Peculiar velocity field: Constraining the tilt of the Universe

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The peculiar velocity field: constraining the tilt of the Universe

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A large bulk flow, which is in tension with the Lambda Cold Dark Matter (ΛCDM) cosmological model, has been observed. In this paper, we provide a physically *plausible* explanation of this bulk flow, based on the assumption that some fraction of the observed dipole in the cosmic microwave background is due to an intrinsic fluctuation, so that the subtraction of the observed dipole leads to a mismatch between the cosmic microwave background (CMB) defined rest frame and the matter rest frame. We investigate a model that takes into account the relative velocity (hereafter the tilted velocity) between the two frames, and develop a Bayesian statistic to explore the likelihood of this tilted velocity.

By studying various independent peculiar velocity catalogs, we find that: (1) the magnitude of the tilted velocity u is around 400 km/s, and its direction is close to what is found from previous bulk flow analyses; for most catalogs analysed, $u = 0$ is excluded at about the 2.5σ level; (2) constraints on the magnitude of the tilted velocity can result in constraints on the duration of inflation, due to the fact that inflation can neither be too long (no dipole effect) nor too short (very large dipole effect); (3) Under the assumption of a super-horizon isocurvature fluctuation, the constraints on the tilted velocity require that inflation lasts at least 6 e-folds longer (at the 95% confidence interval) than that required to solve the horizon problem. This opens a new window for testing inflation and models of the early Universe from observations of large scale structure.

I. INTRODUCTION

The bulk flow, i.e. the streaming motion of galaxies or clusters, is a sensitive probe of the density fluctuation on very large scales. Recently there have been several observations of a large amplitude of the bulk flow on hundred Mpc scales, which are in conflict with the predictions of the ΛCDM model [1, 2]. The bulk flow is measured with respect a frame in which the CMB temperature dipole vanishes. We define this to the *CMB rest frame*. It is usually assumed that the CMB rest frame coincides with the *matter rest frame* which we define to be the frame in which the velocities of matter in our horizon volume are isotropic.

It is possible that the two reference frames actually do not coincide with each other, resulting in a "tilted universe" [3–5]. If the inflationary epoch lasted just a little more than the 60 or so e-folds needed to solve the horizon problem, the observable "remnants" of the pre-inflationary Universe may still exist on very large scales of the CMB. As a result, there could be some fraction of the CMB dipole due to the intrinsic fluctuations rather than observer's kinetic motion. Therefore, when the observed dipole is subtracted from the galaxy peculiar velocity data, the subtraction induces a mismatch between the CMB rest frame and matter rest frame. We define the *intrinsic* CMB dipole to be the CMB dipole measured in the matter rest frame. The recent finding of such a mismatch between the directions of the observed CMB dipole and reconstructed velocity dipole [6, 7] suggested the possible existence of the intrinsic CMB dipole on the sky.

Pre-inflationary fluctuations in a scalar field may produce an intrinsic dipole anisotropy. In this scenario, the observed CMB dipole would be a sum of the dipole from our motion in the matter rest frame and the intrinsic CMB dipole caused by a large scale perturbation. This intrinsic dipole can be produced by a large scale isocurvature perturbation [1], but not a large scale adiabatic perturbation [8]. Note that our local motion relative to the matter rest frame is caused by small scale inhomogeneity (up to about the 100 Mpc scale) and will be negligibly affected by the very large scale ($\gg$ 10 Gpc) perturbation that would cause a tilt effect.

In this paper, we estimate the tilted velocity ($\mathbf{u}$) between the two rest frames using galaxy peculiar velocity data. This opens a new window on testing early-Universe models from observations of large scale structure.

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[1] In our context, an isocurvature perturbation is distinguished from an adiabatic perturbation in that the ratios of the number of photons to baryons and cold dark matter particles is not spatially invariant.

