Interacting N=1 vector-spinor multiplet in 3D

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Interacting N=1 Vector - Spinor Multiplet in 3D

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Abstract

We present an $N = 1$ supersymmetric multiplet with a vector-spinor field in three dimensions. We call this the vector-spinor multiplet with the field content $(\psi_\mu, A_\mu, \lambda)$, where ψ_μ is a vector-spinor, A_μ is a vector, while λ is a gaugino. Based on on-shell component field formulation, we can accommodate $N = 1$ supersymmetric Dirac-Born-Infeld (SDBI) interactions consistently with supersymmetry. This is possible even in the presence of the vector-spinor. The ψ_μ-field equation contains non-trivial interaction term with A_μ. Moreover, it turns out that in the presence of mass terms, one physical degree of freedom in the original λ is transferred to that of ψ_μ, making the latter propagating. In other words, our model presents non-trivial rewriting of SDBI interaction in terms of (ψ_μ, A_μ) instead of (A_μ, λ).

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Key Words: N=1 Supersymmetry, Three Dimensions, Vector-Spinor Field, Spin 3/2, Vector Multiplet, Gaugino, Supersymmetric Dirac-Born-Infeld Action.

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1. Introduction

In four dimensions (4D), there have been supersymmetric formulations for multiplets with a vector-spinors ψ_μ with the minimal spin content $(3/2, 1)$ [1][2]. Here the important point is that the vector-spinor has *no* spin 2 superpartner, but has only spin 1 counterpart. In other words, the system has only global supersymmetry, but *no* local supersymmetry or supergravity. One of the ultimate aims is to establish the foundation for more general supersymmetric higher-spin interactions [3].

However, there have been so far no consistent interactions introduced for the $(3/2, 1)$ multiplet, at least in terms of component fields. This situation is understandable through the conventional wisdom that once spin 3/2 is introduced, there should be spin 2 graviton, resulting necessarily in supergravity for consistency. Another technical reasons is that the auxiliary field structure with off-shell superfields [1][2][4] is so involved that the corresponding formulations in component fields is impractically complicated. This is despite the fact that off-shell superfield formulation is powerful enough to present interaction lagrangians in superspace.

This problem appears to be easily solved in superspace. In superspace, the $(3/2, 1)$ multiplet is represented by a spinor superfield Ψ_α and a real scalar superfield V [1][2][4]. We can conjecture, for example, a total action to be $I \equiv I_1 + I_2$, where the free action I_1 [1] and the supersymmetric Dirac-Born-Infeld (SDBI) action I_2 [5][6] can be written down as

$$\begin{aligned} I_1 \equiv & \int d^8 z \left[(D^\alpha \overline{\Psi}^{\dot\beta})(\overline{D}_{\dot\beta} \Psi_\alpha) + \tfrac{1}{4}(\overline{D}^{\dot\beta} \Psi^\alpha)(\overline{D}_{\dot\beta} \Psi_\alpha) + \tfrac{1}{4}(D_\alpha \overline{\Psi}_{\dot\beta})(D^\alpha \overline{\Psi}^{\dot\beta}) + \Psi^\alpha W_\alpha + \overline{\Psi}^{\dot\alpha} \overline{W}_{\dot\alpha} \right] \\ & + \int d^6 z \left(\frac{1}{4} W^\alpha W_\alpha + \frac{1}{4} \overline{W}^{\dot\alpha} \overline{W}_{\dot\alpha} \right) \quad , \qquad (1.1) \end{aligned}$$

$$I_2 \equiv \int d^8 z \left[1 - \tfrac{1}{2}(K + \overline{K}) + \sqrt{1 - (K + \overline{K}) + \tfrac{1}{4}(K - \overline{K})^2} \right]^{-1} W^2 \, \overline{W}^2 \quad , \qquad (1.2)$$

where $K \equiv 2D^2(W^2)$, and we use the notation in [7]. The two actions I_1 and I_2 are invariant under the following Λ, K and Ω-gauge transformations [1]:

$$\begin{gathered} \delta_{\Lambda, K} \Psi_\alpha = \Lambda_\alpha + i \partial_{\alpha\dot\beta} D^2 \overline{D}^{\dot\beta} K \quad , \qquad \delta_\Omega V = +i(\Omega - \overline{\Omega}) \quad , \\ K = \overline{K} \quad , \quad V = \overline{V} \quad , \quad \overline{D}_{\dot\alpha} \Lambda_\beta = 0 \quad , \quad D_\alpha \overline{\Lambda}_{\dot\beta} = 0 \quad , \quad D_\alpha \overline{\Omega}_{\dot\beta} = 0 \quad , \quad \overline{D}_{\dot\alpha} \Omega_\beta = 0 \quad . \qquad (1.3) \end{gathered}$$

