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Effects of biasing on the galaxy power spectrum at large scales

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In this paper we study the effect of biasing on the power spectrum at large scales. We show that even though non-linear biasing does introduce a white noise contribution on large scales, the $P(k) \propto k^n$ behavior of the matter power spectrum on large scales may still be visible and above the white noise for about one decade. We show, that the Kaiser biasing scheme which leads to linear bias of the correlation function on *large* scales, also generates a linear bias of the power spectrum on rather small scales. This is a consequence of the divergence on small scales of the pure Harrison-Zeldovich spectrum. However, biasing becomes k -dependent when if we damp the underlying power spectrum on small scales. We also discuss the effect of biasing on the baryon acoustic oscillations.

PACS numbers:

I. INTRODUCTION

One of most promising future observations for cosmology is the precise determination of the galaxy power spectrum. So far, the emphasis has been on the anisotropies and polarization of the cosmic microwave background (CMB) but in the future we also want to determine the matter distribution of the Universe with much better precision. The advantage of the matter distribution if compared to the CMB is that while the latter represents only a two-dimensional data set, it has been emitted from the surface of last scattering, the former is three-dimensional and therefore contains, in principle much more information. The disadvantage is that we can only observe galaxies and it is not clear how the observed galaxy distribution is related to the underlying matter distribution which we calculate using cosmological perturbation theory (see e.g. [1]) and numerical N-body simulations (see e.g. [2]).

This problem goes under the name of 'biasing' and has been studied in many works. Among the first influential papers on the topic are [3–5]. They are based on the idea that galaxies form at the peaks of the matter distribution or above a certain threshold. A first interpretation of the 'Kaiser biasing model' was that bias is linear at least on large scales [3]. But data have shown that bias is not linear, see e.g. [6, 7], and this is actually also the case in the full 'Kaiser biasing model' [8, 9]. Subsequently, a fitting formula for bias has been proposed [10], which is often used, but is not derived from physical considerations and leads to cosmological parameters which are not in good agreement with other observations [11]. Most modern versions of biasing have replaced the 'peaks' of the density distribution by so called 'dark matter halos' inside which galaxies are supposed to form [12–17]. Even though these models do qualitatively agree with observations [18], they are very model dependent. One has to specify a so called halo occupation distribution (HOD). Even simple models of this have of the order 10 free parameters [19]. Furthermore, simulations suggest that the HOD probably depends on time [20, 21] and on the de-

tails of the formation process (e.g. whether the halo under consideration has been ejected from another halo [22] or undergone mergers [23]). An alternative idea, where galaxies are considered as the fundamental objects and halos are attached to them is put forward in Ref. [24].

Another modern approach to biasing is perturbation theory. It is based on the assumption that the galaxy distribution can be expanded in a Taylor series of the density fluctuation at the same point (local bias) [25, 26]. This can be generalized by allowing also a dependence on the divergence of the velocity field and on shear [27]. The advantage of this approach is that on large scales, where fluctuations are small, one can reduce it to only a few free parameters. The disadvantage is that one has no a priori information on the amplitude of these unknown parameters and one has to fit them together with other cosmological parameters. Furthermore, the white noise contribution in the power spectrum on large scales is also present in this biasing scheme as soon as one allows for non-linear contributions. This has already been noticed in [9, 28]. In addition, the results depend on whether one uses Lagrangian or Euclidean perturbation theory [29, 30]. A systematic comparison between higher order Eulerian and Lagrangian perturbation theory with numerical simulations can be found in [31].

In this paper we do not develop any detailed biasing model, but we want to address two main issues which we believe are quite model independent. First, we want to illustrate the fact that even though the galaxy correlation function may differ from the matter correlation function by more than a simple multiplicative constant (non-linear biasing) only on small scales, this can significantly modify the galaxy power spectrum also on very large scales.

Secondly we want to estimate the effects of biasing on intermediate scales which are relevant for baryon acoustic oscillations (BAO's). These are the remains of the oscillations in the baryon-photon plasma generated prior to recombination which then contribute to the matter power spectrum. These acoustic peaks have been measured in detail in the CMB anisotropy spectrum [33], and there is evidence of their presence also in the galaxy

power spectrum [34]. Even though there are some doubts about the reality of these features [35], there are now new claims of the detection of BAO's even above 4-sigma [36]. In any case, substantial progress is expected with future surveys like DES [37], BOSS [38] or the satellite project Euclid [39].

In this brief paper, we are not concerned to use the most sophisticated or realistic model of biasing, since we mainly want to make a point of principle. In subsequent work we shall investigate the ideas presented here in detail, using a more realistic biasing scheme e.g. the one studied in Ref. [26].

In the next section we determine the biased correlation function and power spectrum for a simple non-linear biasing model and discuss the main features. In section III we conclude.

II. THE BIASED POWER SPECTRUM

Non-linear Newtonian clustering does not induce an inverse cascade, but rather moves power from large to smaller scales (direct cascade). This effect flattens the density power spectrum on small scales which according to linear perturbation theory should behave like $P(k) \propto k^{-3} \ln^2(k)$ to a behavior roughly like $P(k) \approx k^{-2}$ found in numerical simulations, see e.g. [2]. On large scale, inflation predicts that $P(k) \propto k^n$, and the spectral index $n = 0.963 \pm 0.014$ has been measured with the WMAP satellite [33]. This is a very special power spectrum with $P(0) = 0$. Since the power spectrum is the Fourier transform of the correlation function,

$$P(k) = \sqrt{\frac{2}{\pi}} \int_0^\infty \xi(r) j_0(kr) r^2 dr, \quad j_0(x) = \frac{\sin x}{x}, \quad (1)$$

$$P(0) = 0 \quad \text{implies} \quad \int_0^\infty \xi(r) r^2 dr = 0. \quad (2)$$

This integral constraint is very unusual and implies that the matter distribution represents a 'super-homogeneous' system in a precise statistical sense [40, 41].

For example, this requires that there is absolutely no white noise in the system because this would add a constant to the power spectrum. From Eq. (2) it is clear that if biasing is not everywhere linear, leading to a galaxy correlation function which is not simply $\xi_g(r) = b\xi(r)$, there is a high chance that the very subtle and non local integral constraint (2) will be violated and $P_g(0) \neq 0$. This actually happens in *all* the non-linear biasing models mentioned in the introduction. Of course, if one subtracts the mean galaxy density in an observation to obtain $\delta(\mathbf{x})$ one has, by construction $\delta(\mathbf{k} = 0) = 0$ and hence also $P(k = 0) = \langle |\delta(0)|^2 \rangle = 0$. However, one, in principle has to test whether this mean density is well defined in the sense that it is the same in half the observation volume as in the entire volume. If this is not the case, one has

to be cautious of the fact that one is subtracting a fictitious mean density which is not truly well defined. In this sense, here by $P(k = 0)$ we actually mean the theoretical infinite volume limit $\lim_{k \rightarrow 0} P(k)$ in which the power spectrum of a sample with no correlations (Poissonian) tends to a non-zero constant (white noise).

