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CMB acoustic scale in the entropic-like accelerating universe

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We consider generalizations of the entropic accelerating universe recently proposed in Ref. [4, 5] and show that their background equations can be made equivalent to a model with a dark energy component with constant parameter of state $w_X = -1 + 2\gamma/3$, where γ is related to the coefficients of the new terms in the Friedman equations. After discussing all the Friedman equations for an arbitrary γ , we show how to recover the standard scalings for dust and radiation. The acoustic scale ℓ_A , related to the peak positions in the pattern of the angular power spectrum of the Cosmic Microwave Background anisotropies, is also computed and yields the stringent bound $|\gamma| \ll 1$. We then argue that future data might be able to distinguish this model from pure Λ CDM (corresponding to $\gamma = 0$).

PACS numbers:

I. INTRODUCTION

Modern theoretical cosmology includes an early period of accelerated expansion named *inflation* [1], whose driving force is commonly modeled using a scalar field (the *inflaton*) of uncertain nature. A similarly accelerated phase is undergoing now [2] and has led to conceive the existence of an equally unspecified *dark energy* component in the matter content of the Universe [3].

An alternative scenario has been recently proposed in Refs. [4, 5], based on the idea of entropic gravity introduced in Ref. [6]. In this context, the equations governing the time-evolution of the cosmic scale factor contain terms proportional to the Hubble function squared H^2 and its time derivative \dot{H} originating at the boundary of spatial sections of our universe. According to the authors of Refs. [4, 5], such terms could explain the acceleration occurring both in the early stages and at present. Boundary terms, whose nature is well-known in General Relativity [7], have indeed been analyzed in various contexts, for example in Refs. [8].

In this work, we will not analyze how these terms emerge from an action principle, nor if a unique Lagrangian can be defined at all. We shall instead assume general modifications of the form considered in Refs. [4, 5] and then try to constrain their possible effectiveness by comparing the corresponding Cosmic Microwave Background (CMB) acoustic scale (see, e.g., Refs. [9, 11]) with the most recent available WMAP data [12]. Note that a standard Monte-Carlo Markov Chain analysis (usually employed to extract the cosmological parameters by comparison with available observations) is not feasible if the model is unknown at the linear order. On the contrary, the CMB acoustic scale can be computed directly from

the background equations, and this will allow us to obtain a constraint for the free parameters of the model by comparing with the most recent CMB data.

The paper is organized as follows: in Section II we obtain the complete set of Friedman equations for a generalization of the entropic models introduced in Refs. [4, 5]. This, in Section III, will allow us to regard the model in terms of an effective dark energy contribution depending on one parameter γ . In particular, bounds on γ will be obtained by comparing the CMB acoustic scale with the 7yr WMAP data in Section III B, after computing the deceleration parameter in Section III A. Conclusions will be drawn in Section IV.

II. MODIFIED FRIEDMAN EQUATIONS

In the flat Friedman-Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t) d\vec{x} \cdot d\vec{x}, \quad (1)$$

with the scale factor $a(t)$ normalized so that $a(t_{\text{now}}) = 1$, the model of universe considered in Refs. [4, 5] features a Friedman equation given by

$$\frac{\ddot{a}}{a} = -\frac{4\pi\tilde{G}}{3} \sum_i (\tilde{\rho}_i + 3\tilde{p}_i) + C_H H^2 + C_{\dot{H}} \dot{H}, \quad (2)$$

where $H = \dot{a}/a \equiv a^{-1} da/dt$, \tilde{G} is the “bare” Newton constant, $\tilde{\rho}_i$ and \tilde{p}_i are the “bare” energy density and pressure of the i -th fluid filling the universe, while C_H and $C_{\dot{H}}$ are constants coming from the boundary terms. As already stated in the Introduction, we take such terms as given and do not derive them from the Einstein-Hilbert action on a manifold with boundaries. Instead, we shall determine the full set of cosmological (and continuity) equations consistent with Eq. (2) without *a priori* fixing C_H and $C_{\dot{H}}$.

