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Cosmic Acceleration and the Helicity-0 Graviton

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Abstract

We explore cosmology in the decoupling limit of a non-linear covariant extension of Fierz-Pauli massive gravity obtained recently in arXiv:1007.0443. In this limit the theory is a scalar-tensor model of a unique form defined by symmetries. We find that it admits a self-accelerated solution, with the Hubble parameter set by the graviton mass. The negative pressure causing the acceleration is due to a condensate of the helicity-0 component of the massive graviton, and the background evolution, in the approximation used, is indistinguishable from the ΛCDM model. Fluctuations about the self-accelerated background are stable for a certain range of parameters involved. Most surprisingly, the fluctuation of the helicity-0 field above its background decouples from an arbitrary source in the linearized theory.

We also show how massive gravity can remarkably screen an arbitrarily large cosmological constant in the decoupling limit, while evading issues with ghosts. The obtained static solution is stable against small perturbations, suggesting that the degravitation of the vacuum energy is possible in the full theory. Interestingly, however, this mechanism postpones the Vainshtein effect to shorter distance scales. Hence, fifth force measurements severely constrain the value of the cosmological constant that can be neutralized, making this scheme phenomenologically not viable for solving the old cosmological constant problem. We briefly speculate on a possible way out of this issue.

1 Introduction and summary

The observed late-time acceleration of the Universe [1], and the Cosmological Constant problem (see reviews [2, 3]), remain two of the most tantalizing, mutually connected puzzles at the interface of particle physics and cosmology.

A promising approach to the late-time acceleration enigma is to invoke new degrees of freedom, belonging to the gravitational field itself (as in massive gravity), that give rise to the cosmic speed-up. This framework postulates the existence of a new energy scale – set by the graviton mass – which is very low; nevertheless, this scale is technically natural in the quantum-field theoretical sense. This approach, as is known by now, is challenging theoretically (hence, is interesting), and happens to have robust observational predictions.

Such a scenario was first worked out in a context of the DGP model [4] in Refs. [5, 6], where the cosmic acceleration is due to the helicity-0 component of a five-dimensional graviton. Hence, the solution is said to be self-accelerating.

Regretfully, in the context of DGP, the self-accelerating solution is plagued by negative energy ghost-like states in the perturbative approach [7, 8, 9], and despite the issue of whether or not the negative energy perturbations could be continued in the full nonlinear theory [10], the existence of non-perturbative negative-energy solutions [7, 11, 12] makes the self-accelerating branch unsatisfactory (in spite of the interesting finding of Ref. [13] that the quasi-classical approach does not seem to reveal the instabilities of this solution).

Certain generalizations of the DGP model, however, allow for stable self-accelerating solutions, either by constructing an explicit braneworld model [14] where the negative energy ghost disappears, or by extending the decoupling limit of DGP to the "galilean" invariant interactions, [15].

In this work, we show for the first time that a theory of massive gravity may produce a self-accelerated geometry while being free of the problems that arise in the self-accelerating branch of DGP. In particular, we will work in a certain approximation in which the helicity ± 2, ± 1, and helicity-0 modes of the massive graviton decouple from each other in the linearized theory, while the nonlinear self-interactions, and interactions between them, are captured by a few leading higher-dimensional terms in the Lagrangian; this approximation constitutes the decoupling limit.

In this approximation, we will show the existence of the self-accelerated solution, around which small fluctuations are stable. The acceleration is due to a condensate of the helicity-0 field, which in the decoupling limit is reparametrization invariant. On the other hand, since the helicity-0 is not an arbitrary scalar, but descends from a full-fledged tensor field, it has no potential, but enters the Lagrangian via very specific derivative terms fixed by symmetries [16]. These terms generate the negative pressure density which causes the accelerated expansion with stable fluctuations, as will be discussed below.

From the observational point of view, the obtained self-accelerating background is indistinguishable, in the approximation used, from that of the ΛCDM model. As to the fluctuations, however, the helicity-0 could have introduced some differences. For instance, at cosmological distance scales it could have given an additional force leading to, *e.g.*, changes in the growth of structure [17, 18], while at shorter scales still being strongly screened via the Vainshtein mechanism [19], guaranteeing the recovery of General Relativity with tiny departures [19, 20], which may also be measurable [21, 22] in high-precision Laser Ranging experiments [23] (for recent detailed studies of the Vainshtein mechanism see Refs. [24]). All the above takes place in the DGP model. However, this is not what happens on the self-accelerated background in the massive theory: Surprisingly-enough, the fluctuation of the helicity-0 on this background decouples in the linearized approximation from an arbitrary source! Thus, the astrophysical sources need not excite this fluctuation, in which case one recovers exactly the ΛCDM results. It is likely, however, that this similarity of the self-accelerated solution and its fluctuations to the ΛCDM results will not hold beyond the decoupling limit (*i.e.* will not hold for the horizon-size scales).

Furthermore, if we wish to tackle the Cosmological Constant problem (CCP), S. Weinberg's no-go theorem makes it impossible to find dynamical solutions within General Relativity (GR) without involving fine-tuned parameters, [2]. The idea of infrared (IR) modification of gravity, however addresses this puzzle by accepting a large vacuum energy and modifying instead the gravitational sector in the IR, so that vacuum energy gravitates very weakly [25]. Such a source would not manifest itself as strongly as naively anticipated in GR, *i.e.* it would be *degravitated*, while all the astrophysical sources would exhibit the GR behavior [26]. As shown in Ref. [25, 26], one can think of degravitation as a promotion of Newton's constant to a high pass filter operator thereby modifying the effect of long wavelength sources such as a CC while recovering GR on shorter wavelengths. In particular, theories of massive and resonance gravitons exhibiting the high pass filter behavior to degravitate the CC [25]. Moreover, it was shown in Ref. [27] that any causal theory that can degravitate the CC is a theory of massive and resonance gravitons.

It is important to emphasize that in theories of massive gravity degravitation is a causal process (unlike more general theories considered in [26]). The real measure of whether or not a source is degravitated is given by its time evolution. During inflation for instance, the vacuum energy driving the acceleration of the Universe will not be degravitated for a long time. It is only after long enough periods of time that the IR modification of gravity kicks in and can effectively slow down an accelerated expansion [25, 26]. Hence, a crucial ingredient for the degravitation mechanism to work is the existence of a (nearly) static solution in the presence of a cosmological constant towards which the geometry can relax at late time (or after some long period of time). Indeed, Ref. [27] studied linearized massive gravity demonstrating that in this approximation degravitation takes place after a long enough period of time.

In this paper, we focus on the hard mass case using the generalized Fierz-Pauli theory of massive gravity, as derived in [16]. We show that this model allows for static solutions while evading any ghost issues at least in the decoupling limit. In this framework an arbitrary vacuum energy can be neutralized by the effective stress-tensor of the helicity-0 component of the massive graviton. Small fluctuations around this solution are shown below to be stable, as long as this static solution exists.

Moreover, we find that the energy scale at which the interactions of the helicity-0 modes become nonlinear is affected by the scale of the degravitated cosmological constant – the interaction scale being higher for larger values of the CC[1].

On the one hand, it is intriguing that the interactions of the helicity-0 can be kept linear up to the energy scale which is significantly higher than what it would have been in a theory without the CC. However, this very same phenomenon also creates a problem by postponing Vainshtein's recovery of GR to shorter and shorter distance scales. As a result, the tests of gravity impose a stringent upper bound on the vacuum energy that can be degravitated in this framework without conflicting measurements of gravity. Disappointingly, this upper bound turns out to be of the order the critical energy density of the present-day Universe, $(10^{-3}\text{ eV})^4$ – the value that does not need to be degravitated.

A possible way out of this difficulty may be to envisage a cosmological scenario in which degravitation of the vacuum energy takes place before the Universe enters the radiation dominated epoch – say during the inflationary period, or even earlier. By the end of that epoch then the cosmology should reset itself to continue evolution along the other branch of the solutions that exhibits the standard early behavior followed by the self-acceleration, found in the present work. The existence of such a transition would depend on properties of the degravitating solution in the full theory. Since we have no detailed knowledge of this solution at the time of this writing, we have no concrete mechanism to substantiate the above scenario. Therefore, in what follows we will not rely on it. Instead, we emphasize that there still are two important virtues of the degravitating solution with the low value of the degravitated CC: (I) It is a concrete example of how degravitation could work in four-dimensional theories of massive gravity without giving rise to ghost-like instabilities. (II) As we will show, the degravitated solution with small values of CC can be combined with the self-accelerated solution discussed above, to give a satisfactory solution that is in agreement with the existing cosmological and astrophysical data.