We illustrate the point that the biased power spectrum acquires such a 'white noise' contribution by using the simple biasing scheme which has been proposed by Kaiser [3]: be $\delta(\mathbf{x})$ the density fluctuations with variance $\langle \delta(\mathbf{x})^2 \rangle = \sigma^2 = \xi(0)$. We assume that galaxies form when the density fluctuation is larger than a threshold ν , $\delta(\mathbf{x}) > \nu\sigma$. The correlation function of galaxies forming according to this prescription is given by the correlation function of the threshold sets θ_ν defined by [3, 4]

$$\theta_\nu(\mathbf{x}) \equiv \theta(\delta(\mathbf{x}) - \nu\sigma) = \begin{cases} 1 & \text{if } \delta(\mathbf{x}) \geq \nu\sigma \\ 0 & \text{else.} \end{cases} \quad (3)$$

$\langle \theta(\mathbf{x}) \rangle = \langle \theta(\mathbf{x})^2 \rangle = Q(\nu)$ gives the fraction of the volume in which $\delta(\mathbf{x}) > \nu\sigma$. Note that this biasing scheme is local, $\theta_\nu(\mathbf{x})$ depends only on $\delta(\mathbf{x})$, but, as we shall see, it is non-linear. By construction, the amplitude of θ_ν never exceeds 1 and therefore also its correlation function,

$$\xi_\nu(r) = \langle \theta_\nu(\mathbf{x}) \theta_\nu(\mathbf{y}) \rangle \leq 1 \quad \forall \quad r = |\mathbf{x} - \mathbf{y}|.$$

This overall normalization is arbitrary and so is the overall normalization of the corresponding power spectrum P_ν . We shall therefore not comment on the overall amplitude of the power spectrum in the following. It has no physical significance and could e.g. be multiplied with σ^2 or $(\nu\sigma)^2$.

If we assume that $\delta(\mathbf{x})$ is a Gaussian field with vanishing mean, the one point distribution is

$$P(\delta) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\delta^2}{2\sigma^2}},$$

and the 2-point function is given by

$$P(\delta_1, \delta_2, r) = \frac{\exp\left(-\frac{\sigma^2(\delta_1^2 + \delta_2^2) - 2\xi(r)\delta_1\delta_2}{2(\sigma^4 - \xi^2(r))}\right)}{2\pi\sqrt{\sigma^4 - \xi(r)^2}}, \quad (4)$$

where $\delta_1 = \delta(\mathbf{x}_1)$, $\delta_2 = \delta(\mathbf{x}_2)$ and $r = |\mathbf{x}_1 - \mathbf{x}_2|$. From this it is easy to derive [3, 9] that the correlation function for the biased field θ_ν is given by

$$\xi_\nu(r) = \frac{\int_\nu^\infty dx e^{-x^2/2} \int_{\mu(r)}^\nu dy e^{-y^2/2}}{[\int_\nu^\infty dx e^{-x^2/2}]^2}, \quad (5)$$

where $\mu(r) = (\nu\sigma^2 - \xi(r)r)/\sqrt{\sigma^4 - \xi^2(r)}$. Clearly, if $\xi(r) = 0$, $\mu(r) = \nu$ and hence also $\xi_\nu(r) = 0$, both ξ and ξ_ν pass through zero at the same scale r_0 . Furthermore, it can be shown that on large scales, where $\xi(r) \ll \sigma^2$ and $\xi_\nu(r) \ll 1$ the correlation functions satisfy the linear relation $\xi_\nu(r) \simeq \nu^2 \xi(r)$. For this reason, this biasing

scheme is often called 'linear bias'. However, as has been pointed out in Ref. [9], the biased power spectrum,

$$P_\nu(k) = \sqrt{\frac{2}{\pi}} \int_0^\infty \xi_\nu(r) j_0(kr) r^2 dr \quad (6)$$

does no longer vanish at the origin,

$$P_\nu(0) = \sqrt{\frac{2}{\pi}} \int_0^\infty \xi_\nu(r) r^2 dr \neq 0. \quad (7)$$

Even though $\xi_\nu(r) = \nu^2 \xi(r)$ on large scales, we do not have $P_\nu(k) = \nu^2 P(k)$ for small k . We now want to estimate the amplitude $P_\nu(0)$ in order to decide whether the turn over to the $P(k) \propto k^n$ behavior can be seen in the galaxy power spectrum or it is completely masked by this biasing effect. Note also, that we have not introduced any ad hoc white noise component, but just violated the super-homogeneity condition (2). As mentioned above, this happens for all generic non-linear biasing schemes.

A. Very large scales

We have calculated the biased power spectrum $P_\nu(k)$ from an underlying Λ CDM matter power spectrum as approximated in [42] for different values of the biasing parameter ν . This calculation has already been performed in Ref. [9], but there an exponential cutoff on small scales has been introduced for convenience. The reason for using such a cutoff was to improve the convergence properties of the integrals involved in the computation of the biased power spectrum given the underlying one. However, as we shall see, such a cutoff can hide some effects coming from the small scales that do impact on the results obtained in Ref. [9]. Here we use the full spectrum given in Ref. [42] which does, however neglect baryons and makes use of the fitting formula for the transfer function obtained in [5]. Also, this power spectrum neglects non-linearities which are in principle relevant on small scales. The linear CDM transfer function is given by [5]:

$$T(x) = \frac{\ln(1 + 0.171x)}{0.171x} [1 + 0.284x + (1.18x)^2 + (0.399x)^3 + (0.490x)^4]^{-1/4} \quad (8)$$

with $x \equiv k/k_{eq}$ and $k_{eq} = \sqrt{2(1+z_{eq})}\Omega_M H_0$. The matter power spectrum is given in terms of the transfer function by $P(k) = T^2(k)P_0(k)$, with $P_0(k) \propto k^n$ the primordial power spectrum. This fitting formula does not contain non-linear correction to the matter power spectrum that would enhance the small scale power and, with it, σ^2 . This would therefore even enhance the difference between the result presented here and in Ref. [9].

