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# Chromoelectric dipole moment of the top quark in models with vectorlike multiplets <br> Tarek Ibrahim and Pran Nath 

Phys. Rev. D 84, 015003 - Published 5 July 2011
DOI: 10.1103/PhysRevD.84.015003

# The Chromoelectric Dipole Moment of the Top Quark in Models with Vector Like Multiplets 

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#### Abstract

The chromoelectric dipole moment of the top quark is calculated in a model with a vector like multiplet which mixes with the third generation in an extension of the MSSM. Such mixings allow for new CP violating phases. Including these new CP phases, the chromoelectric dipole moment that generates an electric dipole of the top in this class of models is computed. The top chromoelectric dipole moment operator arises from loops involving the exchange of the W , the Z as well as from the exchange involving the charginos, the neutralinos, the gluino, and the vector like multiplet and their superpartners. The analysis of the chromoelectric dipole moment operator of the top is more complicated than for the light quarks because the mass of the external fermion, in this case the top quark mass, cannot be ignored relative to the masses inside the loops. A numerical analysis is presented and it is shown that the contribution to the top EDM could lie in the range $\left(10^{-19}-10^{-18}\right)$ ecm consistent with the current limits on the EDM of the electron, the neutron and on atomic EDMs. A top EDM of size $\left(10^{-19}-10^{-18}\right)$ ecm could be accessible in collider experiments such as at the LHC and at the ILC.


[^0]
## 1 Introduction

The electric dipole moment (EDM) of elementary particles provide an important window to possible new sources of CP violation (For recent reviews see[1]). This is so because in the Standard Model the EDM of an elementary particle is rather small. Thus for the top quark the EDM in the Standard Model is estimated to be less than $10^{-30} \mathrm{ecm}[2,3,4]$ and outside the realm of experiment in the foreseeable future (For a review of CP violation in top physics see [5]). However, much larger EDMs for elementary particles can arise in new physics models. One such model considered recently was where one has extra vector like generations which can mix with the third generation $[6,7,8]$. Extra vector like generations can arise in many unified theories of particle physics $[9,10]$ and if their masses lie in the TeV range they could mix with the third generation and produce observable effects. Such mixings are consistent with the current precision electroweak data[11] and thus the implications of such vector like multiplets have been analyzed in a number of works $[12,13,14,15,16,17,18,19,20]$. In [8] an analysis of the electric dipole operator for the top quark was given arising from the exchange of the extra vector like generations in the loops and it was found that a significantly larger EDM than in the Standard Model can arise for the top quark from such exchanges. In this work we analyze the contribution to the chromoelectric dipole operator (CEDM) from the exchange of the vector like generations in the loops. Our analysis is done in an extension of the minimal supersymmetric standard model (MSSM) including the extra vector like multiplets. The analysis shows that a top EDM as large as $\left(10^{-19}-10^{-18}\right)$ ecm can arise from a constructive interference between the electric dipole moment and the chromoelectric dipole moment. A top EDM of this size lies within the realm of future experiment [21, 22, 23, 24]. The role of EDMs in a variety of processes such as $e^{+} e^{-} \rightarrow t \bar{t}, \gamma \gamma \rightarrow t \bar{t}$ and other phenomena have been investigated by a number of authors [22, 25, 26, 27, 27, 28, 29] and thus the EDM of the top is of significant interest.

The outline of the rest of the paper is as follows: In Sec.(2) we define the chromoelectric dipole moment of the quark and its connection with the electric dipole moment. In Sec.(3) we give an analysis of the EDM of the top allowing for mixing between the vector like multiplet and the third generation quarks in the underlying model discussed in [8]. These mixings contain new sources of CP violation. Here we compute the loops involving the exchanges of the $W$ and the $Z$, of the charginos, of the neutralinos, of the gluino as well as exchanges involving the vector like multiplets and their superpartners. In Sec.(4) we discuss
the parameter space of the model and list the new CP violating phases that enter in the analysis. A numerical analysis of the size of the EDM of the top is given in Sec.(5). In this section we also display the dependence of the top EMD on the CP phases arising from the mixings of the third generation quarks with the extra vector like generations. Conclusions are given in Sec.(6).

## 2 Chromoelectric dipole moment of the top quark

The chromoelectric dipole moment $\tilde{d}^{C}$ is defined in the effective dimension 5 operator

$$
\begin{equation*}
\mathcal{L}_{I}=-\frac{i}{2} \tilde{d}^{C} \bar{q} \sigma_{\mu \nu} \gamma_{5} T^{a} q G^{\mu \nu a} \tag{1}
\end{equation*}
$$

where $T^{a}$ are the $S U(3)$ generators and $G^{\mu \nu a}$ is the gluon field strength. The contribution of this operator to the EDM of quarks can be computed using dimensional analysis[30]. This technique can be expressed using the "reduced" coupling constant rule. Thus the contribution of chromoelectric dipole moment operator to the EDM of the quarks is given as follows

$$
\begin{equation*}
d^{C}=\frac{e}{4 \pi} \tilde{d}^{C} \tag{2}
\end{equation*}
$$

The alternative technique to estimate contributions of the chromoelectric operator is to use the QCD sum rules[31]. We note that the analysis of the top EDM is more complicated relative to EDM of the light quarks and of the light leptons (see e.g., $[32,33]$ ) because we cannot ignore the mass of the external fermion (i.e., of the top quark in this case) compared to the masses that run inside the loops. So the form factors that enter the analysis of the top EDM are more complicated relative to the form factors that enter the EDM of the light quarks, since for the case of the top the loop integrals are functions of more than just one mass ratio.

