see that for $|f_{NL}|$ to be at least of order unity, the multi-field effects must satisfy two requirements:

1. The total amount of sourcing of curvature modes by isocurvature modes ($T_{\mathcal{RS}}$) must be extremely sensitive to a change in the initial conditions perpendicular to the inflaton trajectory. In other words, neighboring trajectories must experience dramatically different amounts of sourcing.
2. The total amount of sourcing must be non-zero (*i.e.*, $\sin \Delta_N \neq 0$) and sufficiently sized so that the trigonometric terms do not prevent the bound in equation (60) from being satisfied.

The first condition, that $T_{\mathcal{RS}}$ be extremely sensitive to the initial conditions, makes sense on an intuitive level. In order to produce a large degree of skew in the primordial fluctuations, perturbations off the classical trajectory must move the inflaton onto neighboring trajectories that produce very different values for the curvature perturbation at the end of inflation.

³ Observational constraints force the magnitudes of $n_{\mathcal{R}}$ and n_T to be much less than unity, and the turn rate at horizon exit must be at least somewhat less than unity to avoid violating scale-invariance and causing a complete breakdown of the SRST approximation at horizon exit.

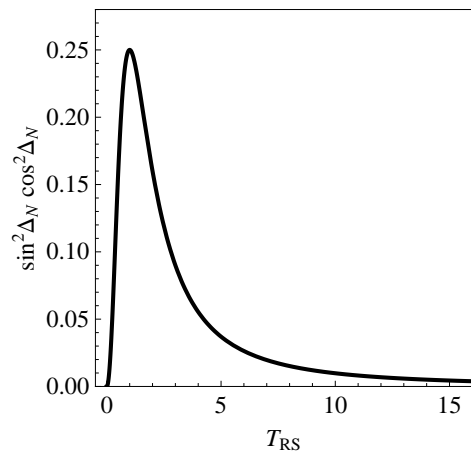


FIG. 1: The trigonometric factor $\sin^2 \Delta_N \cos^2 \Delta_N$ as a function of the total amount of mode sourcing, $T_{\mathcal{RS}}$. For reference, when $T_{\mathcal{RS}} = 1$, half of the curvature (scalar) power spectrum at the end of inflation is due to the sourcing of curvature modes by isocurvature modes.

The second condition reflects the fact that in the limit of no sourcing — which corresponds to single-field behavior — the bound in equation (60) can never be satisfied. And more likely than not, the sourcing will be moderate, but this is not a requirement. Rather, if the total sourcing is tiny ($\sin \Delta_N \ll 1$) or is very large ($\cos \Delta_N \ll 1$), then the trigonometric terms will make it even that much harder to satisfy the bound in equation (60). Figure 1 illustrates this point by showing the value of the factor $\sin^2 \Delta_N \cos^2 \Delta_N$ as a function of the total mode sourcing, $T_{\mathcal{RS}}$. Its maximum value is 0.25, which occurs at $T_{\mathcal{RS}} = 1$.

While we provided a preview of the conditions for large non-Gaussianity in this section, we will revisit this topic and treat it in much greater detail in Section (IV C). But before doing that, we show how $\mathbf{e}_{\perp}^* \cdot \nabla T_{\mathcal{RS}}$ can be explicitly found for some analytically solvable two-field scenarios.

B. Calculation of $\mathbf{e}_{\perp}^* \cdot \nabla_* T_{\mathcal{RS}}$

In this section, we will proceed to take the calculation of $\mathbf{e}_{\perp}^* \cdot \nabla_* T_{\mathcal{RS}}$ as far as possible. The work we present here is mostly applicable to analytically solvable models. As examples, we find $\mathbf{e}_{\perp}^* \cdot \nabla_* T_{\mathcal{RS}}$ for four different classes of tractable models, and we show how strikingly similar the results are if they are expressed in terms of ϵ , the entropy mass, and the transfer functions. This underscores the fact that only a few key quantities appear to control the level of non-Gaussianity in two-field inflation.

Before we proceed, we note that we will indicate when certain quantities are to be evaluated; in particular, we use the superscript e to denote that a quantity is to be evaluated at the end of inflation. Like in the previous sections, the transfer functions are to be evaluated at the

end of inflation (which we take to be when $\epsilon = 1$), except when explicitly indicated otherwise.

Calculating the term $\mathbf{e}_\perp^* \cdot \nabla_* T_{\mathcal{R}S}$ is difficult for two reasons: (1) the field values at the end of inflation depend on the field values at horizon exit and (2) $T_{\mathcal{R}S}$ is not necessarily a conservative function. To act the operator ∇_* on the expression for $T_{\mathcal{R}S}$ in equation (32), we change variables and rewrite the equation as a line integral expression of the fields. We can convert equation (32) into a line integral by working in the SRST limit and by replacing the function $\alpha = 2\frac{\gamma_\perp}{v}$ with its SRST counterpart $-2M_{\parallel\perp}$, which yields

$$T_{\mathcal{R}S} = -2 \int_{\phi_*}^{\phi_e} T_{SS}(\phi_*, \phi) \frac{\mathbf{e}_\perp^T \mathbf{M} d\phi}{\sqrt{2\epsilon}}. \quad (61)$$

$$\nabla_* T_{\mathcal{R}S} = \left[\frac{2\mathbf{M} \mathbf{e}_\perp}{\sqrt{2\epsilon}} + \gamma \mathbf{e}_\perp \right]_* - \mathcal{X} \left[\frac{2\mathbf{M} \mathbf{e}_\perp}{\sqrt{2\epsilon}} + \gamma \mathbf{e}_\perp \right]_e T_{SS} + T_{\mathcal{R}S} \nabla_* (\ln T_{SS})_{\phi_e = \text{const}}, \quad (62)$$

where

$$\mathcal{X}_j^i \equiv \frac{\partial C}{\partial \phi_i^*} \frac{d\phi_j^e}{dC}. \quad (63)$$

The variable C in equation (63) parametrizes motion orthogonal to the given trajectory, as only changes in the initial field vector that are off the trajectory will affect the final field values. The last new term that we introduce is $\nabla_* (\ln T_{SS})_{\phi_e = \text{const}}$, which means to take the gradient of $\ln T_{SS}$ while holding the amplitude of the isocurvature modes at the end of inflation constant. It can be thought of as some sort of measure of the sensitivity of T_{SS} to the initial conditions. This term arises from the fact that acting ∇_* on $T_{\mathcal{R}S}$ in equation (61) involves differentiating under the integral, which is necessary since $T_{SS}(\phi_*, \phi)$ depends on ϕ_* . We emphasize that the form we assume for $\nabla_* T_{\mathcal{R}S}$ in equation (62) is best applicable to analytically solvable models and models where the coupling term $|\frac{\partial_1 \partial_2 V}{\partial_1 V \partial_2 V}| \ll 1$ or is approximately constant, as will become clearer later.

The matrix \mathcal{X} that arises in the above expression captures how a change in the initial conditions at horizon exit affects the final values of the fields at the end of inflation. As it turns out, the matrix \mathcal{X} has a model-independent form, which we will now prove. First, since C is constant along a given trajectory,

$$C' = \phi' \cdot \nabla C = 0. \quad (64)$$

Therefore, ∇C must be orthogonal to the inflaton trajectory — that is,

$$\nabla C = |\nabla C| (\mathbf{e}_\perp)^T. \quad (65)$$

If the integrand of $T_{\mathcal{R}S}$ is the gradient of a function, then operating ∇_* on $T_{\mathcal{R}S}$ simply returns the integrand evaluated both at horizon exit and at the end of inflation, with the latter being times a matrix representing the sensitivity of the final field values to the initial field values. However, in general, the integrand of $T_{\mathcal{R}S}$ will not be the gradient of a function. To account for this, we introduce a model-dependent function γ to represent how much the integrand of $T_{\mathcal{R}S}$ in equation (61) deviates from being the gradient of a function. Now operating ∇_* on $T_{\mathcal{R}S}$ and using the new function γ , we obtain

Next, consider $\frac{d\phi_e}{dC}$. Since a change in C corresponds to motion orthogonal to the trajectory and since the δN formalism is applied to surfaces of constant energy density (see, for example, [89]), $\frac{d\phi_e}{dC}$ is parallel to \mathbf{e}_\perp^e . Now combining this fact with equation (65) and with

$$1 = \frac{dC}{dC} = \frac{d\phi_e}{dC} \cdot \nabla_e C \quad (66)$$

implies that $\frac{d\phi_e}{dC} = |\nabla C|_e^{-1} \mathbf{e}_\perp^e$. Substituting this result and equation (65) into equation (63) yields

$$\mathcal{X} = \frac{|\nabla C|_*}{|\nabla C|_e} \mathbf{e}_\perp^* (\mathbf{e}_\perp^e)^T. \quad (67)$$

Finally, we express the ratio of the norms of the gradients of C at horizon exit and the end of inflation in terms of a physical quantity: the relative amplitude of the entropy modes. Since $\delta C = \delta \phi \cdot \nabla C$, then for a given variation in the trajectory, δC , we have

$$|\nabla C|_* \delta \phi_\perp^* = |\nabla C|_e \delta \phi_\perp^e. \quad (68)$$