Last but not least, the solutions found in the decoupling limit do not necessarily imply the existence of the solutions with identical properties in the full theory. Nevertheless, the decoupling limit solutions should capture the local dynamics at scales well within the present-day Hubble four-volume, as argued in [15]. On the other hand, at larger scales the full solutions may be very different from our ones. These differences would kick in at scales comparable to the graviton Compton wavelength. Therefore, our solutions should manifest themselves at least as transients lasting long cosmological times.

Organization of the paper is as follows. In section 2, we review the generalized

[1]In this work we will use interchangeably the notions of vacuum energy and CC, although there could be a big difference between the two when it comes to IR modified gravity [25].

Fierz-Pauli theory of massive gravity and discuss its ghostless decoupling limit. We then start by focusing on self-accelerating solutions in section 3, first deriving the background solutions, then testing their stability, and finally studying the implications for late-time cosmology. We then explore the cosmology in the presence of a cosmological constant in section 4, proving the existence of a stable degravitating branch of solutions, and analyzing the stability of the de Sitter branch. Brief discussions of the degravitating solution are given at the end of section 4.

2 The Formalism

Search for a consistent theory of a massive spin-2 field goes back to the original work of Fierz and Pauli [28]. Whereas any massive gravity should reduce to the Fierz-Pauli (FP) theory at the quadratic level [29], a generic nonlinear extension exhibits the sixth degree of freedom – the so-called Boulware-Deser (BD) ghost [30]. This sixth mode produces severe instabilities on cosmological backgrounds [31], as well as on locally nontrivial asymptotically flat backgrounds (such as that of a point source, for instance) [32, 33, 34].

This problem is usually related to the helicity-0 sector of massive theories [32]. The latter can efficiently be studied in the decoupling limit, where the sixth mode is hidden in higher-derivative nonlinear terms for the helicity-0 [32, 33, 34]. Such terms make the Cauchy problem ill-defined, unless additional initial data are supplied. This corresponds to an additional, sixth, degree of freedom which shows up as a ghost-like linear mode on various backgrounds mentioned above.

Up until recently it was thought that the cancellation of the higher-derivative nonlinear terms for the helicity-0 was not possible [33]. However, recently an explicit construction was given in Ref. [16] in which all the nonlinear terms for the helicity-0 with more that two time derivatives cancel. Below we briefly review these results and recast them in a more convenient form. We refer to Ref. [16] for more detailed discussions.

Consider a 4D covariant theory of a spin-2 field [32], which, once expanded on Minkowski space-time gives a graviton of mass m:

$$\mathcal{L} = M_{\rm Pl}^2\sqrt{-g}R - \frac{M_{\rm Pl}^2 m^2}{4}\sqrt{-g}\left(U_2(g,H)+U_3(g,H)+U_4(g,H)+U_5(g,H)\cdots\right)\,. \quad (1)$$

Here U_i's denote the mass and potential terms of $i^{\rm th}$ order in $H_{\mu\nu}$

$$\begin{aligned}
U_2(g,H) &= H_{\mu\nu}^2 - H^2\,, && (2)\\
U_3(g,H) &= c_1 H_{\mu\nu}^3 + c_2 H H_{\mu\nu}^2 + c_3 H^3\,, && (3)\\
U_4(g,H) &= d_1 H_{\mu\nu}^4 + d_2 H H_{\mu\nu}^3 + d_3 H_{\mu\nu}^2 H_{\alpha\beta}^2 + d_4 H^2 H_{\mu\nu}^2 + d_5 H^4\,, && (4)\\
U_5(g,H) &= f_1 H_{\mu\nu}^5 + f_2 H H_{\mu\nu}^4 + f_3 H^2 H_{\mu\nu}^3 + f_4 H_{\alpha\beta}^2 H_{\mu\nu}^3 \\
&\quad + f_5 H (H_{\mu\nu}^2)^2 + f_6 H^3 H_{\mu\nu}^2 + f_7 H^5\,. && (5)
\end{aligned}$$

Index contractions are performed using the inverse metric $g^{\mu\nu}$; the coefficients c_i, d_i and f_i are *a priori* arbitrary. The tensor $H_{\mu\nu}$ is not an independent entity; it is related to the metric tensor as $H_{\mu\nu} = g_{\mu\nu} - \eta_{ab}\partial_\mu\varphi^a\partial_\nu\varphi^b$, where $a, b = 0, 1, 2, 3$, $\eta_{ab} = \mathrm{diag}(-1, 1, 1, 1)$, and $H_{\mu\nu}$ is a covariant tensor as long as the four fields φ^a transform as scalars under a change of coordinates [32]. Hence, the potential terms in (1) can be rewritten as functions of the metric g and the specific combination of the four scalars φ^a, as $U(g, \Sigma)$, where $\Sigma_{\mu\nu} = \eta_{ab}\partial_\mu\varphi^a\partial_\nu\varphi^b$. However, we will not be exploiting the latter representation in the present work. Instead, following [32] we expand φ^a in terms of the coordinates x^α, and the field π^α, as $\varphi^a = (x^\alpha - \pi^\alpha)\,\delta^a_\alpha$, and using the convention, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/M_{\mathrm{Pl}}$, we obtain

$$H_{\mu\nu} = \frac{h_{\mu\nu}}{M_{\mathrm{Pl}}} + \partial_\mu\pi_\nu + \partial_\nu\pi_\mu - \eta_{\alpha\beta}\partial_\mu\pi^\alpha\partial_\nu\pi^\beta\,. \tag{6}$$

The π_α's represent the Stückelberg fields that transform under reparametrization to guarantee that the tensor H in (6) transforms covariantly. In particular under linearized diffeomorphism, $x^\mu \to x^\mu + \frac{\xi^\mu}{M_{\mathrm{Pl}}}$, the metric perturbations and the Stückelberg transform respectively as

$$h_{\mu\nu} \to h_{\mu\nu} - \partial_{(\mu}\,\xi_{\nu)} \qquad \text{and} \qquad \pi^\mu \to \pi^\mu + \frac{\xi^\mu}{M_{\mathrm{Pl}}}\,. \tag{7}$$

It is therefore worth pointing out that in the decoupling limit $M_{\mathrm{Pl}} \to \infty$, the Stückelberg field π^μ ends up being gauge invariant under linearized diffeomorphism.

In the unitary gauge one could put $\pi_\alpha = 0$ (or, $\varphi^a = x^\alpha\delta^a_\alpha$), in which case (1) reduces to the standard FP theory extended by a potential for the field $h_{\mu\nu}$. However, this is not a convenient way of dealing with these degrees of freedom. Instead, it is more instructive to retain π_α and fix a gauge for $h_{\mu\nu}$.

The theory (1) was studied in detail in [16, 35], and a two-parameter family of the coefficients was identified for which no sixth (ghost) degree of freedom arises in the decoupling limit[2]. In these theories the higher derivative nonlinear terms either cancel out, or organize themselves into total derivatives. For these ghostless theories, the decoupling limit is defined as follows[3]

$$m \to 0, \quad M_{\mathrm{Pl}} \to \infty, \quad \Lambda_3 = (M_{\mathrm{Pl}}m^2)^{1/3} \ \text{ fixed.} \tag{8}$$

In what follows, we will focus on the helicity-2 and helicity-0 modes, and ignore the helicity-1 modes as they do not couple to a conserved stress-tensor at the linearized level, and, therefore, can be set to zero self-consistently (see, however, important comments on this at the end of section 3.2).

[2]Interestingly, a recently proposed extension of General Relativity by an extra auxiliary dimension [36, 37], automatically generates the coefficients from this family at least up to the cubic order.

[3]By "ghostless" we mean a theory with no ghost at least in the decoupling limit, implying that even if the BD ghost exists in the full theory, it must have a mass larger than the scale Λ_3, [16].

