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Stabilizing All Geometric Moduli in Heterotic Calabi-Yau Vacua

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Abstract

We propose a scenario to stabilize all geometric moduli – that is, the complex structure, Kähler moduli and the dilaton – in smooth heterotic Calabi-Yau compactifications without Neveu-Schwarz three-form flux. This is accomplished using the gauge bundle required in any heterotic compactification, whose perturbative effects on the moduli are combined with non-perturbative corrections. We argue that, for appropriate gauge bundles, all complex structure and a large number of other moduli can be perturbatively stabilized – in the most restrictive case, leaving only one combination of Kähler moduli and the dilaton as a flat direction. At this stage, the remaining moduli space consists of Minkowski vacua. That is, the perturbative superpotential vanishes in the vacuum without the necessity to fine-tune flux. Finally, we incorporate non-perturbative effects such as gaugino condensation and/or instantons. These are strongly constrained by the anomalous $U(1)$ symmetries which arise from the required bundle constructions. We present a specific example, with a consistent choice of non-perturbative effects, where all remaining flat directions are stabilized in an AdS vacuum.

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1 Introduction

In this work, we present a scenario for stabilizing the dilaton and all geometric moduli in smooth, $\mathcal{N} = 1$ supersymmetric vacua of the heterotic string [1, 2] and heterotic M-theory [3, 4, 5, 6]. Heterotic compactifications to four dimensions on Calabi-Yau three-folds with holomorphic, slope-stable vector bundles have produced phenomenologically realistic particle physics models [7, 8, 9], and have stimulated new ideas in cosmology [10, 11, 12]. However, moduli stabilization in this context has been more problematical[1]. In type IIB string theory, moduli stabilization can be achieved with KKLT type vacua [20]. Here, one first fixes some of the moduli, including the complex structure, using flux. The flux is then "tuned" so that the perturbative superpotential in the vacuum is very small. It follows that the fields which are not stabilized by the flux only have a small perturbative contribution to their F-terms. This can then be balanced by non-perturbative effects to form a completely stable supersymmetric vacuum. There are two problems which arise in trying to repeat this approach in heterotic Calabi-Yau three-fold compactifications. First, the Calabi-Yau condition appears to forbid the introduction of topologically non-trivial Neveu-Schwarz flux to stabilize the complex structure moduli[2]. Second, even if one naively allows such field strengths while retaining the Calabi-Yau geometry, the available flux does not allow for a small vacuum value of the perturbative superpotential – see the Appendix for a proof of this in the large complex structure limit. Thus, even if one can stabilize the complex structure in this way, there is a resulting instability in the remaining moduli which is too large to be balanced by non-perturbative effects.

In this paper, instead of using Neveu-Schwarz flux, we will stabilize the complex structure, as well as many of the other geometrical moduli, using fundamental properties of the gauge field strength present in any heterotic compactification [22]-[29]. These effects are perturbative, compatible with the compactification manifold being a Calabi-Yau three-fold, and give rise to $\mathcal{N} = 1$ supersymmetric Minkowski vacua. Because the superpotential vanishes after perturbative stabilization, this naturally avoids a runaway potential for the few remaining moduli. These can then be stabilized with non-perturbative effects, *without the need to tune any flux at all.* We emphasize, however, that although the problem of tuning flux does not arise, stabilizing moduli in our approach requires very specific choices of vector bundles. The relevant gauge field strengths can be in either the hidden or visible sector, or even split between the two. However, since it has less impact on phenomenology, in the generic discussion in the Introduction, and when presenting an explicit example that fixes all moduli, we locate the associated vector bundle in the hidden sector.

Let us now discuss in more detail the perturbative moduli stabilization mechanisms at the heart of our scenario. It is well known that there are contributions to the four-dimensional potential of a heterotic compactification arising from non-vanishing gauge fields in the extra

[1] See [13]-[19] for related work, including stabilization mechanisms in heterotic orbifold models.

[2] However, see [21] for a possible counterexample.

dimensions. The ten-dimensional action of heterotic theories contains the terms

$$S = -\frac{1}{2\kappa_{10}^2}\frac{\alpha'}{4}\int_{\mathcal{M}_{10}} \sqrt{-g}\left\{\mathrm{tr}F^2 - \mathrm{tr}R^2\right\} + \dots \ . \tag{1.1}$$

Using an integrability condition on the Bianchi identity, (1.1) can be rewritten, for the case of a Calabi-Yau compactification, as

$$S = -\frac{1}{2\kappa_{10}^2}\alpha'\int_{\mathcal{M}_{10}} \sqrt{-g}\left\{-\frac{1}{2}\mathrm{tr}(g^{a\bar{b}}F_{a\bar{b}})^2 + \mathrm{tr}(g^{a\bar{a}}g^{b\bar{b}}F_{ab}F_{\bar{a}\bar{b}})\right\} + \dots \ . \tag{1.2}$$

The integrand in (1.2) contains no four-dimensional indices – a, b are holomorphic and $\bar{a}, \bar{b}$ anti-holomorphic indices with respect to a chosen complex structure on the Calabi-Yau three-fold. Hence, upon dimensional reduction, (1.2) gives rise to a potential in the four-dimensional theory. For the low-energy theory to be $\mathcal{N} = 1$ supersymmetric it must be possible to express the potential coming from (1.2) in terms of F- and D-terms. Indeed, the link between supersymmetry and (1.2) is rather direct. To preserve supersymmetry, the gauge fields in a heterotic compactification must satisfy the Hermitian Yang-Mills equations of zero slope; that is,

$$F_{ab} = F_{\bar{a}\bar{b}} = 0 \ \ , \ \ g^{a\bar{b}}F_{a\bar{b}} = 0 \ . \tag{1.3}$$

Clearly, if these equations are satisfied then (1.2) leads to a vanishing potential. If, however, for some values of the moduli, Eqs. (1.3) are not satisfied, then (1.2) gives rise to a positive-definite potential in four dimensions. Thus, the potential (1.2) can stabilize at least some of the moduli in a supersymmetric, Minkowski vacuum. From the point of view of the four-dimensional theory, the expressions $g^{a\bar{a}}g^{b\bar{b}}F_{ab}F_{\bar{a}\bar{b}}$ and $(g^{a\bar{b}}F_{a\bar{b}})^2$ are associated respectively with F- and D-term contributions to the $\mathcal{N} = 1$ potential. In recent work [22]-[29], it has been shown how to calculate these as explicit functions of the moduli fields. This paves the way to using this potential to stabilize moduli in heterotic models.

First, consider the requirement in (1.3) that both the holomorphic and anti-holomorphic components of the gauge field strength must vanish to preserve supersymmetry. This implies that the associated vector bundle must be holomorphic with respect to a given complex structure. It is clear, however, that this field strength need not have zero holomorphic and anti-holomorphic components with respect to a *different* complex structure. If this is the case, it corresponds to the stabilization of some – possibly all – of the complex structure moduli. Explicit examples, together with the associated mathematical and field theoretic formalisms, were presented in [27, 29]. It was shown that these holomorphy "obstructions" are indeed related to non-vanishing F-terms, but with an important subtlety. There are regions of moduli space where the scale of the potential is as large as the compactification scale. In such regimes, the stabilized complex structure moduli should never have been regarded as four-dimensional fields at all – they are fixed at a high scale. For regions of moduli space where this scale is small, however, it was shown in [27, 29] that these complex structure *are* fixed by F-terms.

The second condition for supersymmetry in (1.3) requires the vector bundle to have the geometrical properties of poly-stability and vanishing slope. These properties depend on the

Kähler moduli of the Calabi-Yau three-fold, as can be seen from the appearance of the metric in $g^{a\bar{b}}F_{a\bar{b}} = 0$. Some bundles are only poly-stable with slope zero for a restricted set of Kähler moduli. In addition, due to the warping of the moduli across the M-theory orbifold direction [25] – or, equivalently, to 1-loop corrections in the weakly coupled string [24] – the last equation in (1.3) also involves the four-dimensional dilaton. In favourable cases, these effects can stabilize combinations of the Kähler moduli and dilaton. However, since neither slope nor poly-stability (nor, indeed, holomorphy) depend on the overall size of the compactification, there is always at least one unstabilized modulus remaining. It was shown in [22]-[26] that these effects are associated with non-vanishing D-terms. As with the F-terms, one must be careful in attributing this stabilization mechanism to a D-term potential. The scale of this potential is, once again, often as large as the compactification scale. In such cases, the stabilized dilaton and Kähler moduli should never have been regarded as four-dimensional fields at all – they are fixed at a high scale. However, when this scale is small, it was shown in [22]-[26] that the Kähler moduli and dilaton *are* directly fixed by D-terms.

Given these mechanisms, we propose the following three stage stabilization scenario for heterotic compactifications.