Combining equations (67) and (68), we finally arrive at the model-independent expression

$$\mathcal{X} = \left(\frac{\delta \phi_\perp^e}{\delta \phi_\perp^*} \right) \mathbf{e}_\perp^* (\mathbf{e}_\perp^e)^T = \sqrt{\frac{2\epsilon_e}{2\epsilon_*}} T_{SS} \mathbf{e}_\perp^* (\mathbf{e}_\perp^e)^T. \quad (69)$$

This interesting result shows that the sensitivity of the final field values to the initial field values can be given very simply in terms of the relative growth or decay of entropy modes. In other words, the evolution of entropy modes mirrors whether neighboring trajectories converge or diverge over time. In scenarios where neighboring trajectories converge (“attractor solutions”), the

entropy modes decay. However, when neighboring trajectories diverge, the entropy modes grow. That such a relationship should hold between the convergence/divergence of neighboring trajectories and the evolution of entropy modes makes sense. From a geometrical perspective, we intuitively expect that a positive curvature along the entropic direction focuses neighboring trajectories, whereas a negative curvature creates a hill or ridge in the potential, causing neighboring trajectories to diverge. But we also know that the curvature along the entropic direction determines the evolution of entropy modes. By equation

(28), the entropy modes grow when $M_{\perp\perp} < 0$ and decay when $M_{\perp\perp} > 0$, and how quickly they do so depends on the magnitude of the curvature. Combining these two facts together, we could have concluded that the divergence/convergence of neighboring trajectories must correlate with the growth/decay of entropy modes, without even deriving this result. Nonetheless, equation (69) gives this important relationship explicitly.

Now substituting equation (69) into equation (62) and projecting the result onto \mathbf{e}_\perp^* , we obtain

$$\sqrt{2\epsilon_*} \mathbf{e}_\perp^* \cdot \nabla_* T_{\mathcal{RS}} = (2M_{\perp\perp}^* + \sqrt{2\epsilon_*} \gamma_*) - (2M_{\perp\perp}^e + \sqrt{2\epsilon_e} \gamma_e) T_{SS}^2 + \sqrt{2\epsilon_*} T_{\mathcal{RS}} \mathbf{e}_\perp^* \cdot \nabla_* (\ln T_{SS})_{\phi_e = \text{const}}. \quad (70)$$

The above relation gives us a nice way to understand what geometrical and physical attributes are needed to produce sourcing that is very sensitive to changes in the initial conditions orthogonal to the inflaton trajectory. The above equation shows that the sensitivity of $T_{\mathcal{RS}}$ to the initial conditions is determined by ϵ , $M_{\perp\perp}$, T_{SS} , $T_{\mathcal{RS}}$, and the model-dependent factor γ . Notice how strongly T_{SS} controls $\sqrt{2\epsilon_*} \mathbf{e}_\perp^* \cdot \nabla_* T_{\mathcal{RS}}$.

At this point, it is not possible to proceed any further without specifying the inflationary potential. In the remainder of this section, we discuss how to find the model-dependent terms T_{SS} and γ , which arise in the expression for $\mathbf{e}_\perp^* \cdot \nabla_* T_{\mathcal{RS}}$. We will illustrate the procedure by considering four classes of inflationary models.

First, recall that we defined the model-dependent function γ so that it is zero whenever $T_{\mathcal{RS}}$ is a conservative function. This occurs for product potentials, defined as

$$V = V_1(\phi_1)V_2(\phi_2), \quad (71)$$

and can be attributed to the fact that in these models, the two fields evolve independently of each other. γ is therefore non-zero whenever the evolutions of the two fields influence each other.

We can see that γ is zero for product potentials as follows. For product potentials, the isocurvature mass equals

$$\beta = M_{\parallel\parallel} - M_{\perp\perp} = (\tan \theta - \cot \theta) M_{\parallel\perp}, \quad (72)$$

where θ is the polar coordinate in the (ϕ'_1, ϕ'_2) plane — that is,

$$\tan \theta \equiv \frac{\phi'_2}{\phi'_1}. \quad (73)$$

Using $-M_{\parallel\perp} \approx \frac{\eta_\perp}{v} = \theta'$ and plugging equation (72) into equation (32), one finds that the transfer function T_{SS} for these models can be approximated by

$$T_{SS} = \frac{\sin \theta_e \cos \theta_e}{\sin \theta_* \cos \theta_*}. \quad (74)$$

Substituting $\frac{\eta_\perp}{v} = \theta'$ and equation (74) into equation (32) yields

$$\begin{aligned} T_{\mathcal{RS}} &= \int_{N_*}^{N_e} 2\theta' \frac{\sin \theta \cos \theta}{\sin \theta_* \cos \theta_*} dN, \\ &= \frac{1}{\sin \theta_* \cos \theta_*} \int_{N_*}^{N_e} \frac{d}{dN} (\sin^2 \theta) dN, \end{aligned} \quad (75)$$

which integrates to give

$$\begin{aligned} T_{\mathcal{RS}} &= \frac{1}{\sin \theta_* \cos \theta_*} (\sin^2 \theta_e - \sin^2 \theta_*), \\ &= -\tan \theta_* + \tan \theta_e T_{SS}. \end{aligned} \quad (76)$$

Now we take the gradient of the above transfer function and use that

$$\nabla_{\mathbf{e}_\parallel} = \nabla(\cos \theta, \sin \theta) = -\frac{M \mathbf{e}_\perp}{\sqrt{2\epsilon}} \mathbf{e}_\perp^T \quad (77)$$

in the SRST limit for any two-field model of inflation. Finally, projecting the result onto \mathbf{e}_\perp^* and using equation (72), we find

$$\begin{aligned}
\sqrt{2\epsilon_*} \mathbf{e}_\perp^* \cdot \nabla_* T_{\mathcal{RS}} &= 2M_{\perp\perp}^* - 2M_{\perp\perp}^e (T_{SS})^2 + \sqrt{2\epsilon_*} T_{\mathcal{RS}} \mathbf{e}_\perp^* \cdot \nabla_* (\ln T_{SS})_{\phi_e = \text{const}}, \\
&= 2M_{\perp\perp}^* - 2M_{\perp\perp}^e (T_{SS})^2 + \left(\frac{M_{\perp\perp}^* - M_{\parallel\parallel}^*}{M_{\parallel\perp}^*} \right) M_{\perp\perp}^* T_{\mathcal{RS}}.
\end{aligned} \tag{78}$$

where again T_{SS} for product potentials is given by equation (74). Comparing the first line of equation (78) to equation (70) indeed shows that $\gamma = 0$.

Now for general two-field inflation, we can use a similar procedure to find the model-dependent term γ . Since γ measures how much $T_{\mathcal{RS}}$ deviates from being a conservative function and whether $T_{\mathcal{RS}}$ is conservative depends on T_{SS} , γ depends on T_{SS} . Therefore, we must first find a general expression for the transfer function T_{SS} . Starting from equation (42) in [90], after some algebra, we can show that this implies that T_{SS} takes the form

$$\begin{aligned}
T_{SS} &= \left(\frac{\sin \theta_e \cos \theta_e}{\sin \theta_* \cos \theta_*} \right) \exp \left[\int_{N_*}^{N_e} \frac{M_{12}}{\sin \theta \cos \theta} dN \right], \\
&= \left(\frac{\sin \theta_e \cos \theta_e V_e}{\sin \theta_* \cos \theta_* V_*} \right) s(\phi_*, \phi_e),
\end{aligned} \tag{79}$$

where $M_{12} \equiv \partial_1 \partial_2 \ln V$ and

$$s(\phi_*, \phi_e) \equiv \exp \left[- \int_{\phi_*}^{\phi_e} \left(\frac{V \partial_1 \partial_2 V}{\partial_1 V \partial_2 V} \right) \nabla \ln V \cdot d\phi \right]. \tag{80}$$

Equation (80) shows that whenever $\left(\frac{V \partial_1 \partial_2 V}{\partial_1 V \partial_2 V} \right)$ is constant, $s(\phi_*, \phi_e)$ becomes analytic and hence T_{SS} becomes analytic. This means that there are several instances in which we can find either exact or approximate analytic expressions for the isocurvature modes. For product potentials, $M_{12} = 0$ in equation (79), reproducing the result for T_{SS} that we derived in equation (74). For sum potentials, defined as

$$V = V_1(\phi_1) + V_2(\phi_2), \tag{81}$$

the coupling term $\partial_1 \partial_2 V = 0$, and so

$$T_{SS} = \frac{\sin \theta_e \cos \theta_e V_e}{\sin \theta_* \cos \theta_* V_*}. \tag{82}$$

Equation (82) can also be used to approximate T_{SS} in scenarios where $\left| \frac{V \partial_1 \partial_2 V}{\partial_1 V \partial_2 V} \right| \ll 1$ during all of inflation. Finally, in the more general case where $\left(\frac{V \partial_1 \partial_2 V}{\partial_1 V \partial_2 V} \right)$ is approximately constant during inflation, T_{SS} can be approximated by an analytic function similar to equation

(82), but possessing additional powers of (V_e/V_*) . Importantly, all of the above models share the commonality that either M_{12} or $\left(\frac{V \partial_1 \partial_2 V}{\partial_1 V \partial_2 V} \right)$ is exactly or approximately constant. And whenever this is true, we have that the last term in equation (70) equals

$$\sqrt{2\epsilon_*} \mathbf{e}_\perp^* \cdot \nabla_* (\ln T_{SS})_{\phi_e = \text{const}} = (\cot \theta_* - \tan \theta_*) M_{\perp\perp}^*, \tag{83}$$

since gradients of functions of V do not contribute to $\mathbf{e}_\perp^* \cdot \nabla_* (\ln T_{SS})_{\phi_e = \text{const}}$. Hence for the above mentioned models, the expression encapsulating the sensitivity of T_{SS} to the initial conditions always has the same form. Otherwise, when neither the term M_{12} nor $\left(\frac{V \partial_1 \partial_2 V}{\partial_1 V \partial_2 V} \right)$ is approximately constant, this adds extra terms to $\mathbf{e}_\perp^* \cdot \nabla_* (\ln T_{SS})_{\phi_e = \text{const}}$ that contribute to f_{NL} .

