Anomaly mediation from the Randall-Sundrum to the Dine-Seiberg formalism

David Sanford and Yuri Shirman

Anomaly Mediation from Randall-Sundrum to Dine-Seiberg

David Sanford and Yuri Shirman

Department of Physics, University of California, Irvine, CA 92697.
dsanford@uci.edu, yshirman@uci.edu

Abstract

In this paper we reconsider the derivation of anomaly mediated supersymmetry breaking. We work in a general formalism where the F-term of the conformal compensator superfield is arbitrary. This allows for a continuous interpolation between the original derivation and a more recent Dine-Seiberg derivation of anomaly mediation. We show that the physical soft parameters are independent of the compensator F-term and results of two formalisms agree. Finally, we discuss the role of supersymmetric and non-supersymmetric thresholds in the effective low energy Lagrangian as well as the effects of explicit small mass parameters (such as μ-term) on the superpartner spectrum.

1   Introduction

We are now at the beginning of the era when Large Hadron Collider (LHC) at CERN will probe origin of electroweak symmetry breaking and new physics at the TeV scale. Among the many proposed scenarios of interest in coming years, supersymmetry (SUSY) at the TeV scale represents one of the best motivated extensions of the Standard Model promising a solution to the gauge hierarchy problem and natural unification of gauge couplings. Nevertheless, many supersymmetric models must be fine-tuned to avoid constrants imposed by the existing experimental data. Of these, one of the strongest arises from stringent bounds on flavor changing neutral currents (FCNCs). General SUSY-breaking effects result in FCNCs significantly above current constraints, so suppression of these effects must be addressed in SUSY breaking scenarios. There are several SUSY breaking mechanisms which naturally suppress FCNCs, including gauge, gaugino, and anomaly mediated supersymmetry breaking.

Anomaly mediated supersymmetry breaking (AMSB) in particular has been a subject of active investigation. The minimal version of AMSB is very predictive and insensitive to the details of the ultraviolet (UV) physics, leading to distinct experimental signals [1, 2], however it suffers from negative slepton mass squared problem which is difficult to solve without reintroducing UV sensitivity [3–12]. Finally, the study of formal aspects of AMSB has been motivated by its somewhat mysterious nature. Indeed, in contrast to gauge or gaugino mediated scenarios, one cannot easily draw Feynman diagrams that generate AMSB soft terms in the Lagrangian. This explains why the original papers [1, 2] were followed by a number of attempts to clarify the dynamics underlying the AMSB mechanism, most notably a detailed study of the origin of gaugino masses in [13].

More recently a new perspective on AMSB was provided by Dine and Seiberg [14], who clarified several physical aspects of AMSB. In particular, they demonstrated that mediation of SUSY breaking is not a consequence of any anomaly of the theory – rather it is the reliance of original *derivation* on anomalous symmetries that led to a name for AMSB. At the first glance the Randall-Sundrum (RS) and Dine-Seiberg (DS) formalisms appear to be quite different. For example, the RS derivation is performed within globally supersymmetric effective field theory with SUSY breaking introduced through a non-supersymmetric regulator. In contrast, the origin of SUSY breaking is more intuitive in the DS derivation, with soft parameters arising entirely due to non-vanishing F-terms of light dynamical fields in the theory. Moreover, the RS derivation implicitly neglects all small supersymmetric mass terms in the Lagrangian since the SUSY breaking effects are introduced at the UV cutoff; on the other hand, the DS results are most easily obtained in the Higgs phase and small supersymmetric masses appear to become important because they affect F-terms of the light fields. In fact, it has been argued in the literature [15] that the RS and DS formalisms are not equivalent and at least in some cases lead to different predictions for soft terms.

In this paper we attempt to better understand the physics underlying the AMSB mechanism. To this end we will study the full supergravity Lagrangian with an arbitrary compensator F-term F_ϕ, which will allow us to interpolate between the RS and DS formalisms. We will show that perturbatively generated soft terms are indeed independent of F_ϕ and identical results are obtained in both the RS and DS limits. We will also extend the analysis of the role of non-supersymmetric thresholds [3, 4] to the general formalism in the context of a complete SUGRA Lagrangian, and elaborate on the effects of small supersymmetric mass

parameters (such as a μ-term) on the soft terms in the low energy effective Lagrangian. In particular, we will show that electroweak scale μ-term can only result in small corrections to the AMSB predictions. Even these small corrections are largely accounted for when a careful renormalization group evolution using MSSM Lagrangian is performed from the scales above the gravitino mass down to the TeV scale.

In section Sec. 2 we will begin by reviewing the full supergravity Lagrangian in the presence of a conformal compensator superfield Φ. We will then write down the scalar potential without integrating out Φ. In section Sec. 3 we will derive soft masses with an arbitrary F_ϕ for a toy model with non-abelian gauge group. We calculate gaugino masses in Sec. 3.1, and scalar masses in Sec. 3.2, and show that the results agree with the original AMSB derivation. In Sec. 4 we will review the discussion [3, 4] of thresholds in the effective lagrangian and extend it to a formalism with an arbitrary F_ϕ. In particular we will study supersymmetric versus non-supersymmetric and field-dependent versus field-independent thresholds. We will argue that small mass parameters, even if they have a supersymmetric origin, always lead to non-supersymmetric thresholds and their consequences must be studied in the presence of relevant AMSB soft terms. We will aply these arguments to explain the effects of the MSSM μ-term (whether of supersymmetric or non-supersymmetric origin) on the superpartner spectrum. In Sec. 5 we present our concluding remarks.

2   Conformal Compensator and the Scalar Potential

In this section we will briefly review the super-Weyl symmetry of the supergravity action and, following [16], introduce the conformal compensator $\Phi = \phi + F_\phi \theta^2$ in the Lagrangian. Our goal here is to write the scalar potential in a way which allows us to interpolate between the RS limit ($F_\phi = m_{3/2}$) and the DS limit ($F_\phi = 0$). As a consequence of the super-Weyl symmetry the scalar potential is invariant under shifts in F_ϕ. However, we will demonstrate that the F-terms of matter fields are *not* invariant under shift in F_ϕ, which allows for a transition between the physically equivalent RS and DS limits.

The supergravity Lagrangian has the form [1]

$$\mathcal{L} = \int d\Theta^2 2\mathcal{E} \left[\frac{3}{8} M_{\mathrm{pl}}^2 \left(\bar{\mathcal{D}}^2 - 8\mathcal{R} \right) |\Phi|^2 e^{-K/3M_{\mathrm{pl}}^2} + \Phi^3 W + \tau \mathcal{W}_\alpha \mathcal{W}^\alpha \right] + h.c. \qquad (1)$$

Here K is the Kähler potential, W is the superpotential, $\mathcal{W}$ is the supersymmetric field strength for any gauge multiplet in the theory with coupling τ, $\mathcal{R}$ is the supersymmetric Ricci scalar, $\mathcal{E}$ is the determinant of the supersymmetric vierbein, and Θ is the superspace coordinate. The superfield Φ has been introduced into the supergravity Lagrangian through the Kähler transformation

$$K \to K - 3M_{\mathrm{pl}}^2 \left(\ln \Phi + \ln \bar{\Phi} \right) \qquad W \to \Phi^3 W \ . \qquad (2)$$

The Lagrangian does not contain kinetic terms for Φ, thus it is a purely auxiliary chiral superfield. The role of Φ is to formally restore super-Weyl symmetry of the Lagrangian [16], which is broken both by the anomaly in super-Weyl transformations as well as explicitly by

---

[1]We use the notation of Wess & Bagger [17].

mass terms in the Lagrangian. Thus Φ is usually referred to as a conformal compensator. Explicitly, under a super-Weyl transformation parameterized by an arbitrary chiral superfield ξ,

$$
\begin{aligned}
\mathcal{E} &\rightarrow e^{6\xi}\mathcal{E} \\
\mathcal{E}\left(\bar{\mathcal{D}}^2 - 8\mathcal{R}\right) &\rightarrow \mathcal{E}\left(\bar{\mathcal{D}}^2 - 8\mathcal{R}\right)e^{2\left(\xi+\bar{\xi}\right)} \\
\Phi &\rightarrow e^{-2\xi}\Phi \\
\mathcal{W} &\rightarrow e^{-3\xi}\mathcal{W} \\
X &\rightarrow X \ ,
\end{aligned}
\tag{3}
$$

where X is an arbitrary chiral superfield.

