Thermodynamic origin of the Cardassian universe

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Phys. Rev. D **83**, 123527 — Published 23 June 2011
DOI: 10.1103/PhysRevD.83.123527

Thermodynamic Origin of the Cardassian Universe

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In the Cadassian universe, one can explain the acceleration of the universe without introducing dark energy component. However, it is hard to get the dynamical equations of this model from the action principle. Recently, works on the relation between thermodynamics and gravity indicate that gravity force may not be a fundamental force. In this paper, we shall study the thermodynamics of the Cardassian universe, and it might be the origin of this cosmological model. We find that the corresponding entropy obeys ordinary area law when the area of the trapping horizon is small, and it becomes a constant when the area is going to be large in the original and modified polytropic Cardassian models, while it has a maximum value in the exponential one. It seems that the Cardassian universe only contains finite information according to the holographic principle, which states that all the information in the bulk should be encoded on the boundary of the bulk.

PACS numbers: 98.80.-k

I. INTRODUCTION

Recently, the relation between the laws of thermodynamics and that of gravity has aroused great interest. In Hawking's work [1], it shows that black holes have thermal radiation with a temperature determined by its surface gravity due to the quantum effect, which implies that these two branches of physics may have underly relations, namely, they are unified. The authors in ref. [2] proposed a unified first law of black hole dynamics and relativistic thermodynamics in spherically symmetric general relativity, see also [3–5]. By using the unified law in the framework of Friedmann-Robertson-Walker (FRW) universe, the authors in [6] find the relation between the Friedmann equation and the first law of thermodynamics on the "inner" trapping horizon. They also find that the Clausius relation holds for the Gauss-Bonnet and Lovelock theory by treating the higher derivative terms as an effective energy-momentum tensor, but the Clausius relation never holds for the scalar-tensor theory, which implies that this is a system with non-equilibrium thermodynamics. These results indicate that the gravity may be not one of the fundamental forces and it is only another view of thermodynamics. In this spirit, we can regard the thermodynamics law as the fundamental law and it will give the corresponding gravity theory. Thus, one can study the thermodynamics of some cosmological models to pursue their origins. For related discussions, see also [7–13].

Up to now, there are many kinds of dark energy models and modified gravity theories proposed to explain the current accelerating expansion of the universe, which has been confirmed by the observations like Type Ia supernovae (SNe Ia), CMB and SDSS et al. The dark energy models assume the existence of an energy component with negative pressure in the universe, and it dominates and accelerates the universe at late times. The cosmological constant seems the best candidate of dark energy, but it suffers the fine tuning problem and coincidence problem, and it may even have the age problem [14]. To alleviate these problems, many dynamic dark energy models were proposed, see [15]. However, people still do not know what is dark energy.

Since the Einstein general gravity theory has not been checked in a very large scale, then one does not know whether this gravity theory is suitable or not for studying the observational data like SNe Ia, and maybe the accelerating expansion of universe is due to the gravity theory that differs from the general gravity. Thus, many modified gravity theories like $f(R)$, DGP et al. are proposed to explain the accelerating phenomenology. The Cardassian model is a kind of model in which the Friedmann equation is modified by the introduction of an additional nonlinear term of energy density and in this model one does not need dark energy component also. However, one can not directly find its origin from the first principle.

As we mentioned before, the gravity may be not a fundamental theory, and one can at least study the thermodynamics of a cosmological model to pursue its origin. So, in this paper, we will study the thermodynamics of the Cardassian universe, and regarded it as its origin. We find that the corresponding entropy obeys ordinary area law $S = A/(4G)$ on the trapping horizon when A is small, where A is the the area of the trapping horizon, and it becomes a constant when A is going to be large in the original (OC) [16] and modified polytropic Cardassian model (MPC) [17], while it has a maximum value in the exponential model (EC) [18, 19]. It seems that the Cardassian universe only contains finite information according to the holographic principle, which states that all the information in the bulk should be encoded in the boundary of the bulk.