Now that we have a general solution for T_{SS} , we can proceed to find an expression for γ . To find γ , we plug the expression for T_{SS} into equation (32) for $T_{\mathcal{RS}}$. Integrating by parts again using the fact that $(\sin^2 \theta)' = 2 \sin \theta \cos \theta \theta'$, we obtain the following integral expression:

$$\begin{aligned}
T_{\mathcal{RS}} &= -\tan \theta_* + \tan \theta_e T_{SS} - \\
&\frac{1}{\sin \theta_* \cos \theta_* V_*} \int_{\phi_*}^{\phi_e} d\phi \cdot \nabla [V s(\phi_*, \phi)] \sin^2 \theta.
\end{aligned} \tag{84}$$

The perpendicular component of the gradient of $T_{\mathcal{RS}}$ can be calculated directly from the above equation, where equation (77) comes in handy.⁴ And by cleverly grouping the terms in the resultant expression, we can determine γ .

For example, for sum potentials, the function $s(\phi_*, \phi)$ equals unity, so the integral in equation (84) evaluates to $\int_{\phi_*}^{\phi_e} \sin^2 \theta \nabla V \cdot d\phi = V_2^e - V_2^*$, producing

$$\begin{aligned}
T_{\mathcal{RS}} &= -\tan \theta_* + \frac{V_2^*}{\sin \theta_* \cos \theta_* V_*} \\
&+ \left(\tan \theta_e - \frac{V_2^e}{\sin \theta_e \cos \theta_e V_e} \right) T_{SS}.
\end{aligned} \tag{86}$$

Taking the gradient of the above expression, $\mathbf{e}_\perp^* \cdot \nabla_* T_{\mathcal{RS}}$ for sum potentials can be written as

⁴ From equation (84), we can also derive an upper limit for $T_{\mathcal{RS}}$ for general two-field inflation whenever the SRST limit is a valid approximation. Using the fact that $\sin^2 \theta \leq 1$, we find from equation (84) that

$$T_{\mathcal{RS}} \leq \cot \theta_* - \cot \theta_e T_{SS}. \quad (85)$$

$$\begin{aligned} \sqrt{2\epsilon_*} \mathbf{e}_\perp^* \cdot \nabla_* T_{\mathcal{RS}} &= (2M_{\perp\perp}^* - 2\epsilon_*) - (2M_{\perp\perp}^e - 2\epsilon_e) T_{SS}^2 + \sqrt{2\epsilon_*} T_{\mathcal{RS}} \mathbf{e}_\perp^* \cdot \nabla_* (\ln T_{SS})_{\phi_e = \text{const}}, \\ &= (2M_{\perp\perp}^* - 2\epsilon_*) - (2M_{\perp\perp}^e - 2\epsilon_e) T_{SS}^2 + \left(\frac{M_{\perp\perp}^* - M_{\parallel\parallel}^* - 2\epsilon_*}{M_{\parallel\perp}^*} \right) M_{\perp\perp}^* T_{\mathcal{RS}}, \end{aligned} \quad (87)$$

where T_{SS} is given by equation (82). Hence, by comparing the first line of equation (87) to equation (70), we conclude that $\gamma = -\sqrt{2\epsilon}$ for sum potentials. Similarly, for potentials of the form $V \propto \omega^p$, where $\omega = V_1(\phi_1) + V_2(\phi_2)$, we find that

$$T_{SS} = \frac{\sin \theta_e \cos \theta_e \omega_e}{\sin \theta_* \cos \theta_* \omega_*}, \quad (88)$$

and hence

$$\begin{aligned} T_{\mathcal{RS}} &= -\tan \theta_* + \frac{V_2^*}{\sin \theta_* \cos \theta_* \omega_*} \\ &+ \left(\tan \theta_e - \frac{V_2^e}{\sin \theta_e \cos \theta_e \omega_e} \right) T_{SS}. \end{aligned} \quad (89)$$

Therefore, $\sqrt{2\epsilon_*} \mathbf{e}_\perp^* \cdot \nabla_* T_{\mathcal{RS}}$ for these potentials is identical to the right-hand side of equation (87), except with the substitution of $2\epsilon \rightarrow 2\epsilon/p$ and except that equation (88) is used for T_{SS} . Hence, $\gamma = -\frac{1}{p}\sqrt{2\epsilon}$ for these models.

Finally, we consider one last case, which is broader in scope. For all potentials that satisfy

$$\left(\frac{V \partial_1 \partial_2 V}{\partial_1 V \partial_2 V} \right) = c, \quad (90)$$

where c is either exactly or approximately constant, we can also find T_{SS} and γ . This corresponds to the case where

$$\left(\frac{\partial_1 V}{V^c}, \frac{\partial_2 V}{V^c} \right) = (f(\phi_1), g(\phi_2)), \quad (91)$$

where f and g are arbitrary functions of their arguments. The above two equations allow us to find T_{SS} analytically:

$$T_{SS} = \frac{\sin \theta_e \cos \theta_e V_e^{(1-c)}}{\sin \theta_* \cos \theta_* V_*^{(1-c)}}. \quad (92)$$

Following a procedure similar that which we followed for sum potentials, the above two equations imply that $\gamma = (c-1)\sqrt{2\epsilon}$. Indeed, the product ($c=0$), sum ($c=1$),

and $V \propto [V_1(\phi_1) + V_2(\phi_2)]^p$ ($c = 1 - \frac{1}{p}$) potentials can be seen as subcases of this broader class of models.

In considering four tractable examples with analytic transfer functions T_{SS} , we saw that the results for $\mathbf{e}_\perp^* \cdot \nabla_* T_{\mathcal{RS}}$ were strikingly similar for these models. For each class of models we considered, $\mathbf{e}_\perp^* \cdot \nabla_* T_{\mathcal{RS}}$ depends on ϵ , the entropy mass, and the two transfer functions. Although f_{NL} has been calculated before for three of these four models—for product potentials by [46], sum potentials by [47], and potentials of the form $V \propto [V_1(\phi_1) + V_2(\phi_2)]^p$ by [50]—the results in this and the previous section cast the expressions in what we believe is a more transparent physical form. In particular, this formulation reveals how very strongly the isocurvature modes control the size of f_{NL} .

To find γ for other potentials, if T_{SS} can be found analytically, then following the procedure outlined here may lead to an analytic formula for $\mathbf{e}_\perp^* \cdot \nabla_* T_{\mathcal{RS}}$. For weak coupling among the fields, we expect γ to be of order the slow-roll parameters, as it was for all the models we considered in this section. However, this term may be larger in the limit of strong coupling. Otherwise, if T_{SS} cannot be found analytically, then $\mathbf{e}_\perp^* \cdot \nabla_* T_{\mathcal{RS}}$ can be computed numerically.

C. Conditions for Large $|f_{NL}|$

As we showed in Section (IV A), if the power spectra are nearly scale-invariant, the magnitude of f_{NL} can be greater than unity only if $|\sin^2 \Delta_N \cos^2 \Delta_N \sqrt{-n_T} \mathbf{e}_\perp^* \cdot \nabla_* T_{\mathcal{RS}}| \gtrsim 1$. Satisfying this bound requires that two conditions be met: (1) that $T_{\mathcal{RS}}$ be extremely sensitive to changes in the initial conditions perpendicular to the given trajectory and (2) that the amount of sourcing be non-zero. In this section, we explore these two conditions in more detail, and we investigate what features an inflationary potential needs in order to produce large non-Gaussianity.

We start by considering the second condition, since it is easier to understand. The second condition, that the sourcing must be non-zero, requires that $T_{\mathcal{RS}} \neq 0$.

By equations (32) and (33), this in turn requires that the turn rate not be zero for all of inflation. Typically, we will also find that the sourcing of curvature modes by isocurvature modes will be moderate, although this is not required; rather, weak sourcing ($\sin \Delta_N \ll 1$) or strong sourcing ($\cos \Delta_N \ll 1$) simply makes the bound in equation (60) even that much harder to satisfy. In other words, searching for scenarios where the sourcing is moderate maximizes the chance of finding scenarios that produce large non-Gaussianity. To achieve moderate sourcing, how much of a turn is needed depends on the relative amplitude of the isocurvature modes during the turn. If the isocurvature modes are small at the time, then a larger turn is needed. However, if the isocurvature modes are large, then only a minuscule turn in the trajectory is needed. Conversely, in scenarios where $|f_{NL}|$ is large, but the sourcing is either very weak or very strong, we expect more extensive fine-tuning will be involved.