However, the drawback here is that unless we write down the explicit component total lagrangian after eliminating auxiliary fields, we can not easily see the total consistency of the whole system. There was a superspace lagrangian also proposed by Ogievetsky and

Sokatchev in [2], but its corresponding component total lagrangian has not been presented, to our knowledge. Unless we present the explicit component total lagrangian by eliminating auxiliary fields, we can not easily see the total consistency of the whole system. From this viewpoint, interaction lagrangians in terms of component fields is not just for curiosity, but it is for physical significance. We need to seek some supersymmetric system, in which the role of a vector-spinor is transparent in component language.

In this brief report, instead of addressing the problem directly in 4D, we study an analogous multiplet in three dimensions (3D), taking advantage of simplification of supersymmetry in 3D. We consider the multiplet $(\psi_\mu, A_\mu, \lambda)$,[3)] where ψ_μ is a vector-spinor, A_μ is a vector, and λ is a Majorana spinor.

There are two important ingredients about our vector-spinor ψ_μ in 3D. First, a *massless* vector-spinor ψ_μ has 0 on-shell degree of freedom (DOF), based on the conventional counting: $(D-3)\times 2^{[D/2]-1} = (3-3)\times 2^0 = 0$.[4)] However, as will be seen, when SDBI interactions are introduced, the original field equation $\mathcal{R}_{\mu\nu} \doteq 0$ for the vector-spinor field strength $\mathcal{R}_{\mu\nu}$ is no longer zero, but modified by SDBI interactions. Second, due to mass terms present, the counting should be for a *massive* vector-spinor: $(3-2)\times 2^0 = 1$. We will see in section 5 how this one DOF of ψ_μ is accounted for, transferred from our gaugino field. We will also see that the vector-spinor ψ_μ for a massive case has one propagating DOF.

2. Lagrangian and Transformation Rule

As has been mentioned, our multiplet is $(\psi_\mu, A_\mu, \lambda)$, where λ is a Majorana spinor as the superpartner of A_μ, while ψ_μ is a vector-spinor in the Majorana representation in 3D.

We start with free-fields with the action $I_0 \equiv \int d^3x\, \mathcal{L}_0$, where the lagrangian is:[5)]

$$\begin{aligned}\mathcal{L}_0 = &+ \tfrac{1}{2}\epsilon^{\mu\nu\rho}(\overline{\psi}_\mu \partial_\nu \psi_\rho) - \tfrac{1}{4}(F_{\mu\nu})^2 - \tfrac{1}{2}(\overline{\lambda}\gamma^\mu\partial_\mu\lambda) \\ &- \tfrac{1}{2}m(\overline{\psi}_\mu\gamma^{\mu\nu}\psi_\nu) - m(\overline{\psi}_\mu\gamma^\mu\lambda) + \tfrac{1}{4}m\epsilon^{\rho\sigma\tau}F_{\rho\sigma}A_\tau \quad . \end{aligned} \tag{2.1}$$

The first term is the kinetic term for the ψ_μ in 3D. The second line is for the mass term and related terms under supersymmetry, including the last Chern-Simons term.

[3)] From now on, we use the indices $\mu,\ \nu,\ \cdots = 0,\ 1,\ 2$ for the 3D space-time indices.

[4)] Here $[D/2]$ is the Gauss's symbol for the integer part of the real number $D/2$.

[5)] Our metric is $(\eta_{\mu\nu}) \equiv \text{diag.}\,(-,+,+)$. Accordingly, we have $\epsilon^{012} = +1,\ \gamma^{\mu\nu\rho} = +\epsilon^{\mu\nu\rho},\ \gamma^{\mu\nu} = +\epsilon^{\mu\nu\rho}\gamma_\rho,$ $\gamma^\mu = -(1/2)\epsilon^{\mu\rho\sigma}\gamma_{\rho\sigma},\ I = -(1/6)\epsilon^{\mu\rho\sigma}\gamma_{\mu\rho\sigma}$.