II. LIKELIHOOD AND MOCK CATALOGS

In order to use the galaxy peculiar velocity catalogues (see the following descriptions of the data), we need to model the velocity of galaxies in different rest frames. For each galaxy velocity survey, we first subtract off our local motion with respect to the CMB as estimated from the CMB dipole. However, if there is a non-negligible intrinsic CMB dipole, the CMB defined rest frame will not correspond to the matter rest frame and thus there will be a residual dipole in the galaxy peculiar velocity survey. To test this we estimate the line-of-sight velocity S_n of each galaxy n in the CMB rest frame with measurement noise σ_n. Suppose the CMB rest frame has a tilt velocity $\mathbf{u}$ with respect to the matter rest frame, then the line-of-sight velocity of each galaxy with respect to the matter rest frame becomes $p_n(\mathbf{u}) = S_n - \hat{r}_{n,i}u_i$, where $\hat{r}_{n,i}u_i$ is the projected component of the 3-D Cartesian coordinate $\mathbf{u}$ onto the line-of-sight direction of galaxy n. After subtracting out the "tilted velocity", we model the galaxy line-of-sight velocity with respect to the matter rest frame as $p_n = v_n + \delta_n$, where v_n is the galaxy line-of-sight velocity in the matter rest frame, and δ_n is a superimposed Gaussian random motion with variance $\sigma_n^2 + \sigma_*^2$, where σ_* accounts for the 1-D small scale non-linear velocity. It can also compensate for an incorrect estimation measurement noise σ_n [1, 2]. Therefore, the covariance matrix of $p_n(\mathbf{u})$ becomes

$$
\begin{aligned}
G_{mn} &= \langle v_m v_n \rangle + \delta_{mn}(\sigma_n^2 + \sigma_*^2) \\
&= \langle (\hat{\mathbf{r}}_m \cdot \mathbf{v}(\mathbf{r}_m))\,(\hat{\mathbf{r}}_n \cdot \mathbf{v}(\mathbf{r}_n)) \rangle + \ \delta_{mn}(\sigma_n^2 + \sigma_*^2), \quad (1)
\end{aligned}
$$

in which the cosmic variance term is [9]

$$
\begin{aligned}
&\langle (\hat{\mathbf{r}}_m \cdot \mathbf{v}(\mathbf{r}_m))\,(\hat{\mathbf{r}}_n \cdot \mathbf{v}(\mathbf{r}_n)) \rangle \\
&= \frac{\Omega_{\rm m}^{1.1} H_0^2}{2\pi^2} \int dk P(k) f_{mn}, \quad (2)
\end{aligned}
$$

where

$$
f_{mn}(k) = \int \frac{d^2\hat{k}}{4\pi} \left(\hat{\mathbf{r}}_m \cdot \hat{\mathbf{k}}\right) \left(\hat{\mathbf{r}}_n \cdot \hat{\mathbf{k}}\right) \times \exp\left(ik\hat{\mathbf{k}} \cdot (\mathbf{r}_m - \mathbf{r}_n)\right), \quad (3)
$$

which can be calculated analytically (Appendix A).

Therefore, the likelihood of the tilted vector and the small scale velocity dispersion σ_* can be written as

$$
L(\mathbf{u}, \sigma_*) = \frac{1}{(\det G_{mn})^{\frac{1}{2}}} \exp\left(-\frac{1}{2} p_m(\mathbf{u}) G_{mn}^{-1} p_n(\mathbf{u})\right), \quad (4)
$$

where we fix the cosmological parameters at the WMAP 7-year best-fit values ($\Omega_{\rm b} = 0.0449$, $\Omega_{\rm c} = 0.222$, $h = 0.71$, and $\sigma_8 = 0.801$ [10]).

We parameterize the velocity as $\mathbf{u} = \{u, \cos(\theta), \phi\}$ where in Galactic coordinates $\phi = l$ and $\theta = \pi/2 - b$. In order for our marginalized prior on u to be uniform we set $\text{Prior}(\mathbf{u}, \sigma_*) \propto 1/u^2$. Then the posterior distribution of the parameters $(\mathbf{u}, \sigma_*)$ given the data (D) is $\Pr(\mathbf{u}, \sigma_* | \text{D}) \propto \text{Prior}(\mathbf{u}, \sigma_*) L(\mathbf{u}, \sigma_*)$. Before we perform the likelihood analysis for the real data, we have tested this likelihood from mock catalogs. We input a set of fiducial values of (σ_*,$\mathbf{u}$) and do 300 simulations of the data. The average likelihood of these simulations for each parameter exactly peaks at the input values, with the appropriate width determined by cosmic variance, instrumental noise and small scale velocity dispersion.