We therefore use the following decomposition for $H_{\mu\nu}$ in terms of the canonically normalized helicity-2 and helicity-0 fields after setting $\pi_a = \partial_a \pi/\Lambda_3^3$

$$H_{\mu\nu} \;=\; \frac{h_{\mu\nu}}{M_{\rm Pl}} + \frac{2\partial_\mu\partial_\nu\pi}{\Lambda_3^3} - \frac{\partial_\mu\partial^\alpha\pi\partial_\nu\partial_\alpha\pi}{\Lambda_3^6}. \tag{9}$$

Then, one can show by direct calculations [16] that the Lagrangian (1) reduces in the decoupling limit to the following expression

$$\mathcal{L} = -\frac{1}{2}h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} + h^{\mu\nu}\sum_{n=1}^{3}\frac{a_n}{\Lambda_3^{3(n-1)}}X^{(n)}_{\mu\nu}[\Pi], \tag{10}$$

where the first term represents the usual kinetic term for the graviton, $a_1 = -1/2$, and $a_{2,3}$ are two arbitrary constants, related to the two parameters from the set $\{c_i, d_i\}$ which characterize a given ghostless theory of massive gravity. The expression $(\mathcal{E}h)_{\mu\nu}$ denotes the linearized Einstein operator acting on $h_{\mu\nu}$ defined in the standard way: $\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} = -\frac{1}{2}(\Box h_{\mu\nu} - \partial_\mu\partial_\alpha h^\alpha_\nu - \partial_\nu\partial_\alpha h^\alpha_\mu + \partial_\mu\partial_\nu h - \eta_{\mu\nu}\Box h + \eta_{\mu\nu}\partial_\alpha\partial_\beta h^{\alpha\beta})$.

The three symmetric tensors $X^{(n)}_{\mu\nu}[\Pi]$ are composed of the second derivative of the helicity-0 field $\Pi_{\mu\nu} \equiv \partial_\mu\partial_\nu\pi$. In order to maintain reparametrization invariance of the full Lagrangian the tensors $X^{(n)}_{\mu\nu}[\Pi]$ should be identically conserved. These properties uniquely determine the expressions for $X^{(n)}_{\mu\nu}$ at each order of non-linearity. The obtained expressions agree with the results of the direct calculations of Ref. [16]. A convenient parametrization for the tensors $X^{(n)}_{\mu\nu}$ which we adopt in this work is as follows:

$$\begin{aligned} X^{(1)}_{\mu\nu}[\Pi] &= \varepsilon_\mu{}^{\alpha\rho\sigma}\varepsilon_\nu{}^\beta{}_{\rho\sigma}\Pi_{\alpha\beta}, \\ X^{(2)}_{\mu\nu}[\Pi] &= \varepsilon_\mu{}^{\alpha\rho\gamma}\varepsilon_\nu{}^{\beta\sigma}{}_\gamma\Pi_{\alpha\beta}\Pi_{\rho\sigma}, \\ X^{(3)}_{\mu\nu}[\Pi] &= \varepsilon_\mu{}^{\alpha\rho\gamma}\varepsilon_\nu{}^{\beta\sigma\delta}\Pi_{\alpha\beta}\Pi_{\rho\sigma}\Pi_{\gamma\delta}\,. \end{aligned} \tag{11}$$

The remarkable property of (10) is that it represents the *exact* Lagrangian (excluding the helicity-1 part) in the decoupling limit: All the higher than quartic terms vanish in this limit, making (10) a unique theory to which any nonlinear, ghostless extension of massive gravity should reduce in the decoupling limit [16].

If external sources are introduced, their stress-tensors then couple to the physical metric $h_{\mu\nu}$. In the basis used in (10) there is no direct coupling of π to the stress-tensors. Hence, the Lagrangian (10) is invariant w.r.t. the shifts, and the "galilean" transformations in the internal space of the π field, $\partial_\mu\pi \to \partial_\mu\pi + v_\mu$, where v_μ is a constant four-vector. The latter invariance guarantees that there is no mass nor potential terms generated for π by the loop corrections.

The tree-level coupling of π to the sources arises only after diagonalization: The quadratic mixing $h^{\mu\nu}X^{(1)}_{\mu\nu}$, and the cubic interaction $h^{\mu\nu}X^{(2)}_{\mu\nu}$, can be diagonalized by a nonlinear transformation of $h_{\mu\nu}$, that generates the following coupling of π [16]

$$\frac{1}{M_{\rm Pl}}\left(-2a_1\eta_{\mu\nu}\pi + \frac{2a_2\partial_\mu\pi\partial_\nu\pi}{\Lambda_3^3}\right)T^{\mu\nu}\,. \tag{12}$$

Moreover, the above transformation also generates all the Galileon terms for the helicity-0 field, introduced in a different context in Ref. [15][4].

Since the Galileon terms are known to exhibit the Vainshtein recovery of GR at least for static sources [15], so does the above theory with $a_3 = 0$. The quartic interaction $h^{\mu\nu}X^{(3)}_{\mu\nu}$, however, cannot be absorbed by any local redefinition of $h_{\mu\nu}$. It is still expected though to admit the Vainshtein mechanism.

However, as we will show in the next section, on the self-accelerated background the fluctuation of the helicity-0 field decouples from an arbitrary source, making the predictions of the theory consistent with GR already in the linearized approximation. This decoupling is a direct consequence of the self-accelerated background and the specific form of the coupling (12).

3 The Self-Accelerated Solution

The universality of the decoupling limit Lagrangian (10) for the class of ghostless massive gravities, suggests the possibility of a fairly model-independent phenomenology of the massive theories that should be captured by the limiting Lagrangian (10). In the present section, we will be interested in the cosmological solutions in these theories. We will directly work in the decoupling limit, which implies scales much smaller than the Compton wavelength of the graviton. In the case of the self-accelerated de Sitter solution for instance, this corresponds to probing physics within the Hubble scale, which as one would expect, is set by the value of the graviton mass.

3.1 The solution in the decoupling limit

Below we look for homogeneous and isotropic solutions of the equations of motion that follow from the Lagrangian (10). The helicity-0 equation of motion reads as follows:

$$\partial_\alpha\partial_\beta h^{\mu\nu}\left(a_1\varepsilon_\mu{}^{\alpha\rho\sigma}\varepsilon_\nu{}^\beta{}_{\rho\sigma}+2\frac{a_2}{\Lambda_3^3}\varepsilon_\mu{}^{\alpha\rho\sigma}\varepsilon_\nu{}^{\beta\gamma}{}_\sigma\Pi_{\rho\gamma}+3\frac{a_3}{\Lambda_3^6}\varepsilon_\mu{}^{\alpha\rho\sigma}\varepsilon_\nu{}^{\beta\gamma\delta}\Pi_{\rho\gamma}\Pi_{\sigma\delta}\right)=0\,, \quad (13)$$

while variation of the Lagrangian w.r.t. the helicity-2 field gives

$$-\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta}+\sum_{n=1}^{3}\frac{a_n}{\Lambda_3^{3(n-1)}}X^{(n)}_{\mu\nu}[\Pi]=0. \quad (14)$$

We are primarily interested in the self-accelerated solutions of the system (13)-(14). This solution is obtained by choosing the configuration for π such that the second factor in (13) vanishes. This has for consequence to kill the first order mixing

[4] In this model the coupling of the Galileon field to matter is not only given by πT as considered in the original Galileon theory [15], but also includes more generic mixing of the form $\partial_\mu\pi\partial_\nu\pi T^{\mu\nu}$.

between $h_{\mu\nu}$ and π and hence the coupling of π to matter at leading order (which arises after diagonalization of the mixing term). As a consequence the perturbations around the self-accelerated solution we obtain here do not couple to matter. This will be presented in more details in what follows.

For an observer at the origin of the coordinate system, the de Sitter metric can locally (*i.e.*, for times t, and physical distances $|\vec{x}|$, much smaller than the Hubble scale H^{-1}) be written as a small perturbation over Minkowski space-time [15]

$$\mathrm{d}s^2 = [1 - \frac{1}{2} H^2 x^\alpha x_\alpha] \eta_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu . \tag{15}$$

The linearized Einstein tensor for the (dimensionless) metric (15) is given by

$$G^{\mathrm{lin}}_{\mu\nu} = \frac{1}{M_{\mathrm{Pl}}} \mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} = -3H^2 \eta_{\mu\nu} . \tag{16}$$

For the helicity-0 field we look for the solution of the following isotropic form

$$\pi = \frac{1}{2} q \Lambda_3^3 x^\alpha x_\alpha + b \Lambda_3^2 t + c \Lambda_3 \, , \tag{17}$$

where q, b and c are three dimensionless constants.