The breaking of super-Weyl invariance in realistic supergravity theories can be parameterized by the vacuum expectation value (vev) of the scalar component of the chiral compensator, $\langle\phi\rangle$, while its θ^2 component is usually set to zero. Instead we will keep the compensator F-term, F_ϕ, as a free parameter to interpolate between the RS and DS limits [2].

Gaugino masses were obtained in [1] with the help of super-Weyl rescaling of chiral superfields in the low energy (global) SUSY Lagrangian,

$$
X\Phi \rightarrow X \ .
\tag{4}
$$

We will perform the same rescaling[3] but will work with the full SUGRA Lagrangian. As a result of this rescaling, matter superfields transform as $X \rightarrow e^{-2\xi}X$. While the transformation in Eq. (4) removes the compensator from all dimension 4 terms in the Lagrangian, it does not eliminate Φ completely. Rather, all the explicit mass parameters are now accompanied by appropriate factors of Φ. For instance, for a superpotential of the form

$$
W(X) = X^3 + M_1 X^2 + M_2^2 X \ ,
\tag{5}
$$

this rescaling produces

$$
\Phi^3 W(X) \rightarrow W(X,\Phi) = X^3 + (M_1\Phi)X^2 + (M_2\Phi)^2 X \ .
\tag{6}
$$

A similar relation holds for the Kähler potential, $|\Phi|^2 K(X,\bar{X}) \rightarrow K(X,\bar{X},\Phi,\bar{\Phi})$, with factors of Φ and $\bar{\Phi}$ appearing alongside mass terms in the Kähler potential.

As shown in the App. A, the transformation in Eq. (4) modifies the superspace derivatives according to

$$
D_i W = W_i + K_i \frac{W}{M_{\text{pl}}^2 |\phi|^2} + e^{-K/3M_{\text{pl}}^2 |\phi|^2}\bar{\phi}F_{\bar{\phi}}\partial_{\bar{\phi}}\left(\frac{K_i}{\bar{\phi}}\right) \ .
\tag{7}
$$

---

[2]We stress that there is no physical significance to different values of F_ϕ.

[3]Strictly speaking in an interacting quantum Lagrangian non-trivial anomalous dimensions of the matter fields must be taken in the account when performing this rescaling.

This introduces both explicit and implicit dependence of the scalar potential on F_ϕ,

$$V_{\text{scalar}} = e\, e^{K/3M_{\text{pl}}^2|\phi|^2} \left[K_{ij^*}^{-1} D_i W \bar{D}_{j^*} \bar{W} - 3\frac{|W|^2}{M_{\text{pl}}^2|\phi|^2} \right]$$
$$+ 3e M_{\text{pl}}^2|\phi|^2 e^{-K/3M_{\text{pl}}^2|\phi|^2} |F_\phi|^2 \partial_\phi \partial_{\bar\phi} \left[\ln|\phi|^2 - \frac{K}{3M_{\text{pl}}^2|\phi|^2} \right] \tag{8}$$
$$- e\left[F_\phi \partial_\phi W - 3W F_\phi \partial_\phi \left(\ln|\phi|^2 - \frac{K}{3M_{\text{pl}}^2|\phi|^2} \right) + h.c. \right] .$$

The implicit dependence on F_ϕ enters through the superspace derivatives $D_i W$, while explicit dependence is present because the conformal compensator is the only auxiliary field that has not been integrated out.

Before using Eq. (8) to interpolate between the RS and DS limits, we must explicitly break super-Weyl invariance by setting $\langle\phi\rangle=1$. This is necessary to produce a realistic low-energy Lagrangian, since the physical theory does not exhibit scale invariance. The superspace derivative becomes

$$D_i W = W_i + K_i \frac{W}{M_{\text{pl}}^2} - e^{-K/3M_{\text{pl}}^2} F_{\bar\phi} K_{ij^*} X^{*j} = D_i^{\langle 0\rangle} W - e^{-K/3M_{\text{pl}}^2} K_{ij^*} X^{*j} F_{\bar\phi} , \tag{9}$$

where $D_i^{\langle 0\rangle} W = W_i + K_i W/M_{\text{pl}}^2$ is the standard expression for the superspace derivative. To obtain canonical kinetic terms for gravity multiplet, a standard Weyl rescaling must be performed [4]. After this rescaling, the full scalar potential takes the form

$$V_{\text{scalar}} = e^{K/M_{\text{pl}}^2} \left[K_{ij^*}^{-1} D_i W \bar{D}_{j^*} \bar{W} - 3\frac{|W|^2}{M_{\text{pl}}^2} \right] - e^{K/3M_{\text{pl}}^2} |F_\phi|^2 K_{ij^*} X^i X^{*j}$$
$$+ e^{2K/3M_{\text{pl}}^2} \left[F_\phi X^i D_i^{\langle 0\rangle} W + h.c. \right] , \tag{10}$$

where we have dropped the factor of $e = \det e_m^\mu$ for simplicity and we used the fact that derivatives of K with respect to Φ can be expressed in terms of K_i, K_{ij^*}, etc (see App. B). Moreover, since the rescaled superpotential is a homogeneous function of chiral superfields (including the compensator Φ) of order three, the relation

$$3W - X^i W_i - \partial_\phi W = 0 \tag{11}$$

holds and was used in the last line. Now a fairly simple calculation (see App. B) confirms that the result is independent of the conformal compensator as expected and reduces to the usual form,

$$V_{\text{scalar}}^{\langle 0\rangle} = e^{K/M_{\text{pl}}^2} \left[K^{ij^*} D_i^{\langle 0\rangle} W \bar{D}_{j^*}^{\langle 0\rangle} \bar{W} - 3\frac{|W|^2}{M_{\text{pl}}^2} \right] . \tag{12}$$

---

[4]Since this final rescaling is performed after $\langle\phi\rangle = 1$ is set, the fermionic matter fields are rescaled accordingly. However, here we are interested only in scalar fields and F-terms.

Let us restrict our attention to a theory with Kähler potential in the sequestered form,

$$K = -3M_{\rm pl}^2|\Phi|^2 \ln\left(1 - \sum_i \frac{|X^i|^2}{3M_{\rm pl}^2|\Phi|^2} + \frac{f^{\rm hid}}{3M_{\rm pl}^2|\Phi^2|}\right)$$

$$W = W_{\rm vis} + W_{\rm hid} + W_0\Phi^3 \,,$$

(13)

where $f^{\rm hid}$ and $W_{\rm hid}$ are functions of the hidden sector superfields, X_i's represent visible sector fields, $W_{\rm vis}$ is a visible sector superpotential, W_0 is the constant term in the superpotential introduced to cancel the cosmological constant, and the presence of Φ is kept explicit. Supersymmetry is assumed to be broken through the hidden sector dynamics. While we stress that the resultant scalar potential will be independent of the choice of F_ϕ, we work in the RS limit to clarify that approach. This requires that F-terms of all the massless chiral superfields without a vev must be given by the global SUSY formula, $D_i W = W_i$. Working with j'th superfield we find that this is achieved by setting

$$F_{\bar\phi} = \left\langle e^{K/3M_{\rm pl}^2} K^{ij^*} K_i \frac{1}{X^{*j}} \frac{W}{M_{\rm pl}^2} \right\rangle = \frac{\langle W \rangle}{M_{\rm pl}^2} + \mathcal{O}\left(\frac{1}{M_{\rm pl}^2}\right) = m_{3/2} + \mathcal{O}\left(\frac{1}{M_{\rm pl}^2}\right) \,,$$

(14)

and we stress that there is no summation over j^* in the above expression. The final result is independent of j^* due to the assumption that the tree level Kähler potential for all the visible sector fields is canonical up to Planck suppressed corrections. As usual, one must assume that there are no Planck-scale vevs even in the hidden sector [13], as the presence of such a vev would shift the value of F_ϕ.