In Fig. 2 we show the biased power spectrum obtained from the underlying Λ CDM matter power spectrum. One clearly sees that, even though the underlying spectrum behaves like k^n at large scales (small k), the biased spectra tend to a constant since the integral constraint (2)

is violated, i.e., there is no longer an exact cancellation between correlations at small scales and anti-correlations at large scales. It is interesting to note that the modification of the power spectrum can be split into two effects: a universal modification in the shape and a vertical shift (a reduction of power in the present case) that depends on the biasing parameter ν . This becomes apparent when we normalize the biased spectra to their maximum value. As shown in Fig 2, the normalized biased power spectra share the same shape irrespectively of the particular value of the parameter ν . Moreover, we see that the only modification when compared to the underlying Λ CDM spectrum is the appearance of the plateau for very large scales, spoiling the $P(k) \propto k^n$ behaviour. Thus, we can conclude that the biased power spectrum can be factorized as $P_\nu(k) = b_1(\nu)P_1(k)$, with $b_1(\nu)$ accounting for the ν -dependent shift and $P_1(k)$ the universal biased power spectrum shown in Fig. 2 that becomes a constant for large scales and tends to the underlying power spectrum $P(k)$ on small scales. We obtain the following analytical expression for the shift function:

$$b_1(\nu) = \frac{\pi}{2\sigma^2} \frac{e^{-\nu^2}}{(1 - \text{erf}(\nu/\sqrt{2}))^2} \quad (9)$$

where $\text{erf}(x)$ denotes the error function. In Ref. [9] it is derived that $b_1(\nu)\xi(r) \simeq \xi_\nu(r)$ in the regime where $\nu\xi(r) < 1$ and $\xi(r)/\sigma^2 = \xi_c(r) \ll 1$, hence on sufficiently large scales. Here we see that $b_1(\nu)P(k)$ is a good approximation to the biased power spectrum, $P_\nu(k)$ on small scales as well. This approximation is good for $k > k_{eq}$ up to $\xi_c(1/k) \ll 1$, hence up to about $k \lesssim 10^5 \text{Mpc}^{-1}$.

In the right panel of Fig. 2, we compare the normalized biased power spectra which all collapse on one line with the underlying Λ CDM power spectrum. Contrary to the standard belief, the Kaiser biasing scheme leads to linear bias of the power spectrum on small scales and non-linear bias on large scales. This might seem to contradict the well-known fact that biasing is non-linear on small scales, as commented in the introduction. However, this is not the case. The reason why we obtain the linear biasing on small scales here is the log-divergence of the pure Harrison-Zel'dovich spectrum on small scales which leads to an infinite value of the correlation function at the origin, i.e., $\sigma^2 = \xi(0) = \infty$. Notice that

$$\sigma^2 = \xi(0) = \int_0^\infty k^2 P(k) dk. \quad (10)$$

For large scales (small k) we have that $P(k) \propto k^n$ so we have no divergences in the lower limit, but for small scales (large k) we have $P(k) \propto (\log k)^2 k^{n-4}$ so the integral has a logarithmic divergence in the upper limit for $n = 1$. We are considering a spectral index slightly smaller than 1, hence σ^2 is not infinity, but very large so that $\xi_c(r) = \xi(r)/\sigma^2$ is small already for relatively small scales, see Fig. 1, and the linear approximation $\xi_\nu \simeq b_1(\nu)\xi(r)$ is valid.

Of course, on very small scales even the underlying CDM power spectrum is damped. Typically, the CDM damping scale is expected to be very small, below 1pc. However, as we shall show below, once we introduce a somewhat larger damping scale which might relevant e.g. if a warm dark matter component is present or might come from Silk damping of baryons, the divergence of the HZ spectrum is regularized, σ is substantially reduced, and the amplification of the spectrum on small scales is no longer linear.

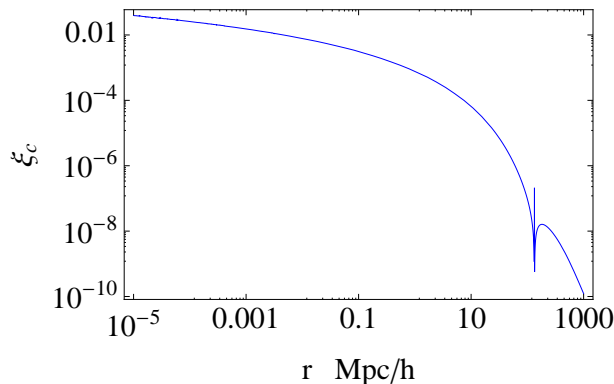


FIG. 1: The normalized correlation function $\xi_c = \xi/\sigma^2$ is shown. Since the spectral index is close to one, σ^2 is very large and $\xi_c(r)$ is small already on small scales.

We now shall consider the case where small scale power has been removed from the underlying power spectrum. To that end, we simply smooth the underlying power spectrum with a Gaussian window function,

$$P = P(k, R) = |\widehat{W}(kR)|^2 P(k) \quad (11)$$

where $\widehat{W}(kR) = \exp(-R^2 k^2/2)$ is the Fourier transform of the normalized window function $W(\mathbf{x}, R) = \frac{1}{V_R} \exp[-\mathbf{x}^2/(2R^2)]$.

This allows us to compare with the previous results obtained in Ref [9], since the cutoff introduced there is equivalent to the damping introduced here. Also, this might be relevant for models with warm dark matter where the primordial power spectrum is damped on small scales. (It is not clear whether this small scale damping survives nonlinear effects which are also relevant on small scales and which are neglected in our treatment.) Otherwise, this may simply be interpreted as an alternative biasing scheme: apply Kaiser biasing not to the true power spectrum but to the one from which the smallest scale structure which is responsible for the very high value of σ is removed. In this case, the shape of the biased power spectrum is no longer universal. It is amplified non-linearly both, on scales smaller than the smoothing scale and on large scales (see Fig. 3). Indeed, for high values of ν or large smoothing scale, the turnover that is present in the underlying power spectrum disappears

completely. This disappearance happens for smaller values of ν as the smoothing scale increases. These results are in line with those obtained in Ref. [9] since, as aforementioned, the smoothing applied here is equivalent to the exponential cutoff for small scales introduced in that work. As explained above, once we introduce smoothing over a scale R , power on scales smaller than R is damped and the UV divergence of the pure Harrison-Zel'dovich spectrum disappears. Notice that, σ^2 , and with it ξ_c strongly depend on the smoothing scale. Therefore, although the Kaiser scheme leads to linear biasing of the non damped CDM power spectrum on small scales, once we remove the small scale power, the expected non-linear biasing on small scales re-appears.

If we choose a very small damping scale, like e.g. 1pc or even 1kpc, the effect of smoothing is not relevant in the observational window from $0.0003h\text{Mpc}^{-1} < k < 10h\text{Mpc}^{-1}$. As can be seen in Fig. 3, damping is only relevant on scales $k \gtrsim R^{-1}$.

It might be surprising at first to see in Fig. 3 that the biased power spectra from larger damping scales are higher at given biasing parameter ν . But this comes simply from the following fact: the larger the damping scale the smaller becomes the variance σ . For fixed ν the probability of $\delta(\mathbf{x})/\sigma > \nu$ then increases and with it the correlation function ξ_ν and the power spectrum P_ν . Alternatively, this can be understood by the increase of the normalized correlation function $\xi_c = \xi/\sigma^2$ on which ξ_ν depends monotonically. Remember however, as stressed above, this amplitude is not physical but simply a consequence of our normalization in Eq. (3), $\theta_\nu \leq 1$.