## 3 Top CEDM from exchange of vector like multiplets

Using the formalism of [8], one can compute the contributions to the chromoelectric dipole moment of the top quark. There are several contribution to it arising from the exchange of the charginos, of the neutralinos, of the gluinos and of the W and Z boson. CP violation in these diagrams enters via the mass matrices involving the third generation and their mirrors and similarly via the mass matrices involving their superpartners and via the interaction


Figure 1: Left: One loop contribution to the chromoelectric dipole moment of the top quark from the exchange of the chargino and from the exchange of sbottoms and mirror sbottoms. Right: Same as the left diagram except that one has chromoelectric dipole moment arising from the exchange of the neutralinos and from the exchange of stops and mirror stops.
vertices. A full description of the CP phases and the dependence of CEDM on them is given in Sec.(4). We discuss now the various contributions to the CEDM of the top.

### 3.1 Chargino exchange contribution

The chargino exchange contribution to the chromoelectric dipole moment of the top quark arises through the left loop diagram of Fig.(1). The relevant part of Lagrangian that generates this contribution is given by

$$
\begin{equation*}
-\mathcal{L}_{t-\tilde{b}-\chi^{+}}=\sum_{k=1}^{2} \sum_{i=1}^{2} \sum_{j=1}^{4} \bar{t}_{k}\left[\Gamma_{L k j i} P_{L}+\Gamma_{R k j i} P_{R}\right] \tilde{\chi}^{+}{ }_{i} \tilde{b}_{j}+H . c . \tag{3}
\end{equation*}
$$

where

$$
\begin{array}{r}
\Gamma_{L k j i}=-g\left[V_{i 2}^{*} \kappa_{t} D_{R 1 k}^{t *} \tilde{D}_{1 j}^{b}-D_{R 2 k}^{t *} V_{i 1}^{*} \tilde{D}_{4 j}^{b}+D_{R 2 k}^{t *} \kappa_{B} V_{i 2}^{*} \tilde{D}_{2 j}^{b}\right], \\
\Gamma_{R k j i}=g\left[U_{i 1} D_{L 1 k}^{t *} \tilde{D}_{1 j}^{b}-D_{L 1 k}^{t *} \kappa_{b} U_{i 2} \tilde{D}_{3 j}^{b}-D_{L 2 k}^{t *} \kappa_{T} U_{i 2} \tilde{D}_{4 j}^{b}\right], \tag{4}
\end{array}
$$

where $\tilde{D}^{b}$ is the diagonalizing matrix of the $4 \times 4$ sbottom mixed with scalar mirrors mass ${ }^{2}$ matrix as defined in the appendix of [8]. These elements contain CP violating phases can also contribute to the chromoelectric dipole moment of the top. The couplings $\kappa_{f}$ are defined as

$$
\begin{equation*}
\left(\kappa_{T}, \kappa_{b}\right)=\frac{\left(m_{T}, m_{b}\right)}{\sqrt{2} M_{W} \cos \beta},\left(\kappa_{B}, \kappa_{t}\right)=\frac{\left(m_{B}, m_{t}\right)}{\sqrt{2} M_{W} \sin \beta} \tag{5}
\end{equation*}
$$

Here $U$ and $V$ are the matrices that diagonalize the chargino mass matrix $M_{C}$ so that

$$
\begin{equation*}
U^{*} M_{C} V^{-1}=\operatorname{diag}\left(m_{\tilde{\chi}_{1}}^{+}, m_{\tilde{\chi}_{2}}^{+}\right) . \tag{6}
\end{equation*}
$$

Using the above interaction, we get from the left loop diagram of Fig.(1) the contribution

$$
\begin{equation*}
\tilde{d^{C}}\left(\chi^{+}\right)=\frac{g_{s}}{16 \pi^{2}} \sum_{i=1}^{2} \sum_{j=1}^{4} \frac{m_{\chi_{i}^{+}}}{m_{\tilde{b_{j}}}^{2}} \operatorname{Im}\left(\Gamma_{L 1 j i} \Gamma_{R 1 j i}^{*}\right) I_{3}\left(\frac{m_{\chi_{i}^{+}}^{2}}{m_{\tilde{b_{j}}}^{2}}, \frac{m_{t_{1}}^{2}}{m_{\tilde{b_{j}}}^{2}}\right), \tag{7}
\end{equation*}
$$

where $I_{3}\left(r_{1}, r_{2}\right)$ is given by

$$
\begin{equation*}
I_{3}\left(r_{1}, r_{2}\right)=\int_{0}^{1} d x \frac{x-x^{2}}{1+\left(r_{1}-r_{2}-1\right) x+r_{2} x^{2}} \tag{8}
\end{equation*}
$$

We note that the limit of $I_{3}\left(r_{1}, r_{2}\right)$ for $r_{2} \sim 0$ is the well known form factors $B\left(r_{1}\right)$ in the case of light quarks [33]. While our analysis is quite general we will limit ourselves for simplicity to the case where there is mixing between the third generation and the mirror part of the vector multiplet. The inclusion of the non-mirror part is essentially trivial as it corresponds to an extension of the CKM matrix from a $3 \times 3$ to a $4 \times 4$ matrix in the standard model sector and similar straightforward extensions in the supersymmetric sector. In the rest of the analysis we will focus just on the mixings with the mirrors which is rather non-trivial.