With this choice of F_ϕ the scalar potential becomes

$$V_{\rm scalar} = e^{2K/3M_{\rm pl}^2} W_i \bar{W}_{i^*} - 3e^{K/M_{\rm pl}^2} M_{\rm pl}^2 m_{3/2}^2 + m_{3/2}^2 \sum_i |X^i|^2$$

$$- m_{3/2}\left(\partial_\phi W_{\rm vis} + \partial_\phi W_{\rm hid} + h.c.\right) + \mathcal{O}\left(\frac{1}{M_{\rm pl}^2}\right) \,,$$

(15)

where Eq. (11) was used to simplify the last line. The explicit $\mathcal{O}(m_{3/2}^2)$ masses in the third term of the equation above is cancelled by expanding the exponents in the first two terms, so any $\mathcal{O}(m_{3/2}^2)$ tree-level contribution to scalar masses vanishes from the potential.

Finally, we expand the Kähler potential to leading order in $1/M_{\rm pl}^2$ and adjust the gravitino mass to cancel the cosmological constant

$$m_{3/2} \equiv \frac{W}{M_{\rm pl}^2} = \left(\frac{\langle K^{ij^*} W_i \bar{W}_{i^*}\rangle}{3M_{\rm pl}^2}\right)^{1/2} + \mathcal{O}\left(\frac{1}{M_{\rm pl}^2}\right) \,.$$

(16)

This allows us to write the scalar potential for the visible sector fields

$$V_{\rm vis} = \partial_i W_{\rm vis} \partial_{i^*} \bar{W}_{\rm vis} - m_{3/2}\partial_\phi W_{\rm vis} - m_{3/2}\partial_{\bar\phi}\bar{W}_{\rm vis} + \mathcal{O}\left(\frac{1}{M_{\rm pl}^2}\right) \,.$$

(17)

We see that at the tree level the full supergravity scalar potential reduces to the globally supersymmetric potential with addition of a $m_{3/2}$-dependent holomorphic soft terms. Note that derivatives with respect to Φ vanish for all dimension four terms in the superpotential. Thus at the tree level all SUSY breaking effects are associated with the explicit mass parameters in the superpotential in agreement with [1].

3 Soft mass terms

Having written scalar potential with the conformal compensator, we will now generalize the formalism of [14] to allow for arbitrary F_ϕ.

We will consider a toy model based on an $SU(N)$ gauge group. The DS formalism in the Higgs phase requires existence of moduli fields whose vevs provide non-supersymmetric thresholds where soft masses are generated. Thus we will include N_f vector-like flavors of quarks Q and $\bar{Q}$ in the fundamental representation of the gauge group as well as a gauge singlet superfield S. We will assume that S couples to one of the flavors through the superpotential

$$W = \lambda_1 S Q_1 \bar{Q}_1 + \frac{M_S}{2} S^2 + \frac{\lambda_S}{3} S^3 \,. \tag{18}$$

For simplicity we will assume that remaining quark flavors ($i = 2, \ldots, N_f$) live at the origin of the moduli space. Depending on the choice of parameters we now have two types of moduli to consider:

- If $\lambda_1 = 0$, S decouples from the gauge dynamics. Fields Q_1 and $\bar{Q}_1$ are massless and in SUSY limit have an arbitrary vev. The gauge group is broken to $SU(N-1)$ and the analysis of the soft masses of the light fields is nearly identical to that of [14] with a simple generalization to non-abelian groups and the addition of F_ϕ-dependent terms.

- If $\lambda_1 \neq 0$ the singlet does not decouple. In the limit $M_S = \lambda_S = 0$ it has an arbitrary vev and provides a non-supersymmetric threshold at which quark multiplets become heavy. As we shall see this limit provides the generization of the DS formalism to a symmetric phase of the gauge theory.

3.1 Gaugino Masses

Let us start by considering the toy model in a broken phase, $\langle Q_1 \rangle = \langle \bar{Q}_1 \rangle = v$. The gauge group is broken to an $SU(N-1)$ subgroup but the modulus $Q_1 \bar{Q}_1$ remains a part of low energy physics and provides a non-supersymmetric threshold for other light degrees of freedom. At this threshold the gauge coupling constant is

$$\tau = \frac{1}{g^2} + \frac{b_0}{32\pi^2} \ln \frac{Q_1 \bar{Q}_1}{\Lambda^2 \Phi^2} \,, \tag{19}$$

where Λ is the UV cutoff.

Let us justify the inclusion of the conformal compensator Φ in Eq. (19). As has been pointed out in [15], the effects of RG evolution between two explicit mass scales in the

Lagrangian are already invariant under super-Weyl transformation. This is because all mass parameters transform identically. For example, in our formalism, all the mass terms must come with the appropriate powers of the conformal compensator but the ratio M_1/M_2 is independent of Φ. On the other hand, IR cutoffs not associated with the explicit mass terms in the Lagrangian are qualitatively different. Such thresholds must be treated as field dependent and must be expressed in terms of the light fields. Indeed, this is what has been done in Eq. (19). The gauge coupling function must be invariant under the super-Weyl transformation but after the rescaling performed in Eq. (4) the modulus transforms non-trivially. Thus super-Weyl invariance requires the inclusion of Φ. Identical arguments imply that wave-function renormalization factors must depend on the compensator, $\mathcal{Z} = \mathcal{Z}(|S|/\Lambda|\Phi|)$.

The effective Lagrangian of the theory at the scale v is given by

$$\mathcal{L}_{\text{gauge}} = \int d^2\theta \tau(Q_1, \bar{Q}_1, \Lambda) \mathcal{W}_\alpha \mathcal{W}^\alpha = \int d^2\theta \left(\frac{1}{g^2} + \frac{b_0}{32\pi^2} \ln \frac{Q_1 \bar{Q}_1}{\Lambda^2 \Phi^2} \right) \mathcal{W}_\alpha \mathcal{W}^\alpha , \qquad (20)$$

and we stress that b_0 is one loop beta-function coefficient of the theory above v, i.e. it includes the contribution of Q_1 and $\bar{Q}_1$ along with those of the remaining light fields. Using the results of the previous section we can obtain a general expression for the moduli F-terms,

$$\begin{aligned}
F_{Q_1} &= -\partial_{Q_1} W - Q_1^* \left(\frac{W}{M_{\text{pl}}^2} - F_\phi \right) + \mathcal{O}\left(\frac{1}{M_{\text{pl}}^2} \right) \\
F_{\bar{Q}_1} &= -\partial_{\bar{Q}_1} W - \bar{Q}_1^* \left(\frac{W}{M_{\text{pl}}^2} - F_\phi \right) + \mathcal{O}\left(\frac{1}{M_{\text{pl}}^2} \right) .
\end{aligned} \qquad (21)$$

Gaugino mass receives the contributions both from the moduli F-terms and factors of F_ϕ associated with the UV cutoff Λ. Performing the superspace integral in Eq. (20) gives:

$$\begin{aligned}
m_{\text{gaugino}} &= \frac{b_0}{32\pi^2} \left\langle \frac{F_{Q_1}}{Q_1} + \frac{F_{\bar{Q}_1}}{\bar{Q}_1} \right\rangle - \frac{b_0 F_\phi}{16\pi^2} \\
&= -\frac{b_0}{32\pi^2} \left\langle \frac{2W}{M_{\text{pl}}^2} + \frac{\partial_{Q_1} W}{Q_1} + \frac{\partial_{\bar{Q}_1} W}{\bar{Q}_1} - 2F_\phi \right\rangle - \frac{b_0 F_\phi}{16\pi^2} \\
&= -\frac{b_0}{32\pi^2} \left\langle \frac{2W}{M_{\text{pl}}^2} + \frac{\partial_{Q_1} W}{Q_1} + \frac{\partial_{\bar{Q}_1} W}{\bar{Q}_1} \right\rangle \simeq -\frac{b_0}{16\pi^2} m_{3/2} ,
\end{aligned} \qquad (22)$$

where in the second equality we assumed that $\langle Q_1 \rangle$ and $\langle \bar{Q}_1 \rangle$ are real and in the last equality we assume $\langle \partial_Q W/Q \rangle \ll m_{3/2}$. Since F_ϕ amounts to a gauge choice soft terms must be independent of it. Indeed we see that gaugino masses are determined by the SUSY breaking order parameter, $m_{3/2}$, as expected. In the DS limit one clearly sees that gaugino mass is associated with F-terms of the dynamical fields. In the RS limit, the soft mass appears to arise from non-supersymmetric regulator. Yet the agreement between the two limits is transparent in our formalism.