Our action I_0 is invariant under global $N=1$ supersymmetry[6)]

$$\delta_Q A_\mu = -(\overline{\epsilon}\psi_\mu) + (\overline{\epsilon}\gamma_\mu\lambda) \quad , \tag{2.2a}$$

$$\delta_Q \psi_\mu = +\tfrac{1}{2}\epsilon_\mu{}^{\rho\sigma} F_{\rho\sigma} = +\epsilon \widetilde{F}_\mu \quad , \tag{2.2b}$$

$$\delta_Q \lambda = -\tfrac{1}{2}(\gamma^{\mu\nu}\epsilon) F_{\mu\nu} = -(\gamma^\mu\epsilon)\widetilde{F}_\mu \quad . \tag{2.2c}$$

We use the Hodge-dual quantities, such as

$$\widetilde{F}_\mu \equiv +\tfrac{1}{2}\epsilon_\mu{}^{\rho\sigma} F_{\rho\sigma} \quad , \qquad \widetilde{\mathcal{R}}_\mu \equiv +\tfrac{1}{2}\epsilon_\mu{}^{\rho\sigma}\mathcal{R}_{\rho\sigma} \quad , \tag{2.3}$$

where $\mathcal{R}_{\mu\nu} \equiv \partial_\mu\psi_\nu - \partial_\nu\psi_\mu$ is the field strength of the vector-spinor ψ_μ. Note that there are two terms in the transformation rule $\delta_Q A_\mu$, as the non-trivial mixture of ψ_μ and λ.

We now consider possible interactions for this system. A typical interaction for the vector field is SDBI interaction [5]. However, in our system, the presence of the vector-spinor makes the superinvariance of the total action non-trivial. We use the real constant parameter α for SDBI terms, so that the total action is now $I \equiv I_0 + I_\alpha \equiv \int d^3x\, \mathcal{L} \equiv \int d^3x\, (\mathcal{L}_0 + \mathcal{L}_\alpha)$, where $\mathcal{L}_0$ is the same as (2.1), while $\mathcal{L}_\alpha$ gives our new interactions at $\mathcal{O}(\alpha)$:

$$\begin{aligned}\mathcal{L}_\alpha \equiv\; & +\tfrac{1}{4}\alpha(F_{\mu\nu}^2)^2 + \alpha\epsilon^{\mu\nu\rho}(\overline{\psi}_\mu\mathcal{R}_{\nu\rho})F_{\sigma\tau}^2 + 4\alpha(\overline{\psi}_\mu\widetilde{\mathcal{R}}^\mu)^2 \\ & + \alpha(\overline{\lambda}\partial\!\!\!/\lambda)(F_{\rho\sigma})^2 + \alpha\widetilde{F}_\mu\Big[\overline{\lambda}\gamma^\mu\partial\!\!\!/(\gamma^\nu\lambda\widetilde{F}_\nu)\Big] + \alpha(\overline{\lambda}\partial\!\!\!/\lambda)^2 + \tfrac{1}{4}\alpha(\overline{\lambda}\lambda)\partial_\mu^2(\overline{\lambda}\lambda) \quad .\end{aligned} \tag{2.4}$$

The first term is proportional to the usual DBI term $(F^4)_\mu{}^\mu - (1/4)(F_{\mu\nu}^2)^2$ [8], because of the particular feature in 3D.

The total action I is invariant up to $\mathcal{O}(\alpha^2)$ and $\mathcal{O}(m\alpha)$-terms, under the modified supersymmetry transformation with the $\mathcal{O}(\alpha)$-terms:

$$\delta_Q A_\mu = -(\overline{\epsilon}\psi_\mu) + (\overline{\epsilon}\gamma_\mu\lambda) - 2\,\alpha\,(\overline{\epsilon}\psi_\mu)(F_{\rho\sigma})^2 - 8\,\alpha\,(\overline{\epsilon}\psi_\mu)(\overline{\psi}_\nu\tilde{\mathcal{R}}^\nu) \quad , \tag{2.5a}$$

$$\delta_Q\psi_\mu = +\epsilon\widetilde{F}_\mu + 8\,\alpha\,\psi_\mu(\overline{\epsilon}\gamma^\rho\partial^\sigma\lambda)F_{\rho\sigma} + 2\alpha\epsilon\Big[\overline{\lambda}\gamma_\mu\partial\!\!\!/(\gamma_\nu\lambda F^\nu)\Big] + 8\alpha\epsilon(\overline{\lambda}\partial\!\!\!/\lambda)\widetilde{F}_\mu \quad , \tag{2.5b}$$

$$\delta_Q\lambda = -\tfrac{1}{2}(\gamma^{\mu\nu}\epsilon)F_{\mu\nu} = -(\gamma^\mu\epsilon)\widetilde{F}_\mu \quad . \tag{2.5c}$$

The λ-transformation rule is *not* modified at $\mathcal{O}(\alpha)$.