This paper is organized as follows: In Section II, we will give a briefly review on the unified first law and its

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application. In Section III, we will study the thermodynamics of the Cardassian universe including the OC, MPC and EC models, for recent works on the Cardassian universe, see [20–23]. In the last section, we will give some discussions and conclusions.

II. BRIEFLY REVIEW ON THE UNIFIED FIRST LAW

Einstein equations can be written in a form called "unified first law" based on the general definition of black hole dynamics on trapping horizon, which was proposed by Hayward [2–5] and developed by Cai et al [6–10]. In the rest of this section, we will give a brief review on this unified first law in the $3+1$-dimensional spherical symmetric spacetime, in which the metric could be locally written in the double-null form as

$$ds^2 = -2e^{-f}d\xi^+ d\xi^- + r^2 d\Omega^2\,, \tag{1}$$

where $d\Omega^2$ is the line element of the 2-sphere with unit radius, r and f are functions of (ξ^+, ξ^-). So, each symmetric sphere has two preferred normal directions, namely the null directions $\partial/\partial\xi^{\pm}$, which will be assumed future-pointing in the following. And also, we will assume the spacetime is time-orientable. The expansions of the radial null geodesic congruence are defined by

$$\theta_{\pm} = 2r^{-1}\partial_{\pm} r\,, \tag{2}$$

where $\partial_{\pm}$ denotes the coordinates derivative along $\xi^{\pm}$. The expansion measures whether the light rays normal to the sphere are diverging ($\theta_{\pm} > 0$) or converging ($\theta_{\pm} < 0$), namely, whether the sphere is increasing or decreasing in the null directions. Note that, although the value of $\theta_{\pm}$ will change with geometries, its sign will not, and the only invariants of the metric and its first derivative are functions of r and $e^f\theta_+\theta_-$, or equivalently $g^{ab}\partial_a r\partial_b r = -\frac{1}{2}e^f\theta_+\theta_-$, which has an important physical and geometrical meaning: a sphere is said to be trapped (untrapped) if $\theta_+\theta_- > 0(\theta_+\theta_- < 0)$, and if $\theta_+\theta_- = 0$, it is a marginal sphere.

Considering non-stationary black holes, Hayward has proposed that the future outer trapping horizons defined as the the closure of a hypersurface foliated by future or past, outer or innner marginal sphere is taken as the definition of black holes, since the horizon possess various properties which are often intuitively ascribed to black hole including confinement of observers and analogues of the zeroth, first and second law of thermodynamics. However, in the case of FRW universe, one should take the future inner trapping horizon defined by

$$\theta_+ = 0\,, \quad \theta_- < 0\,, \quad \partial_-\theta_+ > 0\,, \tag{3}$$

as a system on which the thermodynamics will be established, since the surface gravity is negative on the cosmological horizon.

In a spherical symmetric spacetime, one can obtain the total energy inside the sphere with radius r by calculating the Minsner-Sharp energy given by

$$E = \frac{r}{2G}\left(1 - g^{ab}\partial_a r\partial_b r\right) = \frac{r}{G}\left(\frac{1}{2} - g^{+-}\partial_+ r\partial_- r\right)\,, \tag{4}$$

which is a pure geometric quantity and has much better properties than the other definitions of energy when one consider the case of non-stationary spacetime. The relation between the Minsner-Sharp energy and others could be found in ref. [4] . There are also two invariants constructed from the energy-momentum tensor $T^{\mu\nu}$:

$$W = -\frac{1}{2}g_{ab}T^{ab} = -g_{+-}T^{+-}\,, \tag{5}$$

which are called the work density, and Ψ called the energy flux vector (also called the energy-supply vector), whose components are

$$\Psi_a = T_a^b\partial_b r + W\partial_a r\,. \tag{6}$$

Here and in the following, a, b denotes the two dimension space normal to the sphere. With the help of the definition of Minsner-Sharp energy and the above two quantities, one can write the $(0,0)$ component of Einstein equations as a "unified first law" :

$$dE = A\Psi + WdV\,, \tag{7}$$

where $A = 4\pi r^2$ and $V = \frac{4\pi}{3}r^3$. This unified law contains rich information, e.g. by projecting the unified first law along the trapping horizon, we can obtains the first law of black hole thermodynamics,which has the form [6]