The equations of motion for the helicity-0 and helicity-2 fields (13)-(14), therefore, can be recast in the following form

$$H^2 \left(-\frac{1}{2} + 2a_2 q + 3a_3 q^2 \right) = 0, \tag{18}$$

$$M_{\mathrm{Pl}} H^2 = 2q\Lambda_3^3 \left[-\frac{1}{2} + a_2 q + a_3 q^2 \right] \, . \tag{19}$$

Solving the quadratic equation (18) for q (for $H \neq 0$), we obtain the Hubble constant of the self-accelerated solution from (19). Its magnitude, $H^2 \sim \Lambda_3^3 / M_{\mathrm{Pl}} = m^2$, is set by the graviton mass, as expected (positivity of H^2 is one of the conditions that we will be demanding below). It is not hard to convince oneself that there exists a whole set of self-accelerated solutions, parametrized by a_2 and a_3. This range, however, will be restricted further by the requirement of stability of the solution, which is the focus of the next section.

Before doing so, let us briefly analyze the four scalars φ^a. Using the ansatz, (17), their expression is given by

$$\varphi^a = (1-q) x^\alpha \delta^a_\alpha \, , \tag{20}$$

if we set $b = 0$. Thus the four scalars vanish in the special case of $q = 1$, and the metric is $g_{\mu\nu} = H_{\mu\nu}$, so that the Lagrangian considered in (10) reduces to standard GR plus a CC (at least at the background level). This happens only if the parameters of our theory are such that $a_3 = \frac{1}{6} - \frac{2}{3} a_2$, which is not the regime we will be interested in – we will indeed show in what follow that the stability of the self-accelerated background implies $q \neq 1$.

3.2 Small perturbations and stability

Here we investigate the constraints that the requirement of stability imposes on a possible background. Let us adopt a particular solution of the system (18)-(19) and consider perturbations on the corresponding de Sitter background

$$h_{\mu\nu} = h^b_{\mu\nu} + \chi_{\mu\nu}, \quad \pi = \pi^b + \phi, \tag{21}$$

where the superscript b denotes the corresponding background values. The Lagrangian for the perturbations (up to a total derivative) reads as follows

$$\begin{aligned} \mathcal{L} &= -\frac{1}{2}\chi^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}\chi_{\alpha\beta} + 6(a_2 + 3a_3 q)\frac{H^2 M_{\rm Pl}}{\Lambda_3^3}\phi\Box\phi - 3a_3\frac{H^2 M_{\rm Pl}}{\Lambda_3^6}(\partial_\mu\phi)^2\Box\phi \\ &+ \frac{a_2 + 3a_3 q}{\Lambda_3^3}\chi^{\mu\nu}X^{(2)}_{\mu\nu}[\Phi] + \frac{a_3}{\Lambda_3^6}\chi^{\mu\nu}X^{(3)}_{\mu\nu}[\Phi] + \frac{\chi^{\mu\nu}T_{\mu\nu}}{M_{\rm Pl}}\,, \end{aligned} \tag{22}$$

where Φ denotes the four-by-four matrix with the elements $\Phi_{\mu\nu} \equiv \partial_\mu\partial_\nu\phi$. The first term in the first line of the above expression is the Einstein term for $\chi_{\mu\nu}$, the second term is a kinetic term for the scalar, and the third one is the cubic Galileon. The second line contains cubic and quartic interactions between $\chi_{\mu\nu}$ and ϕ, which are identical in form to the corresponding terms in the decoupling limit on Minkowski space-time (10). None of these interactions therefore lead to ghost-like instabilities [16], as long as the ϕ kinetic term is positive definite.

Most interestingly, however, there is no quadratic mixing term between χ and ϕ in (22). Since it is only $\chi_{\mu\nu}$ that couples to external sources $T_{\mu\nu}$ in the quadratic approximation, then there will not be a quadratic coupling of ϕ to the sources generated in the absence of the quadratic $\chi - \phi$ mixing. Therefore, for arbitrary external sources, there exist consistent solutions for which the fluctuation of the helicity-0 is not excited, $\phi = 0$. On these solutions one exactly recovers the results of the linearized GR. The above phenomenon provides a mechanism of decoupling the helicity-0 mode from arbitrary external sources! This mechanism is a universal property of the self-accelerating solution in ghostless massive gravity.

Hence, there are no instabilities in (22), as long as $a_2 + 3a_3 q > 0$. The latter condition, along with the requirement of positivity of H^2, and the equations of motion (18), requires that the following system be satisfied:

$$\begin{gathered} -\frac{1}{2} + 2a_2 q + 3a_3 q^2 = 0, \\ M_{\rm Pl}H^2 = 2q\Lambda_3^3\left[a_2 q + a_3 q^2 - \frac{1}{2}\right] > 0, \quad a_2 + 3a_3 q > 0, \end{gathered}$$

for the self-accelerating solution to be physically meaningful. The above system can be solved. The solution is given as follows

$$a_2 < 0, \quad -\frac{2a_2^2}{3} < a_3 < -\frac{a_2^2}{2}, \tag{23}$$

while the Hubble constant and q are given by the following expressions

$$H^2 = m^2[2a_2q^2 + 2a_3q^3 - q] > 0, \quad q = -\frac{a_2}{3a_3} + \frac{(2a_2^2 + 3a_3)^{1/2}}{3\sqrt{2}a_3}. \tag{24}$$

It is clear from (23), that the undiagonalizable interaction $h^{\mu\nu}X^{(3)}_{\mu\nu}$ plays a crucial role for the stability of this class of solutions: All theories without this term (*i.e.* the ones with $a_3 = 0$) would have ghost-like instabilities on the self-accelerated background. Notice as well that in the regime (23) none of the scalars φ^a vanish, and our model therefore differs from GR with a CC.

We therefore conclude that there exists a well-defined class of massive theories with the parameters satisfying the conditions (23), which propagate no ghosts on asymptotically flat backgrounds, and also admit stable self-accelerated solutions in the decoupling limit.

As we mentioned before, the helicity-1 field enters only quadratically, or in higher order terms in the Lagrangian, and hence, can consistently be set to be zero (*i.e.* it does not need to be excited by any other fields). Nevertheless, once a background configuration for the helicity-0 field is switched on, the higher-dimensional mixed terms of the helicity-0 and helicity-1 could in principle flip the sign of the Maxwell kinetic term, giving rise to a vector ghost that would only enter the Lagrangian quadratically or in higher powers; this field would couple to other fields at the nonlinear level. This certainly would not be a satisfactory state of affairs.

By restoring back the helicity-1 field in our expressions, and performing direct calculations we have found that in the n^{th} order in nonlinearities, where $n \leq 6$, the coefficient of the Maxwell term on the self-accelerated background is proportional to $(-\frac{1}{2} + 2a_2q + 3a_3q^2)$, up to corrections that are of order $(n+1)$, [38]. Hence, up to these corrections, the Maxwell term vanishes on the self-accelerated background!

If this were the full story we would get a theory of a helicity-1 coupled infinitely strongly to the fluctuations of the helicity-0 in the decoupling limit. However, quantum loop corrections will necessarily generate a nonzero Maxwell term, as it is not protected by any symmetries. In these loops propagate the helicity-0 mode, as well as the matter fields to which the helicity-1 couples nonlinearly (for instance, one of the couplings being, $\partial_\mu A_\alpha \partial_\nu A_\alpha T^{\mu\nu}$). It is worth emphasizing that, in the decoupling limit the theory of tensor, vector and π field represents a theory with independent gauge invariances for the tensor and vector U(1) transformations. One could therefore already consider quantum loops within this theory, which will generate the Maxwell term. Notice that the Maxwell term does not have to be written in terms of the original variables $H_{\mu\nu}$ since in the decoupling limit the original Stückelberg symmetry is split into independent symmetries for $h_{\mu\nu}$ and A_μ.

Then, interpreting the value of the tree-level coefficient of the Maxwell term (which is zero) as an infinite value of the inverse of the running $U(1)$ coupling at some UV scale $\Lambda_{UV} \geq \Lambda_3$, we obtain that at lower scales the coupling constant has a positive value as long as the theory is not asymptotically free (in other words, the $U(1)$ coupling would have a Landau pole at some high scale Λ_{UV})[5]. Hence, the helicity-1 sector would not have a ghost, but the scale at which it would become nonlinearly interacting (the Vainshtein scale) would be parametrically (logarithmically) smaller than Λ_3. Since the helicity-1 field does not have to be excited by any source, this will not be a concern for us.