In real observations, the *galaxy* correlation function, i.e. the biased correlation function, has to be smoothed over some scale R e.g. with a Gaussian window function $W(r/R)$ so that

$$P_\nu^{gal} = P_\nu(k, R) = |\widehat{W}(kR)|^2 P_\nu(k). \quad (12)$$

In Fig. 4 we show the corresponding results for the smoothed biased power spectrum. We see that the linear biasing on small scales disappears, but that the large-scale plateau remains because for scales larger than the smoothing radius the corresponding spectra are unaffected so that the non-linear biasing on large scales that we have found in the previous cases remain here as well.

B. The baryon acoustic oscillations

We now take into account baryons and study the effects of the considered biasing scheme on the BAO's. For that, we shall use the fitting formula given in [43] for the transfer function. Note that the effect of introducing baryons is not only the appearance of the BAO's, but also a suppression on small scales (Silk damping [1]). Also here, we use the linear transfer function.

In Fig. 5 we show our results for this case. In the plot we use a too small value of $\Omega_M = 2\Omega_b = 0.1$ and

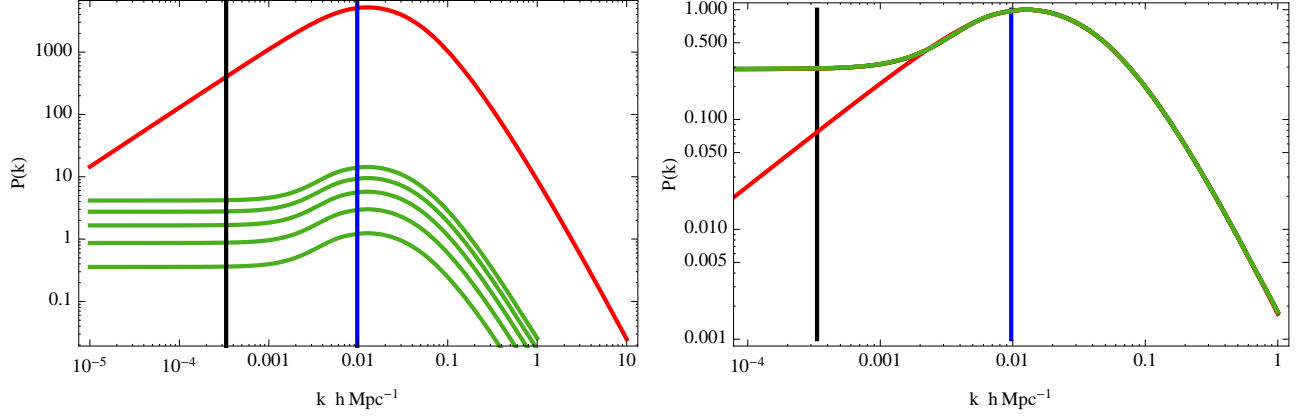


FIG. 2: In the left panel we show the underlying Λ CDM power spectrum (red line) and the corresponding biased power spectra (green lines) for $\nu = 5, 4, 3, 2$ and 1 from top to bottom. As mentioned in the text, the amplitude of the biased power spectra is unphysical. In the right panel we have normalized the power spectra to their maximum value after which they all collapse to $P_1(k)$ as discussed in the main text. We have also indicated the scales corresponding to the horizon scale at equality, k_{eq} (blue vertical line), and the present horizon scale, k_0 (black vertical line).

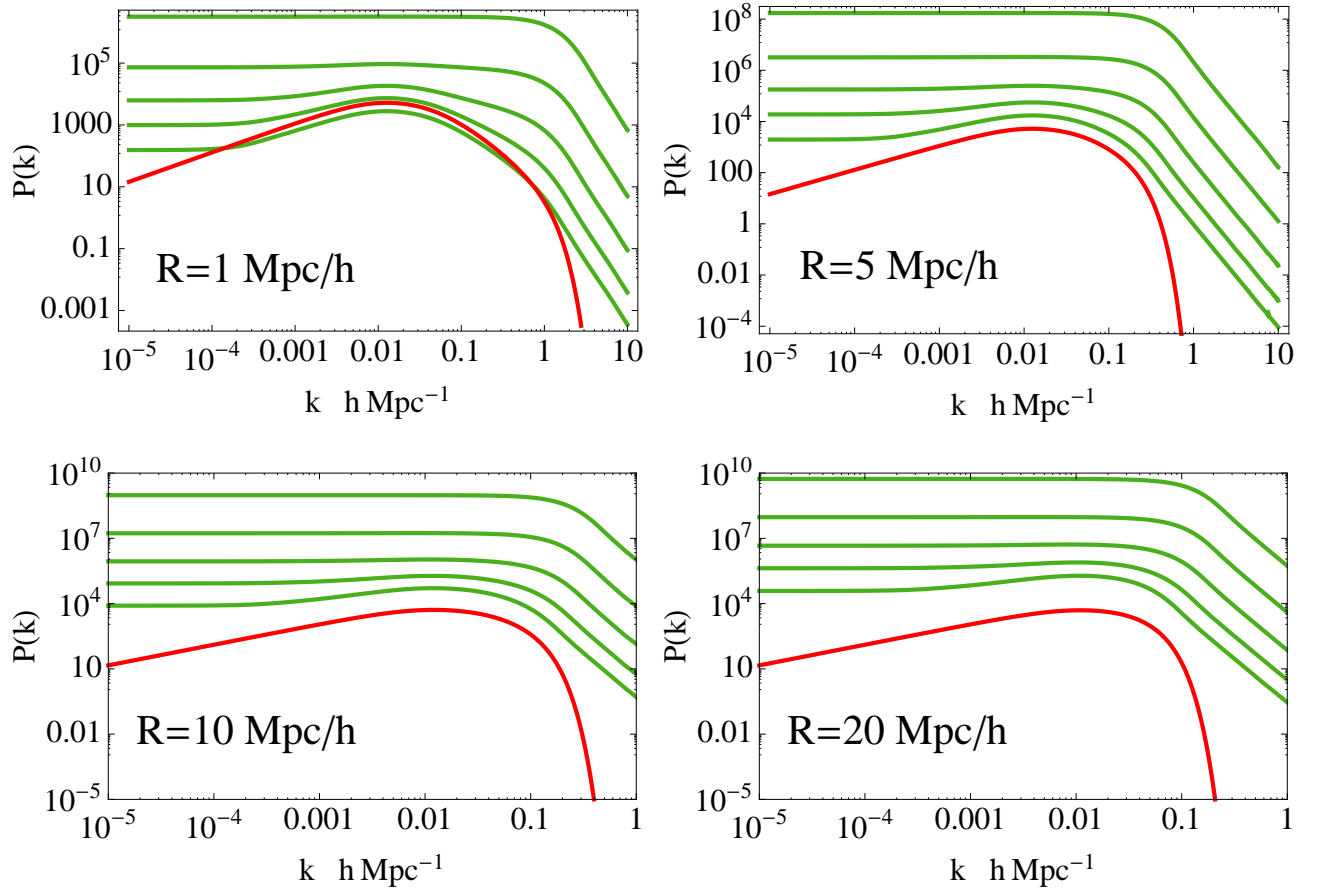


FIG. 3: We show the biased power spectrum for a smoothed underlying power spectrum as explained in the text for several smoothing scales (indicated in the corresponding panels) and for $\nu = 5, 4, 3, 2$ and 1 from top to bottom. Note also here that the amplitude of the biased power spectra is unphysical. Actually, the value $P(0)$ is (up to a factor ν^2) the fraction of the volume for which $\xi_c(r) = \xi(r)/\sigma^2$ is larger than ν which becomes large for larger damping scale R since damping reduces the variance σ^2 .

