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Testing the Bimodal/Schizophrenic Neutrino Hypothesis in Neutrino-less Double Beta Decay and Neutrino Telescopes

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The standard assumption is that all three neutrino mass states are either Dirac or Majorana. However, it was recently suggested by Allaverdi, Dutta and one of the authors (R.N.M.) that mixed, or bimodal, flavor neutrino scenarios are conceivable, and are consistent with all known observations (these were called "schizophrenic" in the ADM paper). In that case each individual mass eigenstate can be either Dirac or Majorana, so that the flavor eigenstates are "large" admixtures of both. An example of this "bimodal" situation is to consider one mass state as a Dirac particle (with a sterile partner), while the other two are of Majorana type. Since only Majorana particles contribute to neutrino-less double beta decay, the usual dependence of this observable on the neutrino mass is modified within this scenario. We study this in detail and in particular generalize the idea for all possible bimodal combinations. Inevitably, radiative corrections will induce a pseudo-Dirac nature to the Dirac states at the one-loop level, and the effects of the pseudo-Dirac mass splitting will show up in the flavor ratios of neutrinos from distant cosmological sources. Comparison of the effective mass in neutrino-less double beta decay as well as flavor ratios at neutrino telescopes, for different pseudo-Dirac cases and with their usual phenomenology, can distinguish the different bimodal possibilities.

I. INTRODUCTION

The observation of neutrino masses and mixings provides the first conclusive evidence for physics beyond the Standard Model (BSM), so that an understanding of this phenomenon will open up one clear direction for the new BSM physics. Knowledge of the nature of the neutrino mass is crucial in order to make progress in this search. Unlike quarks and charged leptons, for which the form of the mass term is unambiguous, neutrinos are electrically neutral and therefore allow several possibilities. The two kinds of mass terms widely discussed are: (i) a Dirac type mass, which requires the theory to have a lepton number symmetry as well as a right-handed (sterile) neutrino degree of freedom; or (ii) a Majorana type mass, which necessitates the breaking of lepton number. Implementing the first possibility in the Standard Model requires a minimal set of assumptions, i.e., simply adding three right-handed neutrinos. In order to understand the small masses one requires that the associated Yukawa couplings are of order 10^{-12} or less. The challenge then becomes one of understanding this tiny Yukawa coupling, or at least connecting it to some other phenomenon that requires it. In the case of Majorana neutrinos, one can write effective dimension five operators of the form $LHLH/M$, where M represents the effect of higher scale physics. While no small couplings need be invoked in this case, one must understand the origin of the high scale M and explore what physics is associated with it. The most widely discussed theories of this type are the seesaw models [1], where the higher scale could come from the breaking of new symmetries such as $B - L$ or possibly a grand unified theory such as $SO(10)$.

An intermediate possibility that has also been discussed in the literature is the pseudo-Dirac scenario, where a tiny Majorana mass is added for either one or both of the two two-component neutrino states that make up the Dirac neutrino [2]. If one considers all three active neutrinos to be pseudo-Dirac, then current observations put very stringent constraints on the magnitude of the Majorana mass [3], i.e., $\lesssim 10^{-10}$ eV, in order to have the pseudo-Dirac mass splitting small enough to remain undetected in solar neutrino oscillation experiments. Roughly speaking, for ν_1 and ν_2, which contain a large amount of ν_e, the pseudo-Dirac mass-squared difference should be smaller than $E/L \sim 10^{-11}$ eV^2 for solar neutrinos, otherwise the associated oscillations would have been observed in solar neutrino

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data. Detailed explanations can be found in Ref. [3]. This tiny splitting makes these neutrinos almost Dirac particles, hence the name pseudo-Dirac.

In a recent paper [4], a new possibility for neutrino masses was pointed out, where some neutrino mass eigenstates are Dirac while the others are Majorana. This is only phenomenologically viable if one defines the Dirac or Majorana nature of neutrinos in terms of the mass eigenstates, rather than the flavor eigenstates. In this case, all neutrino flavors have large admixtures of both Dirac and Majorana type mass, and can be called "bimodal flavor neutrinos" (or "schizophrenic" neutrinos, as in Ref. [4]). One then needs to add as many sterile neutrino states to the Standard Model as there are Dirac mass eigenstates. This is different from the pseudo-Dirac case in the sense that the lepton number violating and conserving terms have comparable magnitude. Another interesting feature of bimodal neutrinos is that unlike the case of pseudo-Dirac flavor neutrinos, where there exist stringent constraints on the Majorana admixture, here the oscillations of solar neutrinos (as well as all other oscillation observations) remain unaffected. In other words, in conventional neutrino oscillation experiments the bimodal flavor case looks the same as the pure Dirac or pure Majorana case.

An obvious place where the bimodal scenario leads to a different effect from both the pure Majorana and pseudo-Dirac possibilities is in the predictions for neutrino-less double beta decay. This was noted for a very specific model in Ref. [4]. In this paper we consider the most general implementation of this idea and present the predictions for neutrino-less double beta decay for the cases of both normal and inverted mass ordering.

It was also pointed out in Ref. [4] that since there is no symmetry guaranteeing the bimodal possibility, one-loop corrections can induce a tiny ($\leq 10^{-14}$ eV) amount of Majorana mass to the mass eigenstate that had a tree level Dirac mass, effectively making this mass eigenstate pseudo-Dirac. Although this value is well within the constraints from solar neutrino observations, there are implications for astrophysical neutrinos. This is one of the new points explored in this paper.

In Section II we provide a brief review of models that could lead to the bimodal flavor neutrino scenario. Sections III and IV contain a discussion of the phenomenology of the bimodal model, as it pertains to astrophysical neutrino flux ratios and neutrino-less double beta decay, respectively. The summary and conclusions are given in Section V.

II. MODELS WITH ONE OR TWO DIRAC MASSES AND ONE LOOP PSEUDO-DIRAC-NESS

There are various possible gauge models in which the bimodal possibility for neutrinos can emerge naturally. In Ref. [4], a model in which only a single mass eigenstate has a Dirac mass was considered. We briefly discuss the key ingredients of this model, and also outline a different model in which there could be two mass eigenstates with Dirac masses. These models illustrate the point that the bimodal scenario leading to a large Dirac and Majorana admixture for flavor neutrino states can be realized within gauge models.

A. One Dirac mass eigenstate

Following the model in Ref. [4], an S_3 symmetry is introduced, permuting the three families of $SU(2)$ lepton doublets (L_e, L_μ, L_τ) among themselves. This reducible representation of S_3 can be decomposed as $\underline{3} = \underline{1} + \underline{2}$, so that the following linear combinations of lepton doublet fields, transforming as one and two dimensional representations of S_3, turn out to be the mass eigenstates:

$$\begin{aligned} L_2 &= \frac{1}{\sqrt{3}}(L_e + L_\mu + L_\tau) \sim \underline{1}\,, \\ (L_1, L_3) &= \left(\frac{1}{\sqrt{6}}(2L_e - L_\mu - L_\tau), \frac{1}{\sqrt{2}}(L_\mu - L_\tau)\right) \sim \underline{2}\,. \end{aligned} \tag{1}$$

The S_3 singlet field couples to the right-handed neutrino field N_μ (assumed to be an S_3 singlet), which is isolated from the other two RH neutrinos by additional quantum numbers. This could either be a Z_n symmetry or may even be the local $B-L$ itself. For example, in Ref. [5], the $B-L$ quantum number is chosen such that N_μ has $B-L=-5$ and $N_{e,\tau}$ each have $B-L=+4$. The $B-L$ breaking Higgs can be chosen to have quantum numbers such that only $N_{e,\tau}$ have large Majorana masses (see Ref. [4] for details). After integrating out the seesaw right-handed neutrinos N_e and N_τ, the effective lepton Yukawa coupling and dimension five terms can be written as

$$\mathcal{L}_\nu = hL_2H_uN_\mu + \frac{h_1^2}{M_{N_e}}(L_1H_u)^2 + \frac{h_3^2}{M_{N_\tau}}(L_3H_u)^2 + \text{H.c.}, \tag{2}$$

with h, h_1 and h_3 dimensionless coupling constants, and H_u the up-type Higgs doublet. After electroweak symmetry breaking, the neutrino sector has one Dirac neutrino corresponding to the mass eigenstate ν_2, two Majorana eigenstates ν_1 and ν_3, as well as tri-bimaximal mixing (TBM) [6].