3.3 Late-time cosmology

In this subsection we discuss the relevance of the results obtained above for the late-time local cosmological evolution of the Universe. As seen from the decoupling limit Lagrangian (10), the helicity-0 mode π provides an effective stress-tensor that is "felt" by the helicity-2 field:

$$T^{\pi}_{\mu\nu} = M_{\rm Pl} \sum_{n=1}^{3} \frac{a_n}{\Lambda_3^{3(n-1)}} X^{(n)}_{\mu\nu}[\Pi] = -6 q M_{\rm Pl} \Lambda_3^3 \left[-\frac{1}{2} + a_2 q + a_3 q^2 \right] \eta_{\mu\nu} \, . \tag{25}$$

It is this stress-tensor that provides the negative pressure density required to drive the acceleration of the Universe. Supplemented by the matter density contribution, it leads to the usual ΛCDM - like cosmological expansion of the background in the sub-horizon approximation used here. This is clear form the fact that the stress-tensor (25) gives rise to a de Sitter background as was shown in the previous subsection. Hence, in the comoving coordinate system – which differs from the one used above – the invariant de Sitter space will be the self-accelerating solution.

All this can be reiterated by performing an explicit coordinate transformation to the comoving coordinates. This will be done in two steps. In the so-called Fermi normal coordinates, the FRW metric can be locally written in space and *for all times*, as a small perturbation over Minkowski space-time:

$$\mathrm{d}s^2 = -[1 - (\dot{H} + H^2)\mathbf{x}^2]\mathrm{d}t^2 + \left[1 - \frac{1}{2} H^2 \mathbf{x}^2\right] \mathrm{d}\mathbf{x}^2 = \left(\eta_{\mu\nu} + h^{\rm FRW}_{\mu\nu}\right) \mathrm{d}x^\mu \mathrm{d}x^\nu, \tag{26}$$

where the corrections to the above expression are suppressed by higher powers of $H^2\mathbf{x}^2$. The Fermi normal coordinates, on the other hand, are related to those used in (15) (in which the FRW metric is a small conformal deformation of Minkowski space-time), by an infinitesimal gauge transformation [15]. The latter does not change the expression (25), since $T^{\pi}_{\mu\nu}$ is invariant under infinitesimal gauge transformations in the decoupling limit. On the other hand, the Fermi normal coordinates can be transformed into the standard comoving coordinates $(t_c, \mathbf{x}_c)$ as follows [15]

$$t_c = t - \frac{1}{2} H(t) \mathbf{x}^2, \qquad \mathbf{x}_c = \frac{\mathbf{x}}{a(t)} \left[1 + \frac{1}{4} H^2(t) \mathbf{x}^2\right]. \tag{27}$$

[5] Alternatively, if the particle content is such that the theory has a negative beta function, then the infinite value of the coupling constant should be attributed to some far IR scale, $\Lambda_{IR} \ll \Lambda_3$, and at any scale greater than Λ_{IR} the helicity-1 theory would have a finite positive coupling square.

The stress-tensor of a perfect fluid, $T_{\mu\nu} = \mathrm{diag}(\rho(t_c), a^2(t_c)p(t_c)\delta_{ij})$, transforms under this change of coordinates (at the leading order in $H^2\mathbf{x}^2$) into the following expression

$$T_{\mu\nu} = \begin{pmatrix} \rho & -H(\rho+p)x^i \\ -H(\rho+p)x^i & p\delta_{ij} \end{pmatrix},$$

where all quantities in the latter expression are evaluated at time t. Note that the off-diagonal entries of the stress-tensor for the cosmological constant vanish in the Fermi normal coordinates, the same is true for $T^{\pi}_{\mu\nu}$ as well. Hence, in all coordinate systems used the expressions for the stress-tensor on the self-accelerated solution is given by (25).

Not surprisingly, the corresponding cosmological equations coincide with the conventional ones for the ΛCDM model, with the cosmological constant set by the mass of the graviton

$$H^2 = \frac{\rho}{3M_{\rm Pl}^2} + \frac{C^2 m^2}{3}, \tag{28}$$

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{1}{6M_{\rm Pl}^2}(\rho + 3p) + \frac{C^2 m^2}{3}. \tag{29}$$

Here ρ and p denote the energy and pressure densities of matter and/or radiation, and $C^2 \equiv 6q\left[-\frac{1}{2} + a_2 q + a_3 q^2\right]$ is a constant that appears in (25).

As already mentioned, irrespective of the completion (beyond the Hubble scale) of the self-accelerated solution, it is locally indistinguishable from the ΛCDM model. At the horizon scales, however, it is likely that these two scenarios will depart from each other: As emphasized in the first section, the solutions found in the decoupling limit do not necessarily imply the existence of full solutions with identical properties. Moreover, the decoupling limit Lagrangian (10) is derived from the full theory by dropping certain total derivative terms (see [16]), implying solutions that decay fast-enough at infinity. On the other hand, the solutions that we found in this section are given in the coordinate system where the fields grow at large distance/time scales. If these solutions are to be continued into the full theory, the latter should have an appropriate large scale behavior in this coordinate system. A given solution in the decoupling limit can just be a transient state of the full solution. Significant deviations of the latter from the former should kick in at distance/time scales comparable to the graviton Compton wavelength.

4 Screening the Cosmological Constant

4.1 Degravitation in generic theories of massive gravity

One explicit realization of degravitation is expected to occur in massive gravity, where gravity is weaker in the IR, and the graviton mass could play the role of a high-pass filter [25]. In the approach of [25], the original theory was formulated as a higher dimensional model, the 4D reduction of which can be thought as massive/resonance gravity with the mass term promoted into a specific differential operators determined by the underlying higher-dimensional construction.

A more general approach was adopted in Ref. [27], where the graviton mass was also promoted to an operator parameterized by a continuous parameter α

$$m^2(\Box) = m_0^{2(1-\alpha)}\Box^\alpha\,, \tag{30}$$

the inverse graviton propagator is typically of the form

$$\mathcal{G}^{-1} \sim \Box - m_0^{2(1-\alpha)}\Box^\alpha\,, \tag{31}$$

so that for $\alpha < 1$, gravity is weaker beyond wavelengths comparable to the graviton Compton wavelength m_0^{-1}.

To take this mechanism a step further, the reliability of this argument within the non-linear regime is hence crucial. A trick to manifest the key interactions that arise in massive gravity is to work in the decoupling limit, where the usual GR interactions are suppressed, while the interactions of the new degrees of freedom are emphasized. This approach was first derived in Ref. [27], which we discuss first before turning to our considerations. As mentioned above, this limit is obtained by taking $M_{\rm Pl} \to \infty$ and $m \to 0$. However unlike in the decoupling limit of the theory discussed in the previous section, the nonlinear dynamics in a generic model of massive gravity is governed by the scale

$$\Lambda_\star^{5-4\alpha} = M_{\rm Pl} m^{4(1-\alpha)}\,. \tag{32}$$

In such models, it has been shown [27] that the helicity-0 (π) and -2 ($\bar{h}_{\mu\nu}$) modes satisfy the following equations in the decoupling limit,

$$-\mathcal{E}^{\alpha\beta}_{\mu\nu}\bar{h}_{\alpha\beta} = -\frac{1}{M_{\rm Pl}}T_{\mu\nu}\,, \tag{33}$$

$$3\Box\pi - \frac{18}{\Lambda_\star^{5-4\alpha}}\left(3\Box(\Box^{1-\alpha}\pi)^2 + \cdots\right) = -\frac{T}{M_{\rm Pl}}\,, \tag{34}$$

where the physical metric is given by $g_{\mu\nu} = \eta_{\mu\nu} + (\bar{h}_{\mu\nu} + \pi\eta_{\mu\nu})/M_{\rm Pl}$. In the presence of a cosmological constant, $T_{\mu\nu} = -\lambda\eta_{\mu\nu}$, the solution for the helicity-2 mode is

$$\bar{h}_{\mu\nu} = -\frac{\lambda}{6M_{\rm Pl}}x_\beta x^\beta\,\eta_{\mu\nu}\,, \tag{35}$$

which is the usual GR solution. One can now check the condition for the existence of a (nearly) static solution towards which the geometry can relax at late times. In the language of the decoupling limit, this would happen if the helicity-0 mode compensates the helicity-2 mode contribution $\pi\eta_{\mu\nu} = -\bar{h}_{\mu\nu}$ to maintain the geometry flat $g_{\mu\nu} = \eta_{\mu\nu}$. However the configuration $\pi = \lambda x^2/6M_{\rm Pl}$ is precisely the solution of (34) when the higher interactions vanish, *i.e.* $6M_{\rm Pl}\Box\pi = -T = 8\lambda$. As shown in [27], such interactions cancel for $\pi \sim x^2$ only if $\alpha < 1/2$, hence implying that a generic theory of massive gravity amended with a nonzero CC can only have a static solution when $\alpha < 1/2$. In particular, in this language the DGP model [4] corresponds to $\alpha = 1/2$ (see Ref. [7], but also [39]) hence explaining why this model does not bear static solutions with a brane tension, while promoting it to higher dimensions corresponds to a theory with $\alpha \to 0$ for which the usual codimension-two conical solutions can accommodate a tension without acceleration, [40, 41, 42, 43, 44].