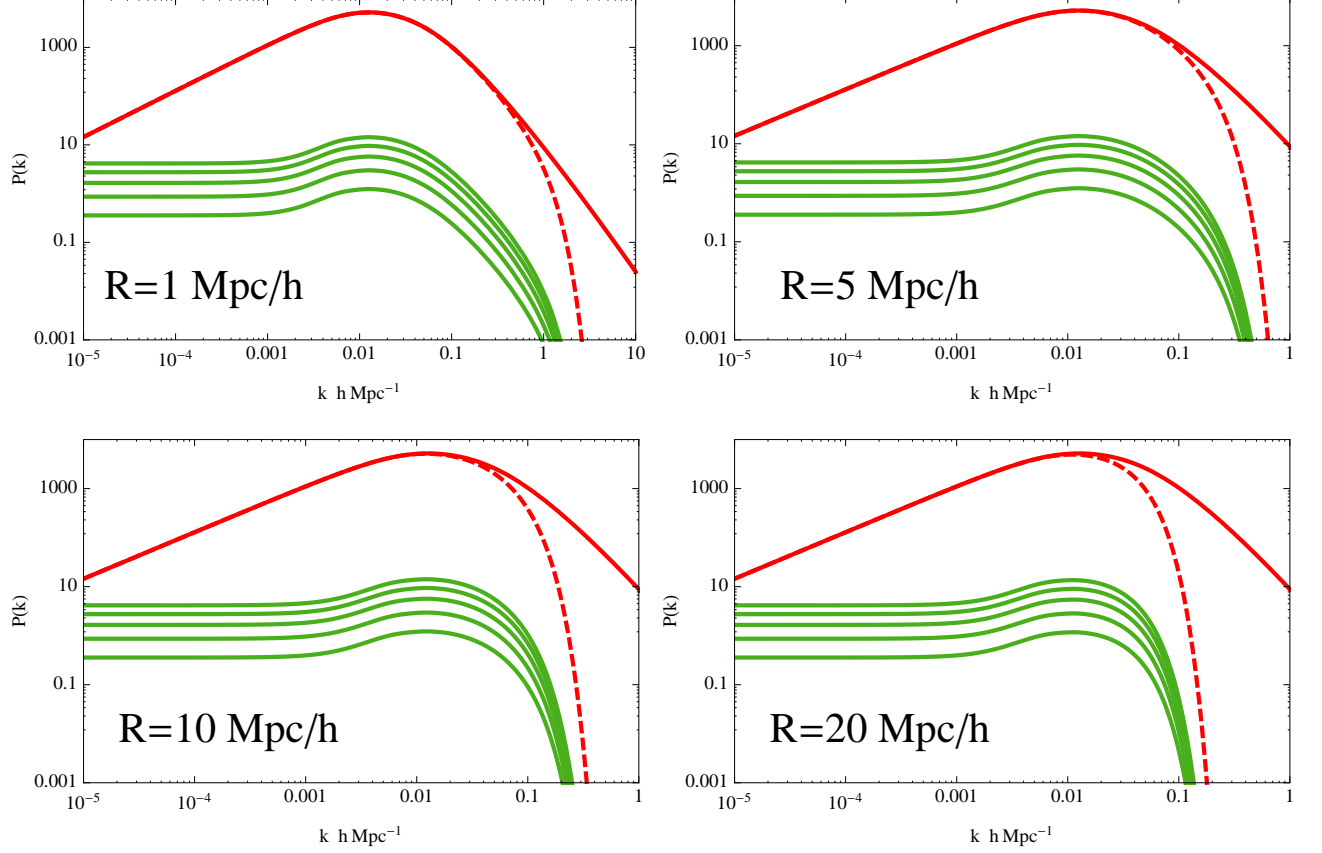


FIG. 4: In this plot we show the biased power spectra after small scale damping. The red line corresponds to the underlying power spectrum, the red-dashed line is the smoothed underlying power spectrum and the green lines show the biased power spectra after applying smoothing for $\nu = 5, 4, 3, 2, 1$ from top to bottom. On large scales, $k < 1/R$, the biased spectra are not affected and correspond to the ones shown in Fig. 2

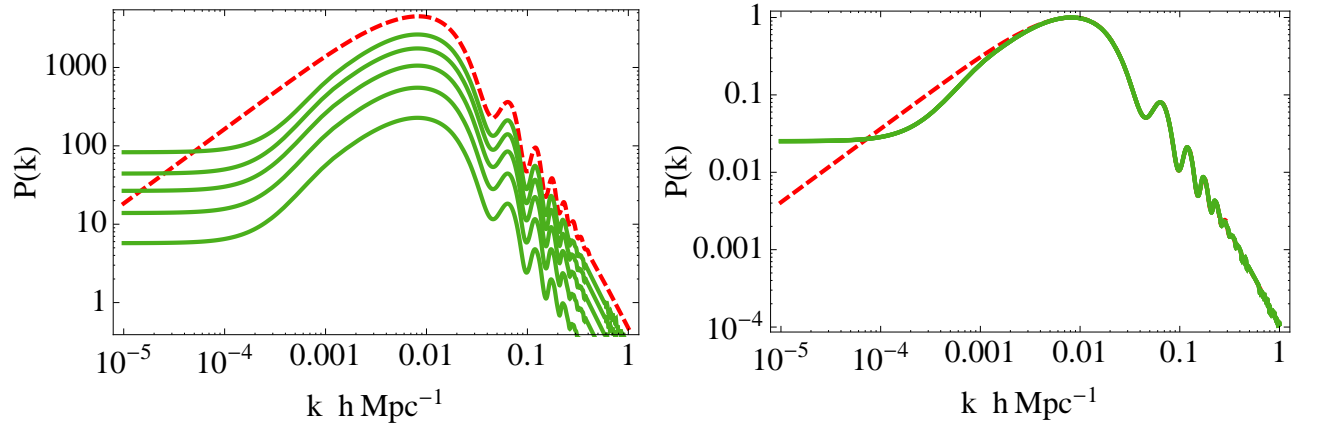


FIG. 5: Biasing in the presence of baryons. For this plot we choose $\Omega_b h^2 = 0.025$, $\Omega_M h^2 = 0.05$ and, $h = 0.5$. With this too large ratio of Ω_b/Ω_M the BAO's are well visible. In the left panel we show the underlying Λ CDM power spectrum (red dashed line) that includes the effects of baryons and the corresponding biased power spectra (green solid lines) for $\nu = 5, 4, 3, 2$ and 1 from top to bottom. In the right panel we have normalized the power spectra to their maximum value that clearly shows the two effects explained in the main text.