$$\langle dE, z\rangle = \frac{\kappa}{8\pi G}\langle dA, z\rangle + \langle WdV, z\rangle\,, \tag{8}$$

where κ defined by

$$\kappa = \frac{1}{2}\nabla^a\nabla_a r\,, \tag{9}$$

is the surface gravity of the trapping horizion. Here $z = z^+\partial_+ + z^-\partial_-$ is a vector tangent to the trapping horizon, and it should be noticed that by definition of the horizon $\partial_+ r = 0$, one has

$$z^a\partial_a(\partial_+ r) = z^+\partial_+\partial_+ r + z^-\partial_-\partial_+ r = 0 \tag{10}$$

on the trapping horizon, then

$$\frac{z^-}{z_+} = -\frac{\partial_+\partial_+ r}{\partial_-\partial_+ r}\,. \tag{11}$$

Also note that by taking the Einstein equations $\partial_+\partial_+ r = -4\pi r T_{++}$, see ref. [4] and the definition of the surface gravity in Eq. (9), one can easily finds

$$\langle A\Psi, z\rangle = \frac{\kappa}{8\pi G}\langle dA, z\rangle\,, \tag{12}$$

which is the Clausius relation in the version of black hole thermodynamics, see the first term on the right side of Eq. (8). The left side of the above equation is nothing but the heat flow δQ, and the right side has the form TdS, if one identities the temperature $T = \kappa/2\pi$ and the entropy $S = A/4G$. So, in Einstein theory, the "unified first law" also implies the Clausius relation, and this relation is also hold in the Gauss-Bonet and Lovelock gravity theories by treating the higher derivative terms as an effective energy-momentum tensor. But, in the scalar-tensor theory, this relation is no longer hold due to some non-equilibrium thermodynamical properties. In the next section, we will study the thermodynamics of the Cardassian universe, and calculate the corresponding entropy.

III. THERMODYNAMICS OF THE CARDASSIAN UNIVERSE

The spacetime of the Cardassian universe is described by the FRW metric, which could be written in the form of

$$ds^2 = h_{ab}dx^a dx^b + \tilde{r}d\Omega^2 \,, \tag{13}$$

where $x^0 = t$, $x^1 = r$ and $\tilde{r} = a(t)r$, which is the radius of the sphere while $a(t)$ is the scale factor. Defining

$$d\xi^{\pm} = -\frac{1}{\sqrt{2}}\left(dt \mp \frac{a}{\sqrt{1-kr^2}}dr\right) \,, \tag{14}$$

where k is the spacial curvature, the metric could be rewritten as a double-null form

$$ds^2 = -2d\xi^+ d\xi^- + \tilde{r}^2 d\Omega^2 \,, \tag{15}$$

then we get the trapping horizon $\tilde{r}_T$ by solving the equation $\partial_+\tilde{r}|_{\tilde{r}=\tilde{r}_T} = 0$ as

$$\tilde{r}_T = \left(H^2 + \frac{k}{a^2}\right)^{-1/2} \,, \tag{16}$$

which has the same form of the apparent horizon. Thus, one can get the surface gravity $\kappa = -(1-\epsilon)/\tilde{r}_T$, where we have defined

$$\epsilon \equiv \frac{\dot{\tilde{r}}_T}{2H\tilde{r}_T} \,. \tag{17}$$

One can also check that $\partial_-\tilde{r}_T < 0$ indicating the trapping horizon is future. By using Eq. (10) and after a direct calculation, one can get $z^- = \epsilon/(1-\epsilon)$ when $z^+ = 1$ is chosen. Then, in the (t, r) coordinates, the project vector is given by $z = \partial_t - (1-2\epsilon)Hr\partial_r$.