The Lagrangian in Eq. (2), together with the S_3 assignments in Eq. (1) and the group multiplication rules allow one to construct the symmetric 4×4 neutrino mass matrix in the flavor basis $(\nu_e, \nu_\mu, \nu_\tau, N_\mu)$, i.e.,

$$M_\nu = \frac{m_2}{\sqrt{3}}\left(\begin{array}{ccc|c} 0 & 0 & 0 & 1 \\ \cdot & 0 & 0 & 1 \\ \cdot & \cdot & 0 & 1 \\ \hline \cdot & \cdot & \cdot & 0 \end{array}\right) + \frac{m_1}{6}\left(\begin{array}{ccc|c} 4 & -2 & -2 & 0 \\ \cdot & 1 & 1 & 0 \\ \cdot & \cdot & 1 & 0 \\ \hline \cdot & \cdot & \cdot & 0 \end{array}\right) + \frac{m_3}{2}\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ \cdot & 1 & -1 & 0 \\ \cdot & \cdot & 1 & 0 \\ \hline \cdot & \cdot & \cdot & 0 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} \frac{2m_1}{3} & -\frac{m_1}{3} & -\frac{m_1}{3} & \frac{m_2}{\sqrt{3}} \\ \cdot & \frac{m_1}{6}+\frac{m_3}{2} & \frac{m_1}{6}-\frac{m_3}{2} & \frac{m_2}{\sqrt{3}} \\ \cdot & \cdot & \frac{m_1}{6}+\frac{m_3}{2} & \frac{m_2}{\sqrt{3}} \\ \hline \cdot & \cdot & \cdot & 0 \end{array}\right), \tag{3}$$

with $m_1 = h_1^2 v_u^2/M_{N_e}$, $m_3 = h_3^2 v_u^2/M_{N_\tau}$ and $m_2 = h v_u$, where $v_u = \langle H_u \rangle$. In order for the Majorana and Dirac mass matrix elements to have comparable magnitudes, the Yukawa coupling h must be of order 10^{-12}. This can be motivated in supersymmetric versions of such models, where the RH sneutrino drives inflation, and a small Dirac coupling is required to give consistent predictions (see Ref. [4] for details). This bimodal scenario is in contrast to the usual pseudo-Dirac [3] models, in which the Majorana masses are much smaller than the Dirac masses, or to the seesaw mechanism, in which case the Dirac masses are much smaller than the right-handed Majorana mass scale. The implication is that despite the large Majorana mass terms, all oscillation results remain unaffected, unlike the conventional pseudo-Dirac case. In fact, it is easy to show that neutrinos described by this mass matrix propagate in matter in the same way as those in the pure Majorana or pure Dirac case. The propagation equation contains the active part of $M_\nu^\dagger M_\nu$, which has the same form for all the scenarios we are contemplating.

The matrix diagonalizing Eq. (3) to $\text{diag}(m_1, m_2, m_3, -m_2)$ is given by

$$V_\nu = \begin{pmatrix} U_{\rm TBM} & 0_3^T \\ 0_3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0_3 \\ 0_3^T & R(\pi/4) \end{pmatrix}, \tag{4}$$

where $0_3 = (0,0,0)$ and $R(\pi/4)$ is the 3×3 unitary rotation matrix

$$R(\pi/4) = \begin{pmatrix} \cos\frac{\pi}{4} & 0 & -\sin\frac{\pi}{4} \\ 0 & 1 & 0 \\ \sin\frac{\pi}{4} & 0 & \cos\frac{\pi}{4} \end{pmatrix}. \tag{5}$$

A loop-induced pseudo-Dirac mass for ν_2 (see Sect. II C), will lead to perturbations to the matrix in Eq. (3). In the simple case where the $(4,4)$ entry of M_ν is perturbed to $\epsilon\, m_2$, the mixing matrix V_ν is modified to

$$V_\nu' = \begin{pmatrix} U_{\rm TBM} & 0_3^T \\ 0_3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0_3 \\ 0_3^T & R(\pi/4+\epsilon/4) \end{pmatrix}. \tag{6}$$

In addition, the full PMNS matrix will include rotations from the charged lepton sector. This can be described by writing down the most general Yukawa superpotential as

$$\begin{aligned} \mathcal{W}_{l,Y} &= \frac{1}{M} h_e H_d (L_e \sigma_e e^c + L_\mu \sigma_\mu \mu^c + L_\tau \sigma_\tau \tau^c) \\ &+ \frac{1}{M} h_\mu H_d (L_\mu \sigma_e e^c + L_\tau \sigma_\mu \mu^c + L_e \sigma_\tau \tau^c) \\ &+ \frac{1}{M} h_\tau H_d (L_\tau \sigma_e e^c + L_e \sigma_\mu \mu^c + L_\mu \sigma_\tau \tau^c) + \text{H.c.}\,, \end{aligned} \tag{7}$$

where $(\sigma_e, \sigma_\mu, \sigma_\tau)$ are gauge singlet superfields, h_α $(\alpha = e, \mu, \tau)$ are the Yukawa couplings and we have assumed three extra Z_n symmetries that "glue" each charged lepton singlet (e^c, μ^c, τ^c) to the corresponding σ_α gauge singlet [7]. The implication is that even though the neutrino mass matrix is diagonalized by the TBM matrix, there are small corrections from the charged lepton sector via the matrix U_ℓ, which diagonalizes it. As a result, the final PMNS matrix $U_{\rm PMNS}$ has a perturbed TBM form, if one assumes the charged lepton contributions to be small. The complete diagonalization matrix takes the same form as Eq. (6), with $U_{\rm TBM}$ replaced by $U_\ell^\dagger U_{\rm TBM}$.

In this model only the ν_2 mass eigenstate has a Dirac mass, but this idea can easily be generalized in the sense that one or both of the other two neutrino mass eigenstates (ν_1 and/or ν_3) could be Dirac. The case in which two of the eigenstates are Dirac is motivated in the next subsection.

B. Model with two Dirac mass eigenstates

A model in which two Dirac mass eigenstates and one Majorana mass eigenstate appear naturally is a minimal $B-L$ extension of the MSSM, where $B-L$ is broken by the vacuum expectation value (VEV) of the right-handed sneutrino [8]. Apart from the fact that there are three right-handed neutrinos (required for anomaly cancellation) and the gauge interactions associated with $B-L$, the model is essentially the same as the MSSM, i.e., two Higgs doublets that have zero $B-L$. This is therefore a minimal extension of the MSSM with local $B-L$. Furthermore, by choosing the gauge group to be $SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L}$, the model preserves gauge coupling unification with $B-L$ breaking at the TeV scale, without any additional fields. Radiative corrections can allow the sneutrino fields to acquire a nonzero VEV, for certain ranges of parameters. As was noted in the first reference of Ref. [8], this leads to a neutrino mass matrix of the form

$$M_\nu = \begin{pmatrix} 0_{3\times 3} & h_\nu v_u & 0_3^T \\ h_\nu^T v_u & 0_{3\times 3} & g_{BL}\langle \tilde{\nu}_\alpha^c \rangle \\ 0_3 & g_{BL}\langle \tilde{\nu}_\alpha^c \rangle^T & \mu \end{pmatrix} , \qquad (8)$$

where the rows and columns correspond to $(\nu_\alpha, \nu_\alpha^c, \tilde{V})$, $\alpha = 1,2,3$ and $\tilde{V}$ is the superpartner of the linear combination of $B-L$ and I_{3R} gauge boson, i.e. $(g_R V_{3R} - g_{BL} V_{BL})/\sqrt{g_R^2 + g_{BL}^2}$. Here g_R, g_{BL} are the gauge couplings of $U(1)_{I_{3R}}, U(1)_{B-L}$ respectively, h_ν is the 3×3 Yukawa coupling matrix for ν^c and $\langle \tilde{\nu}_\alpha^c \rangle$ is a column vector with components given by the three $\tilde{\nu}^c$ VEVs. The parameter μ is the SUSY breaking Majorana mass term of the $B-L$ gaugino $\tilde{V}$. One could in fact redefine the right-handed neutrino and sneutrino states so that the linear combination of right-handed neutrinos that mixes with left-handed neutrinos in the Dirac mass in Eq. (8) is same as the one for sneutrinos that picks up a VEV (we can call this $\tilde{\nu}_e^c$). In that case, only one right-handed neutrino is kept in the 3×3 neutrino mass matrix in the equation above. The other two right-handed neutrinos remain coupled to the left-handed neutrinos only through the Yukawa couplings and are not present in Eq. (8). In the above mass matrix we have neglected small contributions that could arise from induced sneutrino VEVs, which can mix the right-handed neutrinos with Higgsinos.

It is clear that this matrix leads to one linear combination of light neutrinos with Majorana mass given by the inverse seesaw formula [9], while the two other combinations only get a Dirac mass. With additional symmetries, e.g. S_3 as in Ref. [4], one could get the TBM pattern for light neutrinos. However, the main point for our discussion is that this model can naturally lead to one Majorana and two Dirac mass eigenstates.

For illustration, the full symmetric 5×5 mass matrix in the flavor basis (for the TBM version where ν_1, ν_2 are Dirac type) would read:

$$M_\nu = \left(\begin{array}{ccc|cc} 0 & 0 & 0 & \frac{m_2}{\sqrt{3}} & \frac{2m_1}{\sqrt{6}} \\ \cdot & \frac{m_3}{2} & -\frac{m_3}{2} & \frac{m_2}{\sqrt{3}} & -\frac{m_1}{\sqrt{6}} \\ \cdot & \cdot & \frac{m_3}{2} & \frac{m_2}{\sqrt{3}} & -\frac{m_1}{\sqrt{6}} \\ \hline \cdot & \cdot & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & 0 \end{array}\right) . \qquad (9)$$

In analogy to the case treated in the previous subsection, this matrix can have Majorana mass terms that are similar in magnitude to the Dirac masses.

The considerations from this and the preceding subsection indicate that situations in which one or two neutrino mass eigenstates are Dirac and the others (or other) Majorana are possible and arise in simple models. We are therefore motivated to look for experimental implications of these scenarios. However, we wish to note that the small Dirac masses are obtained at the price of tuning the Yukawa couplings. Clearly it will be more desirable to have a theory that can predict these small values.