Now let us examine the first condition for large $|f_{NL}|$. The first condition, that $T_{\mathcal{RS}}$ be very sensitive to a change in initial conditions orthogonal to the trajectory, means that neighboring trajectories must experience very different amounts of sourcing. This requirement can be understood as follows: perturbations off the classical trajectory must move the inflaton onto neighboring trajectories that experience very different dynamics for the curvature perturbation. This instability in the inflaton trajectory is required to produce a large degree of skew in the primordial fluctuations.

After arriving at this qualitative prescription, it is important to explore what features are needed to make the sourcing function $T_{\mathcal{RS}}$ so sensitive to the initial conditions. To start, we can examine equations (32), (33), and (34). These equations show that $T_{\mathcal{RS}}$ is determined by an integral of the turn rate times relative amplitude of isocurvature modes (T_{SS}). *Therefore, to satisfy the first condition for large non-Gaussianity, neighboring trajectories need to have very different turn rate profiles, T_{SS} profiles, or both.*

In Section (IV B), we attempted to explore the first condition from a more quantitative perspective. This can be done in models where $T_{\mathcal{RS}}$ can be approximated by an analytic function. For the collection of models we considered, our goal was to determine the dependence of $\mathbf{e}_\perp^* \cdot \nabla_* T_{\mathcal{RS}}$ on the dynamical and physical attributes of the inflationary scenario. Let us now consider the key result of that section: equation (70). Examining equation (70) reveals that $\sqrt{-n_T} \mathbf{e}_\perp^* \cdot \nabla_* T_{\mathcal{RS}}$ will be large in magnitude for these models if at least one of the following three conditions is met:

1. $2M_{\perp\perp}^* + \sqrt{2\epsilon_*} \gamma_*$ is large in magnitude,
2. $(2M_{\perp\perp}^e + \sqrt{2\epsilon_e} \gamma_e) T_{SS}^2$ is large in magnitude, or
3. T_{SS} is very sensitive to changes in the initial conditions orthogonal to the inflaton trajectory — more specifically, that $\sqrt{2\epsilon_*} \mathbf{e}_\perp^* \cdot \nabla_* \ln(T_{SS})_{\phi_e=\text{constant}}$ is large in magnitude.

In conventional slow-roll, the magnitudes of the entropy mass ($M_{\perp\perp}$) and the other slow-roll parameters at horizon exit are significantly less than unity, so only the latter two conditions can be satisfied. If we assume conventional slow-roll at horizon exit and additionally that the magnitudes of $M_{\perp\perp}^e$ and γ_e not much greater than unity, then for detectably large f_{NL} , the second condition above requires that $T_{SS}^2 \sim O(10)$. If we tighten the constraints even further, strictly requiring that $|M_{\perp\perp}| \ll 1$ during all of inflation, then the second condition becomes even more stringent, requiring that T_{SS}^2 be very large, at least of order $O(100)$. Regardless of the exact constraints imposed on $M_{\perp\perp}^e$, *these conditions suggest that to produce large $|f_{NL}|$ when equation (70) holds, then the isocurvature modes must be very large at the end of inflation and/or their relative amplitude (T_{SS}) needs to be very sensitive to the initial conditions.*

Let us reconcile this statement with the criteria for large bispectra for product potentials and sum potentials, as articulated by Byrnes *et. al.* [48]. For product potentials, Byrnes *et. al.* found that f_{NL} will be large in magnitude when one of the two fields starts with far more kinetic energy than the other (either $\cot \theta_* \gg 1$ or $\tan \theta_* \gg 1$) and when the asymmetry in the kinetic energies of the two fields diminishes significantly by the end of inflation. The reason that these two conditions produce large bispectra in product potentials is that they together guarantee three things: that $T_{SS} \gg 1$ (which follows from equation (74)), that T_{SS} is very sensitive to changes in the initial conditions orthogonal to the inflaton trajectory, and that the turn rate is small yet significant enough to produce moderate sourcing. Interestingly, this means that for product potentials, both the second and third conditions for large $\sqrt{-n_T} \mathbf{e}_\perp^* \cdot \nabla_* T_{\mathcal{RS}}$ are always simultaneously satisfied, as the third condition combined with the requirement that the total sourcing is moderate guarantees the second condition, and vice versa. For sum potentials, the conditions for large bispectra are a bit more complicated to untangle but end up being similar; however, the expression for T_{SS} contains a factor of $\frac{V}{V_*}$, so there may be a way to somehow satisfy condition 3 without requiring that the isocurvature modes be large at the end of inflation. Although we state the conditions for large $|f_{NL}|$ differently and slightly extend them in scope by relaxing constraints on $M_{\perp\perp}$, our conditions otherwise agree with those uncovered by Byrnes *et. al.* [48]. The benefit of our variant of these same conditions is that our version shows that the amplitude of the isocurvature modes at the end of inflation and their sensitivity to perturbations in the classical trajectory are what determines whether non-Gaussianity has any chance of being large.

Let us return to the general case of two-field inflation. Now that we have explored the conditions for large $|f_{NL}|$ separately, we might wonder whether the two most general conditions give us any constraints on the possible ways that the turn rate and/or T_{SS} can vary and still produce large $|f_{NL}|$. The answer to this question is yes.

We can best see this by considering the sourcing function $T_{\mathcal{RS}}$. Consider the case where the amplitude of isocurvature modes never exceeds its value at horizon exit, *i.e.*, $T_{\mathcal{SS}}(N_*, N) \leq 1$. In this case, $T_{\mathcal{RS}}$ can never exceed

$$T_{\mathcal{RS}} \leq 2 \int_{N_*}^{N_e} \theta' dN = 2(\theta_e - \theta_*), \quad (93)$$

where again θ is the polar angle for the field velocity vector. For the most common scenarios, the field velocity vector does not turn through an angle of more than 90° , yielding a bound of

$$T_{\mathcal{RS}} \leq \pi. \quad (94)$$

Let us compare this bound of $T_{\mathcal{RS}} \leq \pi$ with a numerical example. If $\epsilon_* = 0.02$ and we assume nearly scale-invariant scalar and tensor spectra, then we need $\mathbf{e}_\perp^* \cdot \nabla_* T_{\mathcal{RS}} \gtrsim 60$ in order to produce $|f_{NL}| \approx 3$. At a glance, this seems extremely difficult to achieve given the bound in equation (94) and the desirability of moderate sourcing. Now within this set of scenarios where $T_{\mathcal{SS}}(N_*, N) \leq 1$ during all of inflation, let us consider the subset for which $M_{\perp\perp} \geq 0$ both along and in a small neighborhood about the classical inflaton trajectory. If $M_{\perp\perp} \geq 0$ always holds, then neighboring trajectories must converge (or at least not diverge) over time. Since neighboring trajectories remain close to each other during all of inflation and thus must turn through approximately the same angle, $T_{\mathcal{RS}}$ cannot differ widely among neighboring trajectories without discontinuous or other extreme features in the potential that violate the SRST conditions. To be more precise, this would require the speed-up rate, turn rate, and/or entropy mass to be hugely varying in the direction orthogonal to the given trajectory, which effectively constitutes a violation of higher-order SRST parameters. In particular, it is not possible for a neighboring trajectory to have a much larger or smaller turn rate for a substantial period of time without having the two trajectories diverge,⁵ which would violate our assumption that $M_{\perp\perp} \geq 0$. Therefore, we argue that large non-Gaussianity cannot be produced in scenarios where both $T_{\mathcal{SS}}(N_*, N) \leq 1$ and $M_{\perp\perp} \geq 0$ during all of inflation.

Since we argue that it is not possible to achieve large non-Gaussianity during inflation if both $T_{\mathcal{SS}}(N_*, N) \leq 1$ and $M_{\perp\perp} \geq 0$ during all of inflation, then to produce large non-Gaussianity, one or both of these conditions must be violated during inflation. That is, the super-horizon isocurvature modes must grow and/or neighboring trajectories must diverge at least for some time during inflation. If we adopt the common assumptions that $\epsilon_* \ll 1$, ϵ never drops below its value at horizon exit,

and ϵ increases significantly (but not necessarily monotonically) in order to end inflation, then the only way to satisfy $T_{\mathcal{SS}}(N_*, N) > 1$ is for the entropy modes to grow. But this in turn implies that $M_{\perp\perp} < 0$. So under these minimal assumptions on ϵ , $T_{\mathcal{SS}} > 1$ automatically implies that $M_{\perp\perp} < 0$. Therefore, to obtain large non-Gaussianity under these assumptions, we conclude that we must have $M_{\perp\perp} < 0$ during at least part of inflation.