The above results hold true for a generic theory of massive gravity. We now focus the analysis of the ghostless theory [16] reviewed in section 2, which strictly speaking are not captured by the above α parametrization. The key difference in the ghostless case is that interactions for the helicity-0 mode are governed by the larger coupling scale $\Lambda_3 > \Lambda_\star$. The form of these interactions in the ghostless theory, as well as the specific couplings to matter, play a crucial role in accommodating a degravitating branch of solutions, and this without being plagued by any instability at least in the decoupling limit.

4.2 Degravitation in ghostless massive gravity

For convenience we start by recalling the decoupling limit Lagrangian of (10) coupled to an external source

$$\mathcal{L} = -\frac{1}{2} h^{\mu\nu} \mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} + h^{\mu\nu} \sum_{n=1}^{3} \frac{a_n}{\Lambda_3^{3(n-1)}} X^{(n)}_{\mu\nu}[\Pi] + \frac{1}{M_{\rm Pl}} h^{\mu\nu} T_{\mu\nu}\,. \tag{36}$$

The equations of motion for the helicity-0 and 2 modes are then

$$-\mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} + \sum_{n=1}^{3} \frac{a_n}{\Lambda_3^{3(n-1)}} X^{(n)}_{\mu\nu}[\Pi] = -\frac{1}{M_{\rm Pl}} T_{\mu\nu}\,, \tag{37}$$

and

$$\begin{aligned} &\left(a_1 + \frac{a_2}{\Lambda_3^3}\Box\pi + \frac{3a_3}{2\Lambda_3^6}\left([\Pi]^2 - [\Pi^2]\right)\right)\left[\Box h - \partial_\alpha\partial_\beta h^{\alpha\beta}\right] \\ &+\frac{1}{\Lambda_3^3}\left(a_2 \Pi_{\mu\nu} - 3\frac{a_3}{\Lambda_3^3}\left(\Pi^2_{\mu\nu} - \Box\pi\Pi_{\mu\nu}\right)\right)\left[2\partial^\mu\partial_\alpha h^{\alpha\nu} - \Box h^{\mu\nu} - \partial^\mu\partial^\nu h\right] \\ &-\frac{3a_3}{\Lambda_3^6}\left(\Pi_{\mu\alpha}\Pi_{\nu\beta} - \Pi_{\mu\nu}\Pi_{\alpha\beta}\right)\partial^\alpha\partial^\beta h^{\mu\nu} = 0\,. \end{aligned} \tag{38}$$

We now focus on a pure cosmological constant source, $T_{\mu\nu} = -\lambda\eta_{\mu\nu}$, and make use of a similar ansatz as previously,

$$h_{\mu\nu} = -\frac{1}{2} H^2 x^2 M_{\rm Pl}\, \eta_{\mu\nu}\,, \tag{39}$$

$$\pi = \frac{1}{2}\, q\, x^2 \Lambda_3^3\,. \tag{40}$$

The equations of motion then simplify to

$$\left(-\frac{1}{2}M_{\rm Pl}H^2+\sum_{n=1}^{3}a_n\,q^n\Lambda_3^3\right)\eta_{\mu\nu}=-\frac{\lambda}{6M_{\rm Pl}}\eta_{\mu\nu}\,, \tag{41}$$

$$H^2\left(a_1+2a_2q+3a_3q^2\right)=0\,. \tag{42}$$

As we will see below, this system of equations admits two branches of solutions, a "degravitating" one, for which the geometry remains flat (mimicking the late-time part of the relaxation process), and a "de Sitter" branch which is closely related to the standard GR de Sitter solution. We start with the degravitating branch before exploring the more usual de Sitter solution and show that the stability of these branches depends on the free parameters $a_{2,3}$, as well as the magnitude of the cosmological constant.

4.2.1 The degravitating branch

In this formalism, it is easy to check that the geometry can remain flat *i.e.* $H=0$ and $g_{\mu\nu}\equiv\eta_{\mu\nu}$, despite the presence of the cosmological constant. Such solutions are possible due to the presence of the extra helicity-0 mode that "carries" the source instead of the usual metric. With $H=0$, equation (42) is trivially satisfied, while the modified Einstein equation (41) determines the coefficient (which we denote by q_0 here) for the helicity-0 field in (40),

$$a_1q_0+a_2q_0^2+a_3q_0^3=-\frac{\tilde\lambda}{6}\,, \tag{43}$$

in terms of the dimensionless quantity $\tilde\lambda=\lambda/\Lambda_3^3M_{\rm Pl}$. Notice that as long as the parameter a_3 is present, Eq. (43) has always at least one real root. There is therefore a flat solution for arbitrarily large cosmological constant.

Let us now briefly comment on the stability of the flat solution, as this has important consequences for the relaxation mechanism behind degravitation. We consider the field fluctuations above the static solution,

$$\pi = \frac{1}{2}q_0\Lambda_3^3\,x^2-\phi/\kappa\,, \tag{44}$$

$$T_{\mu\nu} = -\lambda\eta_{\mu\nu}+\tau_{\mu\nu}\,, \tag{45}$$

where q_0 is related to λ via (43) and the coupling κ is determined by

$$\kappa=2(a_1+2a_2q_0+3a_3q_0^2)\,. \tag{46}$$

To the leading order, the action for these fluctuations is then simply given by

$$\mathcal{L}^{(2)}=-\frac{1}{2}h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta}-\frac{1}{2}h^{\mu\nu}X^{(1)}_{\mu\nu}[\Phi]+\frac{1}{M_{\rm Pl}}h^{\mu\nu}\tau_{\mu\nu}\,, \tag{47}$$

with $\Phi_{\mu\nu} = \partial_\mu \partial_\nu \phi$. The stability of this theory is better understood when working in the Einstein frame where the helicity-0 and -2 modes decouple. This is achieved by performing the change of variable,

$$h_{\mu\nu} = \bar{h}_{\mu\nu} + \phi \eta_{\mu\nu} \,, \tag{48}$$

which brings the action to the following form

$$\mathcal{L}^{(2)} = -\frac{1}{2} \bar{h}^{\mu\nu} \mathcal{E}^{\alpha\beta}_{\mu\nu} \bar{h}_{\alpha\beta} + \frac{3}{2} \phi \Box \phi + \frac{1}{M_{\rm Pl}} \left(\bar{h}^{\mu\nu} + \phi\, \eta^{\mu\nu} \right) \tau_{\mu\nu} \,. \tag{49}$$

Stability of the static solution is therefore manifest for any region of the parameter space for which κ is real and does not vanish. As already mentioned, if $a_3 \neq 0$ there is always a real solution to (43), which is therefore stable for $\kappa \neq 0$. Furthermore, direct calculations to the $6^{\rm th}$ order show that the helicity-1 fluctuations will have a positive kinetic term as long as $\kappa/(q_0 - 1) > 0$. This suggests the presence of a flat late-time attractor solution for degravitation. The special case $a_3 = 0$ is discussed separately below.