C. One-loop corrections to Dirac mass eigenstates

Here we remark on the observation [4, 10] that one-loop corrections to tree level Dirac state(s) pick up tiny Majorana corrections, if lepton number is not conserved. For simplicity, let us consider the case with one Dirac state; the discussion easily generalizes to the case of two Dirac states. Note that in the effective low energy Lagrangian of Eq. (2), only a specific linear combination of the lepton doublets that are eigenstates of S_3 appear. The charged lepton mass terms break this symmetry, resulting in mixings between different mass eigenstates in the finite wave function renormalization corrections that arise at the one-loop level. In the specific case of our first example, this will mean new terms of the form $\delta_{12}\bar{\nu}_2\gamma^\mu\partial_\mu\nu_1$, where $\delta_{12} \sim \frac{G_F m_\tau^2}{16\pi^2\sqrt{6}} \sim 10^{-7}$. Upon diagonalization of the kinetic terms, the new states become $\nu_1' \approx \nu_1 + \delta_{12}\nu_2$; hence the Majorana mass term for ν_1 in the new basis leads to a Majorana mass of magnitude $\delta_{12}^2 m_1$ for ν_2. The leading pseudo-Dirac contribution to the Dirac eigenstate is therefore of order $10^{-14}\sqrt{\Delta m_{\rm A}^2} \sim 10^{-15}$ eV ($\sqrt{\Delta m_{\rm A}^2}$ is the atmospheric mass-squared difference), corresponding to an oscillation length of $\sim$ 10 kilo parsecs (kpc). This implies that extra galactic neutrinos from sources beyond 10 kpc will have half of their ν_2 component oscillate into sterile neutrinos, thus affecting the observed flavor ratios of extra galactic neutrinos, which we discuss in the next section.

We will no longer specify the magnitude of the mass-squared difference of the pseudo-Dirac neutrino ν_i, but simply call it δm_i^2, and analyze the phenomenological consequences in flavor ratios at neutrino telescopes and in neutrino-less double beta decay.

III. EXTRA-GALACTIC NEUTRINO PHENOMENOLOGY

Extra-galactic neutrinos by definition travel large distances in space and can have different energies, depending on their source. Thus these neutrinos can be a probe of standard and non-standard neutrino properties (see Ref. [11] for reviews). In most cases [12] the neutrinos originate from pion (and kaon) decay, followed by muon decay ($\pi^- \to \mu^- + \bar{\nu}_\mu$ and $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$), giving the initial flavor flux ratios of $\Phi_e^0 : \Phi_\mu^0 : \Phi_\tau^0 = 1 : 2 : 0$. However, in neutron sources the initial ratios are $1 : 0 : 0$, with electron anti-neutrinos originating from β decays [13]; in muon-damped sources they become $0 : 1 : 0$, since the muons (but not pions) lose energy before they decay [14]. Although the latter two sources are presumably less common and harder to measure (there is less total neutrino flux), they allow for interesting comparative studies with the usual pure pion source.

In general, the initial flux composition may be described as [15]

$$(\Phi_e^0 : \Phi_\mu^0 : \Phi_\tau^0) = (1 : n : 0)\,. \tag{10}$$

Here the parameter n distinguishes the different types of neutrino sources: for neutron sources, the initial ratio of $1 : 0 : 0$ is represented by the limit $n \to 0$, whereas in muon-damped sources the initial ratio of $0 : 1 : 0$ is the limit $n \to \infty$. Pure pion sources have $n = 2$. In each case neutrino mixing will affect the final flavor flux ratios at Earth detectors, and these ratios will also depend on whether the neutrinos are pseudo-Dirac or bimodal. It is well known that for the initial ratios of $1 : 2 : 0$, the final ratios turn out to be $1 : 1 : 1$ [12], assuming $\mu - \tau$ symmetry (actually, it suffices to assume that $\mathfrak{Re}(U_{e3}) = 0$ and $\theta_{23} = \pi/4$) and three standard neutrinos. Deviations from this symmetry limit will be discussed below. If some or all of the neutrinos are pseudo-Dirac, the detected flux ratios are modified, and it is possible to study the effects of deviations in each different case.

In the standard three-neutrino scenario (no pseudo-Dirac effects), the flavor conversion probability reads

$$P_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 2\sum_{i>j}\mathfrak{Re}\left(U_{\alpha j}U_{\alpha i}^* U_{\beta j}^* U_{\beta i}\right) = \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2\,. \tag{11}$$

However, it can be shown that if all neutrinos are pseudo-Dirac [16],

$$P_{\alpha\beta} = \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2 \cos^2\left(\frac{\delta m_i^2 L}{4E}\right), \tag{12}$$

where $\delta m_i^2 = (m_i^+)^2 - (m_i^-)^2$ is the small mass-squared difference between the pseudo-Dirac pairs, and it is obvious that this reduces to Eq. (11) for $\delta m_i^2 = 0$. In the spirit of the bimodal flavor neutrino cases discussed above, not all of the three states ν_i could be pseudo-Dirac, but only one or two. If the corresponding $\delta m_i^2 L/4E \gg 1$, in other words if L/E is large enough, the cosine term averages out to 1/2. The standard effects from neutrino mixing are therefore modified and neutrinos from very distant sources could probe the tiny pseudo-Dirac mass-squared differences [16–19]. Recall that mass splittings of less than about 10^{-11} eV^2 have no effect on the solar neutrino flux [3].

If one assumes that only one neutrino is pseudo-Dirac (say ν_2), then the corresponding term ($i = 2$) of the sum in Eq. (12) is modified by a factor of 1/2, leading to the probability

$$P_{\alpha\beta} = |U_{\alpha 1}|^2 |U_{\beta 1}|^2 + \frac{1}{2}|U_{\alpha 2}|^2 |U_{\beta 2}|^2 + |U_{\alpha 3}|^2 |U_{\beta 3}|^2 \ . \tag{13}$$

This can be extended to cases in which different combinations of neutrinos are pseudo-Dirac; the reduction factor of 1/2 is applied to the relevant terms in each case. The measured neutrino flux, Φ_α, is the sum of the product of each initial flux Φ^0_α with the relevant flavor conversion probability,

$$\Phi_\alpha = \sum_\beta P_{\beta\alpha} \Phi^0_\beta \ , \tag{14}$$

so that the presence of one or more pseudo-Dirac neutrinos will change the final detected flux (and flux ratios) compared to the standard case. Table I shows the observable Φ_μ/Φ_e ratio as a function of θ_{12} for the different combinations of pseudo-Dirac neutrinos, for $\mu - \tau$ symmetry and initial fluxes of $1 : 2 : 0$. Note that if all three neutrinos are pseudo-Dirac the observed flux ratio is again $1 : 1$, with an overall reduction in flux of 1/2. In several cases the ratio is independent of θ_{12}. We also observe that if ν_2 or $\nu_{1,3}$ are pseudo-Dirac, then Φ_μ/Φ_e is $1 : 1$ only if $\sin^2\theta_{12} = \frac{1}{3}$, i.e., for exact TBM.

In the most general case one can expect deviations from exact $\mu - \tau$ symmetry, so that the relations $\theta_{13} = 0$ and $\theta_{23} = \frac{\pi}{4}$ are not exact. Defining the deviation parameter

$$\epsilon = \frac{\pi}{4} - \theta_{23} \,, \tag{15}$$

the probability matrix P (without pseudo-Dirac effects) with elements $P_{\alpha\beta}$ can be approximated as

$$P \approx \begin{pmatrix} 1 - 2c^2_{12}s^2_{12} & c^2_{12}s^2_{12} + \Delta & c^2_{12}s^2_{12} - \Delta \\ \cdot & \frac{1}{2}(1 - c^2_{12}s^2_{12}) - \Delta & \frac{1}{2}(1 - c^2_{12}s^2_{12}) \\ \cdot & \cdot & \frac{1}{2}(1 - c^2_{12}s^2_{12}) + \Delta \end{pmatrix} , \tag{16}$$

where $s^2_{12} = \sin^2\theta_{12}$, $c^2_{12} = \cos^2\theta_{12}$ and the universal correction parameter is defined as [21, 22]

$$\Delta \equiv \frac{1}{4}(2\epsilon \sin^2 2\theta_{12} + \theta_{13}\cos\delta \sin 4\theta_{12}) = 2\epsilon s^2_{12}c^2_{12} + \frac{1}{4}\theta_{13}\cos\delta\sin 4\theta_{12} \,. \tag{17}$$

Terms of order $\mathcal{O}(\theta^2_{13})$, $\mathcal{O}(\epsilon^2)$ and $\mathcal{O}(\theta_{13}\epsilon)$ have been neglected in this approximation. In this case it can be shown that the flux ratio evolves as [21, 22]

$$(1 : 2 : 0) \longrightarrow (1 + 2\Delta) : (1 - \Delta) : (1 - \Delta) \,. \tag{18}$$

TABLE I: The observed Φ_μ/Φ_e neutrino flux ratio at the detector for different combinations of pseudo-Dirac neutrinos, assuming the initial flux ratios of $1 : 2 : 0$ and exact $\mu - \tau$ symmetry. Numerical values are calculated from the global fit data in Ref. [20].