We now generalize these results to an unbroken phase of the theory. Thus instead of breaking the gauge group by the modulus vev at the non-supersymmetric threshold, we will assume that one quark flavor becomes heavy due to the superpotential given in Eq. (18). If

$M_S = 0$ and $\lambda_S = 0$, the gauge singlet S is a pseudo-modulus and can acquire large vev [5], $v \gg m_{3/2}$. The singlet remains a part of low energy theory even as the quark fields become heavy and decouple from supersymmetric dynamics. The Lagrangian for the gauge multiplet becomes

$$\mathcal{L}_{\text{gauge}} = \int d^2\theta\, \tau(S, \Lambda) \mathcal{W}_\alpha \mathcal{W}^\alpha = \int d^2\theta \left(\frac{1}{g^2} + \frac{b_0}{16\pi^2} \ln \frac{\lambda_S S}{\Lambda \Phi} \right) \mathcal{W}_\alpha \mathcal{W}^\alpha, \qquad (23)$$

and performing the superspace integral we recover AMSB prediction of gaugino masses which is, once again, independent of the F_ϕ.

Finally, one can ask how AMSB the prediction of gaugino mass can be reproduced in a pure super-Yang-Mills theory when neither (20) nor (23) is directly applicable. As we have argued, IR thresholds must be treated as fluctuations of light fields in the low energy theory. In a theory with broken SUSY such thresholds are accompanied by non-vanishing SUGRA F-terms leading to a correct prediction of gaugino mass. If no such F-terms exist, as in a pure super-Yang-Mills theory, then one must resort ot explicit calculation of non-local counterterms. In fact, in theories where F-terms exist, DS argued that the contribution calculated by using non-local counterterms is equivalent to that produced by the gauge coupling function in the Higgs phase [14], so our results hold using either method of calculation.

Before concluding this section we would like to point out that field-dependent thresholds given by the vevs of Q, $\bar{Q}$, and S represent UV cutoffs in the *Wilsonian* action for light degrees of freedom. Thus soft terms generated at these thresholds should be interpreted as UV effects [14].

3.2 Scalar Masses

Soft gaugino masses arise at one loop due to the gauge coupling renormalization. Thus only the knowledge of the tree level scalar potential is necessary to obtain the leading contribution to gaugino masses while the knowledge of wave-function renarmalization is required to calculate the gaugino masses to higher order in loop expansion. As with gaugino masses, the scalar soft terms arise at the loop level. However, even the leading, two loop, contribution to scalar masses requires the knowledge of the wave function renormalization of the matter fields (albeit at one loop order).

Since the tree level Kähler potential is assumed to be of sequestered form, tree level contributions vanish, as discussed in Sec. 2. We will calculate perturbative contributions to soft scalar masses in the unbroken phase of the theory (the generalization to the Higgs phase is trivial). Thus we assume that S acquires a vev and the renormalized Kähler potential at the field-dependent threshold has the form

$$K = -3M_{\text{pl}}^2 \ln \left(1 - \frac{|S|^2}{3M_{\text{pl}}^2 |\Phi|^2} - \sum_i \frac{|Q_i|^2 + |\bar{Q}_i|^2}{3M_{\text{pl}}^2 |\Phi|^2} \mathcal{Z}_i \left(\frac{|\lambda_S S|}{\Lambda |\Phi|} \right) - \frac{f^{\text{hid}}}{3M_{\text{pl}}^2 |\Phi|^2} \right) \qquad (24)$$

where $\mathcal{Z}_i$ are wave-function renormalization factors arising due to RG evolution between UV and IR thresholds, Λ and $\lambda_S \langle S \rangle$ respectively. Here we are not interested in the mass of S

---

[5]We will not discuss here dynamics responsible for stabilization of S at a finite vev.

itself, so to simplify the formulas we have set $\mathcal{Z}_S = 1$ at the IR threshold. As explained earlier, the inclusion of Φ in $\mathcal{Z}_i$ is required to maintain super-Weyl invariance. Furthermore, we can restrict our attention to the visible sector fields only due to the assumption of the absence of Planck-scale vevs in the hidden sector.

It is possible to perform the full supergravity calculation of soft scalar masses for an arbitrary value of F_ϕ. But since the full scalar potential is independent of F_ϕ we will, in this case, work in the DS limit, $F_\phi = 0$, to simplify the formulas. This limit has the advantage that the visible sector SUSY breaking is encoded in a non-zero F-term for the modulus S [6]. Thus soft scalar masses arise due to the F-terms of the dynamical degrees of freedom. The expressions for all the required derivatives of the Kähler potential are presented in the App. C, but the only relevant term here is the coefficient of $|F_S|^2$,

$$K^{SS^*} = \frac{e^{-K/3M_{\mathrm{pl}}^2}}{1 + \frac{1}{4}\sum \mathcal{Z}_i \dot{\gamma}_i \frac{|Q_i|^2 + |\bar{Q}_i|^2}{|S|^2}} + \mathcal{O}\left(\frac{1}{M_{\mathrm{pl}}^2}\right), \tag{25}$$

where we used the standard expressions for anomalous dimensions,

$$\begin{aligned} \gamma_i &= \frac{\partial \ln \mathcal{Z}_i}{\partial \ln |S|} = 2\frac{\partial \ln \mathcal{Z}_i}{\partial \ln S} = 2\frac{\partial \ln \mathcal{Z}_i}{\partial \ln S^*} \\ \dot{\gamma}_i &= \frac{\partial^2 \ln \mathcal{Z}}{\partial |S|^2} = 4\frac{\partial^2 \ln \mathcal{Z}_i}{\partial \ln S \partial \ln S^*} = \left[\frac{4}{\mathcal{Z}_i}\frac{\partial^2 Z_i}{\partial \ln S \partial \ln S^*} - \gamma_i^2\right], \end{aligned} \tag{26}$$

Due to the presence of the off-diagonal terms in the Kähler metric (see App. C), K^{SS^*} does not depend on anomalous dimensions of the quark superfields, but it does depend on their derivatives. The scalar potential contains the terms of the form

$$V \supset e^{K/M_{\mathrm{pl}}^2} K^{SS^*} D_S W \bar{D}_{S^*}\bar{W} \supset -\frac{1}{4}\sum_i \mathcal{Z}_i \dot{\gamma}_i \left(|Q_i|^2 + |\bar{Q}_i|^2\right) m_{3/2}^2. \tag{27}$$

This result reproduces famous expression for AMSB soft scalar mass squareds

$$\tilde{m}_i^2 = -\frac{1}{4}\dot{\gamma}m_{3/2}^2. \tag{28}$$

We can also obtain expressions for AMSB contributions to holomorphic soft terms. As shown in App. C the following relation holds:

$$K^{iS^*}K_{S^*} = (1 - \frac{1}{2}\gamma_i)Q^i. \tag{29}$$

Substituting this into the formula for scalar potential we have

$$\begin{aligned} V &\supset e^{K/M_{\mathrm{pl}}^2}\left[K^{ij^*}W_i K_{j^*}\frac{\bar{W}}{M_{\mathrm{pl}}^2} - 3W\frac{\bar{W}}{M_{\mathrm{pl}}^2} + h.c.\right] \\ &\supset m_{3/2}\left[\left(1 - \frac{1}{2}\gamma_i\right)\left(Q^i\partial_{Q^i}W + \bar{Q}^i\partial_{\bar{Q}^i}W\right) + SW_S - 3W + h.c.\right] \\ &\supset -m_{3/2}\left[\partial_\phi W + \sum_i \frac{1}{2}\gamma_i\left(Q^i\partial_{Q^i}W + \bar{Q}^i\partial_{\bar{Q}^i}W\right) + h.c.\right] \end{aligned} \tag{30}$$

---

[6]This is not generally true for an arbitrary F_ϕ, as factors of F_ϕ are present in the F-terms of other visible sector fields.