### 3.2 Neutralino exchange contribution

The neutralino exchange contribution to the chromoelectric dipole moment of the top quark through the right loop diagram of Fig.(1). The relevant part of Lagrangian that generates this contribution is given by

$$
\begin{equation*}
-\mathcal{L}_{t-\tilde{t}-\chi^{0}}=\sum_{k=1}^{4} \sum_{i=1}^{4} \sum_{j=1}^{2} \bar{t}_{j}\left[C_{L j k i} P_{L}+C_{R j k i} P_{R}\right] \tilde{\chi}_{i}^{0} \tilde{t}_{k}+\text { H.c. } \tag{9}
\end{equation*}
$$

where

$$
\begin{align*}
C_{L j k i} & =\sqrt{2}\left[\alpha_{t i} D_{R 1 j}^{t *} \tilde{D}_{1 k}^{t}-\gamma_{t i} D_{R 1 j}^{t *} \tilde{D}_{3 k}^{t}+\beta_{T i} D_{R 2 j}^{t *} \tilde{D}_{4 k}^{t}-\delta_{T i} D_{R 2 j}^{t *} \tilde{D}_{2 k}^{t}\right] \\
C_{R j k i} & =\sqrt{2}\left[\beta_{t i} D_{L 1 j}^{t *} \tilde{D}_{1 k}^{t}-\delta_{t i} D_{L 1 j}^{t *} \tilde{D}_{3 k}^{t}+\alpha_{T i} D_{L 2 j}^{t *} \tilde{D}_{4 k}^{t}-\gamma_{T i} D_{L 2 j}^{t *} \tilde{D}_{2 k}^{t}\right] \tag{10}
\end{align*}
$$

The matrix $\tilde{D}^{t}$ is the diagonalizing matrix of the $4 \times 4$ stop mixed with scalar mirrors mass ${ }^{2}$ matrix as defined in the appendix of [8]. The couplings that enter the above equations are
given by

$$
\begin{array}{r}
\alpha_{t j}=\frac{g m_{t} X_{4 j}}{2 m_{W} \sin \beta}, \quad \beta_{t j}=\frac{2}{3} e X_{1 j}^{\prime *}+\frac{g}{\cos \theta_{W}} X_{2 j}^{\prime *}\left(\frac{1}{2}-\frac{2}{3} \sin ^{2} \theta_{W}\right), \\
\gamma_{t j}=\frac{2}{3} e X_{1 j}^{\prime}-\frac{2}{3} \frac{g \sin ^{2} \theta_{W}}{\cos \theta_{W}} X_{2 j}^{\prime}, \quad \delta_{t j}=-\frac{g m_{t} X_{4 j}^{*}}{2 m_{W} \sin \beta} . \tag{11}
\end{array}
$$

Here

$$
\begin{array}{r}
\alpha_{T j}=\frac{g m_{T} X_{3 j}^{*}}{2 m_{W} \cos \beta}, \quad \beta_{T j}=-\frac{2}{3} e X_{1 j}^{\prime}+\frac{g}{\cos \theta_{W}} X_{2 j}^{\prime}\left(-\frac{1}{2}+\frac{2}{3} \sin ^{2} \theta_{W}\right), \\
\gamma_{T j}=-\frac{2}{3} e X_{1 j}^{\prime *}+\frac{2}{3} \frac{g \sin ^{2} \theta_{W}}{\cos \theta_{W}} X_{2 j}^{\prime *}, \quad \delta_{T j}=-\frac{g m_{T} X_{3 j}}{2 m_{W} \cos \beta}, \tag{12}
\end{array}
$$

where

$$
\begin{equation*}
X_{1 j}^{\prime}=\left(X_{1 j} \cos \theta_{W}+X_{2 j} \sin \theta_{W}\right), X_{2 j}^{\prime}=\left(-X_{1 j} \sin \theta_{W}+X_{2 j} \cos \theta_{W}\right) \tag{13}
\end{equation*}
$$

and where the matrix $X$ diagonlizes the neutralino mass matrix so that

$$
\begin{equation*}
X^{T} M_{\tilde{\chi}^{0}} X=\operatorname{diag}\left(m_{\chi^{0}{ }_{1}}, m_{\chi^{0}{ }_{2}}, m_{\chi^{0}{ }_{3}}, m_{\chi^{0}{ }_{4}}\right) . \tag{14}
\end{equation*}
$$