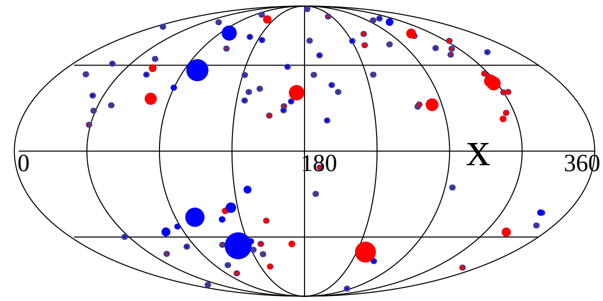

FIG. 1. The SN data plotted in Galactic coordinates.. The red points are moving away from us and the blue ones are moving towards us. The size of the points is proportional to the magnitude of the line-of-sight peculiar velocity. "X" is our estimate of the direction of the tilted velocity estimated from the SN data.

III. DATA ANALYSIS

We studied five different catalogs of galaxy peculiar velocities to constrain the tilted velocity $\mathbf{u}$ and the velocity dispersion σ_*. The five data sets are (see [1] for the procedure of excluding outliers):

- ENEAR is a survey of Fundamental Plane (FP) [29] distances to nearby early-type galaxies [11]. After the exclusion of 4 outliers, there are distances to 698 field galaxies or groups. For single galaxies, the typical distance error is 20%. The characteristic depth of the sample is 29 h^{-1}Mpc.

- SN are 103 Type Ia supernovae distances [12], limited to a distance of $\lesssim$150 h^{-1}Mpc. SN distances are typically precise to 8%. The characteristic depth is 32 h^{-1}Mpc. See Fig. 1.

- SFI++ [13], based on the Tully-Fisher (TF) [29] relation, is the largest and densest peculiar velocity survey considered here. After rejection of 38 (1.4%) field and 10 (1.3%) group outliers, our sample consist of 2720 field galaxies and 736 groups, so we divide it into two sub-samples, field samples SFI++$_{\rm F}$, and group samples SFI++$_{\rm G}$. The characteristic depth is 34 h^{-1}Mpc.

- SMAC [14] is an all-sky Fundamental Plane (FP) [29] survey of 56 clusters, with characteristic depth 65 h^{-1}Mpc.

- COMPOSITE is the combined catalogs (4536 data in total, compiled in [1, 2]) which has the characteristic depth of 33 h^{-1}Mpc. It is a combination of SN, SFI++, SMAC, ENEAR and also the samples from SBF [15] (69 field and 23 group galaxies, characteristic depth 17 h^{-1}Mpc), EFAR [16] (85 clusters and groups, characteristic depth 93 h^{-1}Mpc), SC [17] (TF-based survey of spiral galaxies in 70 clusters, characteristic depth 57 h^{-1}Mpc), and Willick [18] (Tully-Fisher based survey of 15 clusters, characteristic depth 111 h^{-1}Mpc).

In Fig. 2, we show the marginalized 1-D posterior probability distribution functions of the small scale velocity and intrinsic dispersion σ_*, and the velocity vector $\mathbf{u}$. The best fit and marginalized error bars are listed in Table I. In panel (a) of Fig. 2, we see that different surveys prefer different σ_*. For the SN catalog, there is a smaller σ_*, because the supernovae light curves can be used to calibrate the distance measurement very precisely, so the scatter of the line-of-sight velocity is small. In the COMPOSITE catalog, $\sigma_* \sim 450$ km/s reflects the average value of σ_* of all of the catalogs, which can be treated as the average value of the small scale velocity and the intrinsic dispersion. Using a single σ_* should not have a significant effect, in general, because the catalogs are of typical depth > 40 h^{-1}Mpc or in better units > 4000 km/s, and the distance indicators have typically 20% error in the measurement which means that typically $\sigma_n > 800$ km/s. So whether σ_* is 400 km/s or 600 km/s is not of much consequence if $\sigma_n > \sigma_*$.

In panel (b) of Fig. 2, we show the marginalized 1-D distribution of the magnitude of the $\mathbf{u}$. Also, from Table I we can see that the SFI++ and ENEAR catalogs can provide fairly tight upper bounds on u, and ENEAR peaks at zero while the SFI++ catalogs contain zero velocity within 2σ. It should be noticed that the SN and the larger COMPOSITE catalog can provide a non-zero 2σ lower bound on u, in which the latter provides the tightest constraints. In panels (c) and (d) of Fig. 2, we show the marginalized 1-D probability distribution of the direction of the relative velocity $(\cos(\theta), \phi)$. We see that the distribution of $\cos(\theta)$ and ϕ are very close to Gaussian, and the four catalogs predict a very similar direction of the velocity (Fig. 3 and Table I). We should also notice that the various catalogs give consistent constraints on the tilted velocity.

