Stability, ghost, and strong coupling in nonrelativistic general covariant theory of gravity with λ≠1

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Phys. Rev. D **83**, 104012 — Published 5 May 2011

DOI: 10.1103/PhysRevD.83.104012

Stability, ghost, and strong coupling in nonrelativistic general covariant theory of gravity with $\lambda \neq 1$

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In this paper, we investigate three important issues: stability, ghost and strong coupling, in the Horava-Melby-Thompson setup of the Horava-Lifshitz theory with $\lambda \neq 1$, generalized recently by da Silva. We first develop the general linear scalar perturbations of the Friedmann-Robertson-Walker (FRW) universe with arbitrary spatial curvature, and find that an immediate by-product of the setup is that, in all the inflationary models described by a scalar field, the FRW universe is necessarily flat. Applying them to the case of the Minkowski background, we find that it is stable, and, similar to the case $\lambda = 1$, the spin-0 graviton is eliminated. The vector perturbations vanish identically in the Minkowski background. Thus, similar to general relativity, a free gravitational field in this setup is completely described by a spin-2 massless graviton even with $\lambda \neq 1$. We also study the ghost problem in the FRW background, and find explicitly the ghost-free conditions. To study the strong coupling problem, we consider two different kinds of spacetimes all with the presence of matter, one is cosmological and the one is static. We find that the coupling becomes strong for a process with energy higher than $M_{pl}|c_\psi|^{5/2}$ in the flat FRW background, and $M_{pl}|c_\psi|^3$ in a static weak gravitational field, where $|c_\psi| \equiv |(1-\lambda)/(3\lambda-1)|^{1/2}$.

PACS numbers: 04.60.-m; 98.80.Cq; 98.80.-k; 98.80.Bp

I. INTRODUCTION

Recently, Horava proposed a quantum gravity theory [1], motivated by the Lifshitz theory in solid state physics [2]. The Horava-Lifshitz (HL) theory is based on the perspective that Lorentz symmetry should appear as an emergent symmetry at long distances, but can be fundamentally absent at high energies [3]. Along this vein of thinking, Horava considered systems whose scaling at short distances exhibits a strong anisotropy between space and time,

$$\mathbf{x} \to \ell \mathbf{x}, \quad t \to \ell^z t, \tag{1.1}$$

where $z \geq 3$, in order for the theory to be power-counting renormalizable in (3 + 1)-dimensional spacetimes [4]. At low energies, high-order curvature corrections become negligible, and the theory is expected to flow to $z = 1$, whereby the Lorentz invariance is "accidentally restored." Such an anisotropy between time and space can be easily realized when one writes the metric in the Arnowitt-Deser-Misner (ADM) form [5],

$$\begin{aligned} ds^2 \; &= \; -N^2c^2dt^2 + g_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right), \\ & \qquad\qquad (i,\; j = 1,2,3). \end{aligned} \tag{1.2}$$

Under the rescaling (1.1) with $z = 3$, a condition we shall assume in this paper, the lapse function N, the shift vector N^i, and the 3-metric g_{ij} scale as,

$$N \to N,\; N^i \to \ell^{-2}N^i,\; g_{ij} \to g_{ij}. \tag{1.3}$$

The gauge symmetry of the theory is the foliation-preserving diffeomorphism,

$$\tilde{t} = t - f(t), \quad \tilde{x}^i = x^i - \zeta^i(t, \mathbf{x}), \tag{1.4}$$

often denoted by Diff($M,\ \mathcal{F}$), for which N, N^i and g_{ij} transform as

$$\begin{aligned} \delta N \; &= \; \zeta^k \nabla_k N + \dot{N} f + N \dot{f}, \\ \delta N_i \; &= \; N_k \nabla_i \zeta^k + \zeta^k \nabla_k N_i + g_{ik}\dot{\zeta}^k + \dot{N}_i f + N_i \dot{f}, \\ \delta g_{ij} \; &= \; \nabla_i \zeta_j + \nabla_j \zeta_i + f \dot{g}_{ij}, \end{aligned} \tag{1.5}$$

where $\dot{f} \equiv df/dt$, ∇_i denotes the covariant derivative with respect to the 3-metric g_{ij}, $N_i = g_{ik}N^k$, and $\delta g_{ij} \equiv \tilde{g}_{ij}\left(t, x^k\right) - g_{ij}\left(t, x^k\right)$, etc. From these expressions one can see that N and N^i play the role of gauge fields of the Diff($M,\ \mathcal{F}$) symmetry. Therefore, it is natural to assume that N and N^i inherit the same dependence on spacetime as the corresponding generators, that is,

$$N = N(t), \quad N^i = N^i(t, x). \tag{1.6}$$

It is clear that this is preserved by Diff($M,\ \mathcal{F}$), and usually referred to as the *projectability condition* (Note that the dynamical variables g_{ij} in general depend on both time and space, $g_{ij} = g_{ij}(t, x)$.).

Due to the restricted diffeomorphisms (1.4), one more degree of freedom appears in the gravitational sector - a spin-0 graviton. This is potentially dangerous, and needs to decouple in the IR, in order to be consistent with observations. Whether this is possible or not is still an open question [6]. In particular, it was shown that the spin-0 mode is not stable in the original version of the HL theory [1] as well as in the Sotiriou, Visser and Weinfurtner (SVW) generalization [7–9]. But, these instabilities were all found in the Minkowski background.

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In the de Sitter spacetime, it was shown that it is stable [10]. So, one may take the latter as its legitimate background [11]. However, the strong coupling problem still exists [12, 13], although it might be circumvented by the Vainshtein mechanism [14], as recently showed in the spherical static [6] and cosmological [12] spacetimes.

To cure the above problems, various versions of the theory were proposed recently [15, 16]. In particular, Horava and Melby-Thompson (HMT) showed that one can eliminate the spin-0 graviton by introducing two auxiliary fields, the $U(1)$ gauge field A and the Newtonian pre-potentail φ, by extending the Diff($M,\ \mathcal{F}$) symmetry to include a local $U(1)$ symmetry [17],

$$U(1) \ltimes \text{Diff}(M,\ \mathcal{F}). \tag{1.7}$$

Under this extended symmetry, the special status of time maintains, so that the anisotropic scaling (1.1) with $z > 1$ is still realized, whereby the UV behavior of the theory can be considerably improved. Under the local $U(1)$ symmetry, the gravitational and gauge fields transform as

$$\begin{aligned} \delta_\alpha A &= \dot{\alpha} - N^i \nabla_i \alpha, \quad \delta_\alpha \varphi = -\alpha, \\ \delta_\alpha N_i &= N \nabla_i \alpha, \quad \delta_\alpha g_{ij} = 0 = \delta_\alpha N, \end{aligned} \tag{1.8}$$

where α is the generator of the local $U(1)$ gauge symmetry. Under the Diff($M,\ \mathcal{F}$), A and φ transform as,

$$\begin{aligned} \delta A &= \zeta^i \nabla_i A + \dot{f} A + f \dot{A}, \\ \delta \varphi &= f \dot{\varphi} + \zeta^i \nabla_i \varphi. \end{aligned} \tag{1.9}$$

For details, we refer readers to [17, 18].

As shown explicitly in [19], the $U(1)$ symmetry pertains specifically to the case $\lambda = 1$, where λ is a coupling constant that characterizes the deviation of the kinetic part of action from the corresponding one given in general relativity (GR). It is exactly because of this deviation that causes all the problems, including ghost, instability and strong coupling. Therefore, it was considered as a remarkable feature of this nonrelativistic general covariant theory, in which λ is forced to be one. However, this claim was soon challenged by da Silva [20], who argued that the introduction of the Newtonian pre-potential is so powerful that action with $\lambda \neq 1$ also has the $U(1)$ symmetry [1].

Once the coupling constant λ can be different from one, the issues of instability, ghost and strong coupling plagued in other versions of the HL theory all rise again. In this paper, we investigate these important questions in detail in the framework of da Silva's generalization of the HMT setup. Specifically, in Sec. II we briefly review the theory by presenting all the field equations and conservation laws when matter is present. In Sec. III we study the Friedmann-Robertson-Walker (FRW) universe with any given spatial curvature, and derive the generalized Friedmann equation and conservation law of energy. An immediate by-product of the setup is that, in all the inflationary models described by a scalar field, the FRW universe is necessarily flat. In Sec. IV, we develop the general formulas for the linear scalar perturbations of the FRW universe. Applying these formulas to the Minkowski background in Sec.V, we study the stability problem, and show explicitly that it is stable and the spin-0 graviton is eliminated even for $\lambda \neq 1$. This conclusion is the same as that obtained by da Silva for the maximal symmetric spacetimes with detailed balance condition, in which the Minkowski spacetime is not a solution of the theory [20]. In Sec. VI, we study the ghost and strong coupling problems, and derive the ghost-free conditions in terms of λ. To study the strong coupling problem, we consider two different kinds of spacetimes all filled with matter [2]: (a) spacetimes in which the flat FRW universe can be considered as their zero-order approximations; and (b) spherical statics spacetimes in which the Minkowski spacetime can be considered as their zero-order approximations. We find that the strong coupling problem indeed exists in both kinds of spacetimes. It should be noted that strong coupling itself is not a problem, as long as the theory is consistent with observations. In fact, several well-known theories are strong coupling [22]. Interestingly enough, the strong coupling in the Dvali-Gabadadze-Porrati braneworld model even helps to screen the spin-0 mode so that the models are consistent with solar system tests [23]. Finally, in Sec. VII we present our main conclusions and discussing remarks. An appendix is also included, in which, among other things, the kinetic part of the action and coupling coefficients are given.