Using the above interaction, we get from the right loop diagram Fig.(1) the neutralino contributions to the top chromoelectric dipole moment to be

$$
\begin{equation*}
\tilde{d}^{C}\left(\chi^{0}\right)=\frac{g_{s}}{16 \pi^{2}} \sum_{i=1}^{4} \sum_{k=1}^{4} \frac{m_{\chi_{i}^{0}}}{m_{\tilde{t}_{k}}^{2}} \operatorname{Im}\left(C_{L 1 k i} C_{R 1 k i}^{*}\right) I_{3}\left(\frac{m_{\chi_{i}^{0}}^{2}}{m_{\tilde{t}_{k}}^{2}}, \frac{m_{t_{1}}^{2}}{m_{\tilde{t_{k}}}^{2}}\right) . \tag{15}
\end{equation*}
$$

### 3.3 Gluino exchange contribution

The gluino contribution to the chromoelectric dipole moment of the top comes from the two loop diagrams of Fig.(2). The relevant part of Lagrangian that generates this contribution is given by

$$
\begin{equation*}
-\mathcal{L}_{t \tilde{t} \tilde{g}}=\sqrt{2} g_{s} \sum_{a=1}^{8} \sum_{j, k=1}^{3} \sum_{n=1}^{2} \sum_{m=1}^{4} T_{j k}^{a} \bar{t}_{n}^{j}\left[K_{L_{n m}} P_{L}+K_{R_{n m}} P_{R}\right] \tilde{g}_{a} \tilde{t}_{m}^{k}+\text { H.c. } \tag{16}
\end{equation*}
$$

where

$$
\begin{gather*}
K_{L_{n m}}=e^{-i \xi_{3} / 2}\left[D_{R_{2 n}}^{t *} \tilde{D}_{4 m}^{t}-D_{R_{1 n}}^{t *} \tilde{D}_{3 m}^{t}\right] \\
K_{R_{n m}}=e^{i \xi_{3} / 2}\left[D_{L_{1 n}}^{t *} \tilde{D}_{1 m}^{t}-D_{L_{2 n}}^{t *} \tilde{D}_{2 m}^{t}\right] \tag{17}
\end{gather*}
$$



Figure 2: Left: One loop contribution to the chromoelectric dipole moment of the top quark from gluino exchange and from the exchange of stops and mirror stops. Here the external gluon line connects to the stops and mirror stops in the loop. Right: Same as the left diagram except that the external gluon line connects to gluinos in the loop, i.e., one has a gluino-gluino-gluon vertex in this case.
where $\xi_{3}$ is the phase of the gluino mass.
The above Lagrangian gives a contribution

$$
\begin{equation*}
\tilde{d^{C}}(\tilde{g})=\frac{g_{s} \alpha_{s}}{12 \pi} \sum_{j=1}^{4} \frac{m_{\tilde{g}}}{m_{\tilde{t_{j}}}^{2}} \operatorname{Im}\left(K_{L_{1 j}} K_{R_{1 j}}^{*}\right) I_{5}\left(\frac{m_{\tilde{g}}^{2}}{m_{t_{j}}^{2}}, \frac{m_{t_{1}}^{2}}{m_{\tilde{t}_{j}}^{2}}\right), \tag{18}
\end{equation*}
$$

where $I_{5}\left(r_{1}, r_{2}\right)$ is given by

$$
\begin{equation*}
I_{5}\left(r_{1}, r_{2}\right)=\int_{0}^{1} d x \frac{x+8 x^{2}}{1+\left(r_{1}-r_{2}-1\right) x+r_{2} x^{2}} \tag{19}
\end{equation*}
$$

We note that the limit of $I_{5}\left(r_{1}, r_{2}\right)$ for $r_{2} \sim 0$ is the well known form factors $3 C\left(r_{1}\right)$ in the case of light quarks [33].

### 3.4 W and Z exchange contributions

The W boson exchange contribution to the chromoelectric dipole moment of the top quark arises through the left loop diagram of Fig.(3). The relevant part of Lagrangian that generates this contribution is given by

$$
\begin{equation*}
\mathcal{L}_{C C}=-\frac{g}{\sqrt{2}} W_{\mu}^{+} \sum_{i} \sum_{j} \bar{t}_{j} \gamma^{\mu}\left[D_{L 1 j}^{t *} D_{L 1 i}^{b} P_{L}+D_{R 2 j}^{t *} D_{R 2 i}^{b} P_{R}\right] b_{i}+\text { H.c. } \tag{20}
\end{equation*}
$$

where $i, j$ run over the set of quarks and mirror quarks including those from the third generation and from the vector multiplet, $t_{1}$ is the physical top quark, and $D_{L, R}^{t, b}$ are the


Figure 3: Left: One loop contribution to the chromoelectric dipole moment of the top quark from $W^{+}$exchange and from the exchange of the bottom quark and from the mirror bottom. Right: Same as the left diagram except that one has chromoelectric dipole moment arising from Z exchange and from the exchange of the top and from the mirror top.
diagonalizing matrices defined in the appendix of [8]. These matrices contain phases, and these phases generate the chromoelectric dipole moment of the top quark. Using the above interaction, we get from the left loop diagram of Fig.(3), the contribution

$$
\begin{equation*}
\tilde{d^{C}}\left(W^{+}\right)=\frac{g_{s}}{16 \pi^{2} M_{W}^{2}} \sum_{i=1}^{2} m_{b_{i}} \operatorname{Im}\left(\Gamma_{i}^{t b}\right) I_{1}\left(\frac{m_{b_{i}}^{2}}{M_{W}^{2}}, \frac{m_{t_{1}}^{2}}{M_{W}^{2}}\right) . \tag{21}
\end{equation*}
$$