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We first note that Eq. (2) can be rewritten as

$$\dot{H} + \gamma H^2 = -\frac{4\pi G}{3} \sum_i (\tilde{\rho}_i + 3\tilde{p}_i), \quad (3)$$

where

$$\gamma = \frac{1 - C_H}{1 - C_{\dot{H}}}, \quad (4)$$

and we rescaled ¹

$$G = \tilde{G}/(1 - C_{\dot{H}}). \quad (5)$$

Noting that $d/dt = (aH) d/da$ and assuming

$$\tilde{\rho}_i = \tilde{\rho}_i^{(0)} a^{-k_i}, \quad \tilde{p}_i = w_i \tilde{\rho}_i, \quad (6)$$

with $\tilde{\rho}_i^{(0)}$ and w_i constant, Eq. (3) can be integrated exactly and yields

$$H^2 = \frac{8\pi G}{3} \left(\sum_i c_i \tilde{\rho}_i + \frac{C}{a^{2\gamma}} \right), \quad (7)$$

where the coefficients

$$c_i = \frac{1 + 3w_i}{k_i - 2\gamma} \quad (8)$$

are well-defined only for $k_i \neq 2\gamma$ and C is a constant of integration. Further, on deriving Eq. (7) with respect to time and using Eqs. (3) and (8), we obtain the continuity equation

$$\dot{\tilde{\rho}}_i + \frac{H}{c_i} [(2\gamma c_i + 1) \tilde{\rho}_i + 3\tilde{p}_i] = \dot{\tilde{\rho}}_i + H k_i \tilde{\rho}_i = 0, \quad (9)$$

which is identically satisfied for the fluids (6). For example, for dust we have $w_{\text{dust}} = 0$ and requiring $k_{\text{dust}} = 3$ yields $c_{\text{dust}} = 1/(3 - 2\gamma)$. Likewise, radiation has $w_{\text{rad}} = 1/3$ and requiring $k_{\text{rad}} = 4$ results in $c_{\text{rad}} = 1/(2 - \gamma)$. Specifying these parameters and assuming that the matter content of the universe is a mixture of dust and radiation, the Friedman equations (3) and (7) can be rewritten as (see also Appendix A)

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left[\frac{\rho_{\text{dust}}^{(0)}}{a^3} + 2 \frac{\rho_{\text{rad}}^{(0)}}{a^4} - 2(1 - \gamma) \frac{C}{a^{2\gamma}} \right] \quad (10)$$

$$H^2 = \frac{8\pi G}{3} \left[\frac{\rho_{\text{dust}}^{(0)}}{a^3} + \frac{\rho_{\text{rad}}^{(0)}}{a^4} + \frac{C}{a^{2\gamma}} \right], \quad (11)$$

where $\rho_{\text{dust}}^{(0)} = c_{\text{dust}} \tilde{\rho}_{\text{dust}}^{(0)}$ and $\rho_{\text{rad}}^{(0)} = c_{\text{rad}} \tilde{\rho}_{\text{rad}}^{(0)}$ are the present matter and radiation densities involved in observations. Note that the (bare) densities $\tilde{\rho}_{\text{rad}}$ and $\tilde{\rho}_{\text{dust}}$

and the corresponding constant and dimensionless coefficients c_{rad} and c_{dust} are not observable separately, since only their products appear in the equations (as we remark in Appendix A).

Eqs. (10) and (11) are precisely the standard Friedman equations for a universe filled with dust and radiation that scale in the usual way, namely

$$\rho_{\text{dust}} = \rho_{\text{dust}}^{(0)}/a^3, \quad \rho_{\text{rad}} = \rho_{\text{rad}}^{(0)}/a^4, \quad (12)$$

corrected by terms proportional to C . Note also that, in the limit $\gamma \rightarrow 1$, we recover the standard cosmological equations (with no dark energy component!) with the C -term playing the role of an effective curvature contribution. However, for $\gamma \neq 1$, there is a region where the corrections can be interpreted as an effective dark energy component if $C > 0$. This is the case we consider in the following, with $\rho_{\text{dust}}^{(0)}$ and $\rho_{\text{rad}}^{(0)}$ equal to the present dust and radiation densities. For example, $\rho_{\text{rad}}^{(0)}$ is the present energy density of the black-body radiation with temperature $T = 2.725$ K (multiplied by the contribution from neutrinos).