In the Cardassian universe, the Friedmann equation is modified as

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}g(\rho_m) = \frac{8\pi G}{3}(\rho_m + \rho_e) \,, \tag{18}$$

where g is some function of the energy density of matter and we have defined the effect energy density $\rho_e = g(\rho_m) - \rho_m$. Using the continuity equations, we can obtain the effective pressure corresponds to the effective energy density

$$p_e = (\rho_m + p_m)g'(\rho_m) - g(\rho_m) - p_m \,. \tag{19}$$

where the prime denotes the derivative with respect to ρ_m. Then, we get the associated work density W_e and energy-supply vector Ψ_e as

$$W_e = \frac{1}{2}\Big[2g - \rho_m + p_m - (\rho_m + p_m)g'\Big] \,, \tag{20}$$

$$\Psi_e = \frac{1}{2}(g'-1)(\rho_m + p_m)(-H\tilde{r}_T dt + adr) \,. \tag{21}$$

Therefore, we obtain

$$\delta Q_e = \langle A\Psi_e, z\rangle = \frac{\kappa AH\epsilon}{2\pi G}\left(\frac{g'-1}{g'}\right) = T\left(\frac{g'-1}{4Gg'}\right)\langle dA, z\rangle \,, \tag{22}$$

where we have used the relation

$$\dot{H} - \frac{k}{a^2} = -\frac{2\epsilon}{\tilde{r}_T^2} = -4\pi G g'(\rho_m + p_m) \,. \tag{23}$$

Here, we have also identified $T = \kappa/2\pi$. And, for the heat flow of pure matter, we also have

$$\delta Q_m = \frac{\kappa}{8\pi G}\langle dA, z\rangle - \langle A\Psi_e, z\rangle = \frac{T}{4Gg'}\langle dA, z\rangle \,, \tag{24}$$

So, when $g_m = \rho_m$, the above equation reduces to the Clausius relation in the unmodified Friedmann model. In the following, we will focus on some concrete Cardassian models, namely, the original Cardassian model (OC), the modified polytropic Cardassian model (MPC) and the exponential model (EC). In these models, the function $g(\rho_m)$ takes different forms, which will reduce to ρ_m in the early universe and differ from the unmodified Friedmann universe at redshift $z < \mathcal{O}(1)$, during which it will gives rise to accelerated expansion.

A. OC model

In this model, the function $g(\rho_m)$ is given by

$$g(\rho_m) = \rho_m\left[1 + \left(\frac{\rho_m}{\rho_c}\right)^{n-1}\right] = \rho_m\big[1 + f_o(\rho_m)\big] \,, \tag{25}$$

where ρ_c is a character energy density in the Cardassian universe and the parameter n is assumed to satisfy $n < 2/3$ to give rise to a acceleration of the universe. Here we have defined the function $f_o(\rho_m) = (\rho_m/\rho_c)^{n-1}$. Thus, by using Eq. (24), the heat flow of pure matter is given by

$$\delta Q_m = \frac{T}{4G(1+nf_o)}\langle dA, z\rangle = T\langle dS_m, z\rangle \,, \tag{26}$$

where the entropy is obtained by

$$dS_m = \frac{dA}{4G(1+nf_o)}\,, \tag{27}$$

which reduces to the usual relation $dS = dA/4G$ in the limit of $f_o \to 0$. However, when f_o is large enough, the entropy becomes a constant, which means there is no heat flow of pure matter on the trapping horizon, but this time $dS_e = dA/4G$. From Friedmann equation (18), we also have the following constraint equation

$$f_o^{\frac{1}{n-1}}(1+f_o) = \frac{3}{2GA\rho_c}\,. \tag{28}$$

Therefore, we get the entropy by integrating Eq. (27)

$$\begin{aligned} S_m &= -\frac{3}{8G^2\rho_c(n-1)}\int f_o^{-\frac{n}{n-1}}(1+f_o)^{-2}df_o \\ &= \frac{A}{4G}\left(1+f_o\right){}_2F_1\left[2,\frac{-1}{n-1},\frac{n-2}{n-1},-f_o\right], \end{aligned} \tag{29}$$

up to some integration constant. Here ${}_2F_1$ is the hypergeometric function and Eq. (28) gives the relation between f_o and A.