Here $\Gamma_{i}^{t b}$ is given

$$
\begin{equation*}
\Gamma_{i}^{t b}=\frac{g^{2}}{2} D_{L 11}^{t *} D_{L 1 i}^{b} D_{R 21}^{t} D_{R 2 i}^{b *}, \tag{22}
\end{equation*}
$$

and $I_{1}\left(r_{1}, r_{2}\right)$ is given by

$$
\begin{equation*}
I_{1}\left(r_{1}, r_{2}\right)=\int_{0}^{1} d x \frac{\left(4+r_{1}-r_{2}\right) x-4 x^{2}}{1+\left(r_{1}-r_{2}-1\right) x+r_{2} x^{2}} \tag{23}
\end{equation*}
$$

Finally we consider the right loop of Fig.(3) which produces the chromoelectric dipole moment of the top quark through the interaction with the Z boson. The relevant part of Lagrangian that generates this contribution is given by

$$
\begin{equation*}
\mathcal{L}_{N C}=-Z_{\mu} \sum_{i=1}^{2} \sum_{j=1}^{2} \bar{t}_{j} \gamma^{\mu}\left[S_{L j i} P_{L}+S_{R j i} P_{R}\right] t_{i} \tag{24}
\end{equation*}
$$

where

$$
\begin{align*}
S_{L j i} & =-\frac{g}{6 \cos \theta_{W}}\left[-3 D_{L 1 j}^{t *} D_{L 1 i}^{t}+4 \sin ^{2} \theta_{W}\left(D_{L 1 j}^{t *} D_{L 1 i}^{t}+D_{L 2 j}^{t *} D_{L 2 i}^{t}\right)\right] \\
S_{R j i} & =-\frac{g}{6 \cos \theta_{W}}\left[-3 D_{R 2 j}^{t *} D_{R 2 i}^{t}+4 \sin ^{2} \theta_{W}\left(D_{R 1 j}^{t *} D_{R 1 i}^{t}+D_{R 2 j}^{t *} D_{R 2 i}^{t}\right)\right] \tag{25}
\end{align*}
$$

Using the above interaction, we get from the right loop of Fig.(3), the contribution

$$
\begin{equation*}
\tilde{d}^{C}(Z)=\frac{g_{s}}{16 \pi^{2} M_{Z}^{2}} \sum_{i=1}^{2} m_{t_{i}} \operatorname{Im}\left(S_{L 1 i} S_{R 1 i}^{*}\right) I_{1}\left(\frac{m_{t_{i}}^{2}}{M_{Z}^{2}}, \frac{m_{t_{1}}^{2}}{M_{Z}^{2}}\right) . \tag{26}
\end{equation*}
$$

The total chromoelectric dipole moment of the top in the model is then given by the sum of the contributions computed in this section so that

$$
\begin{equation*}
\tilde{d}^{C}=\tilde{d}^{C}\left(\chi^{+}\right)+\tilde{d}^{C}\left(\chi^{0}\right)+\tilde{d}^{C}(\tilde{g})+\tilde{d}^{C}\left(W^{+}\right)+\tilde{d}^{C}(Z) \tag{27}
\end{equation*}
$$

## 4 Parameter space of the model and CP phases

The mass matrices for quarks and mirrors including their mixings are diagonalized using bi-unitary transformations $D_{L}^{b}$ and $D_{R}^{b}$ for the bottom quarks and mirrors and $D_{L}^{t}$ and $D_{R}^{t}$ for the diagonalization of the top quarks and mirrors. We parametrize $D_{L}^{t}$ and $D_{R}^{t}$ as follows

$$
D_{L}^{t}=\left(\begin{array}{cc}
\cos \theta_{L} & -\sin \theta_{L} e^{-i \chi_{L}}  \tag{28}\\
\sin \theta_{L} e^{i \chi_{L}} & \cos \theta_{L}
\end{array}\right), D_{R}^{t}=\left(\begin{array}{cc}
\cos \theta_{R} & -\sin \theta_{R} e^{-i \chi_{R}} \\
\sin \theta_{R} e^{i \chi_{R}} & \cos \theta_{R}
\end{array}\right) .
$$

Thus the mixing between $t$ and $T$ is parameterized by the angles $\theta_{L}, \theta_{R}, \chi_{L}$ and $\chi_{R}$ where the angles $\theta_{L}, \theta_{R}$ are given by

$$
\begin{equation*}
\tan 2 \theta_{L}=\frac{2\left|m_{t} h_{5}^{*}-m_{T} h_{3}\right|}{m_{t}^{2}+\left|h_{3}\right|^{2}-m_{T}^{2}-\left|h_{5}\right|^{2}}, \quad \tan 2 \theta_{R}=\frac{2\left|-m_{t} h_{3}+m_{T} h_{5}^{*}\right|}{m_{t}^{2}+\left|h_{5}\right|^{2}-m_{T}^{2}-\left|h_{3}\right|^{2}}, \tag{29}
\end{equation*}
$$

and $\chi_{L}$ and $\chi_{R}$ are the CP violating phases defined by

$$
\begin{equation*}
\chi_{R}=\arg \left(-m_{t} h_{3}+m_{T} h_{5}^{*}\right), \chi_{L}=\arg \left(m_{t} h_{5}^{*}-m_{T} h_{3}\right) . \tag{30}
\end{equation*}
$$