The first term in the last expression is already present at tree level in Eq. (17) while the second term gives one loop corrections to A and B-terms in agreement with [1].

Before concluding this section we note that both gaugino and scalar masses depend on λ_1 and $\langle S \rangle$ only through the anomalous dimensions of the matter fields. Thus we can take the limit $\lambda_1 \to 0$. In this limit S decouples, the quarks Q_1 and $\bar{Q}_1$ become light but the prediction for the AMSB mass terms remains unchanged. This explains why AMSB results still depend on the contribution of the "heavy" fields Q_1 and $\bar{Q}_1$, in particular why the gaugino mass is proportional to the β-function of the full theory.

4 Threshold Effects

In this section we extend discussion of supersymmetric and non-supersymmetric thresholds found in [3, 4] to the formulation of AMSB with an arbitrary F_ϕ. Once again we will take SUSY QCD as our toy model. Let us assume first that one of the quark flavors is given a large mass term, $M \gg m_{3/2}$. Thus the minimum of the scalar potential is found at $Q_1 = \bar{Q}_1 = 0$. For sufficiently large M, heavy squarks obtain non-holomorphic soft mass squareds, $m_{3/2}M$ while their F-terms vanish in both the RS and DS limits. The inclusion of small soft non-holomorphic masses ($\tilde{m} \lesssim m_{3/2}$) does not shift the ground state. Thus threshold associated with the mass of the heavy quarks is supersymmetric and does not contribute to gaugino masses; heavy flavor decouples from the low energy physics.

As a next step, consider a model where the quark mass term is given by a vev of a heavy field. We take the superpotential Eq. (18) in the limit of $M_S \gg m_{3/2}$. At the minimum of the potential

$$\langle S \rangle = -[M + m_{3/2} + \mathcal{O}(m_{3/2}^2/M_S)]/\lambda_S \,, \qquad \langle Q_1 \bar{Q}_1 \rangle = 0 \,, \tag{31}$$

and quark superfields become heavy. While the quark F-terms still vanish in both the RS and DS limits, F_S depends on F_ϕ. In particular, in the DS limit $F_S = 0$, thus the threshold at $\langle S \rangle$ is supersymmetric and does not contribute to soft terms. In the RS limit, however, $F_S = m_{3/2}\langle S \rangle$. The supersymmetric nature of the threshold can be seen instead in the fact that the shift in the ground state due to SUSY breaking is small [7], $\mathcal{O}(m_{3/2}/M)$. The decoupling of the supersymmetric threshold is a bit more subtle in the RS limit, where soft masses receive contributions both from the field-dependent threshold at $\langle S \rangle$ and from the regulator. The former is proportional to $F_S/\langle S \rangle \approx m_{3/2}$ while the latter is proportional to $F_\phi = -m_{3/2}$, and we easily see the cancellation of the two contributions.

We now take a limit $M_S \to 0$ and $\lambda_S \to 0$ while keeping $\langle S \rangle$ fixed. This is the model of section 3 and we already know that soft masses will be generated. In the DS limit it is easy to see that the origin of the soft terms lies in the non-supersymmetric nature of the threshold at $\langle S \rangle$, since $F_S \approx \langle S \rangle m_{3/2}$. On the other hand, in the RS limit $F_S = 0$. Nevertheless, since S is light the location of the vacuum depends sensitively on the small SUSY breaking terms in the potential and the threshold is non-supersymmetric. The vanishing of F_S in the RS limit implies that the contribution of the regulator can not be canceled by the field-dependent threshold. Thus despite the apparent generation of AMSB soft terms by the regulator in the RS limit, the origin lies in the non-supersymmetric nature of an IR threshold.

---

[7]This is true in both the RS and DS limits.

We now turn to a generalization of our toy model with several thresholds. Consider the toy model with the superpotential

$$W = \lambda_1 S_1 Q_1 \bar{Q}_1 + \lambda_2 S_2 Q_2 \bar{Q}_2 + \lambda_3 S_3 Q_3 \bar{Q}_3 + W(S_i) \,. \tag{32}$$

We will assume that the gauge singlet fields acquire hierarchical vevs $S_1 < S_2 < S_3$ due to $W(S_i)$ so that quark superfields decouple from the low energy theory and the the threshold at $\langle S_1 \rangle$ is non-supersymmetric. Let us consider gaugino masses for concreteness. The kinetic terms for the gauge multiplet can be written as

$$\int d^2\theta \left(\frac{1}{g^2} + \frac{b_0 - 2}{16\pi^2} \ln \frac{\lambda_1 S_1}{\lambda_2 S_2} + \frac{b_0 - 1}{16\pi^2} \ln \frac{\lambda_2 S_2}{\lambda_3 S_3} + \frac{b_0}{16\pi^2} \ln \frac{\lambda_3 S_3}{\Lambda \Phi} \right) \mathcal{W}_\alpha \mathcal{W}^\alpha \,, \tag{33}$$

where b_0 is the β-function coefficient of the theory in the UV. If thresholds S_2 and S_3 are supersymmetric, in the DS limit one finds $F_2 = F_3 = F_\phi = 0$; thus the second and third terms of the Eq. (33) do not contribute to gaugino masses. It is worth noting that gaugino masses are proportional to $b_0 - 2$, the β-function of the theory above non-supersymmetric threshold. In other words, gaugino masses receive contributions from all the light fields in the theory as well as one supermultiplet, Q_1 and $\bar{Q}_1$, that becomes heavy at a non-supersymmetric threshold.

Let us now assume that the threshold at S_2 is also non-supersymmetric. In this case $F_1/S_1 = F_2/S_2 = F_\phi$ and the contribution of the first term in Eq. (33) vanishes. On the other hand, the second term in Eq. (33) results in gaugino masses proportional to $b_0 - 1$.

Finally, assume that S_1 and S_3 are non-supersymmetric thresholds while S_2 is supersymmetric. We can see that there is a non-vanishing contribution at each of the thresholds. However, the combined effect of all the thresholds leads to gaugino mass proportional to $b_0 - 1$. We conclude that in all the cases the fields which become massive at supersymmetric thresholds decouple from the low energy physics while the fields which become massive at non-supersymmetric thresholds continue to contribute to soft terms [3, 4]. We also note that one may construct more complicated models where heavy fields decouple partially.

The discussion of the field-dependent as well as non-supersymmetric thresholds has prepared us for an analysis of the role of small explicit mass parameters in a supersymmetric Lagrangian, for instance a μ-term in the MSSM Higgs sector. In the EWSB vacuum, where both the μ-term and Higgs vev are non-vanishing, one finds $\partial_H W \neq 0$. One then might conclude that the presence of a μ-term in a supersymmetric Lagrangian may modify AMSB prediciton [15]. In particular, in the limit of large $\tan\beta$ it is possible to have $\langle \partial_H W/H \rangle \sim m_{3/2}$ which naively leads to $\mathcal{O}(1)$ corrections to gaugino mass. As we will now argue, a more careful analysis shows that such corrections are subleading. Moreover, they are automatically taken into account when the renormalization group evolution below the gravitino mass is performed consistently.

Recall that in the presence of SUSY breaking, the μ-term implies the existence of the B-term $B = \mu m_{3/2}$. For small $\mu \ll m_{3/2}$ the stable minimum of the potential only exists in the presence of additional contributions to the soft masses in the Higgs sector. Thus any threshold associated with the μ-term is necessarily non-supersymmetric. As explained in this section corrections to F_H due to non-supersymmetric thresholds are small compared to $m_{3/2}\langle H \rangle$ and thus lead to a subleading correction to gaugino mass.