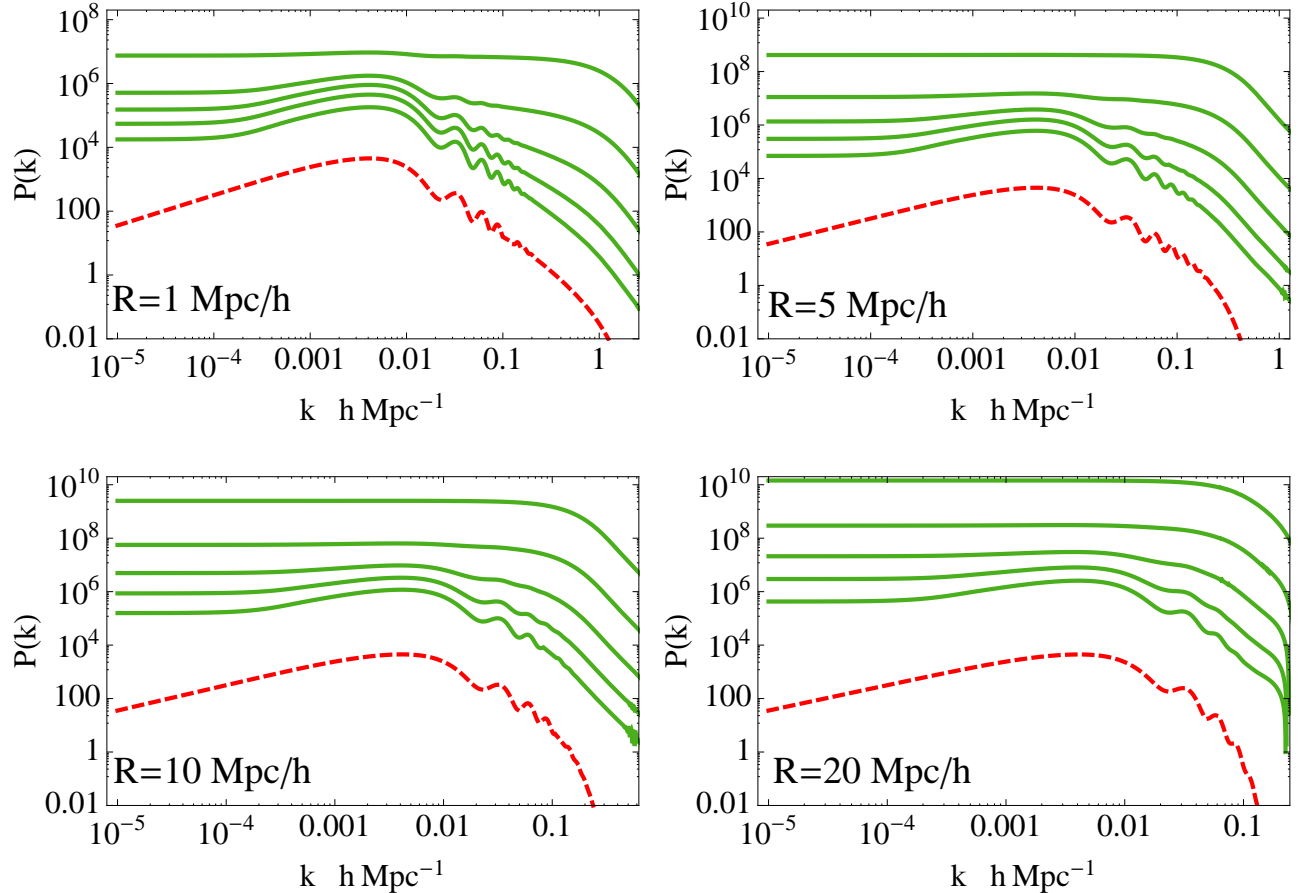


FIG. 6: The parameters are like for Fig. 5. We show the underlying Λ CDM power spectrum smoothed with a Gaussian window function (red dashed line) and the corresponding biased power spectra (green solid lines) for $\nu = 5, 4, 3, 2$ and 1 from top to bottom for the different smoothing scales indicated in the panels.

$h = 0.5$ in order to make the BAO's well visible. We have performed the calculation also for realistic values of the cosmological parameters and found the same results: the biased power spectrum flattens at the largest scales whereas the shape of the small scales of the spectrum, relevant for BAO's, remains unaffected and, therefore, the BAO's are not altered. Once again, this becomes more apparent when we normalize the spectra to their maximum value as shown in the right panel of Fig. 5. As before, this result implies that the biased power spectra can be factorized as $P_\nu(k) = b_1(\nu)P_1(k)$ with $b_1(\nu)$ the function given in (9) and P_1 the universal shape of the biased power spectrum (which of course is different from the one without baryons). Hence Silk damping is not sufficient to alter the linear amplification of the power spectrum on small scales. However, we do find a difference here with respect to the case without baryons: the power spectrum flattens at a larger scale and its decay towards the white noise plateau on large scales is even steeper than the $P(k) \propto k^n$ behaviour of the underlying power spectrum. This difference can only be due to the different shape of the underlying power spectrum

on small scales, the baryon acoustic oscillation and Silk damping. It is, however not clear whether this finding survives in a more realistic biasing scheme.

Also for this case, we have studied the effect of small scale damping of the underlying power spectrum with a Gaussian window function. The corresponding results are shown in Fig. 6. As before, the shape of the damped power spectrum is not conserved after biasing and the turnover tends to disappear for high values of the biasing parameter ν and for large smoothing scale. It is interesting to note that, unlike in the non-smoothed case, the BAO's are now distorted by biasing and they are completely washed out for high values of ν . We find interesting that smoothing over a scale of only $1h^{-1}\text{Mpc}$ already has such a significant effect on the power spectrum on large scales. As in the previous section without BAO's, we have linear biasing of the power spectrum on small scales which becomes non-linear once we introduce smoothing. Again, the reason for this is the high value of σ^2 of the non-smoothed power spectrum which is regularized by smoothing.