Similarly $D_{L}^{b}$ and $D_{R}^{b}$ are given by

$$
D_{L}^{b}=\left(\begin{array}{cc}
\cos \theta_{L} & -\sin \phi_{L} e^{-i \xi_{L}}  \tag{31}\\
\sin \phi_{L} e^{i \xi_{L}} & \cos \phi_{L}
\end{array}\right), D_{R}^{b}=\left(\begin{array}{cc}
\cos \phi_{R} & -\sin \phi_{R} e^{-i \xi_{R}} \\
\sin \phi_{R} e^{i \xi_{R}} & \cos \phi_{R}
\end{array}\right)
$$

where the mixing between $b$ and $B$ is parametrized by the angle $\phi_{L}, \phi_{R}, \xi_{L}$ and $\xi_{R}$. Here the angles $\phi_{L}$ and $\phi_{R}$ are given by

$$
\begin{equation*}
\tan 2 \phi_{L}=\frac{2\left|m_{b} h_{4}^{*}+m_{B} h_{3}\right|}{m_{b}^{2}+\left|h_{3}\right|^{2}-m_{B}^{2}-\left|h_{4}\right|^{2}}, \quad \tan 2 \phi_{R}=\frac{2\left|m_{b} h_{3}+m_{B} h_{4}^{*}\right|}{m_{b}^{2}+\left|h_{4}\right|^{2}-m_{B}^{2}-\left|h_{3}\right|^{2}} \tag{32}
\end{equation*}
$$




Figure 4: (Color online) Left: An exhibition of the dependence of $d_{t}$ on $\alpha_{B}$ when $\tan \beta=5, m_{T}=250,\left|h_{3}\right|=70,\left|h_{4}\right|=80$, $m_{B}=120,\left|h_{5}\right|=90, m_{0}=220,\left|A_{0}\right|=200, \tilde{m}_{1}=50, \tilde{m}_{2}=100, \mu=150, \tilde{m}_{g}=350, \chi_{4}=0.3, \chi_{5}=-0.8, \alpha_{T}=0.4$, and $\chi_{3}=0.4$. (The six curves correspond to the contributions from the Z, W, neutralino, chargino, gluino and total CEDM. They are shown in ascending order at $\alpha_{B}=0$ ). Here and in subsequent figures all masses are in GeV and all angles are in rad. Right: An exhibition of the dependence of $d_{t}$ on $\alpha_{T}$ when $\tan \beta=25, m_{T}=200,\left|h_{3}\right|=85,\left|h_{4}\right|=75, m_{B}=150,\left|h_{5}\right|=85, m_{0}=200$, $\left|A_{0}\right|=200, \tilde{m}_{1}=50, \tilde{m}_{2}=100, \mu=150, \tilde{m}_{g}=400, \chi_{4}=0.5, \chi_{5}=0.7, \chi_{3}=0.8$, and $\alpha_{B}=0.2$. (The six curves correspond to the contributions from the neutralino, Z , chargino, W , total CEDM and gluino. They are shown in ascending order at $\alpha_{T}=0$ ).
and the phases $\xi_{L, R}$ arise from the couplings $h_{4}$ and $h_{3}$ through the relations

$$
\begin{equation*}
\xi_{R}=\arg \left(m_{b} h_{3}+m_{B} h_{4}^{*}\right), \xi_{L}=\arg \left(m_{b} h_{4}^{*}+m_{B} h_{3}\right) \tag{33}
\end{equation*}
$$

For the case of top and bottom masses arising from hermitian matrices, i.e., when $h_{5}=-h_{3}^{*}$ and $h_{4}=h_{3}^{*}$ we have $\theta_{L}=\theta_{R}, \phi_{L}=\phi_{R}, \chi_{L}=\chi_{R}=\chi$ and $\xi_{L}=\xi_{R}=\xi$. Further, here we have the relation $\xi=\chi+\pi$ and thus the W -exchange and the Z-exchange terms in the EDM for the top vanish. However, more generally the top and the bottom mass matrices are not hermitian and they generate non-vanishing contributions to the EDMs. Thus the input parameters for this sector of the parameter space are $m_{t 1}, m_{T}, h_{3}, h_{5}, m_{b 1}, m_{B}, h_{4}$ with $h_{3}, h_{4}$ and $h_{5}$ being complex masses with the corresponding CP violating phases $\chi_{3}, \chi_{4}$ and $\chi_{5}$. For the sbottom and stop mass ${ }^{2}$ matrices we need the extra input parameters of the susy breaking sector, $\tilde{M}_{q}, \tilde{M}_{B}, \tilde{M}_{b}, \tilde{M}_{Q}, \tilde{M}_{t}, \tilde{M}_{T}, A_{b}, A_{T}, A_{t}, A_{B}, \mu, \tan \beta$. The chargino, neutralino and gluino sectors need the extra parameters $\tilde{m}_{1}, \tilde{m}_{2}$ and $m_{\tilde{g}}$. We will assume that the only parameters that have phases in the above set are $A_{T}, A_{B}, A_{t}$ and $A_{b}$ with the corresponding phases given by $\alpha_{T}, \alpha_{B}, \alpha_{t}$ and $\alpha_{b}$.