Pseudo-Dirac neutrinos	Φ_μ/Φ_e General case	Best-fit	3σ
None & all	$1 : 1$	1.00	1.00
ν_1	$1 - \frac{1}{4}\sin^2\theta_{12} : \frac{1}{2}(1 + \sin^2\theta_{12})$	1.40	1.32−1.46
ν_2 & ν_3	$\frac{1}{4}(2 + \sin^2\theta_{12}) : 1 - \frac{1}{2}\sin^2\theta_{12}$	0.67	0.66−0.73
ν_2	$\frac{1}{4}(3 + \sin^2\theta_{12}) : 1 - \frac{1}{2}\sin^2\theta_{12}$	0.99	0.95−1.04
ν_1 & ν_3	$\frac{1}{4}(3 - \sin^2\theta_{12}) : \frac{1}{2}(1 + \sin^2\theta_{12})$	0.58	0.55−0.60
ν_3	$\frac{3}{4} : 1$	0.75	0.75
ν_1 & ν_2	$\frac{3}{2} : 1$	1.50	1.50

The parameters ϵ and Δ lie in the ranges $-0.148 \le \epsilon \le 0.166$ and $-0.10 \le \Delta \le 0.11$ for the current 3σ ranges [20] of the oscillation parameters.[1]

This general framework can be applied to the pseudo-Dirac scenario: if one or more neutrinos are pseudo-Dirac, the probability matrix in Eq. (16) will be modified, leading to different final flux ratios in each case. These probabilities can be written (to first order in θ_{13} and ϵ) in terms of θ_{12}, θ_{13} and the deviation parameters ϵ, Δ and Γ, where

$$\Gamma \equiv \frac{1}{8}\theta_{13}\sin 2\theta_{12}\cos\delta = \frac{1}{4}\theta_{13}s_{12}c_{12}\cos\delta\,. \tag{19}$$

The parameter Γ is constrained to the range $-0.026 \le \Gamma \le 0.026$. For each case, the flavor conversion probabilities are given by:[2]

- ν_1 **pseudo-Dirac:**

$$\begin{aligned}
P_{ee}^{\nu_1} &= \frac{1}{2}c_{12}^4 + s_{12}^4\,,\\
P_{e\mu}^{\nu_1} &= \frac{3}{4}(c_{12}^2 s_{12}^2 + \Delta) - \Gamma\,,\\
P_{\mu\mu}^{\nu_1} &= \frac{1}{2} - \frac{1}{8}s_{12}^2\left(1 + 3c_{12}^2 + 4\epsilon\right) - \frac{3}{4}\Delta - \Gamma\,,\\
P_{e\tau}^{\nu_1} &= \frac{3}{4}(c^2 12 s_{12}^2 - \Delta) + \Gamma\,,\\
P_{\mu\tau}^{\nu_1} &= \frac{1}{4} + \frac{1}{4}c_{12}^4 + \frac{1}{8}s_{12}^4\,,
\end{aligned} \tag{20}$$

- ν_2 **and** ν_3 **pseudo-Dirac:**

$$\begin{aligned}
P_{ee}^{\nu_{2,3}} &= c_{12}^4 + \frac{1}{2}s_{12}^4\,,\\
P_{e\mu}^{\nu_{2,3}} &= \frac{3}{4}(c_{12}^2 s_{12}^2 + \Delta) + \Gamma\,,\\
P_{\mu\mu}^{\nu_{2,3}} &= \frac{1}{4} + \frac{1}{8}s_{12}^2(1 - 3c_{12}^2 + 4\epsilon) - \frac{3}{4}\Delta + \Gamma\,,\\
P_{e\tau}^{\nu_{2,3}} &= \frac{3}{4}(c_{12}^2 s_{12}^2 - \Delta) - \Gamma\,,\\
P_{\mu\tau}^{\nu_{2,3}} &= \frac{1}{8} + \frac{1}{8}c_{12}^4 + \frac{1}{4}s_{12}^4\,,
\end{aligned} \tag{21}$$

- ν_2 **pseudo-Dirac:**

$$\begin{aligned}
P_{ee}^{\nu_2} &= c_{12}^4 + \frac{1}{2}s_{12}^4\,,\\
P_{e\mu}^{\nu_2} &= \frac{3}{4}(c_{12}^2 s_{12}^2 + \Delta) + \Gamma\,,\\
P_{\mu\mu}^{\nu_2} &= \frac{1}{2} - \frac{1}{8}c_{12}^2(1 + 3s_{12}^2 + 4\epsilon) - \frac{3}{4}\Delta + \Gamma\,,\\
P_{e\tau}^{\nu_2} &= \frac{3}{4}(c_{12}^2 s_{12}^2 - \Delta) - \Gamma\,,\\
P_{\mu\tau}^{\nu_2} &= \frac{1}{4} + \frac{1}{8}c_{12}^4 + \frac{1}{4}s_{12}^4\,,
\end{aligned} \tag{22}$$

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[1] "NNLO" terms of second order in ϵ and θ_{13} have been discussed in [23, 24].
[2] The ratio $P_{\tau\tau}$ is omitted, as it is not needed to calculate flux ratios.

- **ν_1 and ν_3 pseudo-Dirac:**

$$\begin{aligned}
P_{ee}^{\nu_{1,3}} &= \frac{1}{2}c_{12}^4 + s_{12}^4\,,\\
P_{e\mu}^{\nu_{1,3}} &= \frac{3}{4}(c_{12}^2 s_{12}^2 + \Delta) - \Gamma\,,\\
P_{\mu\mu}^{\nu_{1,3}} &= \frac{1}{4} + \frac{1}{8}c_{12}^2(1 - 3s_{12}^2 + 4\epsilon) - \frac{3}{4}\Delta - \Gamma\,,\\
P_{e\tau}^{\nu_{1,3}} &= \frac{3}{4}(c_{12}^2 s_{12}^2 - \Delta) + \Gamma\,,\\
P_{\mu\tau}^{\nu_{1,3}} &= \frac{1}{8} + \frac{1}{4}c_{12}^4 + \frac{1}{8}s_{12}^4\,,
\end{aligned} \tag{23}$$

- **ν_3 pseudo-Dirac:**

$$\begin{aligned}
P_{ee}^{\nu_3} &= c_{12}^4 + s_{12}^4\,,\\
P_{e\mu}^{\nu_3} &= c_{12}^2 s_{12}^2 + \Delta\,,\\
P_{\mu\mu}^{\nu_3} &= \frac{3}{8} + \frac{1}{2}\left(c_{12}^2 s_{12}^2 + \epsilon\right) - \Delta\,,\\
P_{e\tau}^{\nu_3} &= c_{12}^2 s_{12}^2 - \Delta\,,\\
P_{\mu\tau}^{\nu_3} &= \frac{3}{8} - \frac{1}{2}c_{12}^2 s_{12}^2\,,
\end{aligned} \tag{24}$$

- **ν_1 and ν_2 pseudo-Dirac:**

$$\begin{aligned}
P_{ee}^{\nu_{1,2}} &= \frac{1}{2}(c_{12}^4 + s_{12}^4)\,,\\
P_{e\mu}^{\nu_{1,2}} &= \frac{1}{2}(c_{12}^2 s_{12}^2 + \Delta)\,,\\
P_{\mu\mu}^{\nu_{1,2}} &= \frac{3}{8} - \frac{1}{4}c_{12}^2 s_{12}^2 - \frac{1}{2}\epsilon - \frac{1}{2}\Delta\,,\\
P_{e\tau}^{\nu_{1,2}} &= \frac{1}{2}(c_{12}^2 s_{12}^2 - \Delta)\,,\\
P_{\mu\tau}^{\nu_{1,2}} &= \frac{3}{8} - \frac{1}{4}c_{12}^2 s_{12}^2\,.
\end{aligned} \tag{25}$$

These expressions can be used to calculate the final flux ratios in each case. We discuss the most straightforwardly measurable flux ratio Φ_μ/Φ_e, and also display results for the ratio Φ_e/Φ_τ, which is harder to measure.

The plots in Figs. 1 and 2 show the variation in the flux ratios Φ_μ/Φ_e and Φ_e/Φ_τ with $\sin^2\theta_{12}$ for the different possible combinations of one or two pseudo-Dirac neutrinos, assuming the standard case of an initial flux ratio of 1 : 2 : 0 (a pure pion source). For comparison, the standard case without any pseudo-Dirac nature is also shown, for which the ratios can be approximated by

$$\frac{\Phi_\mu}{\Phi_e} \approx \frac{1-\Delta}{1+2\Delta} \approx 1 - 3\Delta \quad \text{and} \quad \frac{\Phi_e}{\Phi_\tau} \approx \frac{1+2\Delta}{1-\Delta} \approx 1 + 3\Delta\,, \tag{26}$$

using Eq. (18) and neglecting quadratic terms. One can see from the plots in Figs. 1 and 2 that the two cases in which ν_3 and $\nu_{1,2}$ are pseudo-Dirac show very little dependence on θ_{12} (compare with Table I), even with deviations applied. lowering (decreasing the allowed range) the flux ratio for TBM (with deviations). The ratio Φ_μ/Φ_e differs considerably from the standard case if either ν_1 or both ν_1 and ν_2 are pseudo-Dirac, and can be approximated by

$$\frac{\Phi_\mu^{\nu_1}}{\Phi_e^{\nu_1}} \approx \{1 - 3\Delta\} + \frac{2 - s_{12}^2 - 3s_{12}^4}{2(1+s_{12}^2)^2} - \frac{2\epsilon(s_{12}^2 + s_{12}^4)}{(1+s_{12}^2)^2} - \frac{3\Delta(3 - 4s_{12}^2 - 2s_{12}^4)}{2(1+s_{12}^2)^2} + \frac{2\Gamma(1 - 4s_{12}^2)}{(1+s_{12}^2)^2}\,, \tag{27}$$

$$\frac{\Phi_\mu^{\nu_{1,2}}}{\Phi_e^{\nu_{1,2}}} \approx \{1 - 3\Delta\} + \frac{1}{2} - 2\epsilon - \Delta\,. \tag{28}$$