Before proceeding further, we would like to note that in [18] we studied the HMT setup where $\lambda = 1$. In addition, static spacetimes were also studied recently [24, 25], while its Hamiltonian structure and some possible generalizations were investigated in [21]. In all of these investigations $\lambda = 1$ was assumed. Thus, in this paper we are mainly concerned with $\lambda \neq 1$.

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[1] It should be noted that even in the tree level we could have $\lambda = 1$, it is still subjected to quantum corrections. This is in contrast to the relativistic case, where $\lambda = 1$ is protected by the Lorentz symmetry, $\tilde{x}^\mu = \tilde{x}^\mu(t, \mathbf{x}),\ (\mu = 0, 1, 2, 3)$, even in the quantum level.

[2] Note that, to count the number of the degrees of the propagating gravitational modes, one needs to consider free gravitational fields. Another way to count the degrees of the freedom is to study the structure of the Hamiltonian constraints [17, 21]. On the other hand, to study the ghost and strong coupling problems, one needs to consider the cases in which the gravitational perturbations are different from zero. In this paper, this is realized by the presence of matter fields. That is, it is the matter that produces the gravitational perturbations. Clearly, this does not contradict to the conclusion that the spin-0 mode is eliminated in such a setup. A similar situation also happens in GR, in which gravitational scalar perturbations of the FRW universe in general do not vanish, although the only degrees of the freedom of the gravitational sector are the spin-2 massless gravitons.

Moreover, cosmology and black hole physics in other versions of the HL theory have been intensively studied recently, and it becomes very difficult to review all those important works here. Instead, we simply refer readers to [6, 10, 15, 26] for detail.

II. NONRELATIVISITC GENERAL COVARIANT HL THEORY WITH ANY λ

For any given coupling constant λ, the total action can be written as [17, 18, 20],

$$
\begin{aligned}
S &= \zeta^2 \int dt d^3x N\sqrt{g}\Big(\mathcal{L}_K - \mathcal{L}_V + \mathcal{L}_\varphi + \mathcal{L}_A + \mathcal{L}_\lambda \\
&\qquad + \zeta^{-2}\mathcal{L}_M\Big), \qquad (2.1)
\end{aligned}
$$

where $g = \det g_{ij}$, and

$$
\begin{aligned}
\mathcal{L}_K &= K_{ij}K^{ij} - \lambda K^2, \\
\mathcal{L}_\varphi &= \varphi \mathcal{G}^{ij}\Big(2K_{ij} + \nabla_i\nabla_j\varphi\Big), \\
\mathcal{L}_A &= \frac{A}{N}\Big(2\Lambda_g - R\Big), \\
\mathcal{L}_\lambda &= (1-\lambda)\Big[\big(\nabla^2\varphi\big)^2 + 2K\nabla^2\varphi\Big]. \qquad (2.2)
\end{aligned}
$$

Here Λ_g is a coupling constant, and the Ricci and Riemann terms all refer to the three-metric g_{ij}, and

$$
\begin{aligned}
K_{ij} &= \frac{1}{2N}\left(-\dot{g}_{ij} + \nabla_i N_j + \nabla_j N_i\right), \\
\mathcal{G}_{ij} &= R_{ij} - \frac{1}{2}g_{ij}R + \Lambda_g g_{ij}. \qquad (2.3)
\end{aligned}
$$

$\mathcal{L}_M$ is the matter Lagrangian density, which in general is a function of all the dynamical variables, $U(1)$ gauge field, and the Newtonian prepotential, i.e., $\mathcal{L}_M = \mathcal{L}_M\big(N,\ N_i,\ g_{ij},\ \varphi,\ A;\ \chi\big)$, where χ denotes collectively the matter fields. $\mathcal{L}_V$ is an arbitrary Diff(Σ)-invariant local scalar functional built out of the spatial metric, its Riemann tensor and spatial covariant derivatives, without the use of time derivatives.

Note the difference between the notations used here and the ones used in [17, 20] [3]. In this paper, without further explanations, we shall use directly the notations and conventions defined in [8] and [18], which will be referred, respectively, to as Paper I and Paper II. However, in order to have the present paper as independent as possible, it is difficult to avoid repeating the same materials, although we shall try to limit it to its minimum.

In [7], by assuming that the highest order derivatives are six, the minimum in order to have the theory to be power-counting renormalizable [4], and that the theory preserves the parity, SVW constructed the most general form of $\mathcal{L}_V$,

$$
\begin{aligned}
\mathcal{L}_V &= \zeta^2 g_0 + g_1 R + \frac{1}{\zeta^2}\left(g_2 R^2 + g_3 R_{ij}R^{ij}\right) \\
&\quad + \frac{1}{\zeta^4}\left(g_4 R^3 + g_5 R\, R_{ij}R^{ij} + g_6 R^i_j R^j_k R^k_i\right) \\
&\quad + \frac{1}{\zeta^4}\Big[g_7 R\nabla^2 R + g_8\left(\nabla_i R_{jk}\right)\left(\nabla^i R^{jk}\right)\Big], \qquad (2.4)
\end{aligned}
$$

where the coupling constants $g_s\,(s = 0, 1, 2, \dots 8)$ are all dimensionless. The relativistic limit in the IR requires $g_1 = -1$ and $\zeta^2 = 1/(16\pi G)$ [7].

Then, it can be shown that the Hamiltonian and momentum constraints are given respectively by,

$$
\begin{aligned}
&\int d^3x\sqrt{g}\Big[\mathcal{L}_K + \mathcal{L}_V - \varphi\mathcal{G}^{ij}\nabla_i\nabla_j\varphi - (1-\lambda)\big(\nabla^2\varphi\big)^2\Big] \\
&\qquad = 8\pi G\int d^3x\sqrt{g}\, J^t, \qquad (2.5)
\end{aligned}
$$

$$
\nabla^j\Big[\pi_{ij} - \varphi\mathcal{G}_{ij} - \big(1-\lambda\big)g_{ij}\nabla^2\varphi\Big] = 8\pi G J_i, \qquad (2.6)
$$

where

$$
\begin{aligned}
J^t &\equiv 2\frac{\delta\left(N\mathcal{L}_M\right)}{\delta N}, \\
\pi_{ij} &\equiv -K_{ij} + \lambda K g_{ij}, \\
J_i &\equiv -N\frac{\delta\mathcal{L}_M}{\delta N^i}. \qquad (2.7)
\end{aligned}
$$

Variation of the action (2.1) with respect to φ and A yield, respectively,

$$
\begin{aligned}
&\mathcal{G}^{ij}\Big(K_{ij} + \nabla_i\nabla_j\varphi\Big) + \big(1-\lambda\big)\nabla^2\Big(K + \nabla^2\varphi\Big) \\
&\qquad = 8\pi G J_\varphi, \qquad (2.8)
\end{aligned}
$$

$$
R - 2\Lambda_g = 8\pi G J_A, \qquad (2.9)
$$

where

$$
J_\varphi \equiv -\frac{\delta\mathcal{L}_M}{\delta\varphi}, \quad J_A \equiv 2\frac{\delta\left(N\mathcal{L}_M\right)}{\delta A}. \qquad (2.10)
$$

On the other hand, the dynamical equations now read,

$$
\begin{aligned}
&\frac{1}{N\sqrt{g}}\left\{\sqrt{g}\Big[\pi^{ij} - \varphi\mathcal{G}^{ij} - \big(1-\lambda\big)g^{ij}\nabla^2\varphi\Big]\right\}_{,t} \\
&= -2\left(K^2\right)^{ij} + 2\lambda K K^{ij} \\
&\quad + \frac{1}{N}\nabla_k\left[N^k\pi^{ij} - 2\pi^{k(i}N^{j)}\right] \\
&\quad - 2\big(1-\lambda\big)\Big[\big(K + \nabla^2\varphi\big)\nabla^i\nabla^j\varphi + K^{ij}\nabla^2\varphi\Big] \\
&\quad + \big(1-\lambda\big)\Big[2\nabla^{(i}F^{j)}_\varphi - g^{ij}\nabla_k F^k_\varphi\Big] \\
&\quad + \frac{1}{2}\Big(\mathcal{L}_K + \mathcal{L}_\varphi + \mathcal{L}_A + \mathcal{L}_\lambda\Big)g^{ij} \\
&\quad + F^{ij} + F^{ij}_\varphi + F^{ij}_A + 8\pi G\tau^{ij}, \qquad (2.11)
\end{aligned}
$$