In panel (a) of Fig. 3, we plot 2-D contours of σ_* and u, and we find that, in the posterior distribution, σ_* and u are not correlated. The reason is easy to understand: the "tilted Universe" velocity $\mathbf{u}$ describes super-horizon dipole modulation of CMB photons, whereas σ_* describes the sub-horizon modes for small scale velocities and the intrinsic dispersion of each individual galaxy, so they come from completely different origins therefore without much correlation. Thus, if the instrumental noise σ_n has been underestimated, the distribution of σ_* will be shifted to smaller values, but that will not change the constraints on u significantly. Thus, our constraint on u is pretty robust with respect to the estimate of small scale instrumental noise. In panel (b) of Fig. 3, we plot the 1σ contour of $\mathrm{Cos}(\theta)$ and ϕ. We can see that the directions of tilted velocity found by different surveys are very consistent with each other, and also very consistent with the results from Watkins et al. [1].

IV. DISCUSSION

We should notice that, the direction of the large bulk flow velocity found by [1] is within the 1σ confidence level of the tilted velocity here, which therefore can provide a physical origin of the large bulk flow in [1]. In [2] they evaluated that the probability of getting a higher bulk flow within ΛCDM with WMAP7 parameter values to be less than 2%. In our analysis, the bulk flow of a galaxy survey should be accounted for in the error bars by the cosmic variance term (Eq. 2). So an alternative explanation might be that there is a feature in the matter power spectrum which would increase the cosmic variance. However, as the shear and octupole moments of the peculiar velocity field are not anomalously large, this is disfavored by the data [19].

In addition, the direction we find is consistent with that found in [20] which used the dipole of the kinetic Sunyaev-Zeldovich (kSZ) measurements to estimate the bulk flow on Gpc scales, but our magnitude is lower than what they found. However, there is an additional level of uncertainty in converting the kSZ dipole into a bulk flow which makes it difficult to estimate the magnitude of the bulk flow from the kSZ dipole [20]. In the bulk flow approach, Ref. [2] suggests that the bulk flow comes from scales > 300 h^{-1}Mpc. If [20] proves correct it will be strong support for tilt explanation since the tilted velocity should be the same regardless of the scale probed to measure it. A tilted Universe scenario was also proposed in [5] to explain the kSZ measurement.

An intrinsic dipole on the CMB sky caused by a tilted Universe may be explained by a pre-inflationary relic isocurvature inhomogeneity. If inflation lasts for only a few e-folds longer than required to solve the horizon problem, the scales that were super-horizon at the initial point of inflation are not very far outside our current horizon today. Inflation requires that there was initially a fairly smooth region of order of the inflationary Hubble horizon. A physical scale of such a region at the onset of inflation $l = e^p H_i^{-1}$ will have the physical scale $L = e^P H_0^{-1}$ today, where $P = p + N - N_{\rm min}$ ($N_{\rm min}$ is the minimal number of e-folds to solve the horizon problem and is generally assumed to be around 60) [3]. In the short inflationary case where N is not much larger than $N_{\rm min}$, a remnant of a pre-inflationary Universe still exists on super-horizon scales which may be detectable. The quadrupole effect, aka the Grishchuk-Zel'dovich effect [21], is also part of the effect of this large scale inhomogeneity.