In both cases the expressions are given to first order in the deviation parameters and the curly brackets correspond to the standard case [Eq. (26)]. For the ratio Φ_e/Φ_τ, Fig. 2 shows that there are potentially strong effects if either ν_3 or both ν_2 and ν_3 are pseudo-Dirac, in which case

$$\frac{\Phi_e^{\nu_3}}{\Phi_\tau^{\nu_3}} \approx \{1+3\Delta\} + \frac{1}{3} + \frac{13}{9}\Delta\,, \tag{29}$$

$$\frac{\Phi_e^{\nu_{2,3}}}{\Phi_\tau^{\nu_{2,3}}} \approx \{1+3\Delta\} + \frac{8+4s_{12}^2-8c_{12}^2s_{12}^2-18s_{12}^4-3s_{12}^6}{(2+s_{12}^2)^3} + \frac{3\Delta(8+4s_{12}^2-6s_{12}^4-s_{12}^6)}{(2+s_{12}^2)^3} + \frac{32\Gamma(2+s_{12}^2)}{(2+s_{12}^2)^3}\,. \tag{30}$$

As mentioned above, the intial flavor ratios of other interesting neutrino sources, such as neutron or muon-damped sources, can be parameterised as in Eq. (10). Figures 3 and 4 indicate the dependence of the ratios Φ_μ/Φ_e and Φ_e/Φ_τ on n, for the different pseudo-Dirac combinations. It is evident that in certain cases the observed ratio can be much larger than in the standard case. Specifically, in the case of ν_2 being pseudo-Dirac, the ratios Φ_μ/Φ_e and Φ_e/Φ_τ can become large for $n\to\infty$ and $n\to 0$, respectively. Expanding to first order in the deviation parameters, the ratios are given in these cases by

$$\frac{\Phi_\mu^{\nu_2}}{\Phi_e^{\nu_2}} \xrightarrow{n\to\infty} \frac{P_{\mu\mu}^{\nu_2}}{P_{e\mu}^{\nu_2}} \approx \left\{\frac{1-c_{12}^2s_{12}^2}{2c_{12}^2s_{12}^2} - \frac{1+c_{12}^2s_{12}^2}{2c_{12}^4s_{12}^4}\Delta\right\} - \frac{1+4\epsilon}{6s_{12}^2} + \frac{1+12\Gamma}{6c_{12}^2s_{12}^2} + \frac{3\Delta+4\Gamma}{18c_{12}^2s_{12}^4} - \frac{3\Delta+16\Gamma}{18c_{12}^4s_{12}^4}\,, \tag{31}$$

$$\frac{\Phi_e^{\nu_2}}{\Phi_\tau^{\nu_2}} \xrightarrow{n\to 0} \frac{P_{ee}^{\nu_2}}{P_{e\tau}^{\nu_2}} \approx \left\{\frac{1-2c_{12}^2s_{12}^2}{c_{12}^2s_{12}^2} + \frac{1-2c_{12}^2s_{12}^2}{c_{12}^4s_{12}^4}\Delta\right\} + \frac{1}{3c_{12}^2s_{12}^2} - \frac{6\Delta+8\Gamma}{9c_{12}^4s_{12}^2} + \frac{3\Delta+16\Gamma}{9c_{12}^4s_{12}^4}\,. \tag{32}$$

In each case the terms in curly brackets again denote the flux ratios corresponding to the general case, without pseudo-Dirac neutrinos, in the same limit ($n\to\infty$ or $n\to 0$). Additionally, if both ν_1 and ν_2 are pseudo-Dirac, the ratio Φ_μ/Φ_e becomes large for $n\to\infty$,

$$\frac{\Phi_\mu^{\nu_{1,2}}}{\Phi_e^{\nu_{1,2}}} \xrightarrow{n\to\infty} \frac{P_{\mu\mu}^{\nu_{1,2}}}{P_{e\mu}^{\nu_{1,2}}} \approx \left\{\frac{1-c_{12}^2s_{12}^2}{2c_{12}^2s_{12}^2} - \frac{1+c_{12}^2s_{12}^2}{2c_{12}^4s_{12}^4}\Delta\right\} + \frac{1-4\epsilon}{4c_{12}^2s_{12}^2} - \frac{1}{c_{12}^4s_{12}^4}\Delta\,, \tag{33}$$

and if both ν_2 and ν_3 are pseudo-Dirac, the ratio Φ_e/Φ_τ becomes large for $n\to 0$,

$$\frac{\Phi_e^{\nu_{2,3}}}{\Phi_\tau^{\nu_{2,3}}} \xrightarrow{n\to 0} \frac{P_{ee}^{\nu_{2,3}}}{P_{e\tau}^{\nu_{2,3}}} = \frac{P_{ee}^{\nu_2}}{P_{e\tau}^{\nu_2}} \approx \left\{\frac{1-2c_{12}^2s_{12}^2}{c_{12}^2s_{12}^2} + \frac{1-2c_{12}^2s_{12}^2}{c_{12}^4s_{12}^4}\Delta\right\} + \frac{1}{3c_{12}^2s_{12}^2} - \frac{6\Delta+8\Gamma}{9c_{12}^4s_{12}^2} + \frac{3\Delta+16\Gamma}{9c_{12}^4s_{12}^4}\,. \tag{34}$$

The plots in Figs. 3 and 4 could in principle be used to rule out certain cases. If, for instance, measurements of the neutrino flux ratios from a muon-damped source give $\Phi_\mu/\Phi_e \gtrsim 5$, four of the six possibilities would be ruled out, so that either ν_2 or both ν_1 and ν_2 would have to be pseudo-Dirac neutrinos. A similar result applies for the case of $\Phi_e/\Phi_\tau \gtrsim 7$ and neutron sources, where only ν_2 or ν_2 and ν_3 could be pseudo-Dirac.

IV. NEUTRINO-LESS DOUBLE BETA DECAY PHENOMENOLOGY

Another experimental test of the bimodal flavor neutrino scenario is neutrino-less double beta decay (see Ref. [25] for reviews). In the general case with three Majorana neutrino mass eigenstates, the amplitude for this process is proportional to the effective Majorana mass

$$\langle m_{ee}\rangle = \left|\sum_{i=1}^{3} U_{ei}^2 m_i\right| = \left|c_{12}^2c_{13}^2|m_1| + s_{12}^2c_{13}^2|m_2|e^{i\alpha} + s_{13}^2|m_3|e^{i\beta}\right|, \tag{35}$$

with α and β the Majorana phases. Here the decay is mediated by light, active and massive Majorana neutrinos, and one assumes that there are no other new physics contributions, such as heavy neutrino exchange, right-handed currents or R-parity violating SUSY.

The coherent sum $\langle m_{ee}\rangle$ contains 7 out of 9 parameters of the neutrino mass matrix, and is the only observable carrying information about the Majorana phases. It is possible for $\langle m_{ee}\rangle$ to vanish in the case of normal neutrino mass ordering; this is equivalent to a zero in the (1,1) element of the low energy Majorana neutrino mass matrix. In the inverted ordering case $\langle m_{ee}\rangle$ cannot vanish, and the lower limit is given by

$$\langle m_{ee}\rangle \approx \left|(c_{12}^2 + s_{12}^2e^{i\alpha})\sqrt{\Delta m_{\rm A}^2}\right| \gtrsim (c_{12}^2 - s_{12}^2)\sqrt{\Delta m_{\rm A}^2} \approx \frac{\sqrt{\Delta m_{\rm A}^2}}{3} \approx 17\ {\rm meV}, \tag{36}$$

where $\Delta m_{\rm A}^2$ is the mass-squared difference of atmospheric neutrinos. The final extraction of the decay half-life is affected by uncertainties in the nuclear matrix elements, so that an indisputable measurement of this process requires improved precision in both particle and nuclear physics parameters. Future experiments such as GERDA and SuperNEMO aim to reach a sensitivity of order 10 meV, and should thus be able to rule out the inverted mass ordering, as long as the relevant parameter uncertainties are reduced.

The standard picture of neutrino-less double beta decay is modified in the presence of pseudo-Dirac neutrinos. With three pseudo-Dirac neutrinos, the expression in Eq. (35) becomes proportional to $\sum_{i=1}^{3} U_{ei}^2 \frac{\delta m_i^2}{2m_i}$, because the approximately degenerate eigenstates of the pseudo-Dirac pair have opposite CP parities. This contribution is effectively vanishing ($\langle m_{ee}\rangle \lesssim 10^{-4}$ eV), and can be neglected. However, if only one or two neutrino mass eigenstates are pseudo-Dirac (the bimodal scenario), one effectively has a combination of the standard case [Eq. (35)] and the pure pseudo-Dirac case. Those neutrinos that are pseudo-Dirac do not contribute to $\langle m_{ee}\rangle$, whereas the normal Majorana mass eigenstates contribute as in Eq. (35).

Figures 5 and 6 show the allowed ranges in $\langle m_{ee}\rangle - \sum m_i$ parameter space, for different combinations of pseudo-Dirac neutrinos and both normal and inverted neutrino mass ordering. The parameter space in the standard case is included for comparison. We have plotted the effective mass against the sum of masses $\sum m_i$, rather than the smallest mass itself, because the latter is, strictly speaking, not an observable.