It is interesting to consider what happens as one increases the μ-term. When $\mu \sim m_{3/2}$, inclusion of non-holomorphic soft scalar masses in the analysis leads to an $\mathcal{O}(1)$ shift in $\langle H \rangle$ and F_H. A simple proportionality relation $F_H \sim \langle H \rangle m_{3/2}$ does not hold and a more careful calculation is required to obtain gaugino mass[8]. As μ becomes large compared to $m_{3/2}$, the vev of H is well approximated by its supersymmetric value, H decouples from low energy theory and an AMSB prediction becomes valid again (now with no contribution from H).

This discussion allows us to formulate a prescription for a consistent calculation of AMSB soft terms. These terms must be evaluated using AMSB formulas at scales somewhat large compared to gravitino mass[9]. Values of soft parameters obtained in such a way should then be treated as a boundary conditions and detailed RGE calculations with full non-supersymmetric MSSM Lagrangian must be used to obtain low energy predictions. The phenomenological consequences of the μ-term are automatically accounted for In this approach.

5 Conclusion

In this paper we have reviewed anomaly mediated supersymmetry breaking. We have uplifted the original formalism of [1] to the full supergravity Lagrangian and generalized it to allow an arbitrary compensator F-term. This allowed us to interpolate between the RS and DS derivations of anomaly mediation and show that they lead to completely equivalent results. We have also discussed the effects of supersymmetric and non-supersymmetric thresholds in the theory as well as the role of small supersymmetric mass parameters in the visible sector of the theory.

Acknowledgements

A Conformal Compensator and SUGRA Lagrangian

In this appendix we derive the scalar potential in the supergravity formalism with a conformal compensator. This discussion closely follows [17]. The supergravity Lagrangian is

$$\mathcal{L} = \int d\Theta^2 2\mathcal{E} \left[\frac{3}{8} M_{\mathrm{pl}}^2 \left(\bar{\mathcal{D}}^2 - 8\mathcal{R} \right) |\Phi|^2 e^{-K/3M_{\mathrm{pl}}^2} + \Phi^3 W + \tau \mathcal{W}\mathcal{W} \right] + h.c. \qquad (34)$$

where Φ is the conformal compensator which has been introduced through the replacement

$$K \rightarrow K - 3M_{\mathrm{pl}}^2 \left(\ln \Phi + \ln \bar{\Phi} \right) \qquad W \rightarrow \Phi^3 W \ . \qquad (35)$$

---

[8]This effect can be seen in terms of an effective low energy Lagrangian. Such a Lagrangian always contains terms with explicit superspace derivatives and they become important precisely when $\mu \sim m_{3/2}$.

[9]In the absence of explicit $\mathcal{O}(m_{3/2})$ mass parameters in the visible sector, this calculation can be performed at the scale of gravitino mass.

Following [1] we further rescale the chiral superfields $\Phi X^i \to X^i$, which causes the matter fields to transform under super-Weyl transformations. As a result the explicit factors of Φ only appear in association with mass terms in the Lagrangian.

Super-Weyl transformations consist of

$$\mathcal{E} \;\to\; e^{6\xi}\mathcal{E} \tag{36}$$

$$\Phi \;\to\; e^{-2\xi}\Phi \tag{37}$$

$$X^i \;\to\; e^{-2\xi}X^i \tag{38}$$

$$\tag{39}$$

and the Lagrangian becomes

$$\mathcal{L} = \int d\Theta^2 2\mathcal{E}\left[\frac{3}{8}M_{\mathrm{pl}}^2\left(\bar{\mathcal{D}}^2 - 8\mathcal{R}\right)|\Phi|^2 e^{-K(X_i,\bar{X}_i,\Phi,\bar{\Phi})/3M_{\mathrm{pl}}^2|\Phi|^2} + W(X,\Phi)\right] + h.c., \tag{40}$$

In this basis the scalar Lagrangian is

$$
\begin{aligned}
\mathcal{L}_{\mathrm{scalar}} &= \frac{1}{9}e\Omega\left|M^* - 3\left(\ln\Omega\right)_i F_i\right|^2 + e\Omega\left(\ln\Omega\right)_{ij*}F_iF_j^* \\
&\quad + e\left(W_iF_i - WM^* + h.c.\right)
\end{aligned}
\tag{41}
$$

$$
\begin{aligned}
&= \frac{1}{9}e\Omega\left|M^* - 3\left(\ln\Omega\right)_i F_i - 3F_\phi\partial_\phi\left(\ln\Omega\right)\right|^2 \\
&\quad + e\Omega\left(\ln\Omega\right)_{ij*}F_iF_j^* + e\Omega\partial_{\bar\phi}\left(\ln\Omega\right)_i F_iF_{\bar\phi} \\
&\quad + e\Omega\partial_\phi\left(\ln\Omega\right)_{j*}F_j^*F_\phi + e\Omega|F_\phi|^2\partial_\phi\partial_{\bar\phi}\left(\ln\Omega\right) \\
&\quad + e\left(W_iF_i + F_\phi\partial_\phi W - WM^* + h.c.\right),
\end{aligned}
\tag{42}
$$

where

$$\Omega = -3M_{\mathrm{pl}}^2|\Phi|^2\exp\left(-K(X_i,\bar{X}_i,\Phi,\bar{\Phi})/3M_{\mathrm{pl}}^2|\Phi|^2\right). \tag{43}$$

and M is an auxiliary scalar field in the gravity multiplet. Integrating out the combination $N^* = M^* - 3\left(\ln\Omega\right)_i F_i - 3F_\phi\partial_\phi\left(\ln\Omega\right)$ allows us to write the scalar Lagrangian in the form

$$
\begin{aligned}
\mathcal{L}_{\mathrm{scalar}} = &-9e|W|^2\Omega^{-1} - e\Omega^{-1}\left(\ln\Omega\right)_{ij*}^{-1}D_iW\bar{D}_{j*}\bar{W} \\
&+ e\Omega|F_\phi|^2\partial_\phi\partial_{\bar\phi}\left(\ln\Omega\right) + eF_\phi\partial_\phi W + eF_{\bar\phi}\partial_{\bar\phi}\bar{W} \\
&- 3eWF_\phi\partial_\phi\left(\ln\Omega\right) - 3eWF_{\bar\phi}\partial_{\bar\phi}\left(\ln\Omega\right),
\end{aligned}
\tag{44}
$$

where the covariant superspace derivative is defined by

$$D_{j*}\bar{W} = \bar{W}_{j*} + \Omega F_\phi\partial_\phi\left(\ln\Omega\right)_{j*} - 3W\left(\ln\Omega\right)_{j*}. \tag{45}$$

Finally, using Eq. (43) we can write

$$
\begin{aligned}
\mathcal{L}_{\mathrm{scalar}} = &\; e\, e^{K/3M_{\mathrm{pl}}^2|\phi|^2}\left[3\frac{|W|^2}{M_{\mathrm{pl}}^2|\phi|^2} - K_{ij*}^{-1}D_iW\bar{D}_{j*}\bar{W}\right] \\
&- 3eM_{\mathrm{pl}}^2|\phi|^2 e^{-K/3M_{\mathrm{pl}}^2|\phi|^2}|F_\phi|^2\partial_\phi\partial_{\bar\phi}\left[\ln|\phi|^2 - \frac{K}{3M_{\mathrm{pl}}^2|\phi|^2}\right] \\
&+ e\left[F_\phi\partial_\phi W - 3WF_\phi\partial_\phi\left(\ln|\phi|^2 - \frac{K}{3M_{\mathrm{pl}}^2|\phi|^2}\right) + h.c.\right],
\end{aligned}
\tag{46}
$$

where

$$D_i W = W_i + K_i \frac{W}{M_{\mathrm{pl}}^2 |\phi|^2} + e^{-K/3M_{\mathrm{pl}}^2} \bar{\phi} F_{\bar{\phi}} \partial_{\bar{\phi}} \left(\frac{K_i}{\bar{\phi}} \right) \tag{47}$$

In order to get a canonical gravity kinetic term, the vierbein must be rescaled as $e_m^a \rightarrow e_m^a e^{-K/6M_{\mathrm{pl}}^2}$. However, this tranformation cannot be performed in an explicitly supersymmetric manner in the conformal compensator formalism. We therefore set $\langle \phi \rangle = 1$ and then perform a non-supersymmetric Weyl transformation to achieve canonical gravitational kinetic terms. This retains F_ϕ as an arbitrary parameter. We stress that though doing so breaks the super-Weyl invariance, that invariance has already been broken explicitly by giving a non-zero value to the background scalar field ϕ.