An isocurvature perturbation can produce the "tilted

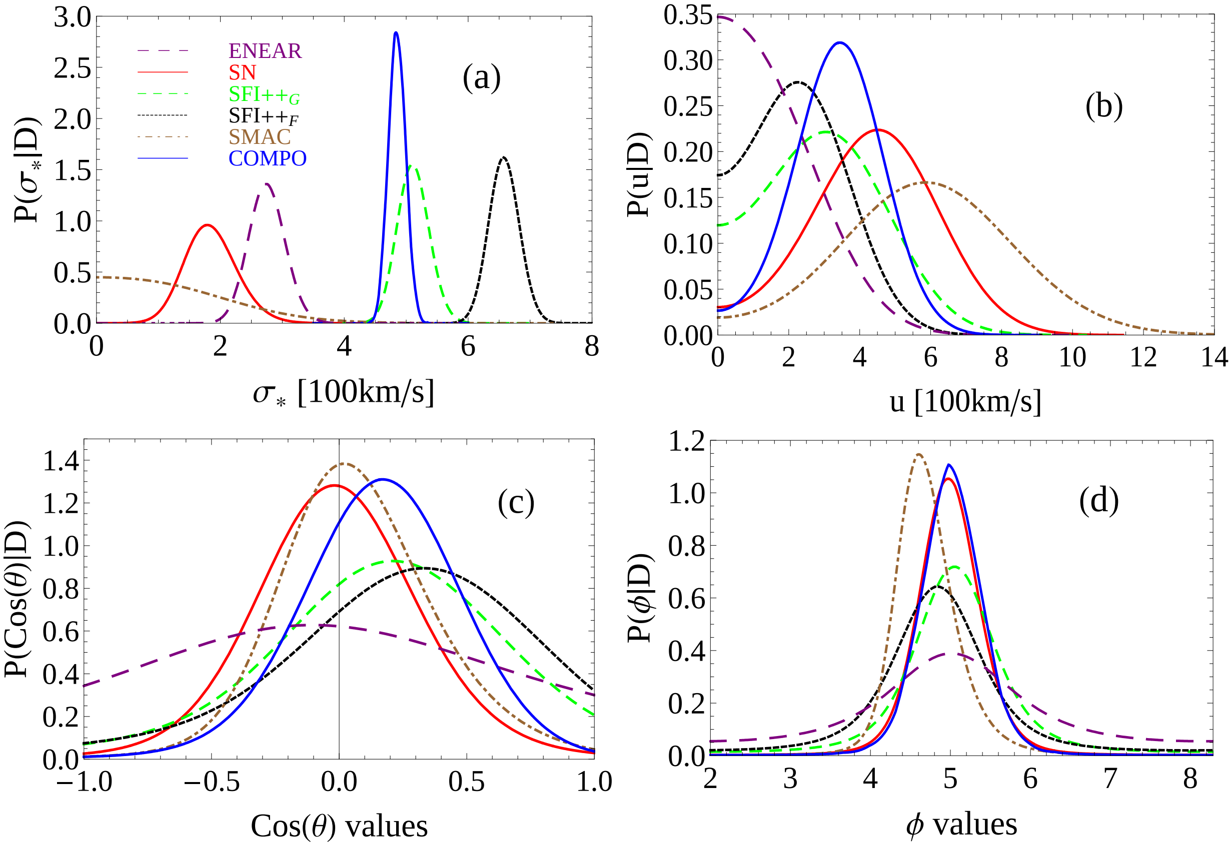

FIG. 2. The 1-D marginalized posterior probability distribution functions of the parameters : (a) σ_*, (b) magnitude of $\mathbf{u}$, (c) Cos(θ), (d) ϕ.