Figure 5: (Color online) Left:An exhibition of the dependence of $d_{t}$ on $\chi_{3}$ when $\tan \beta=10, m_{T}=150,\left|h_{3}\right|=75,\left|h_{4}\right|=90$, $m_{B}=180,\left|h_{5}\right|=80, m_{0}=300,\left|A_{0}\right|=300, \tilde{m}_{1}=50, \tilde{m}_{2}=100, \mu=150, \tilde{m}_{g}=400, \chi_{4}=0.7, \chi_{5}=-0.4, \alpha_{T}=0.2$, and $\alpha_{B}=0.7$. (The six curves correspond to the contributions from the Z , neutralino, W , chargino, gluino and total CEDM. They are shown in ascending order at $\chi_{3}=0$ ). Right:An exhibition of the dependence of $d_{t}$ on $\chi_{4}$ when $\tan \beta=15, m_{T}=350$, $\left|h_{3}\right|=80,\left|h_{4}\right|=70, m_{B}=200,\left|h_{5}\right|=100, m_{0}=400,\left|A_{0}\right|=400, \tilde{m}_{1}=50, \tilde{m}_{2}=100, \mu=150, \tilde{m}_{g}=300, \chi_{3}=0.6, \chi_{5}=0.8$, $\alpha_{T}=0.7$, and $\alpha_{B}=0.2$. (The six curves correspond to the contributions from the Z , neutralino, chargino, W , total CEDM and gluino. They are shown in ascending order at $\chi_{4}=0$ ).


Figure 6: (Color online) An exhibition of the dependence of $d_{t}$ on $\chi_{5}$ when $\tan \beta=20, m_{T}=300,\left|h_{3}\right|=90,\left|h_{4}\right|=85$, $m_{B}=250,\left|h_{5}\right|=95, m_{0}=100,\left|A_{0}\right|=200, \tilde{m}_{1}=50, \tilde{m}_{2}=100, \mu=150, \tilde{m}_{g}=300, \chi_{4}=-0.6, \chi_{3}=0.4, \alpha_{T}=0.7$, and $\alpha_{B}=0.4$. (The six curves correspond to the contributions from the $\mathrm{W}, \mathrm{Z}$, neutralino, chargino, gluino and total CEDM. They are shown in ascending order at $\chi_{5}=0$ ).

## 5 Numerical estimate of the CEDM of the top

To simplify the analysis further we set some of the phases to zero, i.e., specifically we set $\alpha_{t}=$ $\alpha_{b}=0$. With this in mind the only contributions to the chromoelectric dipole moment CEDM
of the top quark arises from mixing terms between the scalars and the mirror scalars, between the fermions - and the mirror fermions and finally among the mirror scalars themselves. Thus in the absence of the mirror part of the lagrangian, the top CEDM vanishes and so we can isolate the role of the CP violating phases in this sector and see the size of its contribution. The $4 \times 4$ mass $^{2}$ matrices of stops and sbottoms are diagonlized numerically. Thus the CP violating phases that would play a role in this analysis are

$$
\begin{equation*}
\chi_{3}, \chi_{4}, \chi_{5}, \alpha_{T}, \alpha_{B} \tag{34}
\end{equation*}
$$

To reduce the number of input parameters we assume $\tilde{M}_{a}=m_{0}, a=q, B, b, Q, T, t$ and $\left|A_{i}\right|=\left|A_{0}\right|, i=T, B, t, b$. In the left panel of $\operatorname{Fig}(4)$, we give a numerical analysis of the top EDM and discuss its variation with the phase $\alpha_{B}$. We note that the only component that varies with this phase is the chargino component. This is expected since $\alpha_{B}$ enters the scalar bottom mass ${ }^{2}$ matrix and the chargino contribution to the EDM is controlled by $\tilde{D}^{b}$ which depends on $\alpha_{B}$ while the other contributions are independent of this phase. Further, the chargino component exhibits a minimum where the different terms of it can have destructive cancellation. In the right panel of $\operatorname{Fig}(4)$, we study the variation of the different components of $d_{t}$ on the phase $\alpha_{T}$. We observe that the components that vary with this phase are the neutralino and the gluino contributions while the $\mathrm{W}, \mathrm{Z}$ and chargino contributions have no dependence on this phase. The reason for the above is that $\alpha_{T}$ enters the scalar top mass ${ }^{2}$ matrix and the EDM arising from $\mathrm{W}, \mathrm{Z}$ and chargino exchanges are independent of $\tilde{D}^{t}$. However, the neutralino and the gluino contributions are affected by it. It is clear that we see here the cancellation mechanism[32, 33, 34, 35]. working since the components are close to each other with different signs, so we have the possibility of a destructive cancellation.