[3] In particular, we have $K_{ij} = -K^{HMT}_{ij}$, $\Lambda_g = \Omega^{HMT}$, $\varphi = -\nu^{HMT}$, $\mathcal{G}_{ij} = \Theta^{HMT}_{ij}$, where quantities with the super-indice "HMT" are those used in [17, 20].

where $\left(K^2\right)^{ij} \equiv K^{il}K_l^j,\ f_{(ij)} \equiv (f_{ij} + f_{ji})/2$, and

$$\begin{aligned}
F^{ij} &\equiv \frac{1}{\sqrt{g}}\frac{\delta\left(-\sqrt{g}\mathcal{L}_V\right)}{\delta g_{ij}} = \sum_{s=0}^{8} g_s\zeta^{n_s}\left(F_s\right)^{ij},\\
F^{ij}_{\varphi} &= \sum_{n=1}^{3} F^{ij}_{(\varphi,n)},\\
F^{i}_{\varphi} &= \Big(K + \nabla^2\varphi\Big)\nabla^i\varphi + \frac{N^i}{N}\nabla^2\varphi,\\
F^{ij}_{A} &= \frac{1}{N}\left[AR^{ij} - \Big(\nabla^i\nabla^j - g^{ij}\nabla^2\Big)A\right],
\end{aligned} \tag{2.12}$$

with $n_s = (2, 0, -2, -2, -4, -4, -4, -4, -4)$. The stress 3-tensor τ^{ij} is defined as

$$\tau^{ij} = \frac{2}{\sqrt{g}}\frac{\delta\left(\sqrt{g}\mathcal{L}_M\right)}{\delta g_{ij}}, \tag{2.13}$$

and the geometric 3-tensors $(F_s)_{ij}$ and $F^{ij}_{(\varphi,n)}$ are given in Paper II.

The matter components $(J^t,\ J^i,\ J_\varphi,\ J_A,\ \tau^{ij})$ satisfy the conservation laws,

$$\begin{aligned}
\int d^3x\sqrt{g}\Bigg[\dot{g}_{kl}\tau^{kl} - \frac{1}{\sqrt{g}}\left(\sqrt{g}J^t\right)_{,t} + \frac{2N_k}{N\sqrt{g}}\left(\sqrt{g}J^k\right)_{,t}&\\
-2\dot{\varphi}J_\varphi - \frac{A}{N\sqrt{g}}\left(\sqrt{g}J_A\right)_{,t}\Bigg] = 0,&
\end{aligned} \tag{2.14}$$

$$\begin{aligned}
\nabla^k\tau_{ik} - \frac{1}{N\sqrt{g}}\left(\sqrt{g}J_i\right)_{,t} - \frac{J^k}{N}\left(\nabla_k N_i - \nabla_i N_k\right)&\\
-\frac{N_i}{N}\nabla_k J^k + J_\varphi\nabla_i\varphi - \frac{J_A}{2N}\nabla_i A = 0.&
\end{aligned} \tag{2.15}$$

III. COSMOLOGICAL MODELS

The homogeneous and isotropic universe is described by,

$$\bar{N} = 1,\ \ \bar{N}_i = 0,\ \ \bar{g}_{ij} = a^2(t)\gamma_{ij}, \tag{3.1}$$

where $\gamma_{ij} = \delta_{ij}\left(1 + \frac{1}{4}kr^2\right)^{-2}$, with $r^2 \equiv x^2 + y^2 + z^2,\ k = 0, \pm 1$. As in Paper I, we use symbols with bars to denote the quantities of background. Using the $U(1)$ gauge freedom of Eq.(1.8), on the other hand, we can always set

$$\bar{\varphi} = 0. \tag{3.2}$$

Then, we find

$$\begin{aligned}
\bar{K}_{ij} &= -a^2 H\gamma_{ij},\quad \bar{R}_{ij} = 2k\gamma_{ij},\\
\bar{F}^{ij}_{A} &= \frac{2k\bar{A}}{a^4}\gamma^{ij},\quad \bar{F}^{ij}_{\varphi} = 0,\quad \bar{F}^{i}_{\varphi} = 0,\\
\bar{F}^{ij} &= \frac{\gamma^{ij}}{a^2}\left(-\Lambda + \frac{k}{a^2} + \frac{2\beta_1 k^2}{a^4} + \frac{12\beta_2 k^3}{a^6}\right),
\end{aligned} \tag{3.3}$$

where $H = \dot{a}/a,\ \Lambda \equiv \zeta^2 g_0/2$, and

$$\beta_1 \equiv \frac{3g_2 + g_3}{\zeta^2},\quad \beta_2 \equiv \frac{9g_4 + 3g_5 + g_6}{\zeta^4}. \tag{3.4}$$

Hence, we obtain

$$\begin{aligned}
\bar{\mathcal{L}}_K &= 3\big(1 - 3\lambda\big)H^2,\ \ \bar{\mathcal{L}}_\varphi = 0 = \bar{\mathcal{L}}_\lambda,\\
\bar{\mathcal{L}}_A &= 2\bar{A}\Big(\Lambda_g - \frac{3k}{a^2}\Big),\\
\bar{\mathcal{L}}_V &= 2\Lambda - \frac{6k}{a^2} + \frac{12\beta_1 k^2}{a^4} + \frac{24\beta_2 k^3}{a^6}.
\end{aligned} \tag{3.5}$$

It can be shown that the super-momentum constraint (2.6) is satisfied identically for $\bar{J}^i = 0$, while the Hamiltonian constraint (2.5) yields,

$$\frac{1}{2}\big(3\lambda - 1\big)H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\bar{\rho} + \frac{\Lambda}{3} + \frac{2\beta_1 k^2}{a^4} + \frac{4\beta_2 k^3}{a^6}, \tag{3.6}$$

where $\bar{J}^t \equiv -2\bar{\rho}$. On the other hand, Eqs.(2.8) and (2.9) give, respectively,

$$H\left(\Lambda_g - \frac{k}{a^2}\right) = -\frac{8\pi G}{3}\bar{J}_\varphi, \tag{3.7}$$

$$\frac{3k}{a^2} - \Lambda_g = 4\pi G\bar{J}_A, \tag{3.8}$$

while the dynamical equation (2.11) reduces to

$$\begin{aligned}
\frac{1}{2}\big(3\lambda - 1\big)\frac{\ddot{a}}{a} \ =\ &-\frac{4\pi G}{3}(\bar{\rho} + 3\bar{p}) + \frac{1}{3}\Lambda - \frac{2\beta_1 k^2}{a^4}\\
&-\frac{8\beta_2 k^3}{a^6} + \frac{1}{2}\bar{A}\left(\frac{k}{a^2} - \Lambda_g\right),
\end{aligned} \tag{3.9}$$

where $\bar{\tau}_{ij} = \bar{p}\,\bar{g}_{ij}$.

The conservation law of momentum (2.15) is satisfied identically, while the one of energy (2.14) reduces to,

$$\dot{\bar{\rho}} + 3H\left(\bar{\rho} + \bar{p}\right) = \bar{A}\bar{J}_\varphi. \tag{3.10}$$

It is interesting to note that the energy of matter is not conserved in general, due to its interaction with the gauge field $\bar{A}$ and the Newtonian pre-potential $\bar{\varphi}$. This might have profound implications in cosmology.

IV. COSMOLOGICAL PERTURBATIONS

As in Papers I and II, when we consider perturbations, we turn to the conformal time η, where $\eta = \int dt/a(t)$. Under this coordinate transformation, the gravitational and gauge fields transfer as,

$$\begin{aligned}
N &= a\tilde{N},\quad N^i = a\tilde{N}^i,\quad g_{ij} = \tilde{g}_{ij},\\
A &= a\tilde{A},\quad \varphi = \tilde{\varphi},
\end{aligned} \tag{4.1}$$

where the quantities with tildes are the ones defined in the coordinates (t, x^i). With these in mind, we write the linear scalar perturbations of the metric in terms of the conformal time η as,

$$\begin{aligned} \delta N &= a\phi, \quad \delta N_i = a^2 B_{|i}, \\ \delta g_{ij} &= -2a^2\big(\psi\gamma_{ij} - E_{|ij}\big), \\ A &= \hat{A} + \delta A, \quad \varphi = \hat{\varphi} + \delta\varphi, \end{aligned} \tag{4.2}$$

where

$$\hat{A} = a\bar{A}, \quad \hat{\varphi} = \bar{\varphi}, \tag{4.3}$$

and $\bar{A}$ and $\bar{\varphi}$ are the gauge fields of the background, given in the last section in the (t, x^i) coordinates. Under the gauge transformations (1.4), we find that

$$\begin{aligned} \tilde{\phi} &= \phi - \mathcal{H}\xi^0 - \xi^{0'}, \quad \tilde{\psi} = \psi + \mathcal{H}\xi^0, \\ \tilde{B} &= B + \xi^0 - \xi', \quad \tilde{E} = E - \xi, \\ \tilde{\delta\varphi} &= \delta\varphi - \xi^0\hat{\varphi}', \quad \tilde{\delta A} = \delta A - \xi^0\hat{A}' - \xi^{0'}\hat{A}, \end{aligned} \tag{4.4}$$