The field equations for our total action $I \equiv \int d^3x\,\mathcal{L}$ are

$$\frac{\delta\mathcal{L}}{\delta\overline{\psi}_\mu} = +\widetilde{\mathcal{R}}^\mu - m(\gamma^\mu\lambda) - m(\gamma^{\mu\nu}\psi_\nu) - 2\alpha\epsilon^{\mu\rho\sigma}\psi_\rho\partial_\sigma(F_{\tau\lambda}^2) \doteq 0 \quad , \tag{2.6a}$$

[6)] This transformation rule will be modified by interaction terms at $\mathcal{O}(\alpha)$ later in (2.5).

$$\frac{\delta \mathcal{L}}{\delta \overline{\lambda}} = -\not{\partial}\lambda + m(\gamma^\mu \psi_\mu)$$

$$- 2\alpha(\gamma^\mu\lambda)\partial_\mu(\widetilde{F}_\nu^2) + 2\alpha\widetilde{F}_\mu\gamma^\mu\not{\partial}(\gamma^\nu\lambda\widetilde{F}_\nu) + 2\alpha\lambda(\partial_\mu\lambda)(\partial^\mu\lambda) \doteq 0 \ , \tag{2.6b}$$

$$\frac{\delta \mathcal{L}}{\delta A_\mu} = -\partial_\nu F^{\mu\nu} + m\widetilde{F}^\mu - 4\alpha\epsilon^{\tau\nu\rho}\partial_\sigma\Big[F^{\sigma\mu}(\overline{\psi}_\tau\mathcal{R}_{\nu\rho})\Big]$$

$$+ \epsilon^{\mu\rho\sigma}\partial_\rho\Big[+4\alpha\widetilde{F}_\sigma\widetilde{F}_\tau^2 - 4\alpha(\overline{\lambda}\not{\partial}\lambda)\widetilde{F}_\sigma + 2\alpha\overline{\lambda}\gamma_\sigma\not{\partial}(\gamma^\nu\lambda\widetilde{F}_\nu)\Big] \tag{2.6c}$$

$$\doteq -\partial_\nu F^{\mu\nu} + m\widetilde{F}^\mu - 4\alpha\epsilon^{\mu\rho\sigma}\widetilde{F}_\rho\partial_\sigma(\widetilde{F}_\tau^2) + 4\alpha(\overline{\lambda}\gamma^{\mu\nu}\partial_\rho\lambda)\partial_\nu\widetilde{F}^\rho$$

$$+ 4\alpha(\partial^\mu\overline{\lambda})(\partial_\nu\lambda)\widetilde{F}^\nu - 4\alpha(\overline{\lambda}\partial^\mu\partial_\nu\lambda)\widetilde{F}^\nu + \mathcal{O}(\alpha^2, \alpha m) \doteq 0 \ , \tag{2.6d}$$

where $\mathcal{O}(\alpha^2, \alpha m)$ are either the terms with α^2 or αm. The latter is ignored, because we have confirmed our action invariance only up to $\mathcal{O}(\alpha^2)$ or $\mathcal{O}(\alpha m)$-terms.

The expression (2.6c) is directly from the lagrangian variation, while (2.6d) is simplification by using field equations. These are equivalent to each other up to field equations at $\mathcal{O}(\alpha, m)$. In other words, they are 'on-shell' equivalent up to $\mathcal{O}(\alpha^2, \alpha m)$-terms. For example, the $\mathcal{R}$-term in the first line (2.6c) with α in front is essentially at $\mathcal{O}(\alpha^2, \alpha m)$, because $\mathcal{R}_{\mu\nu} \doteq \mathcal{O}(\alpha, m)$. The $\mathcal{O}(\alpha^0)$-field equations that can be used for the α-terms are such as

$$\partial_{[\mu}\widetilde{F}_{\nu]} \doteq \mathcal{O}(\alpha, m) \ , \quad \not{\partial}\lambda \doteq \mathcal{O}(\alpha, m) \ , \quad \mathcal{R}_{\mu\nu} \doteq \mathcal{O}(\alpha, m) \ . \tag{2.7}$$

3. Consistency of Interactions

As the consistency confirmation of our total system, we first investigate the supersymmetry transformations of ψ_μ and λ-field equations (2.6a) and (2.6b).

The supersymmetric variation of the vector-spinor field equation is

$$0 \overset{?}{\doteq} + \delta_Q\left(\frac{\delta\mathcal{L}}{\delta\overline{\psi}_\mu}\right) = \delta_Q\left[+\mathcal{R}^\mu - m(\gamma^\mu\lambda) - m(\gamma^{\mu\nu}\psi_\nu) - 2\alpha\epsilon^{\mu\rho\sigma}\psi_\rho\partial_\sigma(F_{\tau\lambda}^2)\right]$$