This conclusion leads to very important implications for the geometry of the inflationary potential. As long as ϵ does not dip below ϵ_* during inflation, then large non-Gaussianity requires $M_{\perp\perp} < 0$, which implies that the curvature of $\ln V$ along the entropic direction is negative. Geometrically, what this means is that the inflaton must roll along a ridge in the potential for some time during inflation. That is, the inflaton trajectory must be unstable, so that neighboring trajectories diverge. Moreover, the potential must not only possess a ridge, but the initial conditions must be fine-tuned so that the inflaton rolls along the ridge for a sufficiently long time and so that the inflaton trajectory turns somewhat during inflation so that $T_{\mathcal{RS}} \neq 0$. Furthermore, since large non-Gaussianity is more likely to be realized when the sourcing is moderate, we will typically find that the steeper the ridge that the inflaton rolls along, the smaller the turn in the trajectory tends to be. However, if ϵ can dip significantly below ϵ_* , then there may be a way to realize large non-Gaussianity in two-field inflation without needing a ridge in the potential.⁶ For the purposes of this paper, we will not focus on this potential exception, as very few models satisfy the requisite $\epsilon \lll \epsilon_*$ necessary to potentially circumvent our general prescription.

In Figure 2, we show an example of a potential with a steep ridge that provides the perfect conditions to make the total sourcing ($T_{\mathcal{RS}}$) so sensitive to the initial conditions. The trajectory of interest (solid lines) rolls along the ridge and turns ever so slightly at the end of infla-

⁵ We make the usual unstated assumption that there are no classical degeneracies in the gradient of $\ln V$, which means that trajectories cannot cross each other.

⁶ If we relax the assumption that ϵ never decreases below its value at horizon exit, then it may be possible to produce large $|f_{NL}|$ in two-field inflation without any negative curvature along the entropic direction. Indeed, Byrnes *et al.* [64] showed that this is possible for a three-field model: a two-component hybrid inflation model, where we are counting the waterfall field as the third field. During the first phase of inflation, which is governed by a vacuum-dominated sum potential and during which only two of the fields are active, ϵ exponentially decays after the given modes exit the horizon. The exponential decay of ϵ causes the isocurvature modes to grow dramatically during the initial phase of inflation, resulting in $T_{\mathcal{SS}} \gg 1$. As a result of the very large isocurvature modes, $T_{\mathcal{RS}}$ greatly exceeds the bound in equation (93) and can vary dramatically among neighboring trajectories even in the SRST limit. Hence, large non-Gaussianity is produced in this scenario, even before the waterfall field comes into play. Here, the large exponential decay in ϵ , which hugely boosts the amplitude of the isocurvature modes, circumvents the absolute need for a negative entropy mass at some point during inflation. However, in this model, a third field, the waterfall field with its associated negative mass, is needed to end inflation.

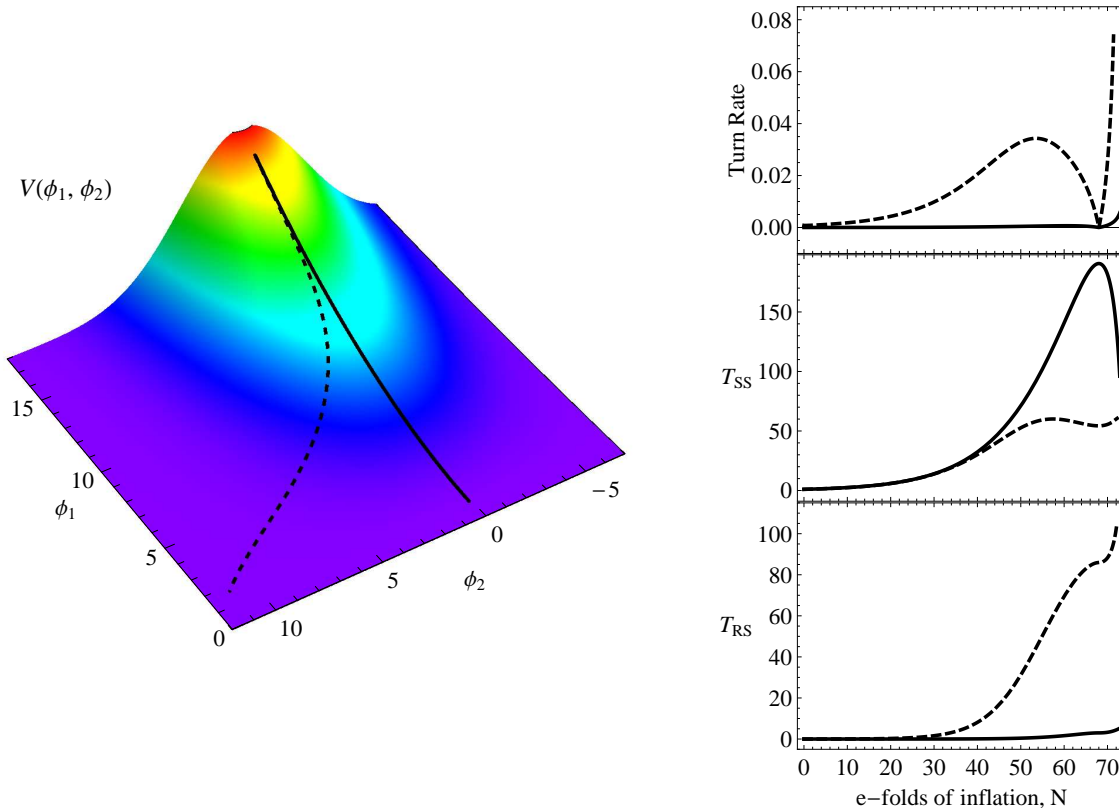


FIG. 2: For $|f_{NL}|$ to be large, the amount of sourcing of curvature modes by isocurvature modes ($T_{\mathcal{RS}}$) must be extremely sensitive to changes in the initial conditions orthogonal to the trajectory and the amount of sourcing must be non-zero (and most likely, it will be moderate). We argue that under some minimal assumptions on ϵ , this implies that the inflaton must roll along a ridge in the potential for some time and that its trajectory must turn at least slightly. Above is an example of a trajectory (solid lines) that meets these criteria. We use the potential $V(\phi_1, \phi_2) = \frac{1}{2}e^{-\lambda\phi_2^2}m^2\phi_1^2$, which Byrnes *et. al.* thoroughly investigated in [48]. We set $\lambda = 0.05$ and illustrate the results for two trajectories: (1) one that starts at $(\phi_1^*, \phi_2^*) = (17, 10^{-4})$, follows along the ridge, and turns slightly at the end of inflation (solid lines), and (2) a neighboring trajectory that starts only $|\Delta\phi_*| = 0.01$ away in field space, but that eventually rolls off the narrow ridge (dashed lines). The plot on the left shows the inflationary potential as a function of the fields, along with the two inflaton trajectories. The plots on the right show the turn rate, the relative amplitude of isocurvature modes (T_{SS}), and the total sourcing (T_{RS}) as a function of N . Here, the approximately 50-fold difference in the total sourcing stems more from the difference in the turn rates for the two trajectories, which both possess large isocurvature modes. Interestingly, the trajectory that rolls along the ridge (solid lines) produces $|f_{NL}| \sim 10^2$, while the neighboring trajectory (dashed lines) corresponds to $|f_{NL}| \approx 1$ [48], visually illustrating the role of fine-tuning in achieving large non-Gaussianity.

tion, resulting in large isocurvature modes and moderate sourcing, and therefore producing $|f_{NL}| \sim 10^2$. By comparison, a neighboring trajectory (dashed lines) eventually rolls off the ridge, producing large isocurvature modes but resulting in very strong sourcing. As a result, this second trajectory produces $|f_{NL}| \approx 1$, which is below the detection threshold for CMB experiments.

Cumulatively, these results explain why it has been so difficult to realize large non-Gaussianity in two-field inflation. First, models with purely attractor solutions (meaning $M_{\perp\perp} \geq 0$ during all of inflation) and that satisfy $\epsilon \geq \epsilon_*$ in the super-horizon regime cannot produce large non-Gaussianity. Second, even in models without

purely attractor solutions, the potential must be fine-tuned enough to possess a steep ridge, while the initial conditions must be fine-tuned enough that the inflaton rolls along the ridge for a significant length of time during inflation, but also slightly turns. Nonetheless, a few two-field scenarios that produce large non-Gaussianity have been identified. For example, Byrnes *et. al.* [48] showed that product potentials where one of the two fields dominates the inflationary dynamics (meaning that the kinetic energy of one field is much larger than the kinetic energy of the other) during all of inflation, but the subdominant field picks up speed logarithmically faster than the dominant field produce large $|f_{NL}|$. Given the results in

this section, we suggest focusing future searches for large non-Gaussianity on models in which the entropy mass is negative during the super-horizon limit.