In each case, the contribution from the pseudo-Dirac pair was assumed to be vanishing, so that

$$\langle m_{ee}\rangle = \left| \sum_{j=1}^{N} U_{ej}^2 m_j \right| , \tag{37}$$

where the index j runs over the neutrinos that are *not* pseudo-Dirac, and $N = 1$ or $N = 2$. For instance, in the case where only ν_2 is pseudo-Dirac, the effective Majorana mass becomes

$$\langle m_{ee}\rangle = \left| c_{12}^2 c_{13}^2 |m_1| + s_{13}^2 |m_3| e^{i\beta} \right| , \tag{38}$$

and there is only one phase, β. One can see from the plots in Figs. 5 and 6 that in the cases of ν_2 and $\nu_{2,3}$ pseudo-Dirac and inverted mass ordering, the lower limit for $\langle m_{ee}\rangle$ is increased by a factor of two [4]. Explicitly, the lower bound for the inverted ordering becomes

$$\langle m_{ee}\rangle \approx c_{12}^2 \sqrt{\Delta m_{\rm A}^2} \gtrsim \frac{2\sqrt{\Delta m_{\rm A}^2}}{3} \approx 34 \text{ meV} , \tag{39}$$

to be compared with the bound for the standard case in Eq. (36).

Due to the fact that $c_{12}^2 - s_{12}^2 \approx s_{12}^2$, the case in which ν_1 is pseudo-Dirac results in $\langle m_{ee}\rangle$ taking its minimal value in the inverted ordering. Another interesting case is when $\nu_{1,3}$ are pseudo-Dirac with normal ordering, where the lower limit of $\langle m_{ee}\rangle$ is given by ($\Delta m_{\rm S}^2$ is the mass-squared difference of solar neutrinos)

$$\langle m_{ee}\rangle \gtrsim s_{12}^2 \sqrt{\Delta m_{\rm S}^2} \approx 2.9 \text{ meV} , \tag{40}$$

and the amplitude for double beta decay can never vanish, in contrast to the usual normal ordering case.

The cases where both ν_1 and ν_2 are pseudo-Dirac obviously lead to small values of $\langle m_{ee}\rangle$, since the only term contributing is $s_{13}^2 |m_3|$. In these cases the effective mass can lie outside the regions in which one expects it in the general case. Another interpretation of this would be that one of the non-standard mechanisms of neutrino-less double beta decay destructively interferes with the usual mass mechanism. The strategy to test this would be to perform multi-isotope investigation, as the cancellation is not expected to be on the same level in different nuclei. However, the pseudo-Dirac suppression discussed here is the same for all nuclei.

In summary, there are several cases for which there is a significant difference from the standard case of pure Majorana neutrinos. If long baseline oscillation experiments establish the neutrino ordering, and/or the neutrino mass scale is pinned down by cosmology or direct searches, neutrino-less double decay can distinguish the different cases. This illustrates the discriminative power of the process.

V. CONCLUSION

In conclusion, we have studied two different ways to test the bimodal (schizophrenic) neutrino hypothesis that one or two of the neutrino mass eigenstates are Dirac particles and the others Majorana. There are in total six non-trivial possible combinations and we have performed a mostly phenomenological analysis of these scenarios. We noted that

(i) flux ratios of extragalactic high energy neutrinos, and (ii) the effective mass for neutrino-less double beta decay are sensitive to the different possibilities, showing non-standard behavior in many cases. Figures 1 to 6 summarize our results. In brief, we found that flux ratios can differ significantly from their standard values, and the effective mass can either lie only in certain regions or even completely outside of its standard parameter space. The examples given show that the many different experimental signatures provide good tests of whether neutrino masses have the bimodal (or pseudo-Dirac) character. We have also discussed simple beyond the Standard Model scenarios in which such bimodal features can arise. Evidence for bimodal nature of neutrino mass will require major changes in our thinking about the physics of neutrino mass. Indeed, the field of neutrino physics has provided many surprising results in the past, and the question of neutrino mass origin is far from settled. If the hypothesis of bimodal neutrinos is supported by the experiments outlined here, it will not only provide a major departure from our current thinking about the nature of neutrino masses, but also its theoretical origin from physics beyond the standard model. As such it will have major impact on the physics at and beyond the TeV scale.

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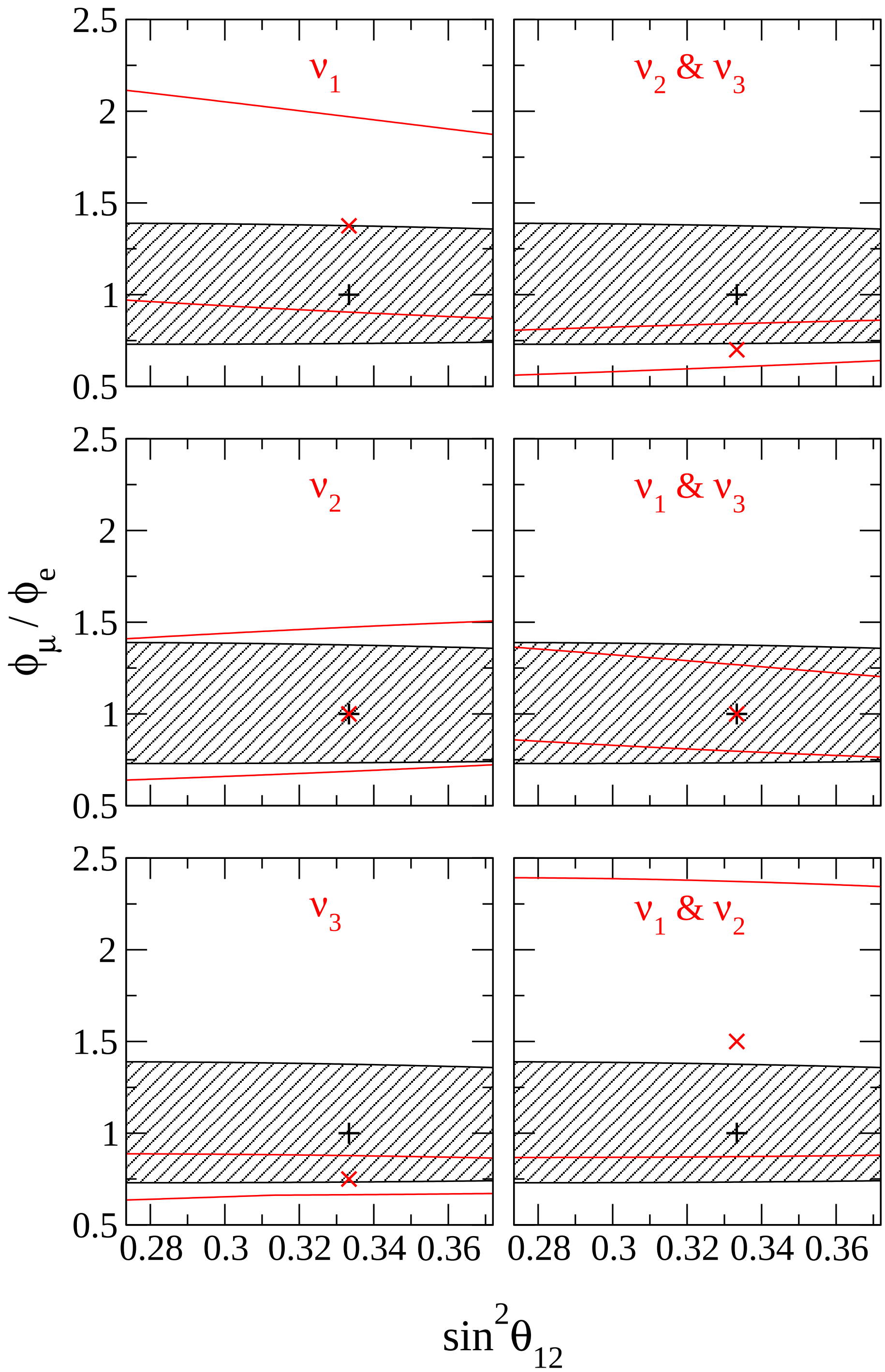

FIG. 1: The observable flux ratio Φ_μ/Φ_e against $\sin^2\theta_{12}$, assuming the initial neutrino flux ratios of $1:2:0$ and different combinations of pseudo-Dirac neutrinos (denoted by red lines), with the parameters θ_{13}, θ_{23} and δ varying in their allowed 3σ ranges. The black hatched region shows the general case with no pseudo-Dirac neutrinos; the red cross (black plus sign) shows the value of Φ_μ/Φ_e in each pseudo-Dirac case (the general case), assuming TBM.