where $f = -\xi^0$, $\zeta^i = -\xi^{|i}$, $\mathcal{H} \equiv a'/a$, and a prime denotes the ordinary derivative with respect to η. Under the $U(1)$ gauge transformations, on the other hand, we find that

$$\begin{aligned} \tilde{\phi} &= \phi, \quad \tilde{E} = E, \quad \tilde{\psi} = \psi, \quad \tilde{B} = B - \frac{\epsilon}{a}, \\ \tilde{\delta\varphi} &= \delta\varphi + \epsilon, \quad \tilde{\delta A} = \delta A - \epsilon', \end{aligned} \tag{4.5}$$

where $\epsilon = -\alpha$. Then, the gauge transformations of the whole group $U(1) \ltimes \text{Diff}(M,\ \mathcal{F})$ will be the linear combination of the above two. Since we have six unknown and three arbitrary functions, the total number of the gauge-invariants of $U(1) \ltimes \text{Diff}(M,\ \mathcal{F})$ is $N = 6 - 3 = 3$. These gauge-invariants can be constructed as,

$$\begin{aligned} \Phi &= \phi + \frac{a}{a - \hat{\varphi}'}\left[\frac{\delta\varphi'}{a} + \mathcal{H}\big(B - E'\big) + \big(B - E'\big)'\right] \\ &\quad + \frac{1}{\big(a - \hat{\varphi}'\big)^2}\left(\hat{\varphi}'' - \mathcal{H}\hat{\varphi}'\right)\Big[\delta\varphi + a\big(B - E'\big)\Big], \\ \Psi &= \psi - \frac{\mathcal{H}}{a - \hat{\varphi}'}\Big[\delta\varphi + a\big(B - E'\big)\Big], \\ \Gamma &= \delta A + \left[\frac{a + \hat{A}}{a - \hat{\varphi}'}\delta\varphi + \frac{a(\hat{A} + \hat{\varphi}')}{a - \hat{\varphi}'}\big(B - E'\big)\right]'. \end{aligned} \tag{4.6}$$

Using the $U(1)$ gauge freedom (4.5), we shall set

$$\delta\varphi = 0. \tag{4.7}$$

This choice completely fixes the $U(1)$ gauge. Then, considering Eq.(3.2), we find that the above expressions reduce to

$$\begin{aligned} \Phi &= \phi + \mathcal{H}\big(B - E'\big) + \big(B - E'\big)', \\ \Psi &= \psi - \mathcal{H}\big(B - E'\big), \\ \Gamma &= \delta A + \Big[\hat{A}\big(B - E'\big)\Big]', \ (\hat{\varphi} = \delta\varphi = 0). \end{aligned} \tag{4.8}$$

The expressions for Φ and Ψ now take precisely the same forms as those defined in Paper I, which are also identical to those given in GR [27]. In Papers I and II, the quasi-longitudinal gauge,

$$\phi = 0 = E, \tag{4.9}$$

was imposed. In this paper, we shall adopt this gauge for the metric perturbations, and the gauge of Eq.(4.7) for the Newtonian pre-potential. We shall refer them as the "generalized" quasi-longitudinal gauge, or simply the quasi-longitudinal gauge.

Then, to first-order the Hamiltonian and momentum constraints become, respectively,

$$\begin{aligned} \int \sqrt{\gamma}d^3x\Bigg[&\left(\vec{\nabla}^2 + 3k\right)\psi - \frac{(3\lambda - 1)\mathcal{H}}{2}\left(\vec{\nabla}^2 B + 3\psi'\right) \\ &- 2k\Big(\frac{2\beta_1}{a^2} + \frac{6\beta_2 k}{a^4} + \frac{3g_7}{\zeta^4 a^4}\vec{\nabla}^2\Big)\left(\vec{\nabla}^2 + 3k\right)\psi \\ &- 4\pi G a^2\delta\mu\Bigg] = 0, \end{aligned} \tag{4.10}$$

$$(3\lambda - 1)\psi' - 2kB - (1 - \lambda)\vec{\nabla}^2 B = 8\pi G a q, \tag{4.11}$$

where $\delta\mu \equiv -\delta J^t/2$ and $\delta J^i \equiv a^{-2}q^{|i}$. On the other hand, the linearized equations (2.8) and (2.9) reduce, respectively, to

$$\begin{aligned} &\Big(\Lambda_g - \frac{k}{a^2}\Big)\Big[\vec{\nabla}^2 B + 3\big(\psi' + 2\mathcal{H}\psi\big)\Big] \\ &\quad + \frac{2\mathcal{H}}{a^2}\Big[\vec{\nabla}^2\psi + 3\big(2k - a^2\Lambda_g\big)\psi\Big] \\ &\quad + \frac{1 - \lambda}{a^2}\vec{\nabla}^2\Big(\vec{\nabla}^2 B + 3\psi'\Big) = 8\pi G a\delta J_\varphi, \end{aligned} \tag{4.12}$$

$$\vec{\nabla}^2\psi + 3k\psi = 2\pi G a^2\delta J_A. \tag{4.13}$$

The linearly perturbed dynamical equations can be divided into the trace and traceless parts. The trace part reads,

$$\begin{aligned} \psi'' &+ 2\mathcal{H}\psi' - \mathcal{F}\psi - \frac{1}{3(3\lambda - 1)}\gamma^{ij}\delta F_{ij} \\ &- \frac{1}{3a(3\lambda - 1)}\Big(2\vec{\nabla}^2 - 3k + 3\Lambda_g a^2\Big)\delta A \\ &+ \frac{2\hat{A}}{3a(3\lambda - 1)}\Big(\vec{\nabla}^2 + 6k - 3\Lambda_g a^2\Big)\psi \\ &+ \frac{1}{3}\vec{\nabla}^2\left(B' + 2\mathcal{H}B\right) = \frac{8\pi G a^2}{(3\lambda - 1)}\delta\mathcal{P}, \end{aligned} \tag{4.14}$$

where

$$\begin{aligned} \mathcal{F} &= \frac{2a^2}{3\lambda - 1}\left(-\Lambda + \frac{k}{a^2} + \frac{2\beta_1 k^2}{a^4} + \frac{12\beta_2 k^3}{a^6}\right), \\ \delta\tau^{ij} &= \frac{1}{a^2}\left[\left(\delta\mathcal{P} + 2\bar{p}\psi\right)\gamma^{ij} + \Pi^{|\langle ij\rangle}\right], \\ f_{|\langle ij\rangle} &\equiv f_{|ij} - \frac{1}{3}\gamma_{ij}\vec{\nabla}^2 f, \end{aligned} \tag{4.15}$$

and $\delta F_{ij} = \sum g_s \zeta^{n_s} \delta(F_s)_{ij}$, with $\delta(F_s)_{ij}$ given by Eq. (A1) in Paper I. The traceless part is given by

$$\Big(B' + 2\mathcal{H}B\Big)_{|\langle ij\rangle} + \delta F_{\langle ij\rangle} - \frac{1}{a}\Big(\delta A - \hat{A}\psi\Big)_{|\langle ij\rangle} = -8\pi G a^2 \Pi_{|\langle ij\rangle}. \tag{4.16}$$

To first order, the conservation laws (2.14) and (2.15), on the other hand, take the forms,

$$\int \sqrt{\gamma} d^3x \Bigg\{ \delta\mu' + 3\mathcal{H}\left(\delta\mathcal{P} + \delta\mu\right) - 3\left(\bar{\rho} + \bar{p}\right)\psi' + \frac{1}{2a^4}\left[\Big(a^3 \bar{J}_A\Big)'\delta A + \hat{A}\Big(a^3\big(\delta A - 3\hat{A}\psi\big)\Big)'\right]\Bigg\} = 0, \tag{4.17}$$

$$q' + 3\mathcal{H}q - a\delta\mathcal{P} - \frac{2a}{3}\left(\vec{\nabla}^2 + 3k\right)\Pi + \frac{1}{2}\bar{J}_A\delta A = 0, \tag{4.18}$$

where $\bar{J}_A$ is given by Eq.(3.8).

This completes the general description of linear scalar perturbations in the FRW background with any spatial curvature in the framework of the HMT setup with any given λ, generalized recently by da Silva [20].

V. STABILITY OF THE MINKOWSKI SPACETIME

The stability of the maximal symmetric spacetimes in the da Silva generalization with $\lambda \neq 1$ was considered in [20] with detailed balance condition. Since the Minkowski is not a solution of the theory when detailed balance condition is imposed, so the analysis given in [20] does not include the case where the Minkowski spacetime is the background. However, for the potential given by Eq.(2.4), the detailed balance condition is broken, and the Minkowski spacetime now is a solution of the theory. Therefore, in this section we study the stability of the Minkowski spacetime with any given λ. The case with $\lambda = 1$ was considered in Paper II, so in this section we consider only the case with $\lambda \neq 1$.