$$\doteq + \epsilon^{\mu\rho\sigma}\partial_\rho\left[\overline{\epsilon}\widetilde{F}_\sigma + m\epsilon\widetilde{F}_\sigma + 8\alpha\psi_\sigma(\overline{\epsilon}\gamma^\tau\partial_\lambda\lambda)F_\tau{}^\lambda + 2\alpha\epsilon\overline{\lambda}\gamma_\sigma\not{\partial}(\gamma_\nu\lambda\widetilde{F}^\nu)\right] + 4\alpha\epsilon^{\mu\rho\sigma}(\overline{\epsilon}\widetilde{F}_\rho)\partial_\sigma(\widetilde{F}_\tau{}^2)$$

$$- 4\alpha\epsilon^{\mu\rho\sigma}\psi_\rho\left[\partial_\sigma(-2\overline{\epsilon}\gamma_\tau\partial_\lambda\lambda)\right]F^{\tau\lambda} - 4\alpha\epsilon^{\mu\rho\sigma}\psi_\rho(-2\overline{\epsilon}\gamma_\tau\partial_\lambda\lambda)\partial_\sigma F^{\tau\lambda} + \mathcal{O}(\alpha^2, \alpha m) \ . \tag{3.1}$$

Here we have used the field equations (2.7) up to $\mathcal{O}(\alpha^2, \alpha m)$-terms. For the A_μ-field equation used for the fist term in (3.1), we have to include $\mathcal{O}(\alpha^0)$, $\mathcal{O}(\alpha)$ and $\mathcal{O}(m)$-terms in (2.6d), while for other terms with α, we need only the field equations (2.7) at $\mathcal{O}(\alpha^0)$. After these manipulations, we get all the remaining terms cancel each other, as desired.

In a similar fashion, we can confirm the vanishing of the supersymmetry transformation of the λ-field equation:

$$
\begin{aligned}
0 \overset{?}{\doteq} & - \delta_Q \left(\frac{\delta \mathcal{L}}{\delta \overline{\lambda}} \right) \\
\doteq & \ \delta_Q \Big[\{ \partial\!\!\!/ \lambda - m(\gamma^\mu \psi_\mu) \} + 2\alpha(\gamma^\mu \lambda)\partial_\mu(\widetilde{F}_\nu{}^2) - 2\alpha \widetilde{F}{}_\mu \gamma^\mu \partial\!\!\!/ (\gamma^\nu \lambda \widetilde{F}_\nu) - 2\alpha\lambda(\partial_\mu \overline{\lambda})(\partial^\mu \lambda) \Big] \qquad (3.2) \\
\doteq & \ + \Big[- 4\alpha(\gamma_\rho{}^\sigma \epsilon)\widetilde{F}{}^\rho \partial_\sigma(\widetilde{F}_\tau{}^2) + 4\alpha(\gamma^\mu\epsilon)(\partial_\mu\overline{\lambda})(\partial_\nu\lambda)\widetilde{F}{}^\nu - 4\alpha(\gamma_\mu\epsilon)(\overline{\lambda}\partial_\mu\partial_\nu\lambda)\widetilde{F}{}^\nu \\
& \qquad + 4\alpha(\gamma^\mu\epsilon)(\overline{\lambda}\gamma_{\mu\nu}\partial_\rho\lambda)\partial^\nu\widetilde{F}{}^\rho \Big] \\
& \ + \Big[+ 2\alpha(\gamma^{\rho\sigma}\epsilon\widetilde{F}_\rho)\partial_\sigma(\widetilde{F}_\tau{}^2) - 2\alpha\epsilon\widetilde{F}{}^\mu\partial_\mu(\widetilde{F}_\nu{}^2) + 4\alpha(\gamma_\mu\epsilon)(\overline{\lambda}\partial^\mu\partial_\nu\lambda)\widetilde{F}{}^\nu - 2\alpha\epsilon(\overline{\lambda}\gamma_\mu\partial_\nu\lambda)\partial_\mu\widetilde{F}_\nu \\
& \qquad + 2\alpha(\gamma_\rho\epsilon)(\overline{\lambda}\gamma^{\mu\rho}\partial_\nu\lambda)\partial_\mu\widetilde{F}{}^\nu + 2\alpha(\gamma^\mu\epsilon)(\overline{\lambda}\partial_\nu\lambda)\partial_\mu\widetilde{F}{}^\nu \Big] \\
& \ + \Big[+ 2\alpha(\gamma^{\rho\sigma}\epsilon)\widetilde{F}_\rho\partial_\sigma(\widetilde{F}_\tau{}^2) + 2\alpha\epsilon\widetilde{F}{}_\mu\partial^\mu(\widetilde{F}_\nu{}^2) - 4\alpha(\gamma^\mu\epsilon)(\partial_\mu\overline{\lambda})(\partial_\nu\lambda)\widetilde{F}{}^\nu \\
& \qquad - 2\alpha(\gamma^\mu\epsilon)(\partial_\nu\overline{\lambda})(\partial^\nu\lambda)\widetilde{F}{}^\mu \Big] \\
& \ + \Big[+ 2\alpha(\gamma^\mu\epsilon)(\partial_\nu\overline{\lambda})(\partial^\nu\lambda)\widetilde{F}_\mu - 2\alpha(\gamma_\mu\epsilon)(\overline{\lambda}\partial_\nu\lambda)\partial^\nu\widetilde{F}{}^\mu \\
& \qquad + 2\alpha(\gamma^\rho\epsilon)(\overline{\lambda}\gamma_{\nu\rho}\partial_\mu\lambda)\partial^\mu\widetilde{F}{}^\nu + 2\alpha\epsilon(\overline{\lambda}\gamma^\mu\partial_\nu\lambda)\partial_\mu\widetilde{F}{}^\nu \Big] + \mathcal{O}(\alpha^2, \alpha m) \ \ , \qquad (3.3)
\end{aligned}
$$