V. TRISPECTRUM

Now we calculate the local form of the trispectrum. The local trispectrum can be expressed in terms of two dimensionless non-linear parameters, τ_{NL} and g_{NL} :

$$T_{\mathcal{R}} = \tau_{NL} [\mathcal{P}_{\mathcal{R}}(|\mathbf{k}_1 + \mathbf{k}_3|) \mathcal{P}_{\mathcal{R}}(k_2) \mathcal{P}_{\mathcal{R}}(k_4) + 11 \text{ perms}] + \frac{54}{25} g_{NL} [\mathcal{P}_{\mathcal{R}}(k_2) \mathcal{P}_{\mathcal{R}}(k_3) \mathcal{P}_{\mathcal{R}}(k_4) + 3 \text{ perms}]. \quad (95)$$

Under the δN formalism, these parameters can be written as [91, 92]

$$\tau_{NL} = \frac{\mathbf{e}_N^T \nabla^T \nabla N \nabla^T \nabla N \mathbf{e}_N}{|\nabla N|^4}, \quad (96)$$

$$\frac{54}{25} g_{NL} = \frac{\mathbf{e}_N^T \nabla^T \nabla \nabla N \mathbf{e}_N \mathbf{e}_N}{|\nabla N|^3}.$$

Since the expression for g_{NL} is less illuminating and cannot be written exclusively in terms of other observ-

ables, we focus on τ_{NL} . Equation (96) for τ_{NL} is equivalent to

$$\tau_{NL} = \left\| \frac{\nabla^T \nabla N \mathbf{e}_N}{|\nabla N|^2} \right\|^2. \quad (97)$$

Using equation (97) and the fact that we already calculated the vector $\frac{\nabla^T \nabla N \mathbf{e}_N}{|\nabla N|^2}$ in equation (54), we can quickly arrive at the answer. Dividing equation (56) by $\cos \Delta_N$, we obtain the \mathbf{e}_{\parallel}^* component of $\frac{\nabla^T \nabla N \mathbf{e}_N}{|\nabla N|^2}$:

$$\frac{(\mathbf{e}_{\parallel}^*)^T \nabla^T \nabla N \mathbf{e}_N}{|\nabla N|^2} = \frac{1}{2} \cos \Delta_N (n_{\mathcal{R}} - n_T). \quad (98)$$

Similarly, dividing equation (57) by $\sin \Delta_N$ gives us the \mathbf{e}_{\perp}^* component:

$$\frac{(\mathbf{e}_{\perp}^*)^T \nabla^T \nabla N \mathbf{e}_N}{|\nabla N|^2} = \cos \Delta_N \times \left[M_{\parallel\perp}^* + \sin \Delta_N \cos \Delta_N \sqrt{2\epsilon_*} \mathbf{e}_{\perp}^* \cdot \nabla_* T_{\mathcal{R}\mathcal{S}} \right]. \quad (99)$$

Substituting the above two equations into equation (97), we find that

$$\tau_{NL} = \frac{1}{4} \cos^2 \Delta_N (n_{\mathcal{R}} - n_T)^2 + \cos^2 \Delta_N \left[M_{\parallel\perp}^* + \sin \Delta_N \cos \Delta_N \sqrt{-n_T} \mathbf{e}_{\perp}^* \cdot \nabla T_{\mathcal{R}\mathcal{S}} \right]^2. \quad (100)$$

Using equations (37), (40), and (58), we can write τ_{NL} completely in terms of observables, giving the following

consistency condition:

$$\tau_{NL} = \frac{1}{4} (1 - r_C^2) (n_{\mathcal{R}} - n_T)^2 + \frac{1}{r_C^2} \left[\frac{6}{5} f_{NL} + \frac{1}{2} (1 - r_C^2) (n_{\mathcal{R}} - n_T) \right]^2, \quad (101)$$

where recall that r_C is the curvature-isocurvature correlation. This gives us a new consistency relation that is unique to two-field inflation and that relates the observables τ_{NL} , f_{NL} , $n_{\mathcal{R}}$, n_T , and r_C .

Examining equation (100), we find that τ_{NL} cannot be large unless $\sqrt{-n_T} \mathbf{e}_{\perp}^* \cdot \nabla T_{\mathcal{R}\mathcal{S}}$ is large in magnitude. So like for $|f_{NL}|$, τ_{NL} cannot be large unless the inflaton trajectory lies along an instability in the potential that causes $T_{\mathcal{R}\mathcal{S}}$ to vary dramatically among neighboring trajectories. Equivalently, by equation (101), τ_{NL} can only be large if $\left(\frac{f_{NL}}{r_C}\right)^2$ is large. Previously, Suyama and Ya-

maguchi [70] showed that

$$\tau_{NL} \geq \left(\frac{6}{5} f_{NL}\right)^2, \quad (102)$$

proving that τ_{NL} will be large whenever f_{NL}^2 is large. But this result still left open the question of whether it is possible for τ_{NL} to be large if f_{NL}^2 is not. This question has since been answered affirmatively for particular models (e.g., [64, 71]). Above we show that it is more generally possible for τ_{NL} to be large even if f_{NL}^2 is not, but only if $\sqrt{-n_T} \mathbf{e}_{\perp}^* \cdot \nabla T_{\mathcal{R}\mathcal{S}}$ is large in magnitude and $r_C^2 \ll 1$. As $r_C = \sin \Delta_N$, this means that the combi-

nation of large τ_{NL} and small f_{NL}^2 can only appear in two-field models where the sourcing effects are weak — that is, when $T_{\mathcal{RS}}$ is much smaller than 1.

Now we consider the limit where $|f_{NL}| \gtrsim 1$. In this limit, equation (101) reduces to

$$\tau_{NL} \approx \frac{1}{r_C^2} \left(\frac{6}{5} f_{NL} \right)^2, \quad (103)$$

and how much τ_{NL} exceeds the Suyama-Yamaguchi bound depends only on r_C . While we might naively expect that making r_C as small as possible would maximize the value of τ_{NL} relative to f_{NL}^2 , this is not necessarily the case. This is because there is a trade-off: scenarios in which the multi-field effects are very small ($r_C \ll 1$) behave in many ways like single-field models and hence they are likely to produce small values for $|f_{NL}|$. Therefore, to further reduce r_C while preserving larger values for $|f_{NL}|$ typically comes at the expense of even more

fine-tuning. Therefore, for detectable non-Gaussianity without excessive fine-tuning, we expect that most models will produce a value for τ_{NL} that is no more than one to two orders of magnitude greater than f_{NL}^2 .

We conclude this section by finding an expression for g_{NL} . However, the result can be expressed only partially in terms of observables. Below, we cast the result in the most simple and transparent way. Starting by operating $\frac{1}{|\nabla N|} \mathbf{e}_N \cdot \nabla$ on the expression for f_{NL} , and then using the definitions of the non-linear parameters and that

$$\begin{aligned} \tau_{NL} &= \frac{\mathbf{e}_N^T \nabla^T (|\nabla N| \mathbf{e}_N^T) \nabla^T \nabla N \mathbf{e}_N}{|\nabla N|^4}, \\ &= \left(\frac{\mathbf{e}_N^T \nabla^T \nabla N \mathbf{e}_N}{|\nabla N|^2} \right)^2 + \frac{\mathbf{e}_N^T (\nabla^T \mathbf{e}_N^T) \nabla^T \nabla N \mathbf{e}_N}{|\nabla N|^3}, \end{aligned} \quad (104)$$

g_{NL} can be written as

$$\boxed{\frac{54}{25} g_{NL} = -2\tau_{NL} + 4 \left(\frac{6}{5} f_{NL} \right)^2 + \sqrt{\frac{r_T}{8}} \mathbf{e}_N \cdot \nabla \left(-\frac{6}{5} f_{NL} \right)}. \quad (105)$$

The last term in equation (105) is equivalent to

$$\sqrt{\frac{r_T}{8}} \mathbf{e}_N \cdot \nabla \left(-\frac{6}{5} f_{NL} \right) = (1 - r_C^2) \left(-\frac{6}{5} f_{NL} \right)' + r_C \sqrt{-n_T (1 - r_C^2)} \mathbf{e}_\perp^* \cdot \nabla \left(-\frac{6}{5} f_{NL} \right), \quad (106)$$

where $\frac{d}{dN} \approx \frac{d}{d \ln k}$. So the last term in equation (105) is a linear combination of the scale-dependence of f_{NL} and of the sensitivity of f_{NL} to changes in the initial conditions orthogonal to the trajectory. (For an entry point into the literature on the scale-dependence of local non-Gaussianity, we refer the interested reader to [93–95].) Taken together, this means that g_{NL} can be large in magnitude only if τ_{NL} is large, f_{NL}^2 is large, and/or if f_{NL} varies dramatically in a small neighborhood about the initial conditions. Therefore, if g_{NL} is large, but neither τ_{NL} and f_{NL} are, then it means that f_{NL} has very strong scale-dependence and/or that the inflaton trajectory is near neighboring trajectories that do produce large f_{NL} .

Finally, we remark that equations (101) (or (103)) and (105) are important because the relationships among the observables f_{NL} , τ_{NL} , and g_{NL} will help us probe the model-dependent nature of inflation. Different scenarios can produce widely different values for these three parameters, which provides another way to test inflationary models. There has already been some interesting work

to classify models by the relationship among f_{NL} , τ_{NL} , and g_{NL} , in particular by Suyama *et al.* [96]. Our work adds to this by showing that it is the degree of sourcing ($T_{\mathcal{RS}}$) that determines the relationship between f_{NL} and τ_{NL} , thereby demystifying what controls the Suyama-Yamaguchi bound in two-field inflation. Also, we have shown that while the relationship among all three parameters is complicated, comparing the values of the parameters with equation (105) will give us some further insight into the nature of inflation.