It is easy to show that the Minkowski spacetime,

$$a = 1, \quad \bar{A} = \bar{\varphi} = k = 0, \tag{5.1}$$

is a solution of the da Silva generalization even with $\lambda \neq 1$, provided that

$$\Lambda_g = \Lambda = \bar{J}_A = \bar{J}_\varphi = \bar{\rho} = \bar{p} = 0. \tag{5.2}$$

Then, the linearized Hamiltonian constraint (4.10) is satisfied identically, while the super-momentum constraint (4.11) yields,

$$\partial^2 B = \frac{3\lambda - 1}{1-\lambda}\dot{\psi}, \tag{5.3}$$

where $\partial^2 = \delta^{ij}\partial_i\partial_j$. Eqs.(4.12) and (4.13) reduce to,

$$\partial^2\Big(\partial^2 B + 3\dot{\psi}\Big) = 0, \tag{5.4}$$

$$\partial^2\psi = 0. \tag{5.5}$$

Then, we have $\delta F_{ij} = -\psi_{,ij}$, and the trace and traceless parts of the dynamical equations reduce, respectively, to

$$\ddot{\psi} - \frac{2}{3(3\lambda-1)}\partial^2\delta A + \frac{1}{3}\partial^2\dot{B} = 0, \tag{5.6}$$

$$\dot{B} = \delta A - \psi. \tag{5.7}$$

It can be shown that Eqs.(5.4) and (5.6) are not independent, and can be obtained from Eqs.(5.3), (5.5) and (5.7). Eq.(5.5) shows that ψ is not propagating, and with proper boundary conditions, we can set $\psi = 0$. Then, Eqs.(5.3) and (5.7) show that B and δA are also not propagating, and shall also vanish with proper boundary conditions. Therefore, we finally obtain

$$\psi = B = \delta A = 0. \tag{5.8}$$

Thus, the scalar perturbations even with $\lambda \neq 1$ vanish identically in the Minkowski background. Hence, the spin-0 graviton is eliminated in the da Silva generalization even for any given coupling constant λ.

VI. GHOST AND STRONG COUPLING

To consider the ghost and strong coupling problems, we first note that they are closely related to the fact that $\lambda \neq 1$. The parts that depend on λ are the kinetic part, $\mathcal{L}_K$, and the interaction part $\mathcal{L}_\lambda(K_{ij}, \varphi)$ between the extrinsic curvature K_{ij} and the Newtonian pre-potential φ. With the gauge choice $\varphi = 0$, we can see that the latter vanishes identically. Then, it is sufficient to consider only the kinetic part S_K, the IR terms R and Λ, and the source term S_M [12, 28, 29],

$$S_{IR} = \int dt d^3x N\sqrt{g}\Big(\mathcal{L}_K + R - 2\Lambda + \mathcal{L}_M\Big). \tag{6.1}$$

Second, the presence of matter is to produce non-zero perturbations. Otherwise, the spacetimes, to zero-order, are the maximally symmetric spacetimes. In these backgrounds, when matter is not present, the corresponding metric and gauge field perturbations, ψ, B and δA, vanish identically, as shown in the last section for the Minkowski spacetime, and in [20] for the (anti-) de Sitter one. On the other hand, $\mathcal{L}_M$ does not depend on λ, so it does not contribute to the strong coupling and ghost problems. Therefore, the only role that $\mathcal{L}_M$ plays here is to produce non-vanishing ψ, B and δA. It is interesting to note that to study the strong coupling problem, in [16] the authors assumed that the background metric has non-vanishing extrinsic and spatial curvatures in the scale L: $\bar{R}_{ij} \sim 1/L^2$ and $\bar{K}_{ij} \sim 1/L$, instead of non-vanishing ψ and B assumed here as well as in [28, 29].

But, the purposes are the same: to provide a environment so that the strong coupling problem can manifest itself properly, if it exists. In the following, we shall consider two different kinds of gravitational fields: one represents spacetimes in which the flat FRW universe with $\Lambda = 0$ can be considered as their zero-order approximations; and the other represents static weak gravitational fields, in which the Minkowski spacetime can be considered as their zero-order approximations.

A. Ghost-free Conditions

In the flat FRW background, the quadratic part of S_{IR} is given by [10],

$$S_{IR}^{(2)} = \zeta^2 \int d\eta d^3x a^2 \Bigg\{ (1-3\lambda)\Big[3\psi'^2 + 6\mathcal{H}\psi\psi' + 2\psi'\partial^2 B + \frac{9}{2}\mathcal{H}^2\psi^2\Big] + 2\big(\partial\psi\big)^2 + (1-\lambda)\big(\partial^2 B\big)^2 \Bigg\}. \tag{6.2}$$

Note that in writing the above expression, we had ignored the term, $\mathcal{L}_M$, as it has no contributions to both the ghost and the strong problems, as mentioned above. Then, from the super-momentum constraint (4.11), we find that

$$\partial^2 B = \frac{3\lambda - 1}{1-\lambda}\psi' - \frac{8\pi G a q}{1-\lambda}. \tag{6.3}$$

Substituting it into Eq.(6.2), we obtain

$$S_{IR}^{(2)} = \zeta^2 \int d\eta d^3x a^{2(1+\delta)} \Bigg\{ -\frac{2}{c_\psi^2}\tilde{\psi}'^2 + 2\big(\partial\tilde{\psi}\big)^2 - \frac{9\lambda(3\lambda-1)}{2}\mathcal{H}^2\tilde{\psi}^2 + \frac{\tilde{q}^2}{c_\psi^2} \Bigg\}, \tag{6.4}$$

where

$$c_\psi^2 = \frac{1-\lambda}{3\lambda-1}, \quad \psi = a^\delta \tilde{\psi}, \quad q = \frac{\sqrt{3\lambda-1}\,\tilde{q}}{8\pi G a^{1-\delta}}, \tag{6.5}$$

and $\delta \equiv -3(1-\lambda)/2$. Thus, the ghost-free condition requires $c_\psi^2 < 0$, or equivalently,

$$i)\ \lambda > 1, \quad \text{or} \quad ii)\ \ \lambda < \frac{1}{3}, \tag{6.6}$$

which are precisely the conditions obtained in Paper I in the SVW setup [8].

It should be noted that the conditions (6.6) also hold in the non-flat FRW backgrounds, as one can easily show by following the above arguments.

In addition, the expression of $S_{IR}^{(2)}$ given by Eq.(6.2) is the same as that given in [30], but different from the one given in [31]. After the typos of [31] are corrected, it can be shown that, in contrast to their claims, the scalar modes are both ghost-free and stable in the ranges of λ defined by Eq.(6.6), when the matter field is a scalar and satisfies the scalar field equations.

B. Strong Coupling Problem

As mentioned previously, we shall consider two different kinds of spacetimes. In the following, let us consider them separately.

1. Flat FRW Background

In this case we adopt the gauge,

$$N = a, \quad N_i = a^2 e^B \partial_i B, \quad g_{ij} = a^2 e^{-2\psi}\delta_{ij}, \tag{6.7}$$

which reduces to the linear perturbations studied in the previous sections to the first order of ψ and B. This gauge is slightly different from the one used in [12, 28, 29]. Then, we find

$$R = \frac{2e^{2\psi}}{a^2}\Big(2\partial^2\psi - \big(\partial\psi\big)^2\Big), \tag{6.8}$$

and the kinetic action S_K is given by Eq.(A.1). Hence, to the third-order of ψ and B, we find that

$$\begin{aligned} S_{IR}^{(3)} = \zeta^2 \int d\eta d^3x a^2 \Bigg\{ & 2\psi\Big[\psi\partial^2\psi + \big(\partial\psi\big)^2\Big] \\ & + \frac{9}{2}\big(3\lambda-1\big)\Big(2\psi\psi'^2 + 6\mathcal{H}\psi^2\psi' + 3\mathcal{H}^2\psi^3\Big) \\ & + (3\lambda-1)\Big[2(\psi' + \mathcal{H}\psi)\big(\psi^{,k}B_{,k}\big) + \psi(2\psi' + \mathcal{H}\psi)\partial^2 B\Big] \\ & - 2\Big[(3\lambda-1)\mathcal{H}B - (\lambda-1)\partial^2 B\Big]\big(\psi^{,k}B_{,k}\big) \\ & - 2\big(3\lambda-1\big)\big(\psi' + \mathcal{H}\psi\big)\Big[B\partial^2 B + \big(\partial B\big)^2\Big] \\ & + 4\psi_{,k}B_{,l}B^{,kl} + (\psi + 2B)\Big[B^{,kl}B_{,kl} - \lambda\big(\partial^2 B\big)^2\Big] \\ & + (3\lambda-1)\mathcal{H}B\Big[B\partial^2 B + 2\big(\partial B\big)^2\Big] \\ & - 2\lambda\big(\partial B\big)^2\partial^2 B + 2B^{,kl}B_{,k}B_{,l} \Bigg\}. \end{aligned} \tag{6.9}$$