where there are four pairs of square brackets, each of which represents the variation of the four terms in (3.2), respectively. (The 'first' term implies the pair of braces.) After using the $\mathcal{O}(\alpha^0)$-field equations (2.7), we see that all the terms in (3.3) cancel each other. A useful lemma here is

$$
\delta_Q F_{\mu\nu} \doteq - 2(\overline{\epsilon}\gamma_{[\mu}\partial_{\nu]}\lambda) + m(\overline{\epsilon}\gamma_{\mu\nu}\lambda) + 2m(\overline{\epsilon}\gamma_{[\mu}\psi_{\nu]}) + \mathcal{O}(\alpha^2, \alpha m) \ \ , \qquad (3.4)
$$

without $\mathcal{O}(\alpha)$-term. In other words, the δ_Q-transformation of $F_{\mu\nu}$ has *no* $\mathcal{O}(\alpha)$ modification. This feature is useful, because essentially the transformation structure of the λ-field equation is exactly the same as the conventional $N = 1$ SDBI system [5].

We next study the consistency of A_μ and ψ_μ-field equations with divergences, *i.e.,* $\partial_\mu(\delta\mathcal{L}/\delta A_\mu) \overset{?}{\doteq} 0$ and $\partial_\mu(\delta\mathcal{L}/\delta\psi_\mu) \overset{?}{\doteq} 0$. The former is just the $U(1)$-gauge invariance, while the latter is similar to the gravitino field equation consistency for supergravity [9]. For the former, we use (2.6d):

$$
\begin{aligned}
0 \overset{?}{\doteq} \partial_\mu \left(\frac{\delta\mathcal{L}}{\delta A_\mu} \right) \doteq & \ + \partial_\mu \Big[- \partial_\nu F^{\mu\nu} + m\widetilde{F}{}^\mu - 4\alpha\epsilon^{\mu\rho\sigma}\widetilde{F}{}_\rho\partial_\sigma(\widetilde{F}_\tau{}^2) \\
& \qquad + 4\alpha(\partial^\mu\overline{\lambda})(\partial_\nu\lambda)\widetilde{F}{}^\nu - 4\alpha(\overline{\lambda}\partial^\mu\partial_\nu\lambda)\widetilde{F}{}^\nu + 4\alpha(\overline{\lambda}\gamma^{\mu\nu}\partial_\rho\lambda)\partial_\nu\widetilde{F}{}^\rho + \mathcal{O}(\alpha^2) \Big] \\
\doteq & \ - 4\alpha\epsilon^{\mu\rho\sigma}(\partial_\mu\widetilde{F}{}_\rho)\partial_\sigma(\widetilde{F}_\tau{}^2)
\end{aligned}
$$

$$
+ 4\alpha(\partial_\mu^2\overline{\lambda})(\partial_\nu\lambda)\widetilde{F}^\nu + 4\alpha(\partial_\mu\lambda)(\partial^\mu\partial_\nu\lambda)\widetilde{F}^\nu + 4\alpha(\partial_\mu\overline{\lambda})(\partial_\nu\lambda)\partial^\mu\widetilde{F}^\nu
$$

$$
- 4\alpha(\partial_\mu\overline{\lambda})(\partial^\mu\partial_\nu\lambda)\widetilde{F}^\nu - 4\alpha(\overline{\lambda}\partial_\mu^2\partial^\nu\lambda)\widetilde{F}_\nu - 4\alpha(\overline{\lambda}\partial_\mu\partial_\nu\lambda)\partial^\mu\widetilde{F}^\nu
$$

$$
+ 4\alpha(\partial_\mu\overline{\lambda})\gamma^{\mu\nu}(\partial_\rho\lambda)\partial_\nu\widetilde{F}^\rho + 4\alpha(\overline{\lambda}\gamma^{\mu\nu}\partial_\mu\partial_\rho\lambda)\partial_\nu\widetilde{F}^\rho + \mathcal{O}(\alpha^2, \alpha m) \ . \tag{3.5}
$$

After using $\mathcal{O}(\alpha^0)$-field equations (2.7), we see that all the terms in (3.5) cancel each other, leaving only $\mathcal{O}(\alpha^2, m)$-terms.