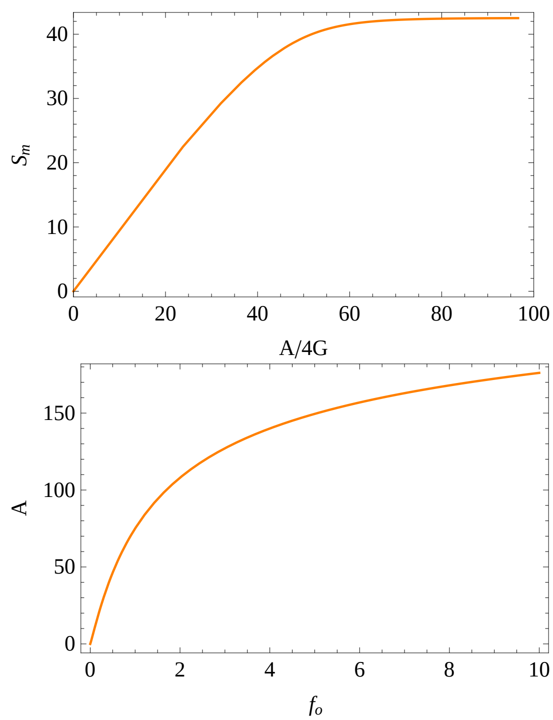

FIG. 1: Top: The entropy of the original Cardassian universe as the function of the surface area with parameter $n = 0.1$. Bottom: The area of the trapping horizon with respect to function f_o with the same parameters.

To illustrate the relation between the entropy and the surface area, we plot an example with parameter $n = 0.1$ in Fig. 1, in which it shows that when f_o is small, the entropy satisfies the usual area law $S = A/4G$, while f_o is large, it becomes a constant as we see in Eq. (27). Actually, from Eq. (29), one can obtain that

$$S_m\big|_{f_o\to\infty} = \frac{3}{8G^2\rho_c}\Gamma\left(\frac{n-2}{n-1}\right)\Gamma\left(\frac{2n-1}{n-1}\right)\,, \tag{30}$$

where Γ denotes Gamma functions.

B. MPC model

The modified polytropic Cardassian model can be obtained by introducing an additional parameter $q > 0$ into the original Cardassian model, in which the function $g(\rho_m)$ is given by

$$g(\rho_m) = \rho_m\left[1+\left(\frac{\rho_m}{\rho_c}\right)^{q(n-1)}\right]^{\frac{1}{q}} = \rho_m\left[1+f_q(\rho_m)\right]^{\frac{1}{q}}\,, \tag{31}$$

where $f_q = (\rho_m/\rho_c)^{q(n-1)}$, and when $q = 1$, it reduces to the original model. Thus, by using Eq. (24), the heat flow of pure matter is given by

$$\delta Q_m = \frac{T}{4G(1+nf_q)(1+f_q)^{\frac{1}{q}-1}}\langle dA, z\rangle = T\langle dS_m, z\rangle\,, \tag{32}$$

where the entropy is obtained by

$$dS_m = \frac{dA}{4G(1+nf_q)(1+f_q)^{\frac{1}{q}-1}}\,, \tag{33}$$

which also reduces to the usual relation $dS = dA/4G$ in the limit of $f_q \to 0$, and the entropy becomes a constant when f_q is large enough. From Friedmann equation (18), we also have the following constraint equation

$$f_q^{\frac{1}{q(n-1)}}(1+f_q)^{\frac{1}{q}} = \frac{3}{2GA\rho_c}\,. \tag{34}$$

Therefore, we get the entropy by integrating Eq. (33)

$$\begin{aligned} S_m &= -\frac{3}{8G^2\rho_c q(n-1)}\int f_q^{-\frac{1}{q(n-1)}-1}(1+f_q)^{-\frac{2}{q}}df_q \\ &= \frac{A}{4G}\left(1+f_q\right)^{\frac{1}{q}}{}_2F_1\left[\frac{2}{q},\frac{-1}{q(n-1)},1-\frac{1}{q(n-1)},-f_q\right], \end{aligned} \tag{35}$$

up to some integration constant. Here Eq. (34) gives the relation between f_q and A.