The superspace derivative and scalar potential can now be written as

$$
\begin{aligned}
D_i W &= W_i + K_i \frac{W}{M_{\mathrm{pl}}^2} - e^{K/3M_{\mathrm{pl}}^2} F_{\bar{\phi}} \left(K_i - \partial_{\bar{\phi}} K_i|_{\phi=1} \right) \\
V_{\mathrm{scalar}} &= e\, e^{K/M_{\mathrm{pl}}^2} \left[K_{ij^*}^{-1} D_i W \bar{D}_{j^*} \bar{W} - 3 \frac{|W|^2}{M_{\mathrm{pl}}^2} \right] \\
&\quad - e\, e^{K/3M_{\mathrm{pl}}^2} |F_\phi|^2 \left[\partial_\phi \partial_{\bar{\phi}} K - \partial_\phi K - \partial_{\bar{\phi}} K + K \right]_{\phi=1} \\
&\quad - e\, e^{2K/3M_{\mathrm{pl}}^2} \left[F_\phi \partial_\phi W - 3 W F_\phi \left(1 + \frac{K}{3M_{\mathrm{pl}}^2} - \frac{\partial_\phi K|_{\phi=1}}{3M_{\mathrm{pl}}^2} \right) + h.c. \right].
\end{aligned}
\tag{48}
$$

B   Physical Scalar Potential and Cancellation of F_ϕ

In this appendix we show explicitly that the scalar potential is independent of F_ϕ. Consider an arbitrary Kähler potential of the form

$$K = -3M_{\mathrm{pl}}^2 |\Phi|^2 \ln \Sigma. \tag{49}$$

Though any Kähler potential may be expressed in this form, the function Σ has the simplest form when the Kähler potential has a sequesterd form. Σ is a function of fields and coupling of the theory and at the same time has mass dimension zero. On the other hand, in the RS formalism chiral superfields have non-trivial Weyl weights. Thus to ensure proper transformations of the Kähler potential under super-Weyl transformations Σ must take the form

$$\Sigma \left(X^i, X^{i*}, \Phi, \bar{\Phi}, \{M\} \right) = \Sigma \left(\frac{X^i}{M\Phi}, \frac{X^{i*}}{M\bar{\Phi}} \right), \tag{50}$$

where X^i are the matter fields in both the visible and hidden sectors and M represents any appropriate mass parameter in the theory.

We can use Eq. (50) to relate the derivatives with respect to ϕ to derivatives with respect

to matter fields,

$$\partial_\phi \Sigma = \frac{\partial \Sigma}{\partial (X^i/\phi)} \frac{\partial (X^i/\phi)}{\phi} = -\frac{\partial \Sigma}{\partial (X^i/\phi)} \frac{X^i}{\phi^2} = -\Sigma_i \frac{X^i}{\phi}$$

$$\partial_\phi \partial_{\bar\phi} \Sigma = \frac{\partial^2 \Sigma}{\partial (X^i/\phi) \, \partial (X^{*j}/\bar\phi)} \frac{X^i X^{*j}}{|\phi|^4} = \Sigma_{ij^*} \frac{X^i X^{*j}}{|\phi|^2} \tag{51}$$

$$\partial_{\bar\phi} \Sigma_i = -\Sigma_{ij^*} \frac{X^{*j}}{\bar\phi} \, .$$

These equations then allow us to simplify several of the terms in Eq. (48),

$$K_i = -3 M_{\rm pl}^2 |\phi|^2 \frac{\Sigma_i}{\Sigma}$$

$$K_{ij^*} = -3 M_{\rm pl}^2 |\phi|^2 \left[\frac{\Sigma_{ij^*}}{\Sigma} - \frac{\Sigma_i \Sigma_{j^*}}{\Sigma^2} \right]$$

$$K^{ij^*} = -\frac{\Sigma}{3 M_{\rm pl}^2 |\phi|^2} \left[\Sigma_{ij^*} - \frac{\Sigma_i \Sigma_{j^*}}{\Sigma} \right]^{-1}$$

$$\partial_\phi K = -3 M_{\rm pl}^2 \bar\phi \left[\ln \Sigma + \phi \frac{\partial_\phi \Sigma}{\Sigma} \right] = \frac{1}{\phi} \left[K - X^i K_i \right] \tag{52}$$

$$\partial_\phi \partial_{\bar\phi} K = -3 M_{\rm pl}^2 \left[\ln \Sigma + \phi \frac{\partial_\phi \Sigma}{\Sigma} + \bar\phi \frac{\partial_{\bar\phi} \Sigma}{\Sigma} + |\phi|^2 \frac{\partial_\phi \partial_{\bar\phi} \Sigma}{\Sigma} - |\phi|^2 \frac{\partial_\phi \Sigma \partial_{\bar\phi} \Sigma}{\Sigma^2} \right]$$

$$= \frac{1}{|\phi|^2} \left[K - X^i K_i - X^{i*} K_{i^*} + K_{ij^*} X^i X^{*j} \right]$$

$$\partial_{\bar\phi} K_i = -3 M_{\rm pl}^2 \phi \left[\frac{\Sigma_i}{\Sigma} + \bar\phi \frac{\partial_{\bar\phi} \Sigma_i}{\Sigma} - \bar\phi \frac{\Sigma_i \partial_{\bar\phi} \Sigma}{\Sigma^2} \right] = \frac{1}{\bar\phi} \left[K_i - K_{ij^*} X^{*j} \right] \, .$$

Then, setting $\langle \phi \rangle = 1$, the scalar potential reduces to

$$D_i W = W_i + K_i \frac{W}{M_{\rm pl}^2} - \Sigma F_{\bar\phi} K_{ij^*} X^{*j} = D_i^{\langle 0 \rangle} W - \Sigma K_{ij^*} X^{*j} F_{\bar\phi}$$

$$V_{\rm scalar} = \frac{1}{\Sigma^3} \left[K_{ij^*}^{-1} D_i W \bar D_{j^*} \bar W - 3 \frac{|W|^2}{M_{\rm pl}^2} \right] \tag{53}$$

$$- \frac{1}{\Sigma} |F_\phi|^2 K_{ij^*} X^i X^{*j} - \frac{1}{\Sigma^2} \left[F_\phi \partial_\phi W - 3 W F_\phi - F_\phi \frac{W}{M_{\rm pl}^2} K_i X^i + h.c. \right] \, .$$

We can further simplify the second line of the potential by noting that, with the inclusion of Φ, W is a homogeneous polynomial function of order three in the fields. Thus, $3W - X^i W_i - \partial_\phi W = 0$, which gives

$$V_{\rm scalar} = \frac{1}{\Sigma^3} \left[K_{ij^*}^{-1} D_i W \bar D_{j^*} \bar W - 3 \frac{|W|^2}{M_{\rm pl}^2} \right]$$

$$- \frac{1}{\Sigma} |F_\phi|^2 K_{ij^*} X^i X^{*j} + \frac{1}{\Sigma^2} \left[F_\phi X^i D_i^{\langle 0 \rangle} W + h.c. \right] \, . \tag{54}$$

15

This, with the replacement $\Sigma = e^{-K/3M_{\mathrm{pl}}^2}$, leads to Eq. (10). It is now fairly simple to show that V is independent of F_ϕ:

$$
\begin{aligned}
V_{\mathrm{scalar}} &= V_{\mathrm{scalar}}^{\langle 0 \rangle} - \frac{1}{\Sigma^2}\left[X^i K_{ik^*} K^{jk^*} F_\phi D_j^{\langle 0 \rangle} W + h.c. \right] + \frac{1}{\Sigma}|F_\phi|^2 K_{ik^*} K^{lk^*} K_{lj^*} X^i X^{*j} \\
&\quad - \frac{1}{\Sigma}|F_\phi|^2 K_{ij^*} X^i X^{*j} + \frac{1}{\Sigma^2}\left[F_\phi X^i D_i^{\langle 0 \rangle} W + h.c. \right] \\
&= V_{\mathrm{scalar}}^{\langle 0 \rangle}.
\end{aligned}
\tag{55}
$$