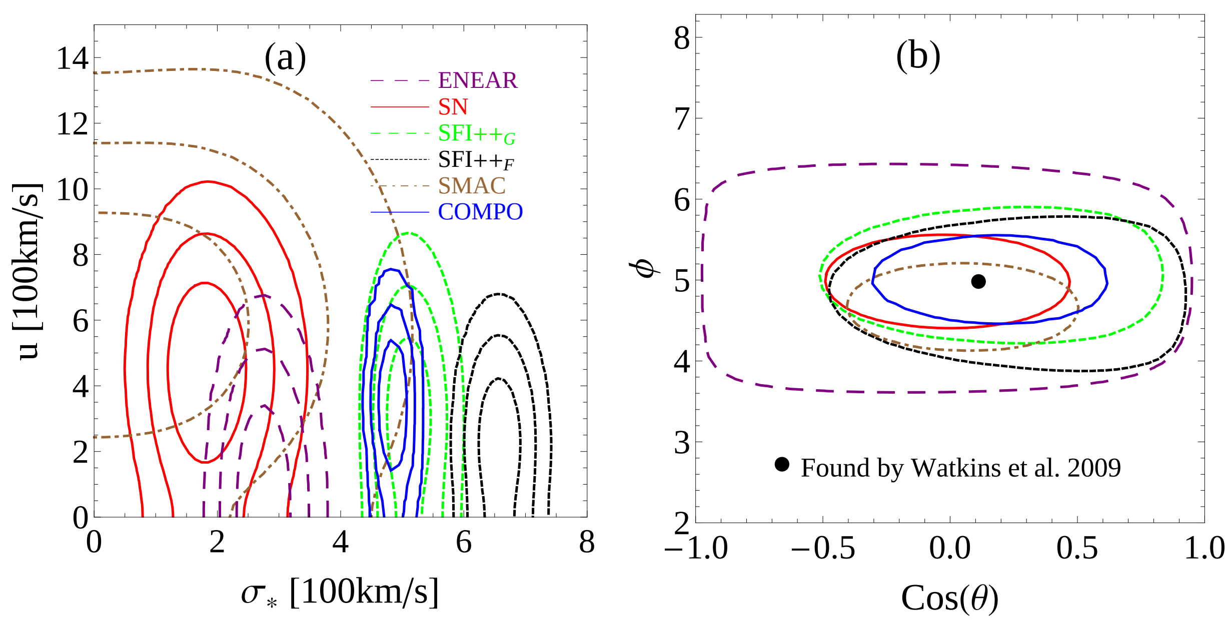

FIG. 3. (a) The 68%, 95% and 99.7% Bayesian confidence interval contours for parameters σ_*-u. (b) The 68% contours for Cos(θ)-ϕ. The black dot is the direction of bulk flow found by [1].

Catalogs	Characteristic Depth (h^{-1}Mpc)	σ_* [100km/s]	u [100km/s]	l (degrees)	b (degrees)	ΔN (2σ)
ENEAR	29	$2.8^{\pm}0.3$	$0^{+2.2}_{\times}$	$287.2^{+68.9}_{-68.3}$	$-3.8^{+37.5}_{-36.0}$	$\Delta N \geq 7$
SN	32	1.8 ± 0.4	$4.5^{+1.8}_{-1.9}$	$284.9^{+22.9}_{-22.1}$	$-1.0^{+18.8}_{-18.3}$	$6 \leq \Delta N \leq 9$
SFI++$_{\rm G}$	34	$5.10^{+0.7}_{-0.3}$	$3.1^{+1.6}_{-1.9}$	289.7 ± 34.9	$10.3^{+26.8}_{-25.5}$	$\Delta N \geq 6$
SFI++$_{\rm F}$	34	$6.6^{+0.3}_{-0.2}$	$2.3^{+1.2}_{-1.6}$	$276.8^{+40.1}_{-39.0}$	$15.8^{+28.4}_{-27.1}$	$\Delta N \geq 6$
SMAC	65	$0.0^{+1.7}_{\times}$	5.9 ± 2.4	$263.8^{+23.6}_{-18.5}$	$1.1^{+18.6}_{-16.0}$	$6 \leq \Delta N \leq 8$
COMPOSITE	33	$4.8^{+0.2}_{-0.1}$	3.4 ± 1.3	$285.1^{+23.9}_{-19.5}$	$9.1^{+18.5}_{-17.8}$	$6 \leq \Delta N \leq 8$

TABLE I. The best-fit and 1σ confidence level for the velocity dispersion σ_*, the magnitude (u) and the direction (l, b) of the tilted velocity, and the 2σ constraints on the number of e-folds of inflation. The "$\times$" means that the value is less than zero.

Universe" effect, which arises in inflationary models due to the perturbations of quantum fields other than the inflaton, such as the axion, whose energy density is subdominant compared to that of the inflaton. There are strong constraints on sub-horizon isocurvature perturbations and thus if they are responsible for the tilt it would require them to be much larger on very large scales than they are on smaller scales. Although multi-field models will not generically produce such a sharp breaking in scale invariance or result in isocurvature modes in the later Universe, there are double inflation models which can achieve this [4].

The dipole anisotropy is associated with the isocurvature fluctuation as follows [3, 4] [2]

$$\frac{u}{c} \simeq \frac{H_0^{-1}}{L}\frac{\delta\varphi}{\varphi_0}, \tag{5}$$

where $\delta\varphi$ is the fluctuation of the quantum field, and φ_0 is the background field. An inflationary scenario that results in isocurvature perturbations in the later Universe, such as in Ref. [4], is required. Assume that at the onset of inflation, the isocurvature quantum fluctuation satisfies $\delta\varphi/\varphi_0 \simeq 1$ (Constants of $\mathcal{O}(1)$ won't affect the results much), thus the Hubble horizon at the onset of the inflation is $p = 0$, $l = H_i^{-1}$, and $L = e^{\Delta N} H_0^{-1}$, where $\Delta N = N - N_{\rm min}$ is the excess number of inflationary e-folds beyond $N_{\rm min}$. The constraints on a "tilted Universe" lead to the constraints on the number of e-folds of inflation $u/c \simeq H_0^{-1}/L \simeq e^{-\Delta N}$. Therefore, an observable "tilted Universe" velocity requires that inflation should last a modest number of e-folds, which should not be too long nor too short— if inflation lasts too long, such perturbation effects would be washed out, if inflation is too short, the dipole effect will be too large, making the Universe over-tilted. We show the constraints on the number of e-folds in the last column of Table I. We find that the required extra number of e-folds is at least 6 for all catalogs, and SN, SMAC and COMPOSITE can also provide a 2σ upper bound on ΔN given their data.