To illustrate the relation between the entropy and the surface area, we plot an example with parameter $n = 0.1$ in Fig. 2, in which it shows that when f_q is small, the entropy satisfies the usual area law $S = A/4G$, while f_q is large, it becomes a constant as we see in Eq. (33). Also, from Eq. (35), one can obtain that

$$S_m\big|_{f_q\to\infty} = \frac{3}{8G^2\rho_c}\frac{\Gamma\left(1-\frac{1}{q(n-1)}\right)\Gamma\left(\frac{2}{q}+\frac{1}{q(n-1)}\right)}{\Gamma\left(\frac{2}{q}\right)}\,, \tag{36}$$

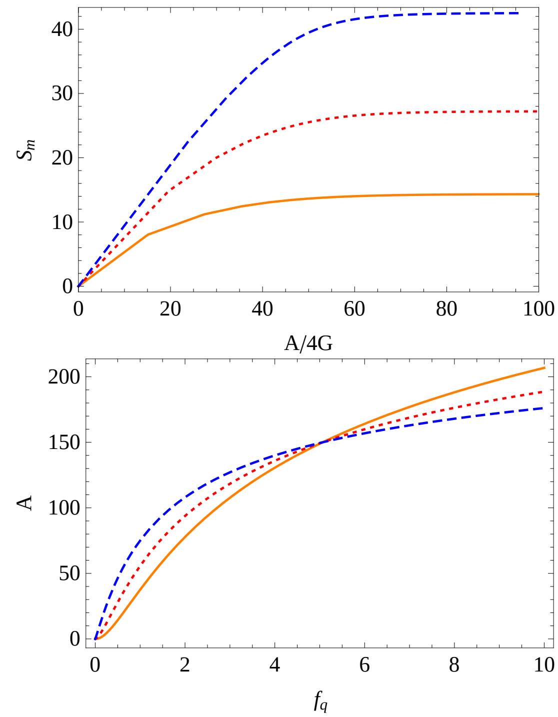

FIG. 2: Top: The entropy of the original Cardassian universe as the function of the surface area with parameter $n = 0.1$ and $q = 0.5$(solid), 0.7(dotted), 1.0(dashed). Bottom: The area of the trapping horizon with respect to function f_q with the same parameters.

so, when $q = 1$, it reduces to Eq. (30). Also, the top figure in Fig. 2 indicates that for a given n, all the curves cross the point $\left(\tilde{f}, \frac{3}{2G\rho_c}\right)$, where $\tilde{f} > 0$ satisfies $\tilde{f}^{\frac{1}{n-1}}(1+\tilde{f}) = 1$.

C. EC model

In this model, the function $g(\rho_m)$ is given by

$$g(\rho_m) = \rho_m \exp\left[\left(\frac{\rho_m}{\rho_c}\right)^{-n}\right] = \rho_m e^{f_e(\rho_m)}\,, \qquad (37)$$

where $f_e(\rho_m) = (\rho_m/\rho_c)^{-n}$. Again, by using Eq. (24), the heat flow of pure matter is given by

$$\delta Q_m = \frac{T}{4Ge^{f_e}(1-nf_e)}\langle dA, z\rangle = T\langle dS_m, z\rangle\,, \qquad (38)$$

where the entropy is obtained by

$$dS_m = \frac{dA}{4Ge^{f_e}(1-nf_e)}\,, \qquad (39)$$

which also reduces to the usual relation $dS = dA/4G$ in the limit of $f_e \to 0$, and the entropy gets its maximum value when $f_e = 1/n$, see the following. From Friedmann equation (18), we also have the following constraint equation

$$e^{f_e} f_e^{-\frac{1}{n}} = \frac{3}{2GA\rho_c}\,. \qquad (40)$$

Thus, the area has a maximum value

$$A_{max} = \frac{3}{2G\rho_c}(ne)^{-\frac{1}{n}}\,, \qquad (41)$$

when $f_e = 1/n$. Therefore, we get the entropy by integrating Eq. (39)

$$\begin{aligned} S_m &= \frac{3}{8G^2\rho_c n}\int e^{-2f_e} f_e^{\frac{1}{n}-1} df_e \\ &= \frac{3}{8G^2\rho_c n 2^{1/n}}\Big[\Gamma(1/n) - \Gamma(1/n, 2f_e)\Big]\,, \end{aligned} \qquad (42)$$

up to some integration constant. Here Eq. (40) gives the relation between f_e and A.