- **Stage 1:** Choose part of the hidden sector vector bundle so that it is holomorphic only for an isolated locus in complex structure moduli space. This corresponds to F-term stabilization of the complex structure moduli.
- **Stage 2:** Choose the remaining part of the hidden sector bundle to be holomorphic for this isolated complex structure. In addition, construct the hidden bundle so that it is poly-stable with zero slope only for restricted values of the dilaton and Kähler moduli. This, we will show, is easily achieved by an appropriate choice of line bundles and corresponds to D-term stabilization of these moduli. It is possible to fix all but one of the remaining geometric moduli in this way. However, as we will see in stage 3, leaving more than one modulus unconstrained at the second stage is desirable.
- **Stage 3:** A crucial point about stages 1 and 2 is that the resulting moduli space of vacua is supersymmetric and Minkowski. That is, the unstabilized fields have no potential and the cosmological constant vanishes. In the final stage of our scenario, we fix these remaining degrees of freedom using a more traditional mechanism – non-perturbative effects such as gaugino condensation and membrane (or string) instantons. The inclusions of such effects is extremely constrained. The D-terms introduced in stage 2 are associated with anomalous $U(1)$ symmetries under which various linear combinations of the axions transform. Any allowed non-perturbative superpotential must be consistent with these $U(1)$ symmetries. We find this restriction sufficiently severe that – if *only one linear combination* of the Kähler moduli and dilaton is left unstabilized in stage 2 – it is not possible to fix this modulus in a controlled regime of field space. If, however, *two* moduli remain to be stabilized, then non-perturbative effects consistent with the $U(1)$ symmetries can fix the

remaining moduli. Moreover, this can be achieved in a region of moduli space where the effective field theory is valid. We will present an explicit example of such a vacuum.

The structure of the paper is as follows. In Section 2, we introduce the perturbative F- and D-terms discussed above. These will be used to carry out the first two stages of our stabilization mechanism in Section 3. This section also includes an explicit example of stage 2 and a demonstration that the moduli can be fixed in a controlled regime of the effective theory. In Section 4, we describe the non-perturbative contributions to the potential. These will be used in Section 5 to discuss the full scenario. Finally, in Section 6, we conclude. In addition, a technical Appendix discussing the perturbative superpotential generated by heterotic Neveu-Schwarz flux is attached.

2 Perturbative contributions to the potential

In this section, we review the perturbative F- and D-term contributions, introduced in [22]-[29], to the four-dimensional potential of heterotic M-theory vacua. These will be important in stages 1 and 2 of our moduli fixing scenario. Specifically, the vacua we consider are smooth Calabi-Yau compactifications of the ten-dimensional $E_8 \times E_8$ heterotic string (or its eleven-dimensional strong-coupling counterpart) with a gauge bundle in each of the two E_8 sectors. These bundles are both of the form $V = \mathcal{U} \bigoplus_I \mathcal{L}_I$. Hence, in each sector, they consist of a non-Abelian, indecomposable piece, $\mathcal{U}$, and a sum of line bundles, $\mathcal{L}_I$.

2.1 F-terms

The F-term contributions, associated with the failure of the gauge bundles to be holomorphic, have been discussed in detail in [27, 29]. It is sufficient for the purposes of this paper, to illustrate our stabilization mechanism within the context of an explicit example.

Consider the complete intersection Calabi-Yau three-fold defined by

$$X = \left[\begin{array}{c|c} \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \\ \mathbb{P}^2 & 3 \end{array} \right]^{3,75} . \tag{2.1}$$

We construct a rank 2 holomorphic bundle $\mathcal{U}$ on this three-fold via the short exact "extension" sequence

$$0 \to \mathcal{L} \to \mathcal{U} \to \mathcal{L}^* \to 0 , \tag{2.2}$$

where $\mathcal{L}$ is the line bundle $\mathcal{O}_X(-2,-1,2)$. At any point in the 75-dimensional complex structure moduli space, with moduli denoted Z^a, the holomorphic extensions correspond to elements of

$$\mathrm{Ext}^1(\mathcal{L}^*, \mathcal{L}) = H^1(X, \mathcal{L}^2) . \tag{2.3}$$

It is well-known that the dimension of a sheaf cohomology, while possessing a generic value, can "jump" at special values of complex structure. For the example discussed here, it was shown in [27, 29] that (2.3) vanishes everywhere in complex structure moduli space *except* on a specific 58-dimensional sub-locus, where $h^1(X, \mathcal{L}^2) = 18$. The dimensions of such cohomologies are computed in this work using techniques and code created in the development of [9, 40]. We choose a point Z_0^a on this sub-locus and a non-vanishing extension class *far* from zero. Corresponding to this choice is a holomorphic, indecomposable $SU(2)$ bundle $\mathcal{U}$. Now move *infinitesimally* to a generic point $Z_0^a + \delta Z^a$ *not* on this sub-locus. Then, $h^1(X, \mathcal{L}^2) = 0$ and the only holomorphic bundle is the direct sum $\mathcal{L} \oplus \mathcal{L}^*$. Since an indecomposable $SU(2)$ bundle cannot split into a direct sum under an infinitesimal change in complex structure, it is clear that $\mathcal{U}$ is not holomorphic at a generic point in moduli space. That is, the holomorphicity of $\mathcal{U}$ is "obstructed" in the $75 - 58 = 17$ directions in complex structure moduli space leading away from the special sub-locus.

As discussed in [27, 29], these obstructions correspond to specific non-vanishing F-terms in the effective theory and, hence, the breakdown of supersymmetry. It is straightforward to determine the zero-mode spectrum of the bundle $\mathcal{U}$ defined in (2.2). As above, consider a point Z_0^a on the sub-locus. For a non-vanishing extension class *far* from zero, there are $h^1(X, \mathcal{U} \otimes \mathcal{U}^*) = h^1(X, \mathcal{L}^2) - 1 = 17$ bundle moduli. However, to discuss the F-term structure it is helpful to first consider bundles near $0 \in \text{Ext}^1(\mathcal{L}^*, \mathcal{L})$. Here, as shown in [22, 25, 26], the low-energy gauge group is enhanced by an anomalous $U(1)$ factor and the bundle moduli are counted by $h^1(X, \mathcal{L}^2) = h^1(X, \mathcal{L}^{*2}) = 18$. We denote these massless fields by C_+^i and C_-^j respectively, with the subscript $\pm$ indicating the $U(1)$ charge. Therefore, to lowest order, the four-dimensional superpotential is

$$W = \lambda_{ij}(Z) C_+^i C_-^j \ . \tag{2.4}$$

The dimension one coefficients $\lambda_{ij}(Z)$ are functions of the complex structure moduli Z^a. The associated F-terms are

$$\begin{aligned} F_{C_+^i} &= \lambda_{ij} C_-^j + K_{C_+^i} W \quad , \quad & F_{C_-^j} &= \lambda_{ij} C_+^i + K_{C_-^j} W, \\ F_{Z_\parallel^a} &= \frac{\partial \lambda_{ij}}{\partial Z_\parallel^a} C_+^i C_-^j + K_{\mathfrak{z}_\parallel^a} W \quad , \quad & F_{Z_\perp^a} &= \frac{\partial \lambda_{ij}}{\partial Z_\perp^a} C_+^i C_-^j + K_{Z_\perp^a} W \end{aligned} \tag{2.5}$$

where we have distinguished between derivatives within the 58-dimensional sub-locus (specified by 58 coordinates $Z_\parallel^a$) and those leaving this sub-locus (specified by 17 coordinates $Z_\perp^a$). Since the fields C_+^i and C_-^j are zero-modes, for Z_0^a *on the sub-locus*, it follows that

$$\lambda(Z_0)_{ij} = 0 \quad \Rightarrow \quad \frac{\partial \lambda_{ij}(Z_0)}{\partial Z_\parallel^a} = 0 \ . \tag{2.6}$$

In the next section, we show how the $Z_\perp^a$-dependence in the superpotential can stabilize the complex structure moduli to the sub-locus where holomorphic, *indecomposable* $SU(2)$ bundles exist. In performing this analysis we will look for supersymmetric Minkowski vacua for which W, as well as the F-terms (2.5), vanishes. Given this we will not need to know the exact form of the Kähler potential in (2.5).