III. EFFECTIVE DARK ENERGY

The next step is to find whether there exist values of γ corresponding to an accelerating universe, i.e., such that $\ddot{a} > 0$. This can be understood analyzing Eq. (10). The effective dark energy term proportional to C will then drive the present acceleration of the universe if it dominates in the r.h.s. of Eq. (10) at recent times. We therefore require that $\gamma < 3/2$. Moreover, since we also assume $C > 0$, $\ddot{a} > 0$ implies

$$\gamma < 1. \quad (13)$$

Hence, when Eq. (13) is satisfied, the C -term mimics the behavior of a dark energy fluid [see Eq. (11)] with constant parameter of state $w_X = -1 + 2\gamma/3$.

A. Deceleration parameter

The deceleration parameter is defined as

$$q = -\frac{\ddot{a}}{a H^2}. \quad (14)$$

Plugging Eqs. (10) and (11) into the above definition and neglecting radiation ² yields

$$q = \frac{1}{2a} \left(\frac{\Omega_C a^{-3} - 2(1 - \gamma) \Omega_\Lambda a^{-2\gamma}}{\Omega_C a^{-3} + \Omega_\Lambda a^{-2\gamma}} \right), \quad (15)$$

¹ This rescaling [17] and Eq. (4) are meaningful only if $C_{\dot{H}} \neq 1$, a condition we assume throughout the paper. If $C_{\dot{H}} = 1$, Eq. (3) does not contain \dot{H} and is therefore not an equation of motion but a constraint.

² Of course, this approximation is valid for recent cosmological times (like in Fig. 1), when the transition from matter to dark energy dominated epochs was taking place and the contribution of radiation was subleading.

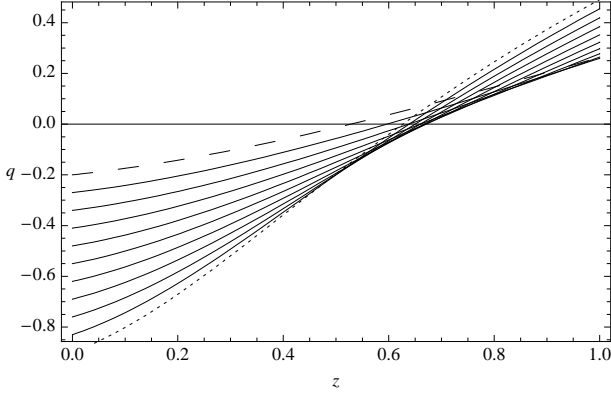


FIG. 1: q vs z for γ from -0.5 to 0.5 with step equal to 0.1 . Dotted line is for $\gamma = -0.5$, dashed line for $\gamma = 0.5$ and solid lines for values in between.

where $\Omega_C = \rho_{\text{dust}}^{(0)}/\rho_c$, $\Omega_\Lambda = C/\rho_c$ and $\rho_c \equiv 3H_0^2/(8\pi G)$ with H_0 the present value of the Hubble function.

In Fig. 1, we show q as a function of the redshift z for various values of γ from -0.5 to 0.5 in steps of 0.1 . Note that, on specializing q at present time, we find

$$q = \frac{1}{2} \Omega_C - (1 - \gamma) \Omega_\Lambda, \quad (16)$$

which turns out to be the standard expression for the Λ CDM model when $\gamma = 0$ [11, 13, 14].