Following [12], we first write the quadratic action (6.4) in its canonical form with order-one coupling constants, by using the coordinate transformations,

$$\eta = \alpha\hat{\eta}, \quad x^i = \alpha|c_\psi|\hat{x}^i, \tag{6.10}$$

and redefinitions of the canonical variables,

$$\begin{aligned} \tilde{\psi} &= \frac{\hat{\psi}}{M_{pl}|c_\psi|^{1/2}\alpha}, \\ \tilde{q} &= \frac{\sqrt{2}\hat{q}}{M_{pl}|c_\psi|^{1/2}\alpha^2}. \end{aligned} \tag{6.11}$$

It must not be confused with the constant α used here and the one used in the previous sections for the $U(1)$

gauge generator. Then, from Eq.(6.3) we find that

$$B = -\frac{1}{|c_\psi|^2\partial^2}\left(\psi' - \frac{8\pi Gaq}{3\lambda-1}\right) = \frac{\hat{B}}{M_{pl}|c_\psi|^{1/2}},$$
$$\hat{B} = -\frac{a^\delta}{\hat{\partial}^2}\left(\hat{\psi}^* + \delta\hat{\mathcal{H}}\hat{\psi} - \sqrt{\frac{2}{3\lambda-1}}\hat{q}\right), \quad (6.12)$$

where $\hat{\psi}^* = \partial\hat{\psi}/\partial\hat{\eta}$, $\hat{\mathcal{H}} = a^*/a$. Inserting Eqs.(6.10)-(6.12) into Eq.(6.9), we obtain

$$\begin{aligned} S^{(3)}_{IR} = \frac{1}{2M_{pl}}\int d\hat{\eta}d^3\hat{x}a^2\Bigg\{&\frac{|c_\psi|^{3/2}}{\alpha}\hat{\mathcal{L}}^{(3)}_1 \\ &+\frac{1}{|c_\psi|^{1/2}\alpha}\Big(\hat{\mathcal{L}}^{(3)}_2 + \hat{\mathcal{L}}^{(3)}_3 + \hat{\mathcal{L}}^{(3)}_4\Big) \\ &+\frac{1}{|c_\psi|^{5/2}\alpha}\hat{\mathcal{L}}^{(3)}_5 \\ &+\frac{1}{|c_\psi|^{1/2}}\Big(\hat{\mathcal{L}}^{(3)}_6 + \hat{\mathcal{L}}^{(3)}_7\Big) \\ &+\frac{1}{|c_\psi|^{5/2}}\Big(\hat{\mathcal{L}}^{(3)}_8 + \hat{\mathcal{L}}^{(3)}_9\Big) \\ &+\frac{\alpha}{|c_\psi|^{1/2}}\hat{\mathcal{L}}^{(3)}_{10}\Bigg\}, \end{aligned} \quad (6.13)$$

where $\mathcal{L}^{(3)}_i$'s are given in Eq.(A.4). Clearly, for any chosen α some of the coefficients of $\mathcal{L}^{(3)}_i$'s always become unbounded as $c_\psi \to 0$, that is, the corresponding theory is indeed plagued with the strong coupling problem.

To study it further, let us consider the rescaling,

$$\hat{\eta} \to s^{-\gamma_1}\hat{\eta}, \quad \hat{x}^i \to s^{-\gamma_2}\hat{x}^i,$$
$$\hat{\psi} \to s^{\gamma_3}\hat{\psi}, \quad \hat{q} \to s^{\gamma_4}\hat{q}. \quad (6.14)$$

Then, $S^{(2)}_{IR}$ given by Eq.(6.4) is invariant for $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4/2 = \gamma$. Without loss of generality, one can set $\gamma = 1$. For such a choice of γ, it can be shown that $\hat{B}$ is scale-invariant,

$$\hat{B} \to \hat{B}. \quad (6.15)$$

Then, in $S^{(3)}_{IR}$ of Eq.(6.13) the first five terms are scaling as s^1, and the sixth to ninth terms all scaling as s^0, while the last term is scaling as s^{-1}. Thus, all the first five terms are irrelevant in the low energy limit, and diverge in the UV, so they are all not renormalizable [22]. The sixth to ninth terms are marginal, and are strictly renormalizable, while the last term is relevant and superrenormalizable. This indicates that the perturbations break down when the coupling coefficients greatly exceed units. To calculate these coefficients, let us consider a process at the energy scale E. Then, we find that the ten terms in the cubic action $S^{(3)}_{IR}$ have, respectively, the magnitudes, $(E, E, E, E, E, E^0, E^0, E^0, E^0, E^{-1})$, for example,

$$\int d\hat{\eta}d^3\hat{x}\hat{\psi}^*\big(\hat{\partial}_i\hat{\psi}\big)\big(\hat{\partial}^i\hat{B}\big) \simeq E. \quad (6.16)$$

Since the action is dimensionless, all the coefficients in (6.13) must have the dimensions E^{-n_s}, where $n_s = (1,1,1,1,1,0,0,0,0,-1)$, $(s = 1,2,3,...,10)$. Writing them in the form,

$$\lambda_s = \left(\frac{\hat{\lambda}_s}{\Lambda_s}\right)^{n_s}, \quad (6.17)$$

where $\hat{\lambda}_s$ is a dimensionless parameter of order one, one finds that Λ_s for $s = 1,2,3,4,5,10$ are given by Eq.(A.5). Translating it back to the coordinates η and x^i, the energy and momentum scales are given by

$$\Lambda^\omega_s = \frac{\Lambda_s}{\alpha}, \quad \Lambda^k_s = \frac{\Lambda_s}{\alpha|c_\psi|}. \quad (6.18)$$

For $s = 6,7,8,9$, the coupling coefficients are given by Eq.(A.6). From these expressions, one can see that the lowest scale of Λ^ω_s and Λ^k_s's is given by

$$\Lambda_{min} = \Lambda^\omega_5 \simeq |c_\psi|^{5/2}M_{pl}, \quad (6.19)$$

as $c_\psi \to 0$. For any process with energy higher than it, the corresponding coupling constants become larger than unit, and then the strong coupling problem rises.

Thus, to be consistent with observations in the IR, λ is required to be closed to its relativistic value $\lambda_{IR} = 1$. On the other hand, to avoid the strong coupling problem, the above shows that it cannot be too closed to it.

2. Static Weak Gravitational Fields

When a static gravitational field produced by a source is weak, such as the solar system, one can treat the problem as perturbations of the Minkowski spacetime. Since the Minkowski background is a particular case of the flat FRW spacetime, one can consider its perturbations still given by Eq.(6.7) but now with $a = 1$. Due to the presence of matter, ψ now is in general different from zero. Then, we find that

$$S^{(2)}_{IR} = \zeta^2\int dtd^3x\left(2\big(\partial\psi\big)^2 - \frac{(8\pi Gq)^2}{\lambda-1}\right), \quad (6.20)$$

where

$$B = \frac{8\pi G}{(\lambda-1)\partial^2}q. \quad (6.21)$$

Setting

$$t = \alpha\hat{t}, \quad x^i = \alpha\hat{x}^i,$$
$$\psi = \frac{\hat{\psi}}{\sqrt{2}\zeta\alpha}, \quad q = \frac{\sqrt{3\lambda-1}|c_\psi|\hat{q}}{8\pi G\zeta\alpha^2}, \quad (6.22)$$

we find that $S^{(2)}$ given by Eq.(6.20) takes its canonical form,

$$S^{(2)}_{IR} = \int d\hat{t}d^3\hat{x}\Big(\big(\hat{\partial}\hat{\psi}\big)^2 - \hat{q}^2\Big). \quad (6.23)$$