2.2 D-terms

The low-energy gauge group arising from a bundle of the form $V = \mathcal{U} \bigoplus_I \mathcal{L}_I$ necessarily includes a number of anomalous $U(1)$ factors, one for each line bundle, $\mathcal{L}_I$. Associated with each anomalous $U(1)$ is a Kähler moduli dependent D-term, whose form is well-known [22]-[26]. These four-dimensional D-terms are the low energy manifestation of the requirement that the internal bundle be poly-stable with zero slope. Here, we simply present these D-terms, using the notation of [25, 26]. Corresponding to each line bundle, $\mathcal{L}_I$, they are

$$D_I^{U(1)} = f_I - \sum_{L\bar{M}} Q_I^L G_{L\bar{M}} C^L \bar{C}^{\bar{M}} \ , \tag{2.7}$$

where C^L are the zero-mode fields with charge Q_I^L under the I-th $U(1)$, G_{LM} is a Kähler metric with positive-definite eigenvalues and

$$f_I = \frac{3}{16}\frac{\epsilon_S \epsilon_R^2}{\kappa_4^2}\frac{\mu(\mathcal{L}_I)}{\mathcal{V}} + \frac{3\pi\epsilon_S^2\epsilon_R^2}{8\kappa_4^2}\frac{\beta_i c_1^i(\mathcal{L}_I)}{2s} \tag{2.8}$$

is a dilaton and Kähler moduli dependent Fayet-Iliopoulos (FI) term [25, 26]. The quantities

$$\mu(\mathcal{L}_I) = d_{ijk} c_1^i(\mathcal{L}_I) t^j t^k, \qquad \mathcal{V} = \frac{1}{6} d_{ijk} t^i t^j t^k \tag{2.9}$$

are the slope of the associated line bundle $\mathcal{L}_I$ and the Calabi-Yau volume respectively. Here t^i are the Kähler moduli relative to a basis of harmonic $(1,1)$ forms ω_i, with the associated Kähler form given by $J = t^i \omega_i$. Furthermore, s is the real part of the dilaton. The quantities $d_{ijk} = \int_X \omega_i \wedge \omega_j \wedge \omega_k$ are the triple intersection numbers of the three-fold and the β_i are the charges on the orbifold plane where the associated line bundle is situated. Explicitly, these charges are

$$\beta_i = \int_X (\mathrm{ch}_2(V) - \frac{1}{2}\mathrm{ch}_2(TX)) \wedge \omega_i \ . \tag{2.10}$$

The parameters ϵ_S and ϵ_R are given by

$$\epsilon_S = \left(\frac{\kappa_{11}}{4\pi}\right)^{2/3} \frac{2\pi\rho}{v^{2/3}} \ , \quad \epsilon_R = \frac{v^{1/6}}{\pi\rho} \ . \tag{2.11}$$

Here v is the coordinate volume of the Calabi-Yau three-fold, ρ is the coordinate length of the M-theory orbifold and κ_{11} is the eleven-dimensional gravitational constant. The four-dimensional gravitational constant κ_4 can be expressed of these 11-dimensional quantities as $\kappa_4^2 = \kappa_{11}^2/(2\pi\rho v)$. In the subsequent discussion we will set $\kappa_{11} = 1$ and further, in order to simplify the FI terms (2.8), choose the coordinate parameters ρ and v such that

$$\frac{3}{16}\frac{\epsilon_S \epsilon_R^2}{\kappa_4^2} = \frac{3\pi\epsilon_S^2\epsilon_R^2}{16\kappa_4^2} = 1 \ . \tag{2.12}$$

Finally, for the explicit vacua discussed in this paper, we choose each line bundle $\mathcal{L}_I$ such that *all* of the C^L fields with non-vanishing charges Q^L_I are absent. Hence, the second term in (2.7) will not appear.

3 Stages 1 and 2: Minimizing the perturbative potential

In this section, we describe the first two stages of our scenario within the explicit context of Section 2. Stage 1 involves fixing the complex structure by setting to zero the F-terms arising from superpotential (2.4). In stage 2, using the expressions given in Subsection 2.2, we fix linear combinations of the Kähler moduli and the dilaton by solving the D-flat constraints. Crucially, both steps lead to a four-dimensional supersymmetric Minkowski vacuum. Hence, by the end of this section, we will have achieved a perturbative stabilization of all but one of the geometrical moduli, with the resulting vacuum space having a vanishing perturbative potential.

3.1 Stage 1: Fixing the complex structure

We will demonstrate stage 1 within the context of the explicit example presented in Subsection 2.1. First, choose the complex structure moduli Z^a_0 to be in the 58-dimensional sub-locus for which an indecomposable bundle $\mathcal{U}$ can be holomorphic. Note from (2.6) that the superpotential (2.4) and the first *three* F-terms in (2.5) always vanish. What are the implications of the fourth term, $F_{Z^a_\perp}$, in (2.5)? The associated potential is

$$V = |F_{Z^a_\perp}|^2 = |\frac{\partial \lambda_{ij}(Z_0)}{\partial Z^a_\perp} \langle C^i_+ \rangle|^2 |C^j_-|^2 + \dots \ , \tag{3.1}$$

where we suppress the multiplicative factor of $e^K G^{a\bar{a}}$ for simplicity.

Now consider a bundle $\mathcal{U}$ defined by a non-vanishing class in $\mathrm{Ext}^1(\mathcal{L}^*, \mathcal{L})$ and, hence, by $\langle C^i_+ \rangle \neq 0$. As mentioned earlier, such a bundle only has C^i_+ fields as zero-modes. Hence, the C^j_- fields must have a non-vanishing mass. It then follows from (3.1) that, in contrast to Eq. (2.6),

$$\frac{\partial \lambda_{ij}(Z_0)}{\partial Z^a_\perp} \neq 0 \ . \tag{3.2}$$

One immediate implication is

$$\langle F_{Z^a_\perp} \rangle = \frac{\partial \lambda_{ij}(Z_0)}{\partial Z^a_\perp} \langle C^i_+ \rangle \langle C^j_- \rangle = 0 \quad \Rightarrow \quad < C^j_- >= 0 \ . \tag{3.3}$$

More interestingly, now consider the potential energy obtained from all four F-terms in (2.5) evaluated at a generic point $Z^a_0 + \delta Z^a_\perp$ *not* on the 58-dimensional sub-locus where non-decomposable bundles $\mathcal{U}$ exist. Then, to quadratic order in the field fluctuations we find, in addition to the C^j_- term in (3.1), that

$$V = |\frac{\partial \lambda_{ij}(Z_0)}{\partial Z^a_\perp} \langle C^i_+ \rangle|^2 |\delta Z^a_\perp|^2 + \dots \ . \tag{3.4}$$

where a sum over index j is implied. It follows from (3.2) that any of the fluctuations in the complex structure away from the special sub-locus has a positive mass and, hence,

$$\langle \delta Z^a_\perp \rangle = 0 \ . \tag{3.5}$$

That is, the complex structure moduli are fixed to be on the sub-locus where an indecomposable bundle $\mathcal{U}$ can be holomorphic!

There are several things to note about the above discussion. First, the dilaton and Kähler moduli have yet to appear in the analysis. Second, the above example is somewhat special in that it is possible to give a four-dimensional description of the stabilization of the complex structure. In general, for the mechanism presented in [27, 29], this stabilization will take place at high scale. Hence, the fixed complex structure should never have been included as fields in the four-dimensional theory in the first place. In such cases, one should simply write down the low-energy $\mathcal{N} = 1$ theory without these fields present[3].

Regardless, for the rest of this paper we simply assume that the complex structure moduli have been stabilized by some appropriate bundle in the theory. For the subsequent stages of our scenario, we will not need to know any more information about what this bundle actually is, other than its second Chern class and how its structure group (times some $U(1)$ factors) is embedded in E_8. Both this topological quantity and the group embedding are required to satisfy certain conditions, as we will discuss below.

3.2 Stage 2: Fixing the Kähler moduli and dilaton

For simplicity, we assume in the following that there are no matter fields C^I which are charged under the anomalous $U(1)$ symmetries[4]. This can be achieved by an appropriate choice of line bundles $\mathcal{L}_I$ and we present an explicit example below. Using the results in the previous section and our choice of conventions, the N D-terms are then given by

$$D_I^{U(1)} = \frac{\mu(\mathcal{L}_I)}{\mathcal{V}} + \frac{\beta_i c_1^i(\mathcal{L}_I)}{s} = c_1^i(\mathcal{L}_I) t_i + \gamma_I s^{-1} \ , \tag{3.6}$$

where we find it convenient to define the "dual" Kähler moduli $t_i = \frac{1}{\mathcal{V}} d_{ijk} t^j t^k$ as well as $\gamma_I = \beta_i c_1^i(\mathcal{L}_I)$.

The D-term equations $D_I^{U(1)} = 0$ for $I = 1, \ldots, N$ form a linear system of equations for the $h^{1,1}(X) + 1$ variables $(t_i, 1/s)$. The system is homogeneous which means that one modulus,

[3] Indeed, this will even be the case in the above example if the mass term in equation (3.4) is of the order of the compactification scale.

[4] The general case, including $U(1)$ charged matter fields, may be interesting and is compatible with our three-stage scenario. However, the detailed analysis is significantly more complicated. The D-terms (2.7) now fix linear combinations of the T-moduli, the dilaton and the matter fields. In addition, the presence of matter fields typically allows for more general non-perturbative contributions consistent with the $U(1)$ symmetries. This will be important for stage 3 of our scenario. We defer a detailed discussion of these possibilities to future work.

corresponding to the overall scaling of the moduli, cannot be fixed. Physically, this occurs because holomorphy and poly-stability/vanishing slope are geometrical properties which do not depend on the overall size of the three-fold. Provided that all of the equations are linearly independent, a non-trivial solution requires that $N \leq h^{1,1}(X)$.