In the left panel of $\operatorname{Fig}(5)$, we show the behavior of the different components of the chromoelectric dipole moment contributions to the top EDM as a function of the phase $\chi_{3}$. We note that $\chi_{3}$ enters $D^{t}, D^{b}, \tilde{D}^{t}$ and $\tilde{D}^{b}$ and as a consequences all diagrams in Fig.(1), Fig. (2) and Fig. (3) that contribute to the top EDM have a $\chi_{3}$ dependence. Further, the various diagrams that contribute to the top EDM may add constructively or destructively as shown in the Z , W, neutralino and chargino contributions. In the case of destructive interference, we have large cancellations again reminiscent of the cancellation mechanism for the EDM of the electron and for the neutron[32, 33, 34, 35]. Of course the desirable larger contributions for the top EDM occur away from the cancellation regions. In the right panel of $\operatorname{Fig}(5)$, we study the variation of the different components of $d_{t}$ as the magnitude of the
phase $\chi_{4}$ varies. The sparticle masses and couplings in the bottom sector and thus the top EDM arising from the exchange of the $W$ and the charginos are sensitive to $\chi_{4}$ and thus only these two contributions to the top EDM have dependence on this parameter. In Fig(6), we study the variation of the different components of $d_{t}$ as the phase $\chi_{5}$ changes. This phase enters the top quark mass matrix and the scalar top mass ${ }^{2}$ matrix and consequently the matrices $D_{L, R}^{t}$ and $\tilde{D}^{t}$. Thus the contributions to the EDM of the top arising from the W, Z, neutralino, chargino and gluino exchanges all have a dependence on $\chi_{5}$ as exhibited in $\operatorname{Fig}(6)$.

A comparison between the contributions of the chromoelectric dipole moment operator of the top EDM and that of the electric dipole moment operator[8], shows that they could be the same order of magnitude with like or unlike signs. That would provide an extra element for constructive or destructive interference of EDM components. To exhibit this, we give in Table 1 the values of EDM for the top quark coming from the electric dipole moment operator and the chromoelectric dipole moment operator. The first entry of Table 1 shows a destructive interference between the electric and the chromoelectric dipole moments while the last two entries show a constructive interference. With constructive interference a value of the top EDM as large as $\sim 6 \times 10^{-19}$ ecm in magnitude (see the middle entry) can be gotten. It is very possible that a full search of the parameter space of phases can lead to a top EDM of size $O\left(10^{-18}\right)$ ecm.

Table 1: Electric and chromoelectric dipole operator contributions.

| $\chi_{3}(\mathrm{rad})$ | $\chi_{4}$ | $\chi_{5}$ | $\alpha_{T}$ | $\alpha_{B}$ | $d_{t}^{E}$ e.cm | $d_{t}^{C}$ e.cm |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| .3 | -.5 | 1.0 | .8 | -.4 | $8.04 \times 10^{-19}$ | $-9.8 \times 10^{-19}$ |
| .8 | .4 | -1.5 | -.6 | .3 | $-1.57 \times 10^{-19}$ | $-4.6 \times 10^{-19}$ |
| -.3 | 1.5 | .1 | .5 | -1.2 | $-1.73 \times 10^{-19}$ | $-9.4 \times 10^{-20}$ |

Table caption: A sample illustration of the electric and chromoelectric dipole operator contributions to the electric dipole moment of the top quark. The inputs are: $m_{T}=350$, $\left|h_{3}\right|=100,\left|h_{4}\right|=175, m_{B}=100,\left|h_{5}\right|=190, m_{0}=200,\left|A_{0}\right|=200, \tilde{m}_{1}=50, \tilde{m}_{2}=100$, $\mu=150, \tilde{m}_{g}=450$ and $\tan \beta=5$ (top row), 30 (middle row), 40 (bottom row). All masses are in units of GeV and all angles are in radian.

Constraints on the top chromo EDM have been obtained using the combined CDF and DØ data and the CMS and ATLAS data on the total $t \bar{t}$ pair production cross section in $[36,37]$. Further, it is shown in $[38,39]$ that with $10 \mathrm{fb}^{-1}$ of data at $\sqrt{s}=14 \mathrm{TeV}$ at the

LHC a $5 \sigma$ statistical sensitivity to a top quark chromo electric dipole moment of about $5 \times 10^{-18} g_{s} . c m$ can be reached.

## 6 Conclusion

Currently the physics at the TeV scale is largely unknown and it is hoped that the LHC will provide us with an insight in this energy regime. It is fully conceivable that this energy regime contains extra anomaly free vector like quark multiplets which can mix with the third generation. In this work we analyze the effect of this mixing on the chromoelectric dipole moment of the top quark. In this case one finds that there are contributions that arise from the exchange of the extra vector like multiplets in the loops. We specifically focus on the exchange of the mirrors since their exchange can produce more dramatic contributions. Several sets of diagrams were computed for this analysis. These include the chargino exchange, the neutralino exchange, the gluino exchange as well as exchange of the W and the Z boson bosons. In the analysis new sources of CP violation enter. They arise from the complex mixing parameters of the third generation with the mirrors and from the soft parameter involving interactions of the third generation with the mirrors. Numerical analysis shows that an EDM as large as $10^{-18} \mathrm{ecm}$ can be obtained for the top quark from the electric and chromoelectric dipole contributions. An EDM of this size could be accessible in future experiments such as at the ILC.
Acknowledgments: We thank German Valencia for bringing to our attention the works of [38, 39]. This research is supported in part by NSF grant PHY-0757959 and PHY-0704067.

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