For the consistency of the ψ_μ-field equation, we have a simpler structure:

$$
\begin{aligned}
0 \overset{?}{\doteq} \partial_\mu \left(\frac{\delta \mathcal{L}}{\delta \overline{\psi}_\mu} \right) &= + \partial_\mu \left[+ \widetilde{\mathcal{R}}^\mu - m(\gamma^\mu\lambda) - m(\gamma^{\mu\nu}\psi_\nu) - 2\alpha\epsilon^{\mu\rho\sigma}\psi_\rho\,\partial_\sigma(\widetilde{F}^{\,2}_{\tau\lambda}) \right] \\
&\doteq + 2\alpha\epsilon^{\mu\rho\sigma}\mathcal{R}_{\mu\rho}\,\partial_\sigma(\widetilde{F}^{\,2}_\tau) \doteq \mathcal{O}(\alpha^2, \alpha m) \ ,
\end{aligned} \tag{3.6}
$$

where due to $\mathcal{R}_{\mu\nu} \doteq \mathcal{O}(\alpha, m)$, all the terms are eventually at $\mathcal{O}(\alpha^2, \alpha m)$, as desired.

Note that the vector-spinor field equation (2.6a) has a non-trivial interaction term which does *not* vanish on-shell by itself, but it satisfies the consistency condition (3.6). Before the discovery of supergravity [9], the possible inconsistency for such divergences was known as Velo-Zwanziger disease [10]. From this viewpoint, it is quite non-trivial that our vector-spinor field equation (2.6a) explicitly satisfies the consistency conditions without local supersymmetry.

In supergravity in 3D [11][12], even though the massless gravitino or graviton field has *no* physical DOF in 3D, their field equations still play important roles, when coupled to matter multiplet [11][12]. In a similar fashion, our vector-spinor field plays a significant role, when coupled to the vector and gaugino, accompanied by SDBI interactions. This situation is further elaborated due to the presence of mass terms, as we will see in section 5.

4. SDBI Interaction in Terms of Vector-Spinor

We have so far not counted the real DOF for the vector-spinor. When a vector-spinor is massless, its physical DOF is $(3-3)\times 1 = 0$, since three components for the index μ is to be subtracted. This is because not only the usual 2 longitudinal components, but also the γ-trace component should be subtracted.

However, for a massive vector-spinor, the counting should be $(3-2)\times 1 = 1$. Hence, there must be one physical degree of freedom carried by ψ_μ. We can understand this in terms of ψ_μ and λ-field equations (2.6a) and (2.6b). We first multiply (2.6a) by γ_μ or applying ∂_μ, getting two equations

$$+ (\gamma_\mu \widetilde{\mathcal{R}}^\mu) - 3m\lambda - 2m(\gamma^\nu \psi_\nu) \doteq \mathcal{O}(\alpha) \ , \tag{4.1a}$$

$$+ m(\partial\!\!\!/ \lambda) + m(\gamma^\mu \widetilde{\mathcal{R}}_\mu) \doteq \mathcal{O}(\alpha) \ . \tag{4.1b}$$

From these equations, we can eliminate $(\gamma_\mu \widetilde{\mathcal{R}}^\mu)$, as

$$+ (\gamma_\mu \widetilde{\mathcal{R}}^\mu) \doteq - (\partial\!\!\!/ \lambda) + \mathcal{O}(\alpha) \ , \tag{4.2a}$$

$$+ (\partial\!\!\!/ \lambda) + 3m\lambda + 2m(\gamma^\mu \psi_\mu) \doteq \mathcal{O}(\alpha) \ . \tag{4.2b}$$

Adding (4.2b) to (2.6b), we get

$$+ \lambda \doteq - (\gamma^\mu \psi_\mu) + \mathcal{O}(\alpha) \ . \tag{4.3}$$

Thus the λ-field is expressed entirely in terms of the vector-spinor ψ_μ. Actually, (4.3) is also consistent with supersymmetry transformation (2.5b) and (2.5c). In other words, it is no longer the λ-field that is fundamental, but ψ_μ becomes the fundamental field.