B. CMB acoustic scale

The characteristic angular scale θ_A of the peaks of the angular power spectrum in CMB anisotropies is defined as [9]

$$\theta_A = \frac{r_s(z_{\text{dec}})}{r(z_{\text{dec}})} = \frac{\pi}{\ell_A}, \quad (17)$$

where $r_s(z_{\text{dec}})$ is the comoving size of the sound horizon at decoupling, $r(z_{\text{dec}})$ the comoving distance at decoupling and ℓ_A the multipole associated with the angular scale θ_A , also called the *acoustic scale*. Let us recall that ℓ_A is not exactly the scale of the first peak. In general, the position of the m -th peak is given by $\ell_m = (m - \phi_m) \ell_A$ where ϕ_m is a phase that depends on other cosmological parameters [9].

In order to make explicit the dependence of ℓ_A on the cosmological parameters, we now consider separately numerator and denominator of Eq. (17). The comoving size of the sound horizon at decoupling can be written as [10]

$$r_s(z_{\text{dec}}) = \frac{4}{3H_0} \sqrt{\frac{\Omega_\gamma}{\Omega_C \Omega_b}} \times \ln \left[\frac{\sqrt{1 + R_{\text{dec}}} + \sqrt{R_{\text{dec}} + R_{\text{eq}}}}{1 + \sqrt{R_{\text{eq}}}} \right], \quad (18)$$

with $R(z) = 3(\Omega_b/(4\Omega_\gamma))/(1+z)$ and where Ω_b and Ω_γ are the present density ratios for baryons and photons respectively [note the index γ in Ω_γ must not to be confused with the parameter γ defined in Eq. (4)]. Moreover, the label “dec” stands for “computed at decoupling”, while “eq” stands for “computed at equivalence” (between radiation and matter). By definition the comoving distance at decoupling reads

$$r(z_{\text{dec}}) = \int_0^{z_{\text{dec}}} \frac{dz'}{H(z')}, \quad (19)$$

where $H(z)$ is given by Eq. (11) and can be recast as

$$H(z) = H_0 [(1+z)^3 \Omega_C + (1+z)^4 \Omega_{\text{rad}} + (1+z)^{2\gamma} \Omega_\Lambda]^{1/2}, \quad (20)$$

where $\Omega_{\text{rad}} = \rho_{\text{rad}}^{(0)}/\rho_c$. We can therefore write the acoustic scale ℓ_A as

$$\ell_A = \frac{3\pi}{4} \sqrt{\frac{\Omega_b}{\Omega_\gamma}} \frac{\int_0^{z_{\text{dec}}} [(1+z)^3 + (1+z)^4 (\Omega_{\text{rad}}/\Omega_C) + (1+z)^{2\gamma} (\Omega_\Lambda/\Omega_C)]^{-1/2} dz}{\ln [\sqrt{1 + R_{\text{dec}}} + \sqrt{R_{\text{dec}} + R_{\text{eq}}}] - \ln [1 + \sqrt{R_{\text{eq}}}]}. \quad (21)$$

Let us remark that Eq. (21) was obtained by neglecting Ω_Λ in $r_s(z_{\text{dec}})$ (the comoving size of the sound horizon at decoupling). However, it was shown in Ref. [11] that this approximation at most leads to $10^{-5}\%$ error, much smaller than the precision of our result below [see Eq. (23)].

Eq. (21) can now be used to constrain the models under study by comparing with the value obtained from the

recent 7yr WMAP data [12]³

$$\ell_A^{\text{WMAP}} = 302.44 \pm 0.8. \quad (22)$$

Note that our choices for $\rho_{\text{dust}}^{(0)}$ and $\rho_{\text{rad}}^{(0)}$ were made in order to minimize deviations from the Λ CDM model. In fact, departures of the background equations (10) and