If any of the coefficients γ_I, for definiteness say γ_1, is different from zero we can proceed by solving the first equations for the dilaton s in terms of the Kähler moduli. This leads to

$$s = -\frac{\gamma_1}{t_i c_1^i(\mathcal{L}_1)} \ . \tag{3.7}$$

Substituting this into the remaining $N-1$ equations, and taking the Calabi-Yau volume $\mathcal{V}$ to be finite, we obtain the linear equations

$$\left(c_1^i(\mathcal{L}_I) - \frac{\gamma_I}{\gamma_1} c_1^i(\mathcal{L}_1)\right) t_i = 0 \ , \quad I = 2, \ldots, N \ , \tag{3.8}$$

which fix a number of directions in Kähler moduli space. In the most restrictive case, that is, if we start with $N = h^{1,1}(X)$ linearly independent D-term equations, we can solve for all of the Kähler moduli in terms of the overall scaling modulus. Then, this scaling modulus is the only flat direction left.

If, on the other hand, all of the coefficients $\gamma_I = 0$, then the dilaton drops out of the D-term equations and remains a flat direction. In this case, the Kähler moduli are constrained by

$$c_1^i(\mathcal{L}_I) t_i = 0 \ , \tag{3.9}$$

and for a non-trivial solution we should have at most $N \leq h^{1,1}(X) - 1$ linearly independent such equations. In the most restrictive case with precisely $N = h^{1,1}(X) - 1$ linearly independent equations all Kähler moduli can be solved for in terms of an overall scaling modulus. Hence, we are left with two flat directions, the scaling modulus and the dilaton.

As a final comment, note that the axions associated with the stabilized combinations of s and t^i are "eaten" by massive anomalous $U(1)$ gauge bosons through the standard supersymmetric Higgs effect [30], albeit involving fields with non-canonical kinetic terms.

3.2.1 An example

As we did for stage 1, we now present an explicit realization of stage 2. This example is intended as a clear example of stage 2 of our scenario and, in particular, as an illustration of how the dilaton can be stabilized. It should be noted that it is *not* compatible with the particular example given for stage 1 of the scenario. However, in Section 5 we will describe how to obtain a single consistent vacuum in which stages 1 and 2 *can* coexist, as well as being compatible with explicit non-perturbative contributions.

Consider the CICY three-fold

$$\left(\begin{array}{c|ccccc} \mathbb{P}^3 & 0 & 1 & 1 & 1 & 1 \\ \mathbb{P}^5 & 2 & 1 & 1 & 1 & 1 \end{array}\right)^{2,50} \ . \tag{3.10}$$

The triple intersection numbers are specified by $d_{111} = 2$, $d_{112} = 8$, $d_{122} = 12$, $d_{222} = 8$. Since $h^{1,1}(X) = 2$, we need to specify two linearly independent D-terms, in the most restrictive case. We accomplish this by choosing one line bundle on each of the two orbifold fixed planes. That is, the vector bundles on the visible and hidden planes are of the form $V_1 = \mathcal{U}_1 \oplus \mathcal{L}_1$ and $V_2 = \mathcal{U}_2 \oplus \mathcal{L}_2$ respectively, where both $\mathcal{U}_1$ and $\mathcal{U}_2$ have rank of at least two. This gives rise to two anomalous $U(1)$ factors in the low-energy gauge group and, hence, two associated D-terms.

On the three-fold (3.10), the line bundle $\mathcal{L}_1 = \mathcal{O}_X(-2,1)$ has no cohomology for a generic complex structure. Thus it gives rise to no C fields. This is also true for $\mathcal{L}_2 = \mathcal{O}_X(3,-2)$. In addition, any other cohomologies which would give rise to fields charged under the two anomalous $U(1)$'s vanish. We use these two line bundles to stabilize the dilaton and one Kähler modulus in stage 2. Given these line bundles, we find that $\gamma_1 = -2\beta_1 + \beta_2$ and $\gamma_2 = -3\beta_1 + 2\beta_2$. Now choose $\mathcal{U}_1$ and $\mathcal{U}_2$ to have second Chern characters

$$\mathrm{ch}_2(\mathcal{U}_1) = -38\nu_1 + 4\nu_2 \ , \quad \mathrm{ch}_2(\mathcal{U}_2) = 15\nu_1 - 36\nu_2 \tag{3.11}$$

respectively, where ν_i is a basis of harmonic four-forms dual to ω_i. It is assumed that $\mathcal{U}_2$ stabilizes the complex structure as in stage 1. In addition we find

$$\mathrm{ch}_2(\mathcal{L}_1) = -6\nu_1 - 4\nu_2 \ , \quad \mathrm{ch}_2(\mathcal{L}_2) = -15\nu_1 - 20\nu_2 \tag{3.12}$$

and

$$\mathrm{ch}_2(TX) = -c_2(TX) = -44\nu_1 - 56\nu_2 \ . \tag{3.13}$$

Combining these results gives

$$\beta = -22\nu_1 + 28\nu_2 \ . \tag{3.14}$$

Note that the charges on the two fixed planes are equal and opposite[5]. We define β to be the fixed plane charge for the locus where the line bundle $\mathcal{L}_1$ is situated. This implies $\gamma_1 = 72$ and $\gamma_2 = 122$.

For this example, equations (3.7) and (3.8) become

$$s = -\frac{72}{3}((t^1)^3 + 12(t^1)^2 t^2 + 18 t^1 (t^2)^2 + 4(t^2)^3)/(4((t^1)^2 - 2t^1 t^2 - 4(t^2)^2)) \tag{3.15}$$

and

$$-151(t^1)^2 + 122 t^1 t^2 + 424 (t^2)^2 = 0 \tag{3.16}$$

respectively. Note that we have expressed the dual Kähler moduli t_i in terms of t^i using the intersection numbers presented above. The above equations can be solved to give the relations

$$t^1 = 2.13 t^2 \ , \quad s = 171 t^2 \tag{3.17}$$

between the moduli in the vacuum. Hence, the only remaining flat direction is the overall scaling of all three moduli.

[5]Here, and in all the examples, we have, for simplicity, chosen vacua where no M5 branes are present.

The D-terms we have been solving are derived (in the language of the strongly coupled theory) for small warping. This approximation will be valid, in our conventions, if the moduli dependent strong-coupling parameters, given by

$$\hat{\epsilon}_S = \frac{\mathcal{V}^{1/3}}{s}\epsilon_S \ , \quad \hat{\epsilon}_R = \frac{s^{1/2}}{\mathcal{V}^{1/3}}\epsilon_R \ , \tag{3.18}$$

are sufficiently small. The Calabi-Yau volume $\mathcal{V}$ was defined in equation (2.9). For the example in this subsection, we find

$$\hat{\epsilon}_S = 0.006 \ll 1 \tag{3.19}$$

and that ϵ_R may be made arbitrarily small by increasing the size of the one remaining modulus.

A number of other consistency checks must also be satisfied. First, the non-Abelian bundles added to each of the fixed planes must be slope stable. A necessary condition for this is that the topological quantities associated with those bundles satisfy the Bogomolov bound [31] for the Kähler moduli evaluated on each fixed plane. We find that this is indeed the case if 1) the rank of the non-Abelian bundle is greater than or equal to 1 on the first fixed plane and 2) greater than or equal to 3 on the second plane. One must also check that the line bundles on each fixed plane are zero slope inside the Kähler cone. Working in terms of the variables t^i, the two line bundles in question are zero slope on the lines of gradient 2 and 3/2 respectively. The Kähler cone, in these variables, is the region between the lines of slope 4 and 2/3, so this test is passed as well.

Thus we have stabilized all but one linear combination of the dilaton and Kähler moduli, in a supersymmetric Minkowski vacuum, in an allowed region of field space.

4 Non-perturbative contributions

Non-perturbative contributions to the superpotential in our scenario are strongly constrained by gauge invariance. To discuss this we first introduce the complexified dilaton and Kähler moduli fields $S = s + i\sigma$ and $T^i = t^i + i2\chi^i$, which include the axions σ and χ^i. The D-terms in stage 2 are associated with Green-Schwarz anomalous $U(1)$ symmetries under which these axions transform non-trivially. Explicitly, these transformations read

$$\delta\chi^i = -\frac{3}{16}\epsilon_S\epsilon_R^2 c_1^i(\mathcal{L}_I)\epsilon \ , \quad \delta\sigma = -\frac{3}{8}\pi\epsilon_S^2\epsilon_R^2 c_1^i(\mathcal{L}_I)\beta_i\epsilon \tag{4.1}$$

for the D-terms as given in Eq. (2.7). Note that there is one such transformation for each D-term.