C   Renormalized Kähler metric

Here we present the Kähler potential derivatives used in Sec. 3.2. The first and second derivatives are

$$
K_i^{\langle \mathrm{vis} \rangle} = e^{K/3M_{\mathrm{pl}}^2}\, \mathcal{Z}_i Q_i^* \qquad\qquad K_S^{\langle \mathrm{vis} \rangle} = e^{K/3M_{\mathrm{pl}}^2}\left(S^* + \frac{1}{2}\sum_i \frac{|Q_i|^2 + |\bar{Q}_i|^2}{S} \mathcal{Z}_i \gamma_i \right)
$$

$$
K_{Sj^*}^{\langle \mathrm{vis} \rangle} = e^{K/3M_{\mathrm{pl}}^2}\, \frac{1}{2}\frac{Q_j}{S}\mathcal{Z}_i \gamma_j \qquad\qquad K_{ij^*}^{\langle \mathrm{vis} \rangle} = e^{K/3M_{\mathrm{pl}}^2}\, \mathcal{Z}_i \delta_{ij^*}
\tag{56}
$$

$$
K_{SS^*}^{\langle \mathrm{vis} \rangle} = e^{K/3M_{\mathrm{pl}}^2}\left(1 + \sum_i \frac{|Q_i|^2 + |\bar{Q}_i|^2}{|S|^2}\frac{\partial^2 \mathcal{Z}_i}{\partial \ln S \partial \ln S^*} \right),
$$

where i and j^* correspond to derivatives with respect to all matter fields besides the modulus S. Note that the anomalous dimension only appears in S-dependent derivatives, since the wave-function renormalization is dependent only on S. Inverting K_{ij^*} gives

$$
K^{\langle \mathrm{vis} \rangle SS^*} = \frac{e^{-K/3M_{\mathrm{pl}}^2}}{1 + \frac{1}{4}\sum_i \mathcal{Z}_i \dot{\gamma}_i \frac{|Q_i|^2 + |\bar{Q}_i|^2}{|S|^2}} + \mathcal{O}\left(\frac{1}{M_{\mathrm{pl}}^2} \right)
$$

$$
K^{\langle \mathrm{vis} \rangle iS^*} = -\frac{e^{-K/3M_{\mathrm{pl}}^2}}{1 + \frac{1}{4}\sum_j \mathcal{Z}_j \dot{\gamma}_j \frac{|Q_j|^2 + |\bar{Q}_j|^2}{|S|^2}}\frac{Q_i \gamma_i}{2S} + \mathcal{O}\left(\frac{1}{M_{\mathrm{pl}}^2} \right)
\tag{57}
$$

$$
K^{\langle \mathrm{vis} \rangle ij^*} = \frac{e^{-K/3M_{\mathrm{pl}}^2}}{1 + \frac{1}{4}\sum_k \mathcal{Z}_k \dot{\gamma} \frac{|Q_k|^2 + |\bar{Q}_k|^2}{|S|^2}}\frac{\delta^{i^* j}}{\mathcal{Z}_i} + \mathcal{O}\left(\frac{|Q|^2}{|S|^2} \right) + \mathcal{O}\left(\frac{1}{M_{\mathrm{pl}}^2} \right).
$$

From this, it is apparent that in the DS limit the only relevant term is K^{SS^*}. Terms involving K^{ij^*} appear to contribute both at tree level and in perturbation theory. However, tree level contributions of these terms vanish cancel due to the sequestered form of the Kähler potential while loop contributions vanish because all matter fields except S are assumed to have no vevs. In models where several light fields have vevs, one must consider interplay of non-supersymmetric thresholds as discussed in section 4.

Finally, to derive holomorphic soft terms we need the following expression:

$$
K^{iS^*} K_{S^*} = \left(1 - \frac{1}{2}\gamma_i \right) Q^i + \mathcal{O}\left(\frac{|Q|^2}{|S|^2} \right) + \mathcal{O}\left(\frac{1}{M_{\mathrm{pl}}^2} \right).
\tag{58}
$$

Once again we have dropped higher-dimensional terms that do not contribute to soft terms.

References

[1] L. Randall and R. Sundrum, "Out of this world supersymmetry breaking,"
Nucl. Phys. **B557** (1999) 79–118, `arXiv:hep-th/9810155`.

[2] G. F. Giudice, M. A. Luty, H. Murayama, and R. Rattazzi, "Gaugino Mass without
Singlets," *JHEP* **12** (1998) 027, `arXiv:hep-ph/9810442`.

[3] A. Pomarol and R. Rattazzi, "Sparticle masses from the superconformal anomaly,"
JHEP **05** (1999) 013, `arXiv:hep-ph/9903448`.

[4] E. Katz, Y. Shadmi, and Y. Shirman, "Heavy thresholds, slepton masses and the mu
term in anomaly mediated supersymmetry breaking," *JHEP* **08** (1999) 015,
`arXiv:hep-ph/9906296`.

[5] Z. Chacko, M. A. Luty, I. Maksymyk, and E. Ponton, "Realistic anomaly-mediated
supersymmetry breaking," *JHEP* **04** (2000) 001, `arXiv:hep-ph/9905390`.

[6] I. Jack and D. R. T. Jones, "Fayet-Iliopoulos D-terms and anomaly mediated
supersymmetry breaking," *Phys. Lett.* **B482** (2000) 167–173, `arXiv:hep-ph/0003081`.

[7] B. C. Allanach and A. Dedes, "R-parity violating anomaly mediated supersymmetry
breaking," *JHEP* **06** (2000) 017, `arXiv:hep-ph/0003222`.

[8] Z. Chacko, M. A. Luty, E. Ponton, Y. Shadmi, and Y. Shirman, "The GUT scale and
superpartner masses from anomaly mediated supersymmetry breaking,"
Phys. Rev. **D64** (2001) 055009, `arXiv:hep-ph/0006047`.

[9] I. Jack and D. R. T. Jones, "R-symmetry, Yukawa textures and anomaly mediated
supersymmetry breaking," *Phys. Lett.* **B491** (2000) 151–156, `arXiv:hep-ph/0006116`.

[10] D. E. Kaplan and G. D. Kribs, "Gaugino-assisted anomaly mediation," *JHEP* **09**
(2000) 048, `arXiv:hep-ph/0009195`.

[11] O. C. Anoka, K. S. Babu, and I. Gogoladze, "Tev-scale horizontal symmetry and the
slepton mass problem of anomaly mediation," *Nucl. Phys.* **B686** (2004) 135–154,
`arXiv:hep-ph/0312176`.

[12] N. Okada, "Positively-deflected anomaly mediation," *Phys. Rev.* **D65** (2002) 115009,
`arXiv:hep-ph/0202219`.

[13] J. A. Bagger, T. Moroi, and E. Poppitz, "Anomaly mediation in supergravity
theories," *JHEP* **04** (2000) 009, `arXiv:hep-th/9911029`.

[14] M. Dine and N. Seiberg, "Comments on Quantum Effects in Supergravity Theories,"
JHEP **03** (2007) 040, `arXiv:hep-th/0701023`.

[15] S. P. de Alwis, "On Anomaly Mediated SUSY Breaking,"
Phys. Rev. **D77** (2008) 105020, `arXiv:0801.0578 [hep-th]`.

[16] V. Kaplunovsky and J. Louis, "Field dependent gauge couplings in locally supersymmetric effective quantum field theories," *Nucl. Phys.* **B422** (1994) 57–124, `arXiv:hep-th/9402005`.

[17] J. Wess and J. Bagger, "Supersymmetry and supergravity,". Princeton, USA: Univ. Pr. (1992) 259 p.