Finally, in order to compare to realistic observations

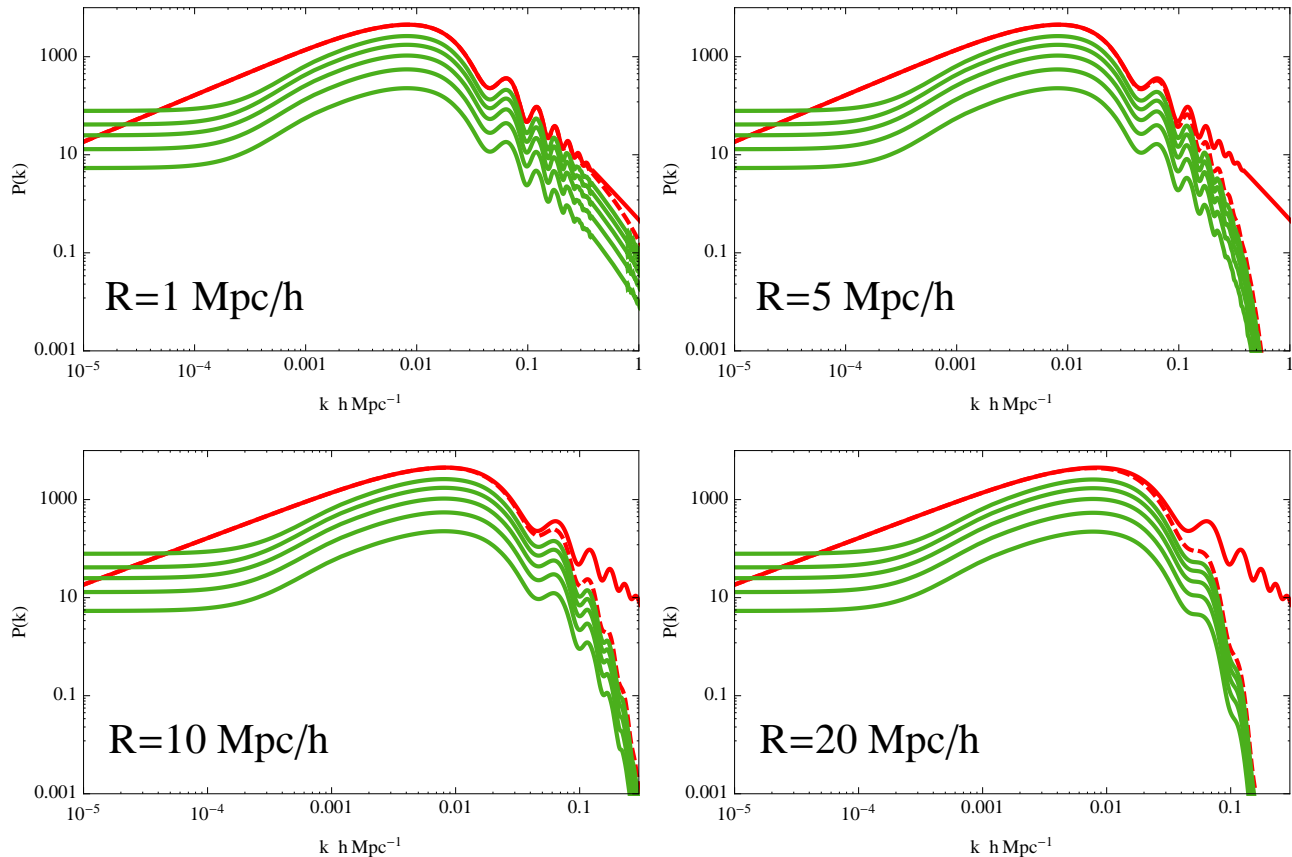


FIG. 7: In this plot we show the biased power spectra after applying smoothing. The red line corresponds to the underlying power spectrum, the red-dashed line is the smoothed underlying power spectrum and the green lines show the biased power spectra after applying smoothing for $\nu = 5, 4, 3, 2, 1$ from top to bottom.

of the galaxy distribution, we have considered the case of introducing smoothing on the biased power spectra. The corresponding results are shown in Fig. 7. As in the previous section, the non-linear biasing on large scales remains unaltered because, as commented above, the scales larger than the smoothing scale are not affected. It is interesting to note however that on scales comparable to the smoothing scale and smaller, the smoothed biased power spectra resemble the smoothed underlying power spectrum at small scales.

III. CONCLUSIONS

In this work we have studied the effects of a simple biasing scheme on the matter power spectrum which has been considered before in Ref. [9]. However, we have used more realistic underlying power spectra, unlike in Ref. [9] where an unrealistic cutoff is introduced. We have studied both, the effect of such a cutoff and the modification of the BAO's due to biasing.

First, we have used the approximate BBKS power spectrum, which neglects baryons and the non-linear con-

tribution, and we have found that the biased power spectrum is modified by two effects: a distortion at large scales where the power spectrum flattens and a vertical shift given by (9). Contrary to the standard claim, in the power spectrum there is linear bias on small scales, large k , $k_{\text{eq}} < k \lesssim 10^5 \text{Mpc}^{-1}$ and a non-linear (k -dependent) modification of the power spectrum on large scales. The linear biasing on small scales is due to the UV divergence of the $n = 1$ Harrison-Zel'dovich spectrum, that makes $\xi_c \ll 1$ already at $r = 10^{-5} \text{Mpc}$ for the realistic value of $n = 0.96$ and, therefore, the linear approximation is valid already above this scale. We have also studied the effect of biasing on a power spectrum where small scale power is damped as e.g. in scenarios with warm dark matter. In that case, the biased power spectrum becomes distorted on all scales. The UV-divergence is no longer present and the turnover on large scales tends to disappear for high values of the biasing parameter or large smoothing scale. Already for a smoothing scale of $1 h^{-1} \text{Mpc}$, the turnover on large scales nearly disappears for $\nu > 2$. If the damping scale is very small, $R \lesssim 1 \text{kpc}$, smoothing has no effect on the power spectrum on the scales considered here.

In real observations, there is of course also smoothing of the galaxy power spectrum due to finite resolution.

We have introduced this smoothing in the biased power spectrum and we have seen that the linear amplification on small scales also disappears and the power spectra flatten on large scales. However, in this case the turnover can be visible.

Finally, we have included baryons and studied the possible effects on the BAO's. We have found the same features as in the case without baryons: the biased power spectrum flattens at large scales, and it is linearly rescaled by the constant factor $b_1(\nu)$ at small scales. There are no effects on the BAO's. They are not smeared out by Kaiser biasing. However, damping of the underlying power spectrum distorts the biased power spectrum at all scales. For a damping scale of $R \simeq 1 \text{ Mpc/h}$ and a bias parameter $\nu \lesssim 2$, the BAO's remain intact. However, they completely disappear for high values of the biasing parameter, $\nu > 3$ and large damping scale, $R > 5 \text{ Mpc/h}$. Again, we have also introduced smoothing of the biased power spectra in order to study the effects in real observations. We have seen that BAO's are not affected if the smoothing scale is small enough. However, we cannot definitively decide on the modification of the BAO's by biasing. The result depends on the details of the underlying biasing scheme and is not robust.

Our findings show that, for the biasing scheme considered in this work, the $P \propto k^n$ slope is in principle observable for nearly a decade before the 'white noise' contribution sets in. When we introduce small scale damping of the underlying power spectrum the situation becomes worse and the slope can become unobservable, even the turn over may disappear. Finally, realistic observations require smoothing of the biased power spectra so we have also studied this and we have seen that the turnover is not affected by smoothing on small scales. This result is less pessimistic than in Ref. [9], but it has to be checked with a more realistic biasing scheme that will be considered in a future project.