To analyze non-perturbative superpotentials, we work, without loss of generality, in the "Kähler frame" – where the superpotential is gauge invariant [30]. Non-perturbative corrections typically depend on linear combinations $n_iT^i + mS$ of the moduli, where, for now, n_i and m are arbitrary coefficients. A particular non-perturbative correction which depends on such a linear combination is allowed only if this linear combination is $U(1)$ invariant[6]. From the transformations (4.1) this implies, given the conventions (2.12), that

$$c_1^i(\mathcal{L}_I)n_i + \gamma_I m = 0 \,. \tag{4.2}$$

We note that this is precisely the same linear system of equations, in variables (n_i, m), as the D-term equations (3.6) which we have used to fix linear combinations of the moduli (t_i, s^{-1}) in stage 2. This means that the number of linear independent combinations $n_i T^i + mS$ on which non-perturbative effects can, in principle, depend equals the number of flat directions left after stage 2. For this reason, there is a tension between our desire to fix as many moduli as possible perturbatively at stage 2 and retaining enough flexibility with non-perturbative effects.

Let us now discuss this in some more detail and ask which, if any, of the known non-perturbative effects can co-exist with our D-terms, that is, with our choice of gauge bundle? We begin with gaugino condensation which is described by a non-perturbative superpotential

$$W_{\text{gaugino}} = Ae^{-\alpha(S-\beta_i T^i)} \,, \tag{4.3}$$

where A, α are constants. In our earlier language this means we have $n_i = -\beta_i$ and $m = 1$. This choice is consistent with gauge symmetry provided that all anomalous $U(1)$ symmetries are located on the orbifold plane opposite the one which carries the condensate. Indeed, in this case we have $\gamma_I = c_1^i(\mathcal{L}_I)\beta_i$ and the conditions (4.2) are obviously satisfied. This fact can be easily understood from Green-Schwarz anomaly cancellation. Given that the anomalous $U(1)$ symmetries and the condensate are on opposing planes, no fields on the condensate plane carry $U(1)$ charge. Hence, there is no triangle anomaly to be cancelled on this plane and, consequently, its gauge kinetic function which appears in the exponent of (4.3) should not transform. If we have anomalous $U(1)$ symmetries on *both* fixed planes they will, in general, forbid gaugino condensates from forming in any gauge group factor. However, this can be avoided for special topological choices. For example, if all line bundles are chosen such that $c_1^i(\mathcal{L}_I)\beta_i = 0$, then the associated $U(1)$ symmetries do not constrain gaugino condensate potentials at all – on either fixed plane.

Membrane instanton superpotentials take the form

$$W_{\text{membrane}} = Be^{-n_i T^i} \,, \tag{4.4}$$

where B and n_i are constants. This means we have to satisfy the conditions (4.2) for $m = 0$. If we stabilize all but one modulus at stage 2 we need at least one of the coefficients γ_I to be non-zero. At the same time, the D-term equations (3.6) as well as the conditions (4.2) have a one-dimensional common solution space which, for finite dilaton $s^{-1} \neq 0$ cannot point into the

[6]Here we assume the absence of singlet matter charged under the anomalous $U(1)$ symmetries, as discussed earlier. If such singlet matter is present additional non-perturbative corrections may be allowed and the discussion becomes more complicated.

$m = 0$ direction. This means that, in this case, instanton corrections are excluded. For two flat directions left at stage 2 we have two linearly independent vectors of the form (n_i, m) solving the invariance conditions (4.2). By taking an appropriate linear combination we see that at least one type of instanton correction is allowed in this case.

Given these facts we should first think about the "maximal" stabilization scenario where we only leave one flat direction at stage 2. As argued above, there is no instanton superpotential in this case. However, if we locate all anomalous $U(1)$ symmetries on one orbifold plane, then gaugino condensates can form on the opposite plane so that we can attempt to stabilize the one remaining modulus by a race-track potential. Unfortunately, this obvious course of action runs into a serious problem. In this case, the solution to the invariance conditions (4.2) is $n_i = -\beta_i$ and $m = 1$ and, hence, the D-term equations (3.6) are solved by $t_i = k\beta_i$ and $s^{-1} = k$ with an arbitrary constant k. Hence, the ratio of the one-loop term $\beta_i t^i$ in the gauge kinetic function relative to the tree-level part s is given by

$$\frac{\beta_i t^i}{s} = 6 \ . \tag{4.5}$$

This means that the expansions defining our four-dimensional theory have broken down and we can not trust any resulting vacuum. For this reason, we will consider models with *two* flat directions left at stage 2 in the subsequent discussion.

5 Stages 1, 2 and 3: Minimizing the full potential

In this section, we combine stages 1 and 2, outlined in Subsections 3.1 and 3.2 above, with a *third stage*, involving the non-perturbative effects discussed in Section 4, to give a complete description of our moduli stabilization scenario. Making the various stages of stabilization compatible is non-trivial. We begin by separating off stage 1. That is, we show that it is possible to stabilize the complex structure using only the perturbative potential described in Subsection 3.1 and, having done so, that we can simply ignore these moduli in the remaining discussion. That this can be done is non-trivial, since there is no separation in scale between the perturbative F-terms of stage 1 and the D-terms used in stage 2.

Once the complex structure has been fixed, we move on to stages 2 and 3 and stabilize the remaining moduli. As we have seen, the allowed non-perturbative effects are restricted by the presence of the D-terms. Conversely, in order to have a stable minimum of the potential, one can view the D-terms one can include as being restricted by the non-perturbative effects. In Subsections 5.2.1 and 5.2.2, we will describe how to fit these competing effects together. We then finish this section by providing an explicit example of our stabilization scenario.

5.1 Separating off Stage 1

We want to extremize the potential of the theory, including all perturbative and non-perturbative effects, with respect to all fields in the problem. Furthermore, to preserve supersymmetry in the vacuum, we set all F-terms and D-terms to zero. In general this means that, in considering the stabilization of the complex structure in stage 1, one should include contributions to the F-terms coming from the non-perturbative effects introduced in stage 3. Since fixing these moduli involves solving $F_{Z^a} = 0$, this would modify the simple perturbative analysis performed in Subsection 3.1. Furthermore, the expectation values for the complex structure moduli must be substituted into the remaining F-terms equations which are solved in stages 2 and 3 to fix some of the remaining fields. Since the F_{Z^a} depend on S and T^i, so will the solutions for Z^a. Thus, substituting these expectation values back into the other F-terms introduces additional S and T^i dependence, which must be taken into account in the remaining analysis.

This effect could, in principle, link perturbative and non-perturbative contributions to the potential in a complicated way. Happily, however, this is not the case for the smooth heterotic vacua discussed in this paper, as we now explain. First, a few facts.

- The superpotential contains two types of contributions – perturbative and non-perturbative. In our theory, these are given by

$$W = W^{(\mathrm{P})}(Z) + W^{(\mathrm{NP})}(Z, S, T^i) \ . \tag{5.1}$$

  The perturbative term, as was described in Section 2.1, does not depend on S or T^i. We emphasize that this is *not* generically the case in string vacua. It arises in our theory precisely because our complex structure is fixed to lie in the image of the Atiyah map discussed in [27, 29]. The non-perturbative term, which contains all fields, is much smaller than the perturbative contribution in any controlled regime of field space.

- The Kähler potential takes the form

$$K = K_{\mathrm{CS}}(Z) + K_{\mathrm{ST}}(S, T^i) \ . \tag{5.2}$$

  As with the superpotential, there are both perturbative and non-perturbative contributions to K. However, the non-perturbative contributions to the Kähler potential are always of higher order in our analysis and, hence, we ignore them in (5.2).

- Using (5.1) and (5.2), it follows that F_{Z^a} is of the form

$$F_{Z^a} = F^{(\mathrm{P})}_{Z^a}(Z) + F^{(\mathrm{NP})}_{Z^a}(Z, S, T^i) \ . \tag{5.3}$$

The discussion of Section 3.1 was concerned with finding a solution to $F^{(\mathrm{P})}_{Z^a} = 0$, that is, the vanishing of the *perturbative* F-term. This resulted in a solution $Z^a = Z^a_0$, which is independent of the S and T^i moduli. The addition of a small correction $F^{(\mathrm{NP})}_{Z^a}$ to this F-term changes this analysis by inducing a similarly small correction $Z^a = Z^a_0 + \delta Z^a$. The crucial point is that, in our theory, if we substitute this perturbed solution for Z^a into the other F-terms and solve for the remaining fields, then it is easy to show that the correction δZ^a only enters into terms which are *second* order in the small non-perturbative quantities. This is due to two important features of our theory; 1) the property that $W^{(\mathrm{P})}$ in (5.1) depends on the complex structure only and 2) the fact the analysis of Section 3.1 resulted in a supersymmetric *Minkowski* vacuum with

$$W^{(\mathrm{P})}(Z_0) = \partial W^{(\mathrm{P})}(Z_0) = 0 \ . \tag{5.4}$$

Hence, to achieve a result accurate to *first* order in small quantities, one need only set $Z^a = Z^a_0$. One can then also forget about the perturbative superpotential in the remaining analysis, as this vanishes for this value of the moduli. This is what we will do in the remainder of the paper.