Combining (4.3) with (4.2b), we get

$$+ (\partial\!\!\!/ \lambda) + m\lambda \doteq \mathcal{O}(\alpha) \ . \tag{4.4}$$

Namely, $\lambda \doteq - (\gamma^\mu \psi_\mu)$ is one propagating DOF. Relevantly, (4.3) with (2.6a) implies that

$$\widetilde{\mathcal{R}}_\mu \doteq - m\psi_\mu + \mathcal{O}(\alpha) \ , \tag{4.5}$$

the divergence of which leads to

$$0 \equiv \partial_\mu \widetilde{\mathcal{R}}^\mu + m \, \partial_\mu \psi^\mu \doteq \mathcal{O}(\alpha, m) \qquad \Longrightarrow \qquad \partial_\mu \psi^\mu \doteq \mathcal{O}(\alpha) \ . \tag{4.6}$$

Eq. (4.6) is analogous to $\partial_\mu A^\mu \doteq 0$ for the massive vector field equation $\partial_\nu F^{\mu\nu} - m^2 A_\mu \doteq 0$.

All of these equations mean that our vector-spinor is now massive and propagating with $(3-2)\times 1 = 1$ DOF, instead of the massless case $(3-3)\times 1 = 0$. Originally, the fields in our multiplet $(\psi_\mu, A_\mu, \lambda)$ have respectively $(0, 1, 1)$ DOF. However, when the mass term is added, the one DOF of λ is transferred to the vector-spinor ψ_μ, making it massive and propagating.

The crucial aspect of our model is that the conventional SDBI action in terms of the vector multiplet (A_μ, λ) has been completely re-written in terms of the new multiplet $(\psi_\mu, A_\mu, \lambda)$.

In particular, the vector-spinor field satisfying the consistency condition without any problem with $N = 1$ supersymmetry.

Moreover, we can rewrite $\lambda = -(\gamma^\mu \psi_\mu) + \mathcal{O}(\alpha)$ everywhere in the field equations (2.6), still maintaining supersymmetric covariance of these field equations. To put it differently, we can present SDBI interaction entirely in terms of the multiplet (ψ_μ, A_μ), instead of the conventional (A_μ, λ).

5. Concluding Remarks

In this brief report, we have shown how to introduce non-trivial interaction to the multiplet $(\psi_\mu, A_\mu, \lambda)$ with a vector-spinor in 3D. We have seen that the SDBI terms can be accommodated into the multiplet consistently with global $N = 1$ supersymmetry.

We have seen highly non-trivial structure for the supersymmetry transformations of ψ_μ and λ-field equations, that are all consistent with all other field equations under $N = 1$ supersymmetry (2.5). We have also confirmed the divergence of the A_μ and ψ_μ-field equations, and they vanish up to $\mathcal{O}(\alpha^2, m)$-terms, as desired. The latter confirmation is well known for the gravitino field equation in supergravity (local supersymmetry). However, we have established similar consistency for our vector-spinor, *without* local supersymmetry.

It has been well-known that a *massless* vector-spinor has *no* DOF in 3D by simple counting $(3-3) \times 1 = 0$. However, as supergravity theories in 3D indicates [11], the gravitino field equation plays important role, when it is coupled to matter fields. By the same token, our vector-spinor field equation plays an important role, when coupled to the vector and gaugino with non-trivial SDBI interactions. This feature is crystalized in the non-trivial coupling in the ψ_μ-field equation (2.6a).

We have shown moreover that the original system can be re-expressed only in terms of (ψ_μ, A_μ), because of the on-shell relationship $\lambda \doteq -(\gamma^\mu \psi_\mu) + \mathcal{O}(\alpha)$, when mass terms are present. In other words, the original 1 DOF of λ is transferred to that of ψ_μ. This is consistent with the counting $(3-2) \times 1 = 1$ for a *massive* vector-spinor in 3D. Consequently, the conventional SDBI interactions in terms of (A_μ, λ) can be re-expressed in terms of (ψ_μ, A_μ), including the vector-spinor ψ_μ.

Our non-trivial interaction of the vector-spinor is to be emphasized, because the conventional wisdom tells us that once a vector-spinor (spin 3/2) is introduced into a system, supergravity or local supersymmetry with graviton (spin 2) is inevitable. Even though our success might be attributed to the special feature of 3D, we can also regard that our results indicate the encouraging aspect of 3D, where we can study the non-trivial couplings of vector-spinor without introducing a graviton or spin 2 field.

Our system in 3D can serve as the testing ground for the study of non-trivial interactions of spin $(3/2, 1)$ multiplet in 4D [1][2], where auxiliary field structure in component language is considerably involved.

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