On the other hand, we have

$$
\begin{aligned}
S_{IR}^{(3)} &= \zeta^2 \int dt d^3x \Big\{ 2\psi\Big[\psi\partial^2\psi + \big(\partial\psi\big)^2\Big] \\
&\quad +2(\lambda-1)\partial^2 B\Big(\psi^{,k}B_{,k}\Big) + 4\psi_{,k}B_{,l}B^{,kl} \\
&\quad +(\psi+2B)\Big[B^{,kl}B_{,kl} - \lambda\big(\partial^2 B\big)^2\Big] \\
&\quad -2\lambda\big(\partial B\big)^2\partial^2 B + 2B^{,kl}B_{,k}B_{,l}\Big\} \\
&= \frac{1}{M_{pl}}\int d\hat{t}d^3\hat{x}\Big\{\frac{L_1^{(3)}}{\alpha} + \frac{2L_2^{(3)}}{\alpha} \\
&\quad + \frac{L_3^{(3)}}{(3\lambda-1)|c_\psi|^2\alpha} \\
&\quad + \left(\frac{2}{3\lambda-1}\right)^{3/2}\frac{L_4^{(3)}}{|c_\psi|^3}\Big\}, \qquad (6.24)
\end{aligned}
$$

where

$$
\begin{aligned}
L_1^{(3)} &= \hat{\psi}\Big[\hat{\psi}\big(\hat{\partial}^2\hat{\psi}\big) + \big(\hat{\partial}\hat{\psi}\big)^2\Big], \\
L_2^{(3)} &= \big(\hat{\partial}^2\hat{B}\big)\big(\hat{\partial}^k\hat{\psi}\big)\big(\hat{\partial}_k\hat{B}\big), \\
L_3^{(3)} &= \hat{\psi}\Big[\big(\hat{\partial}_k\hat{\partial}_l\hat{B}\big)^2 - \lambda\big(\hat{\partial}^2\hat{B}\big)^2\Big] \\
&\quad +4\big(\hat{\partial}^k\hat{\psi}\big)\big(\hat{\partial}^l\hat{B}\big)\big(\hat{\partial}_k\hat{\partial}_l\hat{B}\big), \\
L_4^{(3)} &= \hat{B}\Big[\big(\hat{\partial}_k\hat{\partial}_l\hat{B}\big)^2 - \lambda\big(\hat{\partial}^2\hat{B}\big)^2\Big] - \lambda\big(\hat{\partial}^2\hat{B}\big)\big(\hat{\partial}\hat{B}\big)^2 \\
&\quad +\big(\hat{\partial}^k\hat{B}\big)\big(\hat{\partial}^l\hat{B}\big)\big(\hat{\partial}_k\hat{\partial}_l\hat{B}\big), \qquad (6.25)
\end{aligned}
$$

but now with

$$
B = \frac{1}{\zeta|c_\psi|\sqrt{3\lambda-1}}\left(\frac{1}{\hat{\partial}^2}\hat{q}\right) \equiv \frac{\hat{B}}{\zeta|c_\psi|\sqrt{3\lambda-1}}. \qquad (6.26)
$$

Considering the rescaling (6.14) with $\hat{t} = \hat{\eta}$, we find that $S_{IR}^{(2)}$ given by Eq.(6.23) is invariant, provided that $\gamma_3 = (\gamma_1+\gamma_2)/2$ and $\gamma_4 = (\gamma_1+3\gamma_2)/2$. Without loss of generality, we can set $\gamma_1 = \gamma_2 = 1$, and then $\hat{B}$ scales exactly as that given by Eq.(6.15), while the four terms in $S_{IR}^{(3)}$ of Eq.(6.24) scale, respectively, as s^1, s^1, s^1 and s^0. Then, following the analysis given between Eqs.(6.17) and (6.19), we find that $\Lambda_s^k = \Lambda_s^\omega$ for $s = 1, 2, 3$, where

$$
\begin{aligned}
\Lambda_1^\omega &= 2\Lambda_2^\omega = M_{pl}, \\
\Lambda_3^\omega &= (3\lambda-1)M_{pl}|c_\psi|^2, \qquad (6.27)
\end{aligned}
$$

and

$$
\lambda_4 = \left(\frac{2}{3\lambda-1}\right)^{3/2}\frac{1}{M_{pl}|c_\psi|^3}. \qquad (6.28)
$$

Clearly, as $\lambda \to 1$, the coupling also becomes strong. In particular, since the fourth term scales as s^0, its amplitude remain the same, as the energy scale of the system changes. That is, this term is equally important at all energy scales. The strength of this term gives the lowest energy scale, as $c_\psi \to 0$. Therefore, now we have

$$
\Lambda_{min} \simeq M_{pl}|c_\psi|^3. \qquad (6.29)
$$

It should be noted that, in the above we studied the strong coupling problem only in terms of ψ. Then, one may argue that our above conclusions may be gauge-dependent. In the following, we shall show that this is not true.

Let us first note that in the static case the gauge invariant quantity Ψ is precisely equal to ψ, as one can see from Eq.(4.8). Therefore, in this case the coupling indeed becomes strong when $E > M_{pl}|c_\psi|^3$, even in terms of the gauge-invariant quantity.

On the other hand, in the cosmological case, from Eqs.(4.8) and (6.12) we find that the gauge-invariant quantity Ψ can be written as

$$
\Psi = \frac{a^\delta \hat{\Psi}}{M_{pl}|c_\psi|^{1/2}\alpha}, \qquad (6.30)
$$

where

$$
\hat{\Psi} \equiv \hat{\psi} + \frac{\alpha\mathcal{H}}{\hat{\partial}^2}\left[\alpha\big(\hat{\psi}' + \delta\mathcal{H}\hat{\psi}\big) - \sqrt{\frac{2}{3\lambda-1}}\hat{q}\right]. \qquad (6.31)
$$

Since the lowest energy scale (6.19) is independent of α (as it should be), we can always choose $\alpha \propto |c_\psi|^d, (d > 0)$, so that $\hat{\Psi} \simeq \hat{\psi}$ and $\Psi \simeq \psi$ as $|c_\psi| \to 0$. Then, one can repeat the analysis in terms of Ψ and $\hat{\Psi}$ and finds that the strong coupling problem exists even in terms of Ψ, which is gauge-invariant.

VII. CONCLUSIONS

Recently, Horava and Melby-Thompson [17] proposed a new version of the HL theory of gravity, in which the spin-0 graviton, appearing in all the previous versions of the HL theory, is eliminated by introducing a Newtonian pre-potential φ and a local $U(1)$ gauge field A. Such a setup was orginally believed valid only for $\lambda = 1$. However, da Silva argued that the HMT setup can be easily generalized to the case with $\lambda \neq 1$. With such a generalization, the three challenging questions, ghost, stability and strong coupling, all related with $\lambda \neq 1$ and plagued in most of the previous versions of the HL theory [6, 15], rise again.

In this paper, we addressed these issues, by first developing the linear scalar perturbations of the FRW spacetimes for any given λ in the da Silva generalization. In particular, in Sec. II we derived all the field equations and the corresponding conservation laws, while in Sec. III we studied the cosmological models of the FRW universe with any given spatial curvature k. When $\bar{J}_A = 0$, from Eq.(3.8) we find that $k = 0 = \Lambda_g$, that is, the universe must be flat. When the matter is described by a scalar field, one can see that $\bar{J}_A$ indeed vanishes. Therefore, in all the inflationary models described by a scalar field, the FRW universe is necessarily flat. Thus, the theory naturally gives rise to a flat FRW universe, which is consistent with all the observations carried out so far [32].

Then, in Sec. IV we presented the general formulas for the linear scalar perturbations. By studying the general gauge transformations of $U(1) \ltimes \text{Diff}(M, \mathcal{F})$, we found that there are only three gauge-invaraint quantities, and constructed them explicitly, as given by Eq.(4.8). Applying these formulas to the Minkowski background, in Sec. V we showed explicitly that the Minkowski spacetime is stable, and the corresponding spin-0 graviton is eliminated by the gauge field even for $\lambda \neq 1$.

In Sec. VI, we considered the ghost and strong coupling problems. To study them, we need to consider the cases where the linear perturbations of the metric, described by ψ and B in the quasilongitudinal gauge (4.9), are different from zero, so that these problems can manifest themselves, if they exist. One way to have non-vanishing ψ and B is to assume that the spacetimes are not vacuum. In particular, taking the flat FRW universe as the background, we found that the ghost-free conditions are the same as these found in Paper I in the SVW setup, given explicitly by Eq.(6.6). In such backgrounds, we found that the strong coupling problem also shows up. In particular, for a process with energy E higher than $|c_\psi|^{5/2}M_{pl}$, the corresponding coupling constants become much larger than unit, and then the strong coupling problem rises. In the static case, strong coupling problem also exists for $E > |c_\psi|^3 M_{pl}$. To resolve this problem, one way is to provoke the Vainshtein mechanism [14], similar to what was done previously in spherical static spacetimes [6], as well as in cosmology [12], or use the BPS mechanism [12, 16].

The gauge field A and the Newtonian pre-potentail φ have no contributions to the vector and tensor perturbations, so the results presented in [33] in the SVW setup can be equally applied to the da Silva generalization even with $\lambda \neq 1$. In particular, it was shown that the vector perturbations vanish identically in the Minkowski background. Combining it with the result obtained in this paper, one can see that the only non-vanishing part is the tensor one. As a result, in the Minkowski background the gravitational sector is still described only by the spin-2 massless graviton even in the da Silva generalization ($\lambda \neq 1$).

Finally, we would like to note that, although this new version of the HL theory has several attractive features and solves various important issues plagued in the previous versions, many fundamental issues still need to be addressed, before it is considered as a viable theory. These include the strong coupling problem found above, the RG flow, phenomenological constraints from the solar system tests, the couplings of matter fields to gravity, and so on.

Acknowledgments: The authors thank Robert Brandenberger, Shinji Mukohyama, Tony Padilla and Thomas Sotiriou for valuable comments and suggestions. The work of AW was supported in part by DOE Grant, DE-FG02-10ER41692.