³ <http://lambda.gsfc.nasa.gov/>

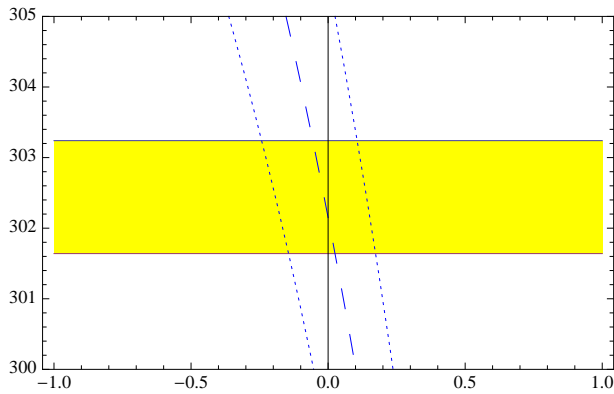


FIG. 2: Acoustic scale for entropic universe as function of γ . Horizontal lines represent $1\text{-}\sigma$ WMAP measurement (colored region displays $1\text{-}\sigma$ contour). Long dashed line is for ℓ_A computed at best fit of $z_{\text{eq}} = 3196$. Short dashed lines are for ℓ_A computed at $1\text{-}\sigma$ level of $z_{\text{eq}} = 3196^{+134}_{-133}$.

(11) from ΛCDM are completely parameterized by the single parameter γ and we are therefore allowed to estimate Eq. (21) with the values of the other parameters that best fit WMAP data. We insert in Eq. (21) the 7yr WMAP best fit values [12] $\Omega_b = 0.0449$, $\Omega_\gamma = 4.89 \times 10^{-5}$, $z_{\text{dec}} = 1088.2$, $z_{\text{eq}} = 3196$, $\Omega_C = 0.266$, $\Omega_\Lambda = 0.734$ and $\Omega_{\text{rad}} = 1.69\Omega_\gamma$. In Fig. 2, we show the acoustic scale (long and short dashed lines) versus γ , along with the $1\text{-}\sigma$ levels of the WMAP measurement (solid horizontal lines). We also display the dependence of the acoustic scale on z_{eq} : the long dashed line stands for the 7yr WMAP best fit ($z_{\text{eq}} = 3196$) whereas the short dashed lines stand for its $1\text{-}\sigma$ values, $z_{\text{eq}} = 3196^{+134}_{-133}$.

As Fig. 2 shows clearly, the parameter γ must be very close to 0 in order to have consistency with the WMAP observations. More precisely, from Eq. (21) computed at the best fit values of the 7yr WMAP parameters⁴, we obtain

$$\gamma = -0.02 \pm 0.04. \quad (23)$$

This result is consistent with Eq. (13) and implies $-1.040 < w_X < -0.986$, so that the added contributions must closely mimic a cosmological constant. Further, from Eq. (4), this implies that

$$|1 - C_H| \ll |1 - C_{\dot{H}}| \quad (24)$$

in Eq. (2). For example, if $0 < C_H, C_{\dot{H}} < 1$, then the strong inequality (24) is satisfied for $C_H \approx 1$.

⁴ This means taking into account only the long dashed line in Fig. 2.

IV. CONCLUSIONS

In the present paper, we have shown how it is possible to recover standard background scalings for radiation and matter and standard effective cosmological equations [see Eqs. (10) and (11)] when the Friedman equation for \ddot{a} is modified by adding terms proportional to H^2 and \dot{H} like in Eq. (2). An example that requires such a modification is given by the entropic accelerating universe of Refs. [4, 5], although our considerations are more general. Moreover we have shown how to obtain the recent cosmological acceleration within the considered model, without adding a dark energy fluid. We note that for the range of parameters considered here, the model under analysis does not modify the evolution of the universe when it was matter or radiation dominated. Therefore, none of the standard cosmological constraints coming from such early epochs, as for instance the Big Bang Nucleosynthesis (BBN), are affected.