[2] There may be some factors of order unity in this equation for different models, but they have negligibly small effect on the ΔN constraints due to the exponential. The same holds for $\delta\varphi/\varphi_0 \simeq 1$.

Note that our study provides a similar constraint to the Grishchuk-Zel'dovich effect (which can arise from either adiabatic or isocurvature perturbations) which requires that the extra number of e-folds should be greater than about 7 at the 95% confidence interval [22]. Also, correlations in the CMB multipoles may be used to estimate, from the PLANCK data, the tilt with an error bar similar to what we obtained here [23]. In the future, there is the potential to use the large number of SNe measured by the Large Synoptic Survey Telescope (LSST) to constrain u with a standard deviation of about 30 km/s [24]. So it may be possible to explore the duration of inflation and the pre-inflationary quantum state at quite a precise level. The peculiar velocity field, is therefore a powerful tool to probe the very early Universe in a manner not accessible by CMB observations alone.

V. CONCLUSION

In this paper, we developed a model and a statistical method to justify whether the apparent bulk flow motion of galaxies in our surveys is due to the subtraction of the intrinsic dipole on the CMB sky. In the conventional bulk flow scenario, the galaxies in the local region ($\lesssim 150$ h^{-1}Mpc) are moving towards direction ($l = 287^o \pm 9^o$, $b = 8^o \pm 6^o$) with $v = 407 \pm 81$ km/s seems in tension with the ΛCDM predictions. However, we point out that some fraction of the CMB dipole can be intrinsic due to large scale inhomogeneities generated by preinflationary isocurvature fluctuations, so that in the CMB rest frame, all of the galaxies have streaming velocity towards the particular direction, resulting in the tilted Universe.

We modeled an intrinsic CMB dipole as a tilted velocity $\mathbf{u}$ and developed a statistical tool to constrain its magnitude and direction. We found that (1) the magnitude of the tilted velocity u is around 400 km/s, and its direction is close to what was found in previous bulk flow studies. For SN, SMAC and COMPOSITE catalogs, $u = 0$ is excluded at about the 2.5σ level, which can explain the apparent flow; (2) there is little correlation between the tilted velocity $\mathbf{u}$ and galaxy's small scale velocity and intrinsic dispersion σ_*, which confirms our assumption.

Furthermore, assuming that primordial isocurvature modes lead to an intrinsic dipole anisotropy, constraints on the magnitude of the tilted velocity can result in constraints on the duration of inflation, due to the fact that inflation can neither be too long (no dipole effect) nor too short (very large dipole effect). Under this assumption, the constraints on the tilted velocity require that inflation lasts at least 6 e-folds longer (at the 95% confidence interval) than that required to solve the horizon problem.

Finally, we should point out that if there is an intrinsic fluctuation, the free electrons in clusters should be able to "see" the modulation on the sky, which can be tested through the kSZ effect. Therefore, the results from South Pole telescope (SPT) [25] and Atacama cosmology telescope (ACT) [26, 27] may shed some light on the intrinsic fluctuations. Such work is in progress.

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Appendix A: Analytic formulae of correlated Window Function $f_{12}(k)$

The correlated window function $f_{12}(k)$

$$f_{12}(k) = \int \frac{d^2\hat{k}}{4\pi} \left(\hat{\mathbf{r}}_1\cdot\hat{\mathbf{k}}\right)\left(\hat{\mathbf{r}}_2\cdot\hat{\mathbf{k}}\right) \times \exp\left(ik\hat{\mathbf{k}}\cdot(\mathbf{r}_1-\mathbf{r}_2)\right), \tag{A1}$$

can be calculated analytically, by transforming it into harmonic space and using the property of spherical harmonics. The final integral should only depend on: (a) the angle α between $\mathbf{r}_1$ and $\mathbf{r}_2$ (therefore $\mathbf{r}_1$ and $\mathbf{r}_2$ should be symmetric); (b) r_1 and r_2 (amplitude of the vector); (c) k ($\mathbf{k}$'s amplitude). Therefore, we can specify $r_1 = (0, 0, 1)$, $r_2 = (0, \sin\alpha, \cos\alpha)$, where α is the relative angle between r_1 and r_2. Therefore,

$$\mathbf{A} = \mathbf{r}_1 - \mathbf{r}_2 = (0, -r_2\sin\alpha, r_1 - r_2\cos\alpha). \tag{A2}$$