This establishes a separation between stage 1 and the remaining two stages. In the following, we will assume that the vector bundles are chosen so that stage 1 is accomplished. Recall that – in each E_8 sector – the vector bundle is of the form $V = \mathcal{U} \bigoplus_I \mathcal{L}_I$. The relevant quantity in stage 1 is the subbundle $\mathcal{U}$ which, via the *perturbative* superpotential $W^{(\mathrm{P})}(Z)$, stabilizes the complex structure moduli which can be integrated out and, henceforth, ignored. That is, for stages 2 and 3 only the Abelian subbundles $\bigoplus_I \mathcal{L}_I$ with $I = 1, \ldots, N$ are relevant. However, certain topological data associated with the full bundles V still appears in stages 2 and 3. Before continuing, we list this data. The bundles and their constituents must be consistent with

- Anomaly cancellation: $\mathrm{ch}_2(TX) = \mathrm{ch}_2(V_1) + \mathrm{ch}_2(V_2)$
- Bogomolov bound:
  $\int_X \left(2\,\mathrm{rk}(\mathcal{U})\mathrm{c}_2(\mathcal{U}) - (\mathrm{rk}(\mathcal{U}) - 1)\mathrm{c}_1^2(\mathcal{U})\right) \wedge J \geq 0$

Furthermore, the charges β_i given by Eq. (2.10) depend on the choice of bundle $\mathcal{U}$ at stage 1 and should be consistent with the values used at later stages. Lastly the rank and embedding of the hidden sector bundle within E_8 must be compatible with the existence of the gaugino condensates which will be employed in stage 3. With this in hand, we continue to the full stabilization scenario.

5.2 Stabilizing the remaining moduli: Stages 2 and 3

In the rest of this section we carry out stages 2 and 3 of our scenario simultaneously, thus stabilizing the remaining geometrical moduli in a supersymmetric vacuum. We will see that, by allowing the two effects – D-terms and non-perturbative F-terms – to coexist, one places considerable constraint on which theories can be considered. Not only does the presence of D-terms restrict the non-perturbative effects one can use, but the non-perturbative potential, together with the requirement that there exist a stable supersymmetric vacuum, restricts the form of the D-terms in stage 2. In particular, we begin by showing that no supersymmetric vacua exist unless the gauge bundle, and thus the D-terms, satisfy specific constraints. When these constraints are satisfied, however, we will find explicit supersymmetric AdS vacua with all of the geometric moduli stabilized at a minimum in a controlled regime of field space.

5.2.1 A no-go result

Previously, we have seen that leaving only one flat direction after stage 2 leads to a breakdown of the expansions defining the four-dimensional heterotic theory. Here, we present an independent reason for why leaving only one modulus un-stabilized after perturbative effects is problematic. Recall that in this case, at least one of the coefficients γ_I, say γ_1, is different from zero so that the associated D-term equation can be solved for the dilaton. This results in

$$s = -\frac{\gamma_1}{t_i c_1^i(\mathcal{L}_1)} \ . \tag{5.5}$$

Following Section 4, one can write the most general non-perturbative superpotential as

$$W = \sum_a A_a e^{-\alpha_a (S - \beta_i T^i)} + \sum_x B_x e^{-n_i^x T^i} \ , \tag{5.6}$$

where n_i^x, A_a, B_x, α_a are constants. To ensure gauge invariance of the instanton terms under the first $U(1)$ symmetry, we require that

$$n_i^x c_1^i(\mathcal{L}_1) = 0 \tag{5.7}$$

for all x. Some of the constants A_a, B_x may be set to zero if required for invariance under all $U(1)$ symmetries. The corresponding F-terms are

$$\begin{aligned} F_S &= -\sum_a A_a \alpha_a e^{-\alpha_a(S-\beta_i T^i)} - \frac{1}{\kappa_4^2}\frac{1}{2s} W & (5.8) \\ F_{T^j} &= \sum_a A_a \alpha_a \beta_j e^{-\alpha_a(S-\beta_i T^i)} - \sum_x B_x n_j^x e^{-n_i^x T^i} + K_{T^j} W & (5.9) \end{aligned}$$

Multiplying Eq. (5.9) by $c_1^j(\mathcal{L}_1)$ and using $\gamma_1 = c_1^j(\mathcal{L}_1)\beta_j$, Eq. (5.7) and $K_{T^j} = -\frac{t_j}{4\kappa_4^2}$, we find

$$c_1^j(\mathcal{L}_1) F_{T^j} = \sum_a A_a \alpha_a \gamma_1 e^{-\alpha_a(S-\beta_i T^i)} - \frac{1}{4\kappa_4^2} t_j c_1^j(\mathcal{L}_1) W \ . \tag{5.10}$$

Substituting Eq. (5.8) into (5.10) and setting $c_1^j(\mathcal{L}_1) F_{T^j} = 0$, we obtain

$$W\left(\gamma_1 + \frac{s}{2} t_j c_1^j(\mathcal{L}_1)\right) = 0 \ . \tag{5.11}$$

There are now two possibilities. If $W = 0$ then we are considering Minkowski vacua. Such vacua, while desirable, require a careful tuning of the constants A_a, B_x. At present we cannot justify this from string theory so we will focus on the case where $W \neq 0$ which leads to AdS vacua. Then, Eq. (5.11) implies that

$$s = -2\frac{\gamma_1}{t_i c_1^i(\mathcal{L}_1)} \ , \tag{5.12}$$

which is clearly inconsistent with the D-flat condition (5.5). We conclude that if any of the anomalous $U(1)$ factors have $c_1^i(\mathcal{L})\beta_i \neq 0$, it is not possible to simultaneously solve the D- and F-flat conditions and, hence, no supersymmetric AdS vacua exist.

5.2.2 Avoiding the no-go result

The no-go result of the previous subsection tells us that, if we are to successfully combine the stabilization mechanisms in stages 2 and 3, we must constrain the gauge bundle such that, for each anomalous $U(1)$,

$$c_1^i(\mathcal{L}_I)\beta_i = 0\ . \tag{5.13}$$

It follows from (3.6) that the dilaton no longer appears in any D-term. Hence, when combining the various effects in our scenario, one can not use the full power of stage 2 to stabilize the dilaton in linear combination with the Kähler moduli. It follows that one need only include $N = h^{1,1} - 1$ D-terms in the four-dimensional theory which will stabilize an equivalent number of Kähler moduli. The overall Kähler modulus as well as the dilaton will remain as flat directions. Non-perturbative effects prevent us from making "optimal" use of the D-term stabilization at stage 2 which would only leave one flat direction.

From Eqs. (3.6) and (5.13), the D-term equations $D_I^{U(1)} = 0$ now take the form

$$c_1^i(\mathcal{L}_I)t_i = 0\,. \tag{5.14}$$

These equations are obviously solved by choosing $t_i \propto \beta_i$. We take the superpotential to be of the general form (5.6). Recall that the gaugino condensation part is automatically gauge-invariant thanks to the condition (5.14) while for the instanton corrections we have to impose Eq. (4.2). For the present case, this along with (5.13) implies that $n_i^x = b^x\beta_i$ for each x. Then, the associated F-terms are

$$F_S = -\sum_a \alpha_a A_a e^{-\alpha_a(S-\beta_i T^i)} - \frac{1}{2\kappa_4^2 s}W \tag{5.15}$$

$$F_{T^j} = \sum_a A_a\alpha_a\beta_j e^{-\alpha_a(S-\beta_i T^i)} - \sum_x B_x b^x \beta_j e^{-b^x\beta_i T^i} - \frac{3}{2}\frac{1}{\kappa_4^2}\frac{\beta_j}{\beta_i t^i}W \tag{5.16}$$

for $j = 1, \dots, h^{1,1}$. In Eq. (5.16) we have used the relation $K_{T^j} = -\frac{3}{2}\frac{1}{\kappa_4^2}\frac{\beta_j}{\beta_i t^i}$ which follows from $t_i \propto \beta_i$. Note that every term in F_{T^j} is proportional to β_j. Therefore, setting all of the Kähler moduli F-terms to zero leads to just one equation. We will look for solutions to our theory where the axion expectation values appearing in the F-terms vanish. For such a choice, we see that this equation and $F_S = 0$ only depend on two variables, s and $\beta_i t^i$. Note that the latter is proportional to the volume of the Calabi-Yau three-fold, that is, $\beta_i t^i \propto \mathcal{V}$ since $t_i \propto \beta_i$. Thus, we end up with two constraints on two real variables from the F-terms. Recalling that the $h^{1,1} - 1$ D-terms constrain the remaining variables, one expects to find isolated solutions to this system. This is indeed the case, as we now demonstrate with an explicit example.