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- [1] R. Durrer *The Cosmic Microwave Background*, Cambridge University Press (2008).
 - [2] V. Springel et al., *Nature* **435**, 629 (2005) [e-print: arXiv:astro-ph/0504097].
 - [3] N. Kaiser, *Astrophys. J.* **284**, L9 (1984).
 - [4] D. Politzer and M. Wise, *Astrophys. J.* **285**, L1 (1984).
 - [5] J.M. Bardeen, J.R. Bond, N. Kaiser and A. Szalay, *Astrophys. J.* **304**, 15 (1986).
 - [6] M.E.C. Swanson et al., *Mon. Not. R. Astron. Soc.* **385**, 1635 (2008) [e-print: arXiv:astro-ph/0702584].
 - [7] W.J. Percival et al., *Astrophys. J.* **657**, 645 (2007) [e-print: arXiv:astro-ph/0608636].
 - [8] A.G. Sánchez and S. Cole *Mon. Not. R. Astron. Soc.* **385**, 830 (2008) [e-print: arXiv:0708.1517].
 - [9] R. Durrer, A. Gabrielli, M. Joyce and F. Sylos Labini, *Astrophys. J.* **585**, L1 (2003) [e-print: arXiv:astro-ph/0211653].
 - [10] S. Cole et al., *Mon. Not. R. Astron. Soc.* **362**, 505 (2005) [e-print: arXiv:astro-ph/0501174].
 - [11] J. Hamann, S. Hannestad, A. Melchiorri, and Y. Wong, *JCAP* **7**, 17 (2008) [e-print: arXiv:0804.1789].
 - [12] J.R. Bond and S.T. Myers, *Astrophys. J. Suppl.* **103**, 1 (1996).
 - [13] H.J. Mo, Y.P. Jing, and S.D.M. White, *Mon. Not. R. Astron. Soc.* **284**, 189 (1997) [e-print: arXiv:astro-ph/9603039].
 - [14] U. Seljak, *Mon. Not. R. Astron. Soc.* **318**, 203 (2000) [e-print: arXiv:astro-ph/0001493].
 - [15] U. Seljak, *Mon. Not. R. Astron. Soc.* **325**, 1359 (2001) [e-print: arXiv:astro-ph/0009016].
 - [16] J.L. Tinker, D.H. Weinberg, Z. Zheng, and I. Zehavi, *Astrophys. J.* **631**, 41 (2005) [e-print: arXiv:astro-ph/0411777].
 - [17] M. White and N. Padmanabhan, *Mon. Not. R. Astron. Soc.* **395**, 2381 (2009) [e-print: arXiv:0812.4288].
 - [18] J.L. Tinker et al., *Astrophys. J.* **686**, 53 (2008) [e-print: arXiv:0707.3445].
 - [19] Z. Zheng and D. H. Weinberg, *Astrophys. J.* **659**, 1 (2007) [e-print: arXiv:astro-ph/0512071].
 - [20] O. Hahn, C. Porciani, A. Dekel, and C. M. Carollo, *Mon. Not. R. Astron. Soc.* **398**, 17421756 (2009). [e-print: arXiv:0803.4211].
 - [21] H.Y. Wang, H.J. Mo and Y.P. Jing, *Mon. Not. R. Astron. Soc.* **375**, 633 (2007) [e-print: arXiv:astro-ph/0608690].
 - [22] H. Wang, H.J. Mo and Y. P. Jing, *Mon. Not. R. Astron. Soc.* **396**, 2249 (2009) [e-print: arXiv:0811.3558].
 - [23] N. Dalal, M. White, J.R. Bond and A. Shirokov, *Astrophys. J.* **687**, 12 (2008) [e-print: arXiv:0803.3453].
 - [24] M.C. Neyrinck, A.J.S. Hamilton, and N.Y. Gnedin, *Mon. Not. R. Astron. Soc.* **362**, 337 (2005) [e-print: arXiv:astro-ph/0504161].
 - [25] J.N. Fry and E. Gaztanaga, *Astrophys. J.* **413**, 447 (1993) [e-print: arXiv:astro-ph/9302009].
 - [26] V. Desjacques *Phys. Rev. D* **78**, 103503 (2008) [e-print: arXiv:0806.0007].
 - [27] P. McDonald and A. Roy, *JCAP* 0908:020 (2009) [e-print arXiv:0902.0991].
 - [28] R.J. Scherrer and D.H. Weinberg, *Astrophys. J.* **504**, 607 (1998) [e-print: arXiv:astro-ph/9712192].
 - [29] T. Matsubara, *Phys. Rev. D* **78**, 083519 (2008) [e-print: arXiv:0807.1733].
 - [30] T. Matsubara, *Phys. Rev. D* **78**, 109901 (2008) [e-print: arXiv:0807.1733].
 - [31] J. Carlson, M. White and N. Padmanabhan, *Phys. Rev. D* **80**, 043531 (2009) [arXiv:0905.0479].
 - [32] P. Coles, *Mon. Not. R. Ast. Soc.* **264**, 1065 (1993).

- [33] D. Larson et al., (2010) e-print: arXiv:1001.4635.
- [34] D. Eisenstein et al., *Astrophys. J.* **633**, 560 (2005) [e-print: arXiv:astro-ph/0501171];
W.J. Percival et al. *Mon. Not. R. Ast. Soc.* **381**, 1053 (2007) [e-print: arXiv:0705.3323].
- [35] F. Sylos Labini, Nikolay L. Vasilyev, Yuriy V. Baryshev, Martin Lopez-Corredoira. *Astron. Astrophys.* **505**: 981-990 (2009); Anna Cabre, Enrique Gaztanaga, arXiv:1011.2729 [astro-ph.CO].
- [36] H.J. Tian, Mark C. Neyrinck, Tamas Budavari, Alexander S. Szalay, *Astrophys. J.* **728**, 34 (2011); P. Arnalte-Mur, A. Labatie, N. Clerc, V.J. Martinez, J.-L. Starck, M. Lachize-Rey, E. Saar, S. Paredes, arXiv:1101.1911 [astro-ph.CO].
- [37] DES (Dark Energy Survey)
<https://www.darkenergysurvey.org/>
- [38] BOSS (Baryon Oscillation Spectroscopic Survey)
<http://cosmology.lbl.gov/BOSS/>
- [39] EUCLID: <http://sci.esa.int/science-e/www/object/index.cfm?fobjectid=42267> or: Euclid assessment study report (SRE-2009-2), ESA.
- [40] A. Gabrielli, M. Joyce and F. Sylos Labini, *Phys. Rev.* **D65**, 083523 (2002) [e-print: arXiv:astro-ph/0110451].
- [41] F. Sylos Labini, A. Gabrielli, M. Joyce and L. Pietronero, *Statistical physics for cosmic structures*, Springer Verlag (Berlin 2004).
- [42] Scott Dodelson, *Modern Cosmology*, Academic Press (2000).
- [43] Daniel J. Eisenstein and Wayne Hu, *Astrophys. J.* **496**, 605 (1998)