4.2.2 de Sitter branch

In the presence of a cosmological constant, the field equations (41) and (42) also admit a second branch of solutions; these connect with the self-accelerating branch presented in section 3, and we refer to them as the de Sitter solutions. The parameters for these solutions should satisfy

$$a_1 + 2a_2 q_{\rm dS} + 3a_3 q_{\rm dS}^2 = 0 \,, \tag{50}$$

$$H_{\rm dS}^2 = \frac{\lambda}{3M_{\rm Pl}^2} + \frac{2\Lambda_3^3}{M_{\rm Pl}} \left(a_1 q_{\rm dS} + a_2 q_{\rm dS}^2 + a_3 q_{\rm dS}^3 \right) \,. \tag{51}$$

This solution is closer to the usual GR de Sitter configuration and only exists if $a_2^2 \geq 3a_1 a_3$. The stability of this solution can be analyzed as previously by looking at fluctuations around this background configuration,

$$\pi = \frac{1}{2} q_{\rm dS}\, \Lambda_3^3\, x^2 + \phi \,, \tag{52}$$

$$h_{\mu\nu} = -\frac{1}{2} H_{\rm dS}^2\, x^2\, \eta_{\mu\nu} + \chi_{\mu\nu} \,, \tag{53}$$

$$T_{\mu\nu} = -\lambda \eta_{\mu\nu} + \tau_{\mu\nu} \,. \tag{54}$$

To second order in fluctuations, the resulting action is then of the form

$$\mathcal{L}^{(2)} = -\frac{1}{2} \chi^{\mu\nu} \mathcal{E}^{\alpha\beta}_{\mu\nu} \chi_{\alpha\beta} + \frac{6 H_{\rm dS}^2 M_{\rm Pl}}{\Lambda_3^3} (a_2 + 3a_3 q_{\rm dS}) \phi \Box \phi + \frac{1}{M_{\rm Pl}} \chi^{\mu\nu} \tau_{\mu\nu} \,. \tag{55}$$

It is interesting to point out again that the helicity-0 fluctuation ϕ then decouples from matter sources at quadratic order (however the coupling reappears at the cubic order). Stability of this solution is therefore ensured if the parameters satisfy one of the following three constrains, (setting $a_1 = -1/2$ and $\tilde{\lambda} > 0$)

$$a_2 < 0 \quad \text{and} \quad -\frac{2a_2^2}{3} \leq a_3 < \frac{1 - 3a_2\tilde{\lambda} - (1 - 2a_2\tilde{\lambda})^{3/2}}{3\tilde{\lambda}^2}\,, \tag{56}$$

or

$$a_2 < \frac{1}{2\tilde{\lambda}} \quad \text{and} \quad a_3 > \frac{1 - 3a_2\tilde{\lambda} + (1 - 2a_2\tilde{\lambda})^{3/2}}{3\tilde{\lambda}^2}\,, \tag{57}$$

or

$$a_2 \geq \frac{1}{2\tilde{\lambda}} \quad \text{and} \quad a_3 > -\frac{2}{3}a_2^2\,. \tag{58}$$

These are consistent with the results (23) found for the self-accelerating solution in the absence of a cosmological constant. Moreover, the requirement of stability of helicity-1 fluctuations does not impose further bounds on the parameters (see, discussions at the end of section 3.2). Notice here that in the presence of a cosmological constant, the accelerating solution can be stable even when $a_3 = 0$. This branch of solutions therefore connects with the usual de Sitter one of GR.

4.2.3 Diagonalizable action

In section 3 we have emphasized the importance of the contribution of $X^{(3)}_{\mu\nu}$ for the stability of the self-accelerating solution. However, in the presence of a nonzero cosmological constant, this contribution is not *a priori* essential for stability of either the degravitating or the de Sitter branches. Furthermore, since the helicity-0 and -2 modes can be diagonalized at the nonlinear level when $a_3 = 0$, as was explicitly shown in [16], we will study this special case separately below. In particular, we will show that it leads to certain special bounds both in the degravitating and de Sitter branches of solution.

Stability: To start with, when $a_3 = 0$, the degravitating solution only exists if

$$2a_2\tilde{\lambda} < 3a_1^2\,. \tag{59}$$

This bound, along with the stability condition for the helicity-1 $\frac{4a_2q_0-1}{q_0-1} > 0$, then also ensures the absence of ghost-like instabilities around the degravitating solution. Assuming that the parameters $a_{1,2} = \mathcal{O}(1)$ take some natural values then the situation $a_2 > 0$ implies a severe constraint on the value of the vacuum energy that can be degravitated. This is similar to the bound in the non-linear realization of massive gravity [37], as well as in codimension-two deficit angle solutions, $\lambda \lesssim m^2 M_{\rm Pl}^2$. The situation $a_2 < 0$ on the other hand allows for an arbitrarily large CC.

On the other hand, the bound $a_2^2 \geq 3a_1a_3$ for the existence of the de Sitter solution is always satisfied if $a_3 = 0$. However, the constraints on the parameters (56) - (58) which guarantee the absence of ghosts on the de Sitter branch imply that

$$2a_2\tilde{\lambda} > 3a_1^2\,. \tag{60}$$

In this specific case then, we infer that when the Sitter solution is stable, the degravitating branch does not exist, and when the degravitating branch exists the de Sitter solution is unstable. Therefore, at each point in the parameter space there is only one, out of these two solutions, that makes sense. In the more general case where $a_3 \neq 0$ the situation is however much more subtle and it might be possible to find parameters for which both branches exist and are stable simultaneously.

Einstein's frame: Finally, to understand how this degravitating branch connects with the arguments in [27] and how it relates with Galileon theories, let us now work instead in the Einstein frame, where the helicity-2 and -0 modes are diagonalized (which is possible as long as $a_3 = 0$). The transition to Einstein's frame is performed by the change of variable [16, 45]

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - 2a_1 \pi \eta_{\mu\nu} + \frac{2a_2}{\Lambda_3^3} \partial_\mu \pi \partial_\nu \pi \,, \tag{61}$$

such that the action takes the form

$$\begin{aligned} \mathcal{L} &= -\frac{1}{2} \bar{h}^{\mu\nu} (\mathcal{E}\bar{h})_{\mu\nu} + 6a_1^2 \pi \Box \pi - \frac{6a_2 a_1}{\Lambda_3^3} (\partial\pi)^2 [\Pi] + \frac{2a_2^2}{\Lambda_3^6} (\partial\pi)^2 \left([\Pi^2] - [\Pi]^2 \right) \\ &+ \frac{1}{M_{\rm Pl}} \left(\bar{h}_{\mu\nu} - 2a_1 \pi \eta_{\mu\nu} + \frac{2a_2}{\Lambda_3^3} \partial_\mu \pi \partial_\nu \pi \right) T^{\mu\nu} \,, \end{aligned} \tag{62}$$

and the structure of the Galileon becomes manifest. Notice however, that the coefficients of the different Galileon interactions are not arbitrary. Furthermore, the coupling to matter includes terms of the form $\partial_\mu \pi \partial_\nu \pi T^{\mu\nu}$, absent in the original Galileon formalism [15]. Both of these distinctions play a crucial role in screening the cosmological constant – the task which was thought impossible in the original Galileon theory. Here, however, as long as the bound (59) is satisfied, the solution for π reads

$$\pi = \frac{1}{2} \, q_0 \, \Lambda_3^3 \, x^2 \qquad \text{with} \qquad a_1 q_0 + a_2 q_0^2 = -\frac{\tilde{\lambda}}{6}, \tag{63}$$

while the helicity-2 mode $\bar{h}_{\mu\nu}$ now takes the form

$$\bar{h}_{\mu\nu} = \left(\frac{\xi}{2} - \frac{\lambda}{6 M_{\rm Pl}} \right) x^2 \eta_{\mu\nu} + \xi \, x_\mu x_\nu \,, \tag{64}$$

with ξ being an arbitrary gauge freedom parameter. Fixing $\xi = -2a_2 q_0^2 \Lambda_3^3$, the physical metric is then manifestly flat:

$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} + \frac{1}{M_{\rm Pl}} \left(\bar{h}_{\mu\nu} - 2a_1 \pi \eta_{\mu\nu} + \frac{2a_2}{\Lambda_3^3} \partial_\mu \pi \partial_\nu \pi \right) \\ &= \eta_{\mu\nu} - \frac{\lambda_3^3}{M_{\rm Pl}} \left(a_1 q_0 + a_2 q_0^2 + \frac{\tilde{\lambda}}{6} \right) x^2 \eta_{\mu\nu} + \frac{1}{M_{\rm Pl}} \left(\xi + 2a_2 q_0^2 \Lambda_3^3 \right) x_\mu x_\nu \\ &\equiv \eta_{\mu\nu} \,. \end{aligned} \tag{65}$$

To reiterate, the specific nonlinear coupling to matter that naturally arises in the ghostless theory of massive gravity is essential for the screening mechanism to work. This allows us to understand why neither DGP nor an ordinary Galileon theory are capable of achieving degravitation.

4.3 Phenomenology

Let us now focus on the phenomenology of the degravitating solution. This mechanism relies crucially on the extra helicity-0 mode in the massive graviton. However tests of gravity severely constrain the presence of additional scalar degrees of freedom. As is well known in theories of massive gravity, the helicity-0 mode can evade fifth force constrains in the vicinity of matter if the helicity-0 mode interactions are important enough to freeze out the field fluctuations, [19].