So its direction and amplitude become

$$\hat{\mathbf{A}} = \frac{1}{A}(0, -r_2\sin\alpha, r_1 - r_2\cos\alpha), \tag{A3}$$

and

$$A = [r_1^2 + r_2^2 - 2r_1r_2\cos\alpha]^{\frac{1}{2}}. \tag{A4}$$

We can set

$$\hat{\mathbf{k}} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta), \tag{A5}$$

therefore,

$$\hat{\mathbf{k}}\cdot\hat{\mathbf{A}} = \frac{1}{A}\left((-r_2\sin\alpha)\sin\theta\sin\phi + (r_1 - r_2\cos\alpha)\cos\theta\right). \tag{A6}$$

Now we can use spherical harmonic function $Y_{lm}(\theta,\phi)$ to decompose the integrand as follows.

$$\begin{aligned}\left(\hat{\mathbf{r}}_1\cdot\hat{\mathbf{k}}\right)\left(\hat{\mathbf{r}}_2\cdot\hat{\mathbf{k}}\right) &= \cos\theta(\sin\alpha\sin\theta\sin\phi + \cos\alpha\cos\theta)\\ &= i\sqrt{\frac{2\pi}{15}}\sin\alpha\left(Y_{2,1}+Y_{2,-1}\right)\\ &+ \frac{4}{3}\sqrt{\frac{\pi}{5}}\cos\alpha Y_{2,0} + \frac{1}{3}\cos\alpha,\end{aligned} \tag{A7}$$

$$\exp(ik\hat{\mathbf{k}}\cdot(\mathbf{r_1}-\mathbf{r_2})) = \sum_l i^l(2l+1)j_l(kA)P_l(\hat{\mathbf{k}}\cdot\hat{\mathbf{A}}), \tag{A8}$$

in which we can just consider $l = 0, 2$ two terms in the summation. The reason is that the mixing angle between $\mathbf{k}$ and $\mathbf{A}$ just causes the mixing between different m modes in the spherical harmonics in Eq. (A8), so the final non $l = 0$ and $l = 2$ terms vanish due to the orthogonality. Therefore, Eq. (A8) becomes

$$\exp(ik\hat{\mathbf{k}}\cdot(\mathbf{r_1}-\mathbf{r_2})) = j_0(kA) - 5j_2(kA)P_2(\hat{\mathbf{k}}\cdot\hat{\mathbf{A}}), \tag{A9}$$

where $j_l(kA)$ is spherical bessel function.

$$\begin{aligned}P_2(\hat{\mathbf{k}}\cdot\hat{\mathbf{A}}) &= \frac{1}{2}\left(3(\hat{\mathbf{k}}\cdot\hat{\mathbf{A}})^2 - 1\right)\\ &= -\frac{3}{2A^2}\sqrt{\frac{2\pi}{15}}(r_2\sin\alpha)^2\left(Y_{2,2}+Y_{2,-2}\right)\\ &+ \frac{2}{A^2}\sqrt{\frac{\pi}{5}}\left((r_1 - r_2\cos\alpha)^2 - \frac{1}{2}(r_2\sin\alpha)^2\right)Y_{20}\\ &- \frac{3}{A^2}(r_2\sin\alpha)(r_1 - r_2\cos\alpha)\\ &\times i\sqrt{\frac{2\pi}{15}}\left(Y_{2,1}+Y_{2,-1}\right).\end{aligned} \tag{A10}$$

Note that there is no zero order term in $P_2(\hat{\mathbf{k}}\cdot\hat{\mathbf{A}})$. Then we use the orthogonality property of Y_{lm} $\int d^2\hat{\mathbf{k}} Y_{lm}Y^*_{l'm'} = \delta_{ll'}\delta_{mm'}$ and $Y^*_{lm} = Y_{l,-m}(-1)^m$ and get the final result

$$f_{12}(k) = \frac{1}{3}\cos\alpha(j_0(kA) - 2j_2(kA)) + \frac{1}{A^2}j_2(kA)r_1r_2\sin^2\alpha. \tag{A11}$$

It is clear that this integration has the three properties we listed above and the window function f_{12} depends only on (r_1,r_2,k,α). This is an independent and simplified but equivalent result to Eq.(9.32) in [28].

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