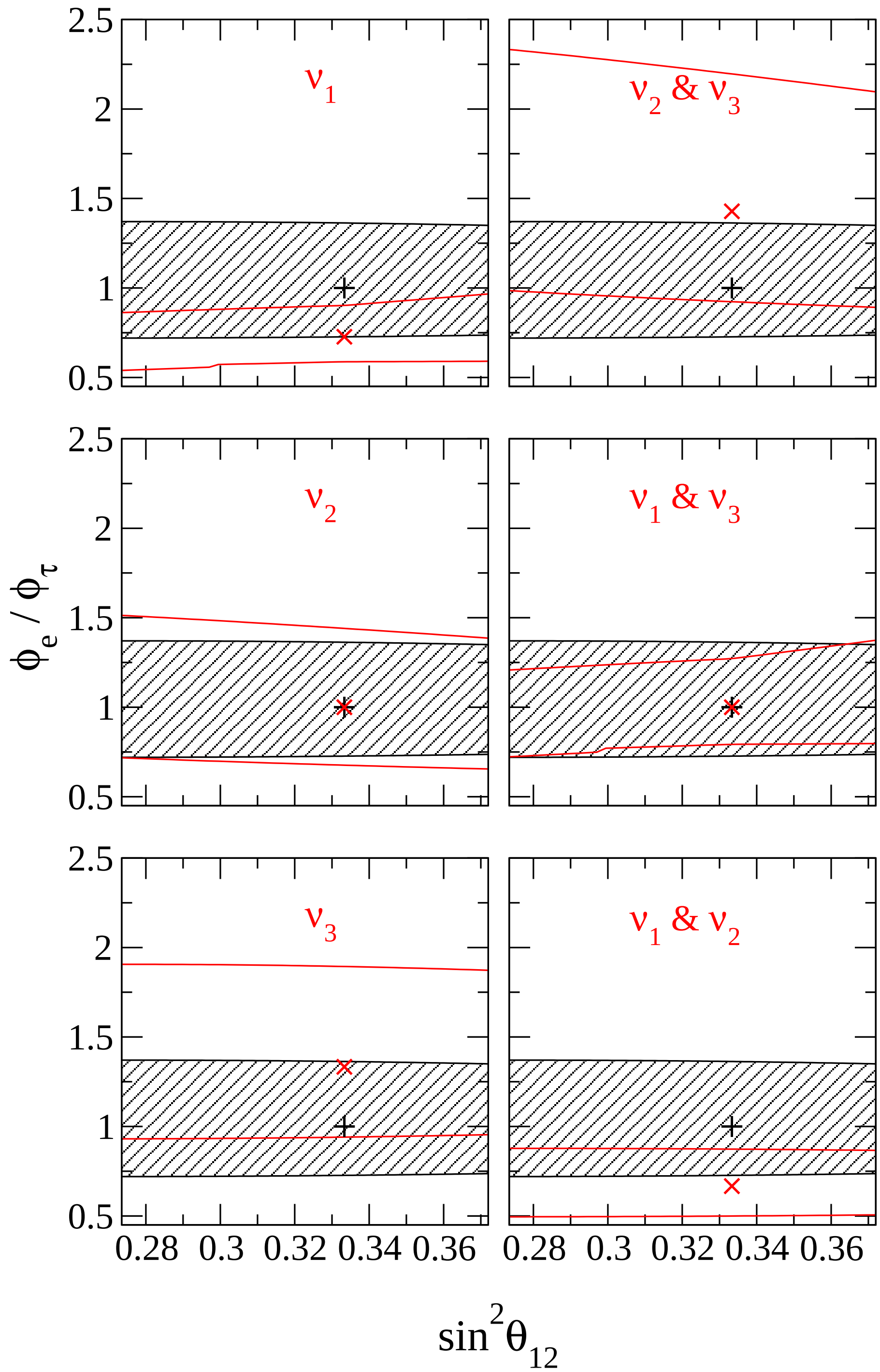

FIG. 2: Same as Fig. 1, for the observable flux ratio Φ_e/Φ_τ.

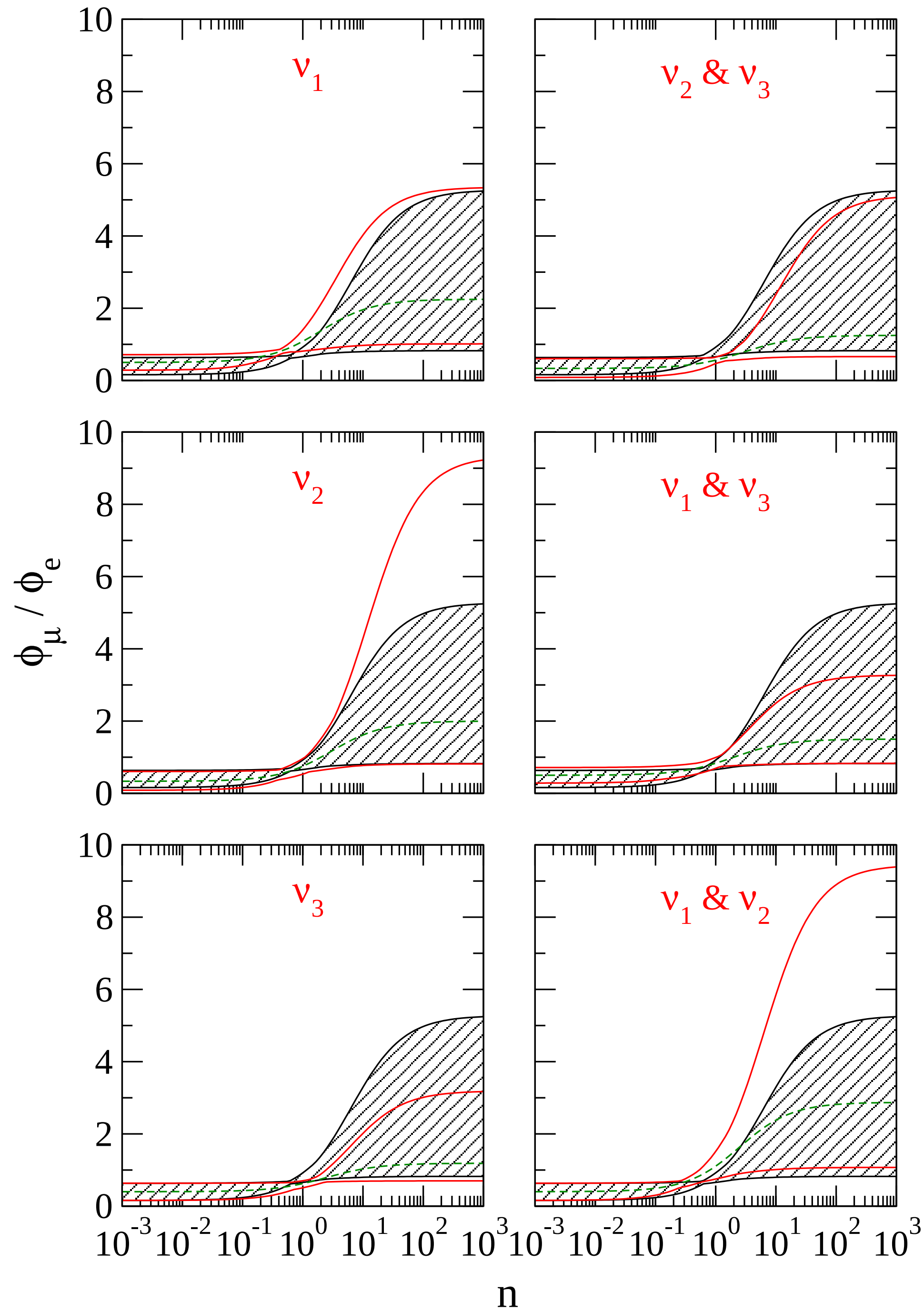

FIG. 3: The observable flux ratio Φ_μ/Φ_e against n, assuming the initial neutrino flux ratios of $1 : n : 0$ and different combinations of pseudo-Dirac neutrinos (denoted by red lines), with the parameters θ_{13}, θ_{23} and δ varying in their allowed 3σ ranges. The black hatched region shows the general case with no pseudo-Dirac neutrinos; the green dashed line shows the value of Φ_μ/Φ_e in each pseudo-Dirac case, assuming TBM.

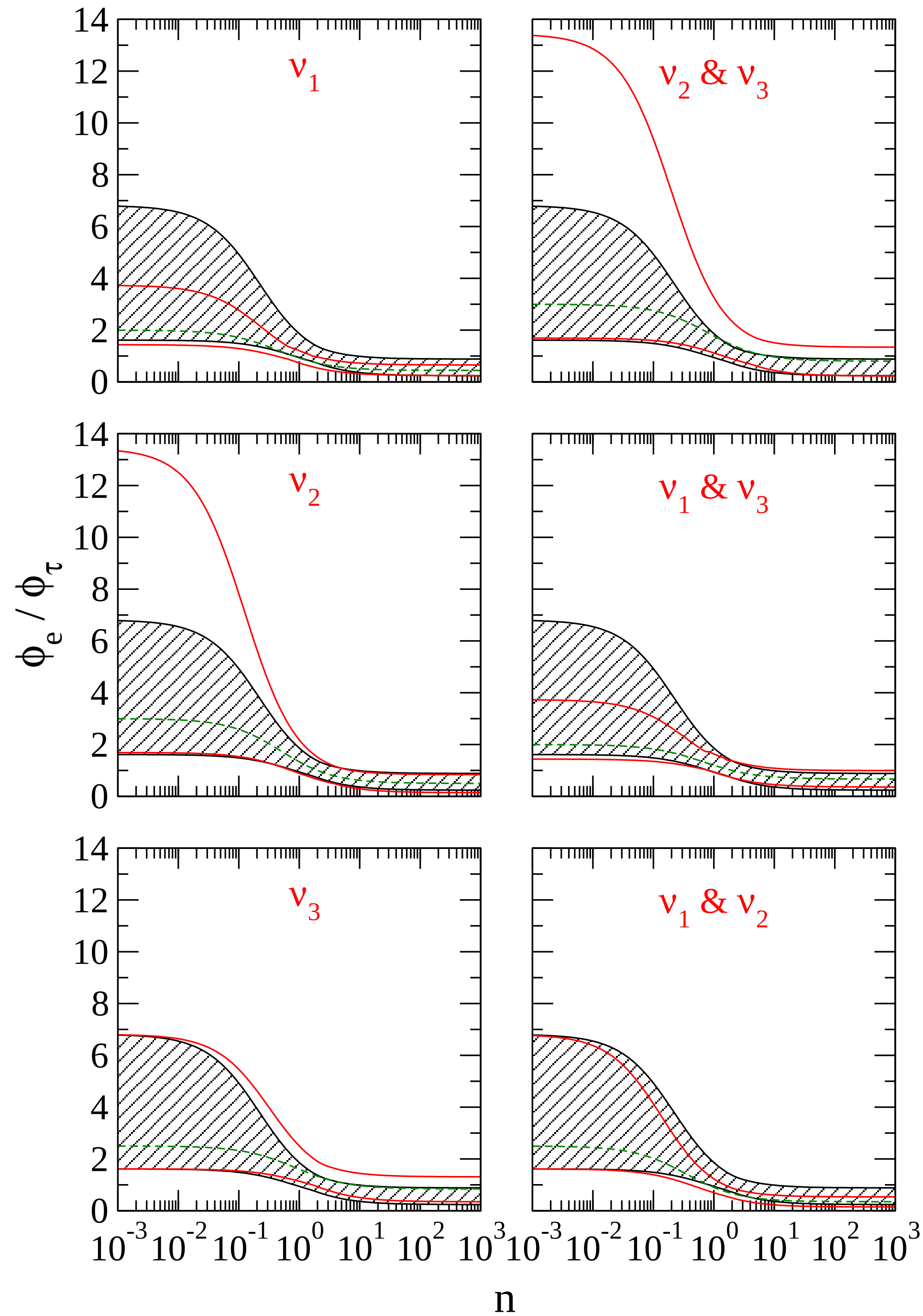

FIG. 4: Same as Fig. 3, for the observable flux ratio Φ_e/Φ_τ.