VI. CONCLUSIONS

In this paper, we considered the local form of the bispectrum and trispectrum in general two-field inflation. We found semi-analytic expressions for f_{NL} , τ_{NL} , and g_{NL} , and we developed a set of novel conditions encapsulating when non-Gaussianity is large. To do so, we worked within the δN formalism, which expresses the

bispectrum and trispectrum in terms of gradients of N , where N is the number of e -folds of inflation. To perform the calculation, we invoked the slow-roll and slow-turn approximations, and we used a unified kinematical framework and the transfer matrix formalism.

We showed that f_{NL} can be written in terms of sines and cosines (related to the degree of sourcing) times $n_{\mathcal{R}} = 1 - n_s$, n_T , the turn rate at horizon exit, and $\mathbf{e}_{\perp}^* \cdot \nabla_* T_{\mathcal{RS}}$, where $T_{\mathcal{RS}}$ is the transfer function that encodes the relative degree of sourcing of curvature modes by isocurvature modes. As the magnitudes of all quantities but the term $\sin^2 \Delta_N \cos^2 \Delta_N \sqrt{-n_T} \mathbf{e}_{\perp}^* \cdot \nabla_* T_{\mathcal{RS}}$ are constrained to be less than unity, $|f_{NL}|$ can only be large when (1) $T_{\mathcal{RS}}$ is extremely sensitive to a change in initial conditions orthogonal to the inflaton trajectory and (2) the total sourcing is non-zero. The former condition makes sense on an intuitive level, as to produce a large amount of skew in the primordial perturbations, fluctuations off the classical inflaton trajectory must result in very different inflationary dynamics for the field perturbations. Now since $T_{\mathcal{RS}}$ is an integral of the turn rate and the relative amplitude of isocurvature modes (T_{SS}), the former condition implies that neighboring trajectories must have dramatically different turn rate profiles, T_{SS} profiles, or both. Though we only presented proofs of these conditions for two-field inflation, similar conditions hold for multi-field inflation as well. Moreover, one of the important benefits of our expression for f_{NL} is that it can be applied either directly or with modifications to other scenarios described by the transfer function formalism.

Next, we found an expression for $\mathbf{e}_{\perp}^* \cdot \nabla_* T_{\mathcal{RS}}$ for four analytically solvable scenarios, and we outlined a prescription to try in the more general case when T_{SS} is analytic. We found that the result depends on the entropy mass, the isocurvature transfer function T_{SS} , and a model-dependent correction γ , which quantifies the coupling between the fields. Invoking minimal assumptions about the terms in equation (70), we showed that for $\sqrt{-n_T} \mathbf{e}_{\perp}^* \cdot \nabla_* T_{\mathcal{RS}}$ to be large requires that the relative amplitude of isocurvature modes (T_{SS}) at the end of inflation be large and/or that T_{SS} be very sensitive to changes in the initial conditions perpendicular to the inflaton trajectory.

We then further explored the conditions for large non-Gaussianity in general two-field inflation. After proving an upper bound for $T_{\mathcal{RS}}$ in the case where $T_{SS} \leq 1$ during all of inflation, we argued that if neighboring trajectories do not diverge, then due to constraints on higher-order SRST parameters, the amount of sourcing cannot vary dramatically among neighboring trajectories and hence non-Gaussianity cannot be large. Therefore, either $T_{SS} > 1$ or $M_{\perp\perp} < 0$ sometime during inflation. Under the assumption that $\epsilon \geq \epsilon_*$ in the super-horizon regime, the first condition implies the second one, and hence we require that $M_{\perp\perp} < 0$ during at least part of inflation. Geometrically, this means that $|f_{NL}|$ will be large only if the inflaton traverses along a ridge in the

inflationary potential at some point during inflation and the inflaton trajectory turns at least slightly.

Unfortunately, though, this implies that some fine-tuning of the potential and/or the initial conditions is usually needed both to produce a steep enough ridge and/or to situate the inflaton on top of the ridge without it falling off too quickly and yet still slightly turning. Inflationary scenarios that are attractor solutions and that satisfy $\epsilon \geq \epsilon_*$ therefore cannot produce large $|f_{NL}|$. This explains why it has been so difficult to achieve large non-Gaussianity in two-field inflation. Moreover, it explains why large non-Gaussianity arises in models such as axionic \mathcal{N} -flation and tachyonic (p)reheating. The common denominator of these models is a significant negative curvature (mass) along the entropic direction that produces the requisite instability.

Finally, we showed that the calculations of τ_{NL} and g_{NL} are very similar to that of f_{NL} . τ_{NL} can be written entirely in terms of the spectral observables f_{NL} , $n_{\mathcal{R}}$, n_T , and r_C , where r_C is the dimensionless curvature-isocurvature correlation. This provides a new consistency relation unique to two-field inflation. Like for f_{NL} , we found that τ_{NL} cannot be large unless $T_{\mathcal{RS}}$ varies dramatically among neighboring trajectories. In addition, we shed new light on the Suyama-Yamaguchi bound $\tau_{NL} \geq \left(\frac{6}{5}f_{NL}\right)^2$, showing that for $|f_{NL}| \gtrsim 1$, $\tau_{NL} = \left(\frac{6f_{NL}/5}{r_C}\right)^2$. Thus, it is the amount of sourcing, $T_{\mathcal{RS}}$, that controls the Suyama-Yamaguchi bound. We also calculated the trispectrum parameter g_{NL} and showed that it can only be large in magnitude if τ_{NL} is large, f_{NL}^2 is large, f_{NL} has strong scale-dependence, or f_{NL} varies dramatically among neighboring trajectories.

Our results for the local bispectrum and trispectrum from inflation allow us to better test and constrain two-field models of inflation using observational data. Our results also provide better guidance for model-builders seeking to find inflationary models with large non-Gaussianity. In the future, it will be interesting to explore the range of shapes of ridges that give rise to large non-Gaussianity and to better understand the degree of fine-tuning needed in the potential and/or initial conditions. Finally, it is important to better understand the model-dependent nature of (p)reheating and the aftermath of inflation, to understand how they affect the primordial non-Gaussianity from inflation.