Around the degravitating solution, the scale for helicity-0 interactions are no longer governed by the parameter Λ_3, but rather by the scale determined by the cosmological constant $\tilde{\Lambda}_3 \sim (\lambda/M_{\rm Pl})^{1/3}$. To see this, let us pursue the analysis of the fluctuations around the degravitating branch (44) and keep the higher order interactions. The resulting Lagrangian is then

$$\mathcal{L}^{(2)} = -\frac{1}{2}h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} - \frac{1}{2}h^{\mu\nu}\left(X^{(1)}_{\mu\nu}[\Phi] + \frac{\tilde{a}_2}{\tilde{\Lambda}^3}X^{(2)}_{\mu\nu}[\Phi] + \frac{\tilde{a}_3}{\tilde{\Lambda}^6}X^{(3)}_{\mu\nu}[\Phi]\right) + \frac{1}{M_{\rm Pl}}h^{\mu\nu}\tau_{\mu\nu}\,, \tag{66}$$

with

$$\frac{\tilde{a}_2}{\tilde{\Lambda}^3} = -2\frac{a_2 + 3a_3 q_0}{\Lambda_3^3\kappa^2}\,, \quad \text{and} \quad \frac{\tilde{a}_3}{\tilde{\Lambda}^6} = -\frac{2a_3}{\Lambda_3^6\kappa^3}\,. \tag{67}$$

Assuming $a_{2,3} \sim \mathcal{O}(1)$, a large cosmological constant $\tilde{\lambda} \gg 1$, implies $q_0 \gg 1$, so that $a_3 q_0^2 \gg a_2 q_0 \gg a_1$ and $\kappa \sim a_3 q_0$ such that

$$\frac{\tilde{a}_2}{\tilde{\Lambda}^3} \sim \frac{1}{\Lambda_3^3 a_3 q_0^3} \sim \frac{1}{\Lambda_3^3\tilde{\lambda}} \sim \frac{M_{\rm Pl}}{\lambda} \tag{68}$$

(notice that this results is maintained even if $a_3 = 0$), and similarly

$$\frac{\tilde{a}_3}{\tilde{\Lambda}^6} \sim \left(\frac{M_{\rm Pl}}{\lambda}\right)^2\,. \tag{69}$$

To evade fifth force constrains within the solar system, the scale $\tilde{\Lambda}$ should therefore be small enough to allow for the nonlinear interactions to dominate over the quadratic contribution and enable the Vainshtein mechanism. In the DGP model this typically imposes the constraint, $\tilde{\Lambda}^3/M_{\rm Pl} \lesssim (10^{-33}\ {\rm eV})^2$, while this value can be pushed by a few orders of magnitude in the presence of Galileon interactions, [15, 46]. Therefore, the allowed value of vacuum energy that can be screened without being in conflict with observations is fairly low, of the order of $(10^{-3}\ {\rm eV})^4$ or so.

Notice that this maximal cosmological constant is at least of the same order of magnitude, if not better, than the tension that can be carried by a codimension-2 brane embedded in six dimension with a Planck scale M_6. In this scenario, the maximal tension is of the order of $\lambda < 2\pi M_6^4$. From a four-dimensional point of view, this model with the brane-induced Einstein-Hilbert term looks like a theory of massive gravity with a graviton mass $m^2 \sim M_6^4/M_{\rm Pl}^2$. Phenomenology imposes the graviton mass to be $m \lesssim 10^{-33}$eV, which therefore implies the upper bound of the brane tension, $\lambda \lesssim (10^{-3}\ \rm eV)^4$.

An alternative would be to impose a hierarchy between the dimensionless coefficients a_i. Since the Galilean interactions satisfy a non-renormalization theorem [47], such a tuning would remain technically natural[6]. To explore this avenue in a simple way, let us set $a_3 = 0$. In that case, the effective strong coupling scale is given by

$$\tilde{\Lambda}^3 = \Lambda_3^3 \frac{\frac{3}{4} - 2a_2\tilde{\lambda}}{a_2}\,. \tag{70}$$

The strong coupling scale can then be tuned to small values by adjusting the parameter a_2 within the very small window

$$|a_2\tilde{\lambda} - \frac{3}{8}| \lesssim \frac{(10^{-33}{\rm eV})^2 M_{\rm Pl}}{\Lambda_3^3}. \tag{71}$$

Therefore even when allowing a hierarchy between the parameters, once they are fixed only very restricted values of the degravitated cosmological constant would be compatible with solar system tests. The previous argument would have been unaffected if we had set $a_3 \neq 0$.

The above constraint on the vacuum energy that can be degravitated makes the present framework not viable phenomenologically for solving the old cosmological constant problem. There may be a way out of this setback though: As mentioned in the first section, one may envisage a cosmological scenario in which the neutralization of vacuum energy takes place before the Universe enters the epoch for which the Vainshtein mechanism is absolutely necessary to suppress the helicity-0 fluctuations. Such an epoch should certainly be before the radiation domination. During that epoch, however, the cosmological evolution should reset itself –perhaps via some sort of phase transition – to continue subsequent evolution along the other branch of the solutions that exhibits the standard early behavior followed by the self-acceleration, found in the present work. This scheme would have to address the cosmological instabilities discussed in Refs. [48, 49]. Moreover, the viability of such a scenario would depend on properties of the degravitating solution in the full theory –which are not known. Therefore, we do not rely on this possibility.

Nevertheless, there are certain important virtues to the degravitating solution with the low value of the degravitated CC. This is an example of high importance in understanding how S. Weinberg's no-go theorem can be evaded in principle. As

[6]We thank the referee for this suggestion.

already emphasized in [41, 42, 43, 44], such mechanisms evade the no-go theorem by employing a field which explicitly breaks Poincaré invariance in its vacuum configuration $\pi \sim x^2$, while keeping the physics insensitive to this breaking. Indeed, physical observables are only sensitive to $\Pi_{\mu\nu} = \partial_\mu \partial_\nu \pi$ which is clearly Poincaré invariant, while the configuration of the π field itself has no direct physical bearing. This is built in the specific Galileon symmetry of the theory, and is a consequence of the fact that π is not an arbitrary scalar field but rather descends as the helicity-0 mode of the massive graviton. More precisely, under a Poincaré transformation, $x^\mu \to \Lambda^\mu_\nu x^\nu + a^\mu$, the configuration for π transforms as $x^2 \to x^2 + v_\mu x^\mu + c$, with $v_\mu = 2 a_\nu \Lambda^\nu_{\ \mu}$ and $c = a^2$ which is precisely the Galileon transformation for π under which the action is invariant. In other words the Poincaré symmetry is still realized up to a Galilean transformation (or, there is a diagonal subgroup of Poincaré and internal "galilean" groups that remains unbroken by the VEV of the π field).

Thus, we have presented here the crucial steps towards a non-linear realization of degravitation within the context of massive gravity, and this, without introducing any ghosts (at least in the decoupling limit). The arguments presented here only rely on the decoupling limit and it is reasonable to doubt their validity beyond that regime. Fortunately, non-linear theories of massive gravity have been explicitly formulated in [36, 37], and static solutions in the fully non-linear regime have been presented in [37]. The absence of the ghost in theories of massive gravity requires the presence of additional symmetry projecting out the usual Boulware-Deser ghost, which can typically be thought as inherited from a higher dimensional fundamental theory. It is therefore only natural to investigate massive gravity as arising in braneworld models embedded in (spurious) extra dimensions. The static solutions presented so far then embrace a much more physical meaning, where the quantity $\Pi_{\mu\nu}$ plays the role of the extrinsic curvature on the brane, describing the brane position along the extra dimension(s). The fact that our model allows for flat solutions while carrying the cosmological constant with $\Pi_{\mu\nu}$ suggests that such models could be understood as flat branes embedded in extra dimensions, similarly as in [36, 37], [40, 41] and [42].

Some interesting work in Refs. [44] appeared during the completion of this manuscript. These have certain overlaps with the ideas of section 4 of the present work. In particular, Refs. [44] emphasize the role of Galileon fields in the context of degravitation. These works differ, however, in several aspects from the present one. In particular, Ref. [44] relies on the existence of two Galileon fields, as would arise in models with two extra dimensions, [41], whilst our model explores the degravitating solutions with a unique extra helicity-0 mode which naturally arises in the 4D theory of massive gravity. Our mechanism is possible thanks to the very specific coupling to matter that arises in a ghostless theory of massive gravity, and differ from the standard Galileon coupling.

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