Specifically, we have shown that the parameter space admits a region (i.e., $\gamma < 1$) where the universe accelerates at recent cosmological times (i.e., $z \sim 0.5$). In fact, the additional terms mimic the behavior of a fluid with a constant parameter of state $w_X = -1 + 2\gamma/3$. This has been studied by computing the deceleration parameter q [see Fig. 1 and Section III A] and stringent constraints have been obtained comparing the CMB acoustic scale ℓ_A with the WMAP 7yr release data. Note that a standard Monte-Carlo Markov Chain analysis (usually employed to extract the cosmological parameters by comparison with available observations) is not feasible if the model is not known at linear order⁵. On the contrary, the CMB acoustic scale can be computed directly from the background equations, and this has allowed us to obtain a constraint for the free parameters of the model by comparing with the most recent CMB data. This comparison has told us that $|\gamma| \ll 1$ (so that $w_X \simeq -1$) and the coefficients of \dot{H} and of H^2 in Eq. (2) must therefore satisfy Eq. (24) for the model to be phenomenologically viable. In particular, the entropic accelerating universe corresponds to a specific choice of the constants C_H and $C_{\dot{H}}$, that is $\gamma_I = 0$ and $\gamma_{II} = 0.68$ for the two cases explicitly mentioned in Ref. [4]. The latter is at odd with the constraint (23), whereas the former is consistent.

Future CMB observations coming from the PLANCK satellite⁶ are expected to improve the error on the acoustic scale by about one order of magnitude [15]. The same improvement is therefore expected for the estimate of the

⁵ In fact, as far as we know, no Lagrangian is known for these models. However, as we stated in the Introduction, we are not debating the theoretical ground the “entropic-like” proposal is based on, but are rather interested in which constraints we can provide for such class of models from what we have at hand, i.e. the background equations.

⁶ Planck (<http://www.esa.int/Planck>) is a project of the European Space Agency, ESA.

parameter γ , which, in principle, should allow us to distinguish these models from the pure Λ CDM model with $\gamma = 0$. We finally mention that our findings are in agreement with those of Ref. [16].

Acknowledgments

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Appendix A: Derivation of Friedman equations

From $\dot{H} = \ddot{a}/a - H^2$, we can rewrite Eq. (3) as

$$\frac{\ddot{a}}{a} + (\gamma - 1) H^2 = -\frac{4\pi G}{3} \sum_i (\tilde{\rho}_i + 3\tilde{p}_i), \quad (\text{A1})$$

in which we note the use of G (instead of the bare \tilde{G}). We then replace H^2 from Eq. (7) into Eq. (A1) and specifying only two fluids (dust and radiation, with equations of state $\tilde{p}_{\text{dust}} = 0$ and $\tilde{p}_{\text{rad}} = \tilde{\rho}_{\text{rad}}/3$) we obtain

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\tilde{\rho}_{\text{dust}} + 2\tilde{\rho}_{\text{rad}} + 2c_{\text{dust}} \tilde{\rho}_{\text{dust}} (\gamma - 1) \right)$$

$$+ 2c_{\text{rad}} \tilde{\rho}_{\text{rad}} (\gamma - 1) + 2(\gamma - 1) \frac{C}{a^{2\gamma}} \Big). \quad (\text{A2})$$

From the definition of c_{dust} and c_{rad} , it is easy to show that

$$2c_{\text{dust}} (\gamma - 1) + 1 = c_{\text{dust}} \quad (\text{A3})$$

and

$$2c_{\text{rad}} (\gamma - 1) + 2 = 2c_{\text{rad}}. \quad (\text{A4})$$

Therefore, Eq. (A2) is equivalent to

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(c_{\text{dust}} \tilde{\rho}_{\text{dust}} + 2c_{\text{rad}} \tilde{\rho}_{\text{rad}} + 2(\gamma - 1) \frac{C}{a^{2\gamma}} \right), \quad (\text{A5})$$

which is exactly Eq. (10) with the densities of Eq. (12), namely

$$\rho_{\text{rad}} = c_{\text{rad}} \tilde{\rho}_{\text{rad}}, \quad \rho_{\text{dust}} = c_{\text{dust}} \tilde{\rho}_{\text{dust}}. \quad (\text{A6})$$

Note that the constant and dimensionless coefficients c_{rad} and c_{dust} are not observable, since they never appear without multiplying the corresponding (bare) densities, and the above rescaling simply reflects the choice of standard units for the densities [as well as Eq. (5) is for the Newton constant].

Finally, Eq. (7) becomes Eq. (11) using the same redefinitions (choice of units) for the densities.

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