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- [1] A. H. Guth, *Phys. Rev. D* **23**, 347 (1981).
- [2] A. D. Linde, *Particle Physics and Inflationary Cosmology* (Harwood: Switzerland, 1990).
- [3] D. H. Lyth and A. Riotto, *Phys. Rep.* **314**, 1 (1999).
- [4] A. R. Liddle and D. H. Lyth, *Cosmological Inflation and Large-Scale Structure* (Cambridge Univ. Press: Cambridge, 2000).
- [5] B. A. Bassett, S. Tsujikawa, and D. Wands, *Rev. Mod. Phys.* **78**, 537 (2006).
- [6] V. F. Mukhanov and G. V. Chibisov, *JETP Lett.* **33**, 532 (1981).
- [7] V. F. Mukhanov and G. V. Chibisov, *Sov. Phys. JETP* **56**, 258 (1982).
- [8] S. W. Hawking, *Phys. Lett. B* **115**, 295 (1982).
- [9] A. A. Starobinsky, *Phys. Lett. B* **117**, 175 (1982).
- [10] A. H. Guth and S.-Y. Pi, *Phys. Rev. Lett.* **49**, 1110 (1982).
- [11] J. M. Bardeen, P. J. Steinhardt, and M. S. Turner, *Phys. Rev. D* **28**, 679 (1983).
- [12] J. Maldacena, *J. High Energy Phys.* 05 (2003) 013.
- [13] D. Babich, P. Creminelli, and M. Zaldarriaga, *J. Cosmol. Astropart. Phys.* 08 (2004) 009.
- [14] J. R. Fergusson and E. P. S. Shellard, *Phys. Rev. D* **80**, 043510 (2009).
- [15] E. Komatsu *et al.*, *Astrophys. J. Suppl.* **192**, 18 (2011).
- [16] E. Komatsu and D. N. Spergel, *Phys. Rev. D* **63**, 063002 (2001).
- [17] L. Verde, L.-M. Wang, A. Heavens, and M. Kamionkowski, *Mon. Not. Roy. Astron. Soc.* **313**, 141 (2000).
- [18] J. Smidt, A. Amblard, A. Cooray, A. Heavens, D. Munshi, and P. Serra, arXiv:1001.5026.
- [19] J. R. Fergusson, D. M. Regan, and E. P. S. Shellard, arXiv:1012.6039.
- [20] T. J. Allen, B. Grinstein and M. B. Wise, *Phys. Lett. B* **197**, 66 (1987).
- [21] A. Gangui, F. Lucchin, S. Matarrese and S. Mollerach, *Astrophys. J.* **430**, 447 (1994).
- [22] L. Wang and M. Kamionkowski, *Phys. Rev. D* **61**, 063504 (2000).
- [23] V. Acquaviva, N. Bartolo, S. Matarrese, and A. Riotto, *Nucl. Phys. B* **667** 119 (2003).
- [24] P. Creminelli, *J. Cosmol. Astropart. Phys.* 10 (2003) 003.
- [25] A. Gruzinov, *Phys. Rev. D* **71**, 027301 (2005).
- [26] P. Creminelli and M. Zaldarriaga, *J. Cosmol. Astropart. Phys.* 10 (2004) 006.
- [27] D. Seery and J. E. Lidsey, *J. Cosmol. Astropart. Phys.* 06 (2005) 003.
- [28] T. Okamoto and W. Hu, *Phys. Rev. D* **66**, 063008 (2002).
- [29] D. Seery, J. E. Lidsey, and M. S. Sloth, *J. Cosmol. Astropart. Phys.* 01 (2007) 027.
- [30] D. Seery and J. E. Lidsey, *J. Cosmol. Astropart. Phys.* 01 (2007) 008.
- [31] G. I. Rigopoulos, E. P. S. Shellard, and B. J. W. van Tent, *Phys. Rev. D* **73**, 083522 (2006).
- [32] G. I. Rigopoulos, E. P. S. Shellard, and B. J. W. van Tent, *Phys. Rev. D* **76**, 083512 (2007).
- [33] S. A. Kim and A. R. Liddle, *Phys. Rev. D* **74**, 063522 (2006).
- [34] T. Battefeld and R. Easther, *J. Cosmol. Astropart. Phys.* 03 (2007) 020.
- [35] D. Battefeld and T. Battefeld, *J. Cosmol. Astropart. Phys.* 05 (2007) 012.
- [36] S. Yokoyama, T. Suyama, and T. Tanaka, *J. Cosmol. Astropart. Phys.* 07 (2007) 013.
- [37] S. Yokoyama, T. Suyama, and T. Tanaka, *Phys. Rev. D* **77**, 083511 (2008).
- [38] A. Misra and P. Shukla, *Nucl. Phys. B* **810**, 174 (2009).
- [39] Q.-G. Huang, *J. Cosmol. Astropart. Phys.* 06 (2009) 035.
- [40] C. T. Byrnes and K.-Y. Choi, *Adv. Astron.* **2010**, 724525 (2010).
- [41] T. Tanaka, T. Suyama, and S. Yokoyama, *Class. Quant. Grav.* **27**, 124003 (2010).
- [42] S. A. Kim, A. R. Liddle, and D. Seery, *Phys. Rev. Lett.* **105**, 181302 (2010).
- [43] N. Bartolo, S. Matarrese, and A. Riotto, *Phys. Rev. D* **65**, 103505 (2002).
- [44] F. Bernardeau and J.-P. Uzan, *Phys. Rev. D* **66**, 103506 (2002).
- [45] F. Bernardeau and J.-P. Uzan, *Phys. Rev. D* **67**, 121301 (2003).
- [46] K.-Y. Choi, L. M. H. Hall, and C. van de Bruck, *J. Cosmol. Astropart. Phys.* 02 (2007) 029.
- [47] F. Vernizzi and D. Wands, *J. Cosmol. Astropart. Phys.* 05 (2006) 019.
- [48] C. T. Byrnes, K.-Y. Choi, and L. M. H. Hall, *J. Cosmol. Astropart. Phys.* 10 (2008) 008.
- [49] A. C. Vincent and J. M. Cline, *J. High Energy Phys.* 10 (2008) 093.
- [50] T. Wang, *Phys. Rev. D* **82**, 123515 (2010).
- [51] J. Meyers and N. Sivanandam, arXiv:1011.4934.
- [52] A. D. Linde and V. F. Mukhanov, *Phys. Rev. D* **56**, 535 (1997).
- [53] D. H. Lyth, C. Ungarelli, and D. Wands, *Phys. Rev. D* **67**, 023503 (2003).
- [54] N. Bartolo, S. Matarrese, and A. Riotto, *Phys. Rev. D* **69**, 043503 (2004).
- [55] A. D. Linde and V. F. Mukhanov, *J. Cosmol. Astropart. Phys.* 04 (2006) 009.
- [56] K. A. Malik and D. H. Lyth, *J. Cosmol. Astropart. Phys.* 09 (2006) 008.
- [57] M. Sasaki, J. Valiviita, and D. Wands, *Phys. Rev. D* **74**, 103003 (2006).
- [58] K. Ichikawa, T. Suyama, T. Takahashi, and M. Yamaguchi, *Phys. Rev. D* **78**, 023513 (2008).
- [59] K. Enqvist and A. Vaihkonen, *J. Cosmol. Astropart. Phys.* 09 (2004) 006.
- [60] L. Alabidi and D. Lyth, *J. Cosmol. Astropart. Phys.* 08 (2006) 006.
- [61] L. Alabidi, *J. Cosmol. Astropart. Phys.* 10 (2006) 015.
- [62] N. Barnaby and J. M. Cline, *Phys. Rev. D* **73**, 106012 (2006).
- [63] N. Barnaby and J. M. Cline, *Phys. Rev. D* **75**, 086004 (2007).
- [64] C. T. Byrnes, K.-Y. Choi, and L. M. H. Hall, *J. Cosmol. Astropart. Phys.* 02 (2009) 017.
- [65] M. Sasaki, *Prog. Theor. Phys.* **120**, 159 (2008).
- [66] A. Naruko and M. Sasaki, *Prog. Theor. Phys.* **121**, 193 (2009).
- [67] Q.-G. Huang, *J. Cosmol. Astropart. Phys.* 05 (2009) 005.
- [68] G. Dvali, A. Gruzinov, and M. Zaldarriaga, *Phys. Rev. D* **69**, 083505 (2004).

- [69] M. Zaldarriaga, *Phys. Rev. D* **69**, 043508 (2004).
- [70] T. Suyama and M. Yamaguchi, *Phys. Rev. D* **77**, 023505 (2008).
- [71] K. Ichikawa, T. Suyama, T. Takahashi, and M. Yamaguchi, *Phys. Rev. D* **78**, 063545 (2008).
- [72] K. Enqvist, A. Jokinen, A. Mazumdar, T. Multamaki, and A. Vaihkonen, *Phys. Rev. Lett.* **94**, 161301 (2005).
- [73] K. Enqvist, A. Jokinen, A. Mazumdar, T. Multamaki, and A. Vaihkonen, *J. Cosmol. Astropart. Phys.* 03 (2005) 010.
- [74] A. Jokinen and A. Mazumdar, *J. Cosmol. Astropart. Phys.* 04 (2006) 003.
- [75] H. R. S. Cogollo, Y. Rodriguez and C. A. Valenzuela-Toledo, *J. Cosmol. Astropart. Phys.* 08 (2008) 029.
- [76] Y. Rodriguez and C. A. Valenzuela-Toledo, *Phys. Rev. D* **81**, 023531 (2010).
- [77] C. M. Peterson and M. Tegmark, *Phys. Rev. D* **83**, 023522 (2011).
- [78] C. Gordon, D. Wands, B. A. Bassett, and R. Maartens, *Phys. Rev. D* **63**, 023506 (2001).
- [79] S. Groot Nibbelink and B. J. W. van Tent, arXiv:hep-ph/0011325 (2000).
- [80] S. Groot Nibbelink and B. J. W. van Tent, *Class. Quant. Grav.* **19**, 613 (2002).
- [81] M. Sasaki, *Prog. Theor. Phys.* **76**, 1036 (1986).
- [82] V. F. Mukhanov, *Soviet Phys. JETP* **68**, 1297 (1988).
- [83] D. Wands, N. Bartolo, S. Matarrese, and A. Riotto, *Phys. Rev. D* **66**, 043520 (2002).
- [84] L. Amendola, C. Gordon, D. Wands, and M. Sasaki, *Phys. Rev. Lett.* **88**, 211302 (2002).
- [85] A. A. Starobinsky, *JETP Lett.* **42**, 152 (1985).
- [86] M. Sasaki and E. D. Stewart, *Prog. Theor. Phys.* **95**, 71 (1996).
- [87] D. H. Lyth, K. A. Malik and M. Sasaki, *J. Cosmol. Astropart. Phys.* 05 (2005) 004.
- [88] D. H. Lyth and Y. Rodriguez, *Phys. Rev. Lett.* **95**, 121302 (2005).
- [89] J. Garcia-Bellido and D. Wands, *PRD*, **53**, 5437 (1996)
- [90] V. F. Mukhanov and P. J. Steinhardt, *Phys. Lett. B* **422**, 52 (1998).
- [91] L. Alabidi and D. Lyth, *J. Cosmol. Astropart. Phys.* 05 (2006) 016.
- [92] C. T. Byrnes, M. Sasaki, and D. Wands, *Phys. Rev. D* **74**, 123519 (2006).
- [93] C. T. Byrnes, M. Gerstenlauer, S. Nurmi, G. Tasinato, and D. Wands, *J. Cosmol. Astropart. Phys.* 10 (2010) 004.
- [94] C. T. Byrnes, K. Enqvist, and T. Takahashi, *J. Cosmol. Astropart. Phys.* 09 (2010) 026.
- [95] A. Becker, D. Huterer, and K. Kadota, *J. Cosmol. Astropart. Phys.* 01 (2011) 006.
- [96] T. Suyama, T. Takahashi, M. Yamaguchi, and S. Yokoyama, arXiv:1009.1979.