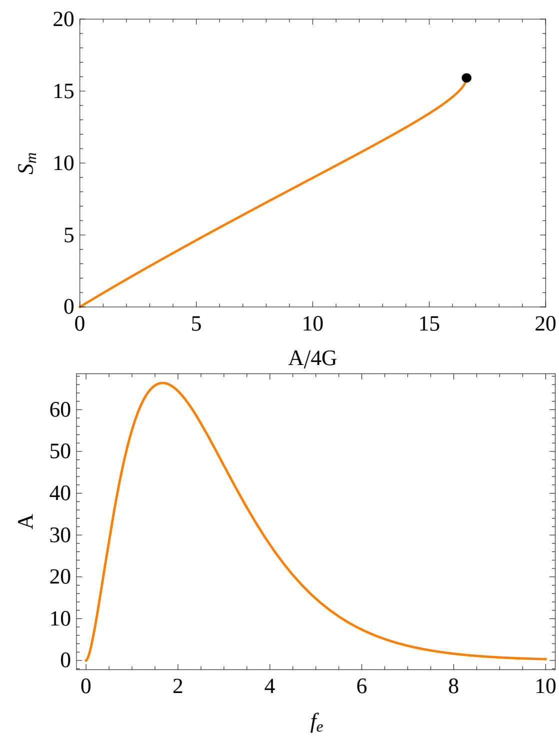

FIG. 3: Left: The entropy of the original Cardassian universe as the function of the surface area with parameter $n = 0.6$ and the black point corresponds to the maximum point of the area.

To illustrate the relation between the entropy and the surface area, we plot an example with parameter $n = 0.6$ in Fig. 3, in which it shows that when f is small, the entropy satisfies the usual area law $S = A/4G$, and it reaches its maximum value at $A = A_{max}$ when $f_e = 1/n$. Actually, from Eq. (42), one can obtain that

$$S_m\big|_{f_q\to 0} \approx \frac{3}{8G^2\rho_c} f_e^{\frac{1}{n}} \approx \frac{A}{4G}\,, \qquad (43)$$

where we have used the relation (40).

IV. DISCUSSION AND CONCULSION

In this paper, we have studied the thermodynamics of the Cardassian universe and calculated its corresponding entropy, in particular, for the OC, MPC and EC Cardassian models and the thermodynamic law might be regarded as the origin of Cardassian model, since there is some kind of correspondence between thermodynamical behavior and gravitational equations. So, if one starts from the basic law of thermodynamics, one can find the dynamics of the Cardassian universe if and only if the entropy-area relation is modified as Eqs.(29), (35) and (42), in which, the entropy obeys ordinary area law on the trapping horizon when the area is small, and it becomes a constant when area is going to be large in the OC and MPC model, while it has a maximum value in the EC model.

As we known that, the holographic principle states that all the information in the bulk should be encoded in the boundary of the bulk, so it seems that the Cardassian universe could only contain finite information. This may lead to a question that does the information in our universe will be infinite or not? Of course, if the ordinary area law $S = A/(4G)$ is always valid, the entropy will blow up when A goes to infinite value, in contrast with the case of that in the Cardassian models, S finally becomes a constant or gets its maximum value, when A is large. We will make a further study on this interesting topic [24].

It should be noticed that, the cosmological constant will not change the area law $S = A/(4G)$, because only the derivative g' emergences in Eq. (24) or (22), but in general, different dynamic dark energy models will change the law differently. So, the method we used in this paper provides a new way to distinguash different kinds of dark energy models.

Acknowledgments

This work is supported by National Science Foundation of China grant No. 11047138, National Education Foundation of China grant No. 2009312711004 , Shanghai Natural Science Foundation, China grant No. 10ZR1422000 and Shanghai Special Education Foundation, No. ssd10004.

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