5.3 An example

Let us consider an example where $h^{1,1} = 2$ and, hence, we need only one line bundle $\mathcal{L}$. Furthermore, take the *moduli fixing* bundle $V = \mathcal{U} \bigoplus \mathcal{L}$ to be located in the *hidden sector*. As discussed above, the subbundle $\mathcal{U}$ is assumed to fix the complex structure moduli and does not enter the rest of the calculation. Now demand that there be *two* gaugino condensates and a *single* membrane instanton present. Note that, although the higher rank subbundle does not enter the remaining calculation, the condition that there be two gaugino condensates requires that the structure group of $\mathcal{U} \bigoplus \mathcal{L}$ be embedded in E_8 in such a way that the commutant has *two* non-Abelian gauge factors. This is easily accomplished. We will specify a Calabi-Yau three-fold and the line bundle $\mathcal{L}$ shortly. However, one can get a surprisingly long way in the analysis without giving this data, as we now show.

Although physically the parameters in the superpotential would be determined by fundamental theory, and one would then solve for the field values at the minimum, it is simpler in practice to proceed in the inverse fashion. That is, we can ask what parameter values are required in the superpotential to give a minimum with specified vacuum expectation values for the fields. Setting the F-terms (5.15) and (5.16) to zero for the case at hand gives us the following result.

$$A_1 = Be^{\alpha_1(s-\beta_i t^i)-b\beta_i t^i}\left(\frac{b\beta_i t^i+\alpha_2(\beta_i t^i+s(3+2b\beta_i t^i))}{(\alpha_1-\alpha_2)(3s+\beta_i t^i)}\right) \quad (5.17)$$

$$A_2 = -Be^{\alpha_2(s-\beta_i t^i)-b\beta_i t^i}\left(\frac{b\beta_i t^i+\alpha_1(\beta_i t^i+s(3+2b\beta_i t^i))}{(\alpha_1-\alpha_2)(3s+\beta_i t^i)}\right) \quad (5.18)$$

Note that the fields that appear in the analysis of the F-terms are exactly those not constrained by the D-term. More precisely, the dilaton, s, does not appear in the D-term since $\beta_i c_1^i(\mathcal{F})=0$. In addition, the D-term constrains a different combination of Kähler moduli than $\beta_i t^i$. If, for example, we ask that the dilaton be stabilized at $s=1000$ and the overall volume be fixed at $\beta_i t^i=100$, we find the following values solve equations (5.17) and (5.18),

$$A_1=-299,\ A_2=734,\ \alpha_1=1/10,\ \alpha_2=10/99,\ b=1,\ B=1000\ . \quad (5.19)$$

Note that these are reasonable parameter choices and that the moduli are stabilized in controlled regions of field space. Also note that the two exponents associated with the gaugino condensates are quite close in value. This is as expected since the dilaton here is being stabilized essentially by the racetrack mechanism [32, 33, 34, 35].

Up to this point, the F-term equations have not depended on the specific choice of Calabi-Yau three-fold, except through the value of $h^{1,1}(X)$. In particular, to discuss the stabilization of the overall volume and the dilaton, we have not needed the intersection numbers of the three-fold in any way. To go further, however, and write down the specific solution for both Kähler moduli, one must introduce this data. We then use the D-term constraint (5.14), that is,

$$c_1^i(\mathcal{L})d_{ijk}t^j t^k=0\ , \quad (5.20)$$

together with the values of s and $\beta_i t^i$ fixed by the F-terms, to determine the stabilized values of the real parts of the Kähler moduli, t^i. To proceed, one must now specify, in addition to the triple intersection numbers d_{ijk} of the Calabi-Yau three-fold, the charges β_i and the explicit anomalous $U(1)$ in the hidden sector. We take the Calabi-Yau three-fold to be that given in equation (3.10), which has non-vanishing intersection numbers

$$d_{111} = 2,\ d_{112} = 8,\ d_{122} = 12,\ d_{222} = 8 \tag{5.21}$$

as well as those related to the above by symmetry of the indices. We choose the anomalous $U(1)$ in the hidden sector to be associated with the line bundle

$$\mathcal{L} = \mathcal{O}_X(-2, 1) \ . \tag{5.22}$$

Finally, let

$$\beta = (1, 2) \ . \tag{5.23}$$

Note that, as required by (5.13), $\beta_i c_1^i(\mathcal{L}_1) = 0$. Having explicitly chosen the Calabi-Yau three-fold, this choice of β_i corresponds to a specification of the second Chern class of the non-Abelian part of the hidden sector gauge bundle, that is, $c_2(\mathcal{U})$. Thus, again, despite the fact that $\mathcal{U}$ does not enter the calculation in stages 2 and 3, the conditions required to solve for the vacuum put further constraints on the choice of $\mathcal{U}$. Given these choices, (5.20) tells us that

$$t^1 = (1 + \sqrt{5})t^2 \ . \tag{5.24}$$

Using the fact that $\beta_i t^i = 100$ and the value of β_i in (5.23), we find

$$t^1 = 61.8 \ , \quad t^2 = 19.1 \quad . \tag{5.25}$$

As stated in the previous subsection, the vacuum we are describing has vanishing vevs for the axionic components of the Kähler modulus and the dilaton stabilized by the F-terms. The remaining axion, associated with the Kähler modulus fixed by the D-term, is a "flat direction" of the potential – as is required by the fact that it will be "eaten" in the process of the associated anomalous gauge boson becoming massive. Putting everything together, we have shown that in this example the vevs of the moduli are

$$\langle s \rangle = 1000,\ \langle \sigma \rangle = 0,\ \langle t^1 \rangle = 61.8,\ \langle t^2 \rangle = 19.1,\ \langle \chi \rangle = 0 \ . \tag{5.26}$$

Finally, it is easily demonstrated that the vacuum presented here has a positive definite mass squared matrix for all fields. That is, it corresponds to a supersymmetric *minimum* of the potential and not merely a saddle point. Some plots of the potential for various slices through field space are presented in Figure 1. We emphasize that stage 1 also results in a minimum of the potential for the $h^{2,1} = 50$ complex structure moduli. Thus, this vacuum is a true minimum of the full theory. The minimum is Minkowski at the perturbative level. However, the non-perturbative effects induce a small non-vanishing superpotential in the vacuum – as can be verified by substituting the vevs (5.26) into the superpotential (5.6) – resulting in a shallow AdS vacuum at the end of stage 3.

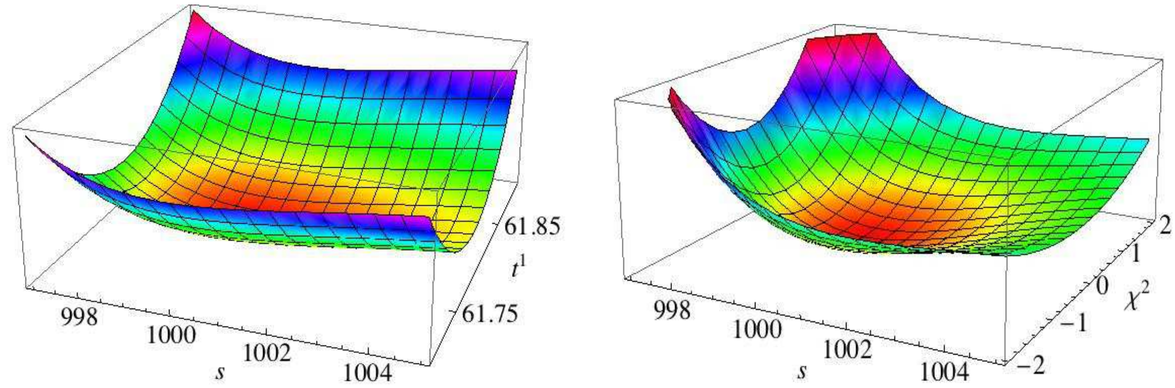

Figure 1: *Plots of the potential, for the example in Section 5.3 of the text, for various slices through field space. The left hand image presents the potential as a function of s and t^1, whereas the right hand image depicts the s, χ^2 plane. The plots are color shaded as a function of the height of the potential. Clearly the vacuum is a minimum of the potential in these directions, as confirmed, for all field directions, by a calculation of the eigenvalues of the mass matrix.*

There are various important consistency conditions that this example should, and does, satisfy. For example, all of the expansion parameters of the four-dimensional theory can be computed and are sufficiently small that the approximations used in the analysis are valid. In addition, the second Chern class of the non-Abelian part of the hidden sector gauge bundle is such that it satisfies the Bogomolov bound for the stabilized values of the Kähler moduli, whatever the rank of that bundle may be. This is required for this Chern class to be consistent with the existence of a supersymmetric bundle stabilizing the complex structure moduli.