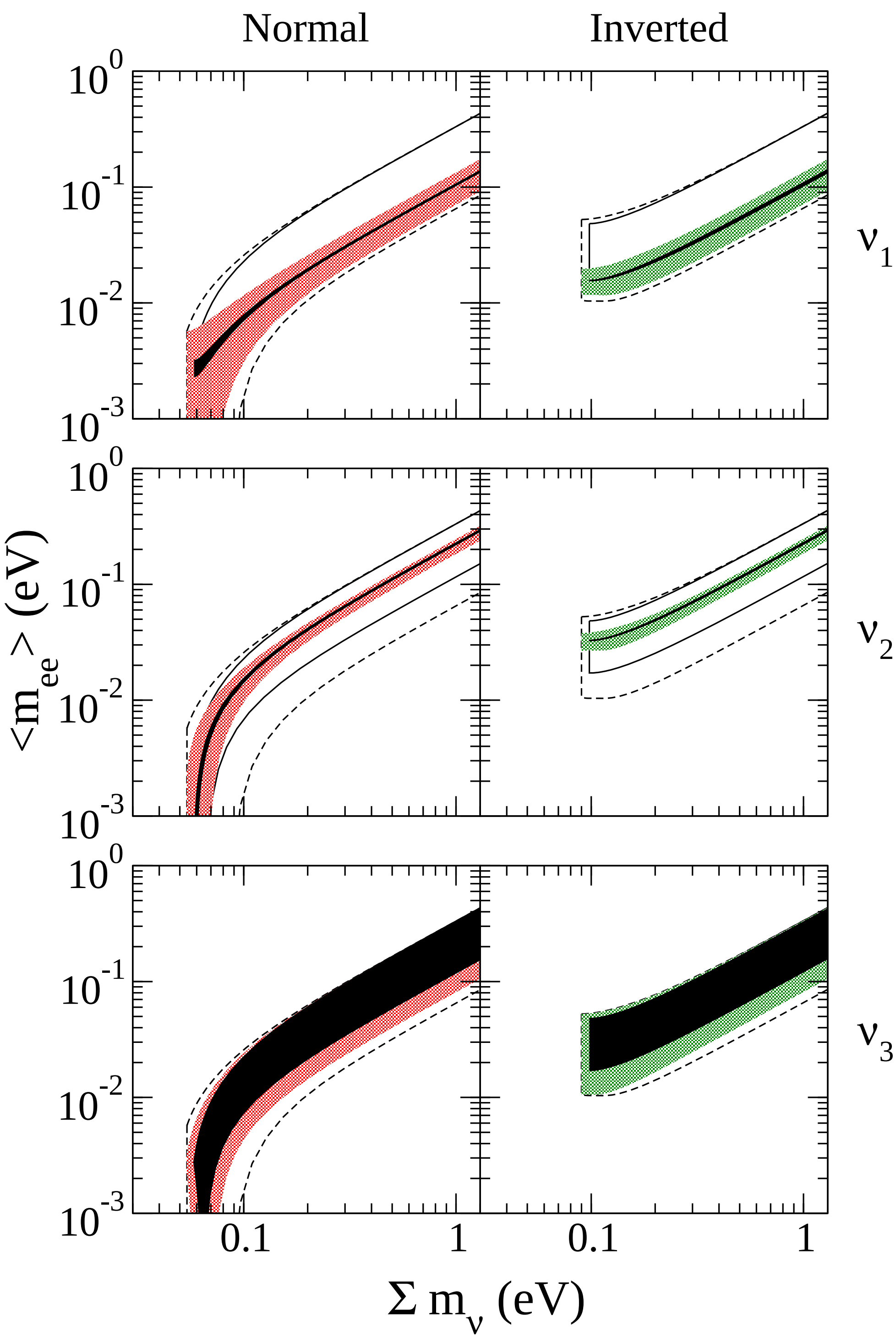

FIG. 5: Allowed regions in the $\langle m_{ee}\rangle - \sum m_i$ plane for the three different cases of one pseudo-Dirac neutrino (indicated on the right of each row). The black regions are for exact TBM, and the light red (green) shaded regions correspond to the 3σ ranges of the oscillation parameters, for normal (inverted) ordering. The solid (dashed) lines indicate the best-fit (3σ) allowed regions in the standard three neutrino scenario.

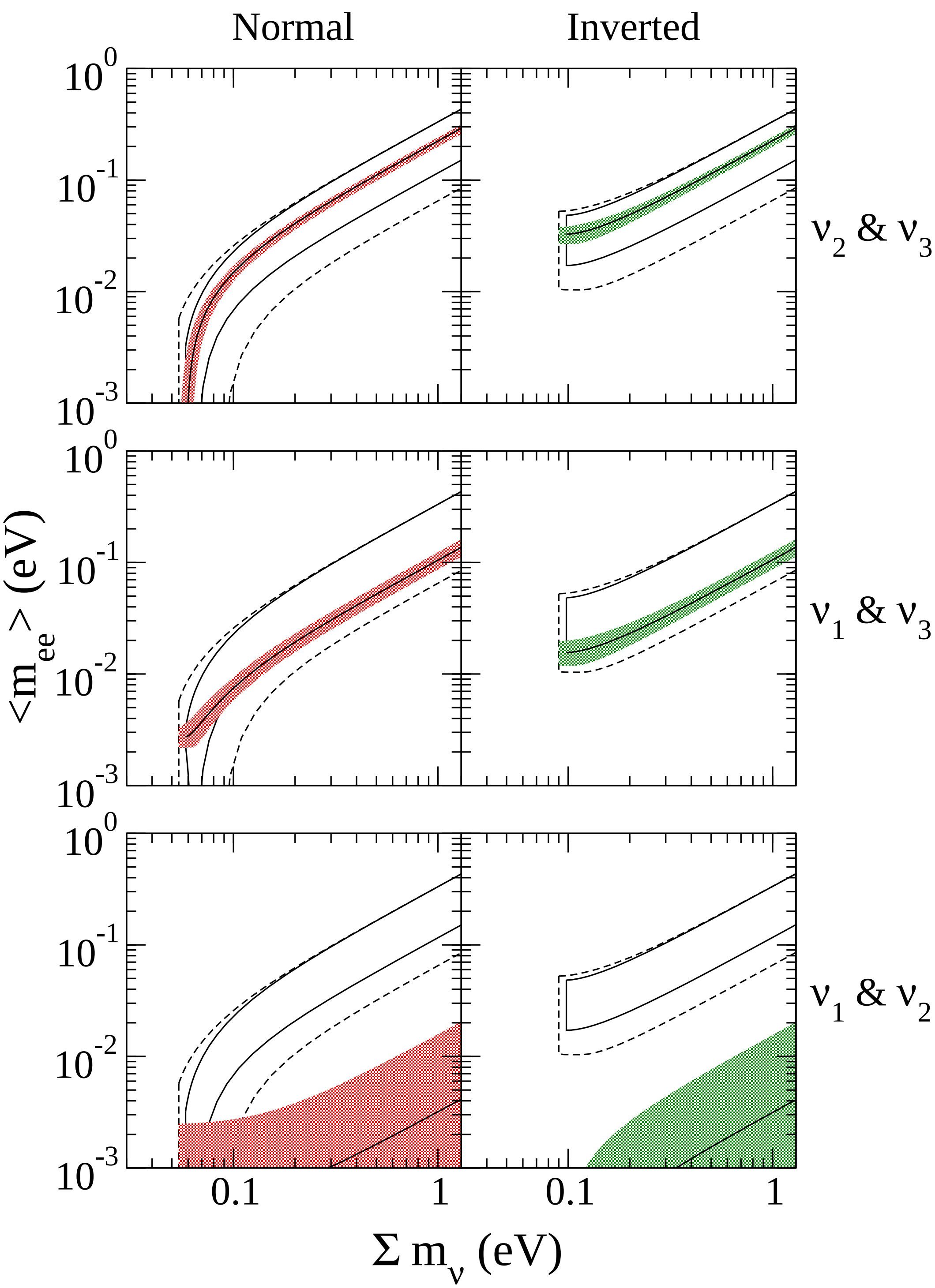

FIG. 6: Same as Fig. 5, for the three different cases of two pseudo-Dirac neutrinos (indicated on the right of each row).