VIII. APPENDIX: THE KINETIC ACTION S_K

For the metric given by Eq.(6.7), it can be shown that the kinetic part of the action is given by

$$\begin{aligned}
S_K &= \zeta^2 \int d\eta d^3x a^2 \Bigg\{ 3\big(1-3\lambda\big)\big(\mathcal{H}-\psi'\big)^2 e^{-3\psi} \\
&\quad + \Big[\big(B^{,ij}+B^{,i}B^{,j}\big)\big(B_{,ij}+B_{,i}B_{,j}\big) \\
&\qquad - \lambda\Big(\partial^2 B + \big(\partial B\big)^2\Big)\Big] e^{2B+\psi} \\
&\quad -\lambda\big(\psi^{,k}B_{,k}\big)^2 e^{2B+\psi} \\
&\quad -2\big(1-\lambda\big)\Big[\partial^2 B + \big(\partial B\big)^2\Big]\big(\psi^{,k}B_{,k}\big)e^{2B+\psi} \\
&\quad -2\big(1-3\lambda\big)\big(\mathcal{H}-\psi'\big)\Big[\partial^2 B + \big(\partial B\big)^2 \\
&\qquad\qquad - \big(\psi^{,k}B_{,k}\big)\Big] e^{B-\psi} \\
&\quad +4\psi_{,i}B_{,j}\big(B^{,ij}+B^{,i}B^{,j}\big)e^{2B+\psi} \\
&\quad +\Big[2\big(\partial\psi\big)^2\big(\partial B\big)^2 + \big(\psi^{,k}B_{,k}\big)^2\Big]e^{2B+\psi}\Bigg\}, \quad \text{(A.1)}
\end{aligned}$$

from which we find that the quadratic part is given by

$$\begin{aligned}
S_K^{(2)} &= \zeta^2 \int d\eta d^3x a^2 \Big\{ \big(1-3\lambda\big)\Big[3\psi'^2 + 18\mathcal{H}\psi\psi' \\
&\quad + 2\psi'\partial^2 B + \frac{27}{2}\mathcal{H}^2\psi^2\Big] + (1-\lambda)B\partial^4 B\Big\}. \quad \text{(A.2)}
\end{aligned}$$

This is different from the expression given by Eq.(6.2). The reason is that, in the calculations of Eq.(6.2), the 3-metric g_{ij} is approximated to the first-orders of ψ and B, as one can see from Eq.(4.2), while g^{ij} to their second orders (So does $\sqrt{g}$). For detail, we refer readers to [34]. However, in the derivation of Eq.(A.2), we practically expanded both g_{ij} and g^{ij} to second orders. It is interesting to note that this difference does not affect the super-momentum constraint (6.3), which can be also obtained by the variation of $S_{IR}^{(2)}$ with respect to B. Since the B-terms in both expressions of Eqs.(6.2) and (A.2) are the same, so is the resulting equation obtained by the variation of $S_{IR}^{(2)}$ with respect to B.

Substituting Eq.(6.3) into Eq.(A.2), we find that

$$\begin{aligned}
S_{IR}^{(2)} &= \zeta^2 \int d\eta d^3x a^{2(1+\delta)} \Bigg\{ -\frac{2}{c_\psi^2}\tilde{\psi}'^2 + 2\big(\partial\tilde{\psi}\big)^2 \\
&\quad - \frac{27(3\lambda-2)(3\lambda-1)}{2}\mathcal{H}^2\tilde{\psi}^2 + \frac{\tilde{q}^2}{c_\psi^2}\Bigg\}, \quad \text{(A.3)}
\end{aligned}$$

where $S_{IR}^{(2)} = S_K^{(2)} + S_R^{(2)}$, and c_ψ, $\tilde{\psi}$ and $\tilde{q}$ are defined by Eq.(6.5) but now with $\delta = 9(\lambda-1)/2$. Then, we find that the ghost-free conditions are the same as that given by Eq.(6.6).

One can show that the conclusions regarding to the strong coupling problem are also independent of the use of either the expression (A.3) or (6.4) for $S_{IR}^{(2)}$.

Inserting Eqs.(6.10)-(6.12) into Eq.(6.9), we find that $S_{IR}^{(3)}$ is given by Eq.(6.13), where

$$\begin{aligned}
\hat{\mathcal{L}}_1^{(3)} &= \frac{9}{2}(3\lambda-1)a^{3\delta}\Big[2\hat{\psi}\big(\hat{\psi}^* + \delta\hat{\mathcal{H}}\hat{\psi}\big)^2 \\
&\quad + 6\hat{\mathcal{H}}\hat{\psi}^2\big(\hat{\psi}^* + \delta\hat{\mathcal{H}}\hat{\psi}\big) + 3\hat{\mathcal{H}}^2\hat{\psi}^3\Big],\\
\hat{\mathcal{L}}_2^{(3)} &= 2a^{3\delta}\Big[\hat{\psi}^2\hat{\partial}^2\hat{\psi} + \hat{\psi}\big(\hat{\partial}\hat{\psi}\big)^2\Big],\\
\hat{\mathcal{L}}_3^{(3)} &= (3\lambda-1)a^{2\delta}\Bigg\{2\Big[\hat{\psi}^* + (1+\delta)\hat{\mathcal{H}}\hat{\psi}\Big]\big(\hat{\partial}_i\hat{\psi}\big)\big(\hat{\partial}^i\hat{B}\big)\\
&\quad + \hat{\psi}\Big[2\hat{\psi}^* + (1+2\delta)\hat{\mathcal{H}}\hat{\psi}\Big]\big(\hat{\partial}^2\hat{B}\big)\Bigg\},\\
\hat{\mathcal{L}}_4^{(3)} &= 2(3\lambda-1)a^{\delta}\big(\hat{\partial}^2\hat{B}\big)\big(\hat{\partial}_i\hat{\psi}\big)\big(\hat{\partial}^i\hat{B}\big),\\
\hat{\mathcal{L}}_5^{(3)} &= a^{\delta}\Big[4\big(\hat{\partial}^k\hat{\partial}^l\hat{B}\big)\big(\hat{\partial}_k\hat{\psi}\big)\big(\hat{\partial}_l\hat{B}\big) + \big(\hat{\partial}^k\hat{\partial}^l\hat{B}\big)^2\\
&\quad - \lambda\big(\hat{\partial}^2\hat{B}\big)^2\Big],\\
\hat{\mathcal{L}}_6^{(3)} &= 2(1-3\lambda)a^{\delta}\Big[\hat{\psi}^* + (1+\delta)\hat{\mathcal{H}}\hat{\psi}\Big]\times\\
&\quad \Big[\hat{B}\big(\hat{\partial}^2\hat{B}\big) + \big(\hat{\partial}\hat{B}\big)^2\Big],\\
\hat{\mathcal{L}}_7^{(3)} &= 2(1-3\lambda)a^{\delta}\hat{\mathcal{H}}\hat{B}\big(\hat{\partial}_i\hat{\psi}\big)\big(\hat{\partial}^i\hat{B}\big),\\
\hat{\mathcal{L}}_8^{(3)} &= 2\hat{B}\Big[\big(\hat{\partial}^k\hat{\partial}^l\hat{B}\big)^2 - \lambda\big(\hat{\partial}^2\hat{B}\big)^2\Big],\\
\hat{\mathcal{L}}_9^{(3)} &= 2\Big[\big(\hat{\partial}^k\hat{\partial}^l\hat{B}\big)\big(\hat{\partial}_k\hat{B}\big)\big(\hat{\partial}_l\hat{B}\big) - \lambda\big(\hat{\partial}\hat{B}\big)^2\big(\hat{\partial}^2\hat{B}\big)\Big],\\
\hat{\mathcal{L}}_{10}^{(3)} &= (3\lambda-1)\hat{\mathcal{H}}\hat{B}\Big[\hat{B}\big(\hat{\partial}^2\hat{B}\big) + 2\big(\hat{\partial}\hat{B}\big)^2\Big]. \qquad (A.4)
\end{aligned}$$

The coupling coefficients of these terms defined by Eq.(6.17) for $s = 1, 2, 3, 4, 5, 10$ are given by

$$\begin{aligned}
\Lambda_1 &= \frac{4M_{pl}\alpha}{9(3\lambda-1)|c_\psi|^{3/2}}, \quad \Lambda_2 = M_{pl}|c_\psi|^{1/2}\alpha,\\
\Lambda_3 &= \frac{2M_{pl}|c_\psi|^{1/2}\alpha}{3\lambda-1}, \quad \Lambda_4 = \frac{M_{pl}|c_\psi|^{1/2}\alpha}{3\lambda-1},\\
\Lambda_5 &= 2M_{pl}|c_\psi|^{5/2}\alpha, \quad \Lambda_{10} = \frac{(3\lambda-1)\alpha}{2|c_\psi|^{1/2}M_{pl}}. \qquad (A.5)
\end{aligned}$$

We also have

$$\begin{aligned}
\lambda_6 &= \lambda_7 = \frac{3\lambda-1}{|c_\psi|^{1/2}M_{pl}},\\
\lambda_8 &= \lambda_9 = \frac{1}{|c_\psi|^{5/2}M_{pl}}. \qquad (A.6)
\end{aligned}$$

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