6 Summary, conclusions, and future directions

The goal of this paper is to provide a new stabilization scenario for the *geometric* moduli – that is, the dilaton, complex structure and Kähler moduli – of smooth heterotic compactifications. Our approach has several novel features. These include using the natural constraints arising in a heterotic theory – namely the holomorphy and slope-stability of the visible and hidden sector gauge bundles – to perturbatively stabilize most of the moduli. The three stages of this scenario are as follows.

First, in *stage 1* the complex structure moduli are stabilized by the presence of a vector bundle which is holomorphic only for an isolated locus in complex structure moduli space. This geometric mechanism can, in concrete examples, be described by explicit F-term contributions to the effective potential. In this approach, the stabilization of the complex structure is achieved without introducing flux. As a result, the compactification remains a Calabi-Yau three-fold, and hence we are able to retain a considerable mathematical toolkit for analyzing such geometries.

In *stage 2*, it is possible to use the remaining perturbative condition of slope-stability to restrict the dilaton and Kähler moduli. This corresponds to partial D-term stabilization of these fields. We demonstrate that the presence of these D-terms is highly constraining to the effective theory. In particular, the D-terms used in stage 2 are associated with gauging various linear combinations of axions. Any non-perturbative superpotential must be consistent with this.

Finally, in *stage 3*, we introduce more familiar non-perturbative effects such as gaugino condensation and membrane instantons. However, a significant feature of our scenario is that the presence of the D-terms in stage 2 highly constrains the possible non-perturbative effects in stage 3. We prove a "no-go" result – namely, if only one linear combination of the Kähler moduli and dilaton is left unstabilized in stage 2, there exists no AdS vacuum of the full theory including non-perturbative effects. However, it is possible to avoid this no-go result by allowing two free moduli to remain at the end of stage 2. We demonstrate explicitly that, in this case, the non-perturbative mechanisms of stage 3 can complete the stabilization.

A crucial aspect of this scenario is that, at the end of stages 1 and 2, the resulting moduli space of vacua is supersymmetric and Minkowski. That is, the unstabilized fields have no potential and the classical cosmological constant is zero. As a result, this scenario does not suffer from a need to "fine-tune" the perturbative potential to be small, as arises in some "KKLT"-like scenarios.

It should be noted that while the geometric and effective field theory arguments given in this paper are complete, the results presented here are still a "scenario" since we have not provided a *complete* example of all three stages on a single Calabi-Yau three-fold. To find such an example, and to couple it to realistic particle physics in the visible sector, would be an important step forward in heterotic model building. A search for such geometries and vacua is currently underway. This will be the subject of future work [36].

Finally, it is essential to stabilize the remaining compactification moduli not considered in this paper – namely, the vector bundle moduli, counted by $h^1(V \otimes V^*)$. Potential mechanisms for such stabilization are already evident in the proceeding sections. While stages 1 and 2 are largely independent of these moduli, the non-perturbative effects considered in stage 3 are inherently bundle moduli dependent. Specifically, the pre-factors of the superpotential contributions of both gaugino condensation and membrane instantons, (4.3) and (4.4) respectively, manifestly depend on the bundle moduli. These pre-factors are complicated, manifold dependent polynomials in these moduli. Their specific form, particularly the bundle moduli dependent pfaffians associated with membrane instantons, has been studied in [37]. We hope to explore this structure and the stabilization of the vector bundle moduli in future work.

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Appendix

A Complex structure moduli and NS flux

In this Appendix, we discuss the complex structure dependent heterotic superpotential W generated by Neveu-Schwarz flux. This topic lies somewhat outside our main line of development. However, as we will see, the negative results presented here can be seen, in part, as the motivation for studying the alternative moduli stabilization mechanisms in heterotic theories discussed in this paper. The analysis of this Appendix assumes one can continue to work on a Calabi-Yau three-fold despite the introduction of NS flux [21].

The heterotic NS superpotential fixes the complex structure. However, it also destabilizes the other moduli, specifically the Kähler moduli and the dilaton. Overall stabilization of the model requires adding non-perturbative effects, such as gaugino condensation or instantons. For this to work, the non-perturbative potential and the flux potential have to be comparable in size so that the perturbative runaway can be balanced by the non-perturbative effects. Since non-perturbative effects are exponentially suppressed, one way to achieve this is by having a small flux superpotential, similar to what is required for the KKLT scenario in type IIB theories. We would like to analyze whether such a small flux superpotential is possible for heterotic NS flux. Given that the parameters in W are quantized flux, this is by no means obvious. In type IIB, this can be achieved by an appropriate "tuning" of the integer NS and RR flux, but in the heterotic case only NS flux is available.

We begin by introducing the projective complex structure fields $\mathcal{Z}^A = (\mathcal{Z}^0, \mathcal{Z}^a)$. The heterotic NS flux potential then takes the form

$$W = n_A \mathcal{Z}^A - m^A \mathcal{F}_A \,, \tag{A.1}$$

where $\mathcal{F}_A = \partial\mathcal{F}/\partial\mathcal{Z}_A$ are the derivatives of the pre-potential $\mathcal{F}$ and n_A, m^A are flux integers. We would like to study this superpotential in the large complex structure limit where the pre-potential is given by

$$\mathcal{F} = \frac{\tilde{d}_{abc}\mathcal{Z}^a\mathcal{Z}^b\mathcal{Z}^c}{6\mathcal{Z}^0} \,, \tag{A.2}$$

with $\tilde{d}_{abc}$ the intersection numbers of the mirror Calabi-Yau manifold. In terms of the physical fields $Z^a = \mathcal{Z}^a/\mathcal{Z}^0$, the associated flux superpotential in the large complex structure limit reads

$$W = n_0 + n_a Z^a - \frac{1}{2}\tilde{d}_{abc}m^a Z^b Z^c + \frac{1}{6}m^0\tilde{d}_{abc}Z^a Z^b Z^c \,. \tag{A.3}$$

It is useful to split the fields into their real and imaginary parts as $Z^a = \zeta^a + iz^a$. Further, we introduce the quantity $\kappa = \tilde{d}_{abc} z^a z^b z^c$, which is proportional to the volume of the mirror manifold and, hence, should be large in the large complex structure limit, as well as its derivatives $\kappa_a = \tilde{d}_{abc} z^b z^c$ and $\kappa_{ab} = \tilde{d}_{abc} z^c$.

What we would like to study, for now at large complex structure, is whether W can be made small at a supersymmetric point, that is, at a solution of the F-equations $W_a \equiv \partial W/\partial Z^a = 0$ [7]. The imaginary parts of the F-equations read

$$\mathrm{Im}(W_a) = \kappa_{ab}(m^0 \zeta^b - m^b) = 0 \; . \tag{A.4}$$

It turns out that the matrix κ_{ab} must be non-singular. This follows because the Kähler metric for the complex structure moduli, given by

$$K_{ab} = -\frac{3}{2}\left(\frac{\kappa_{ab}}{\kappa} - \frac{3}{2}\frac{\kappa_a \kappa_b}{\kappa^2}\right) \; , \tag{A.5}$$

must be non-singular. Consequently, we can solve Eq. (A.4) for $\zeta^a = m^a/m^0$. (Here we can assume that m^0 is non-vanishing. Otherwise, all fluxes except n_0 are forced to zero and no moduli are fixed.) Inserting this result into the real parts of the F-equation gives

$$\mathrm{Re}(W_a) = n_a - \frac{1}{2m^0}\tilde{d}_{abc} m^b m^c - \frac{m^0}{2}\kappa_a \; , \tag{A.6}$$

while the imaginary part of the superpotential can be written as

$$\mathrm{Im}(W) = n_a z^a - \frac{1}{2m^0}\tilde{d}_{abc} z^a m^b m^c - \frac{m^0}{6}\kappa \; . \tag{A.7}$$

Multiplying Eq. (A.6) with z^a and subtracting this from $\mathrm{Re}(W)$, one easily finds

$$\mathrm{Im}(W) = \frac{m^0}{3}\kappa \; . \tag{A.8}$$

Since m^0 is a flux integer and κ needs to be large in the large complex structure limit, this result implies that $|W|$ cannot be made small. Hence, the heterotic flux superpotential is always large in the large complex structure limit.

What happens if we depart from the large complex structure limit? In this case, the prepotential $\mathcal{F}$ becomes a complicated function which was first computed for specific examples in Refs. [38, 39]. While a general analysis covering the complete moduli space is not straightforward, we have looked at another limit, namely the region of moduli space near the conifold point. We have also performed a simple computer scan of the models of Refs. [38, 39] and we again find that $|W|$ cannot be made small at a supersymmetric vacuum. In conclusion, although we cannot show in general that $|W|$ is large for vacua away from the large complex structure limit, we have been unable to find any counterexamples.

[7] While these are the global F-equations, the local ones only differ by a term proportional to W which is negligible if W is small. Hence, absence of solutions with small W at the global level implies their absence at the local level.

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