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Abstract

We observe that a recently proposed supersymmetric model with Q_6 flavor symmetry admits a new CP violating ground state. A new sum rule for the quark mixing parameters emerges, which is found to be consistent with data. Simple extensions of the model to the neutrino sector suggest an inverted hierarchical mass spectrum with nearly maximal CP violation ($|\delta_{\mathrm{MNS}}| \simeq \pi/2$). Besides reducing the number of parameters in the fermion sector, these models also provide solutions to the SUSY flavor problem and the SUSY CP problem. We construct a renormalizable scalar potential that leads to the spontaneous breaking of CP symmetry and the family symmetry.

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1 Introduction

Non–Abelian discrete symmetries have found applications in explaining aspects of the flavor question not addressed by the standard model (SM) of particle physics. Restrictions imposed by such symmetries can lead to predictions for the Cabibbo–Kobayashi–Maskawa (CKM) quark mixing angles in terms of the quark mass ratios [1]. Such symmetries have been employed successfully to generate a geometric structure for the leptonic mixing angles, independent of the lepton mass ratios [2].

The non–Abelian discrete symmetric structure G appears to bode well within the supersymmetric (SUSY) standard model, since the same symmetry can also provide a natural solution to the excessive flavor change that occurs with generic soft SUSY breaking terms [3]. The three families of quarks and leptons will transform as doublets or triplets of the group G, which would result in the degeneracy of their masses. Degenerate squarks (and sleptons) would alleviate the SUSY flavor violation problem. If the non–Abelian symmetry is continuous [4] and gauged [5], there is flavor violation arising from the D–terms of the flavor group [6]. Continuous global symmetries are susceptible to explicit violation from quantum gravity. Non–Abelian discrete symmetries which have a gauge origin are free from these problems and deserve special considerations [7].

In Ref. [8] we presented a SUSY model based on the non–Abelian discrete group Q_6 – the binary dihedral group of order 12. This group has two inequivalent doublet representations, one real doublet, and one pseudo–real doublet, which can be handy for model building. In the flavor sector this symmetry results in one prediction for for a combination of the CKM mixing parameters, which was shown to be consistent with data. The quark and lepton superfields are assigned to doublets and singlets of Q_6, with the singlets identified as belonging to the third family. In the Q_6 symmetric limit the squarks of the first two families would be degenerate, which is sufficient to solve the SUSY flavor problem. Furthermore, by assuming that CP violation has a spontaneous origin, this model also solves the SUSY CP problem. Excessive CP violating processes arising from the SUSY breaking sector are absent, since the parameters are all real. Yet the model admits CP violation in the quark mixing matrix.

One major purpose of the present paper is to show that the Q_6 model studied in Ref. [8] admits a new minimum which violates CP, but leaves a new interchange symmetry in tact. By virtue of this interchange symmetry, we derive a new sum rule among the quark mixing parameters and CP violation, which is found to be consistent with observations. Such an interchange symmetry was present in Ref. [8] as well, but the new one presented here is different, although it arises from the same Higgs potential. We extend this symmetry to the

lepton sector and obtain interesting correlations between the neutrino oscillation parameters. We also compare the predictions of the new minimum with those of the old, and in the process update our old predictions. We use the most recent values of light quark masses where the errors have decreased significantly as a result of improved lattice calculations. We compare the model predictions to the best fit values obtained in the SM as well as by including certain new physics contributions in $B_{d,s} - \overline{B}_{d,s}$ mixings as obtained by the CKFfitter group [9]. These new contributions are motivated by certain discrepancies obtained in the SM CKM fits – such as the differences of order 20% in the CP violation parameter $\overline{\eta}$ obtained from fits to ϵ_K and $B \to J/\Psi K_S$ decay. Small new physics contributions naturally arise in our Q_6 based model. For example, there are contributions to meson–antimeson mixing via SUSY box diagrams, which may be important for the $B_{d,s}$ meson system since the third family squark is not degenerate with the first two family squarks.

We also present a complete Higgs potential that leads to the spontaneous breaking of CP symmetry and the Q_6 flavor symmetry without leading to pseudo–Nambu–Goldstone bosons. In addition to the Q_6, a flavor universal Z_4 symmetry is introduced. Owing to this Z_4, even after spontaneous symmetry breaking, an unbroken interchange symmetry survives in the Higgs potential, for which there are two possible choices, denoted as $\mathcal{P}_{I,II}$. These symmetries, along with Q_6, reduce significantly the number of parameters in the fermion mass matrices. This reduction of parameters leads to a sum rule involving quark masses and mixings [8]. Moreover, CP violation has a spontaneous origin, which is perhaps more satisfying than the usual assumption of explicit CP violation. Nevertheless, the dominant source of CP violation in the quark sector is the Kobayashi-Maskawa mechanism. The sum rule involving quark masses and mixings that has been derived relies on the spontaneous violation of CP. With this, the problem of excessive CP violation that generically exists in the soft SUSY breaking sector can be solved in a rather simple way. Various phenomenological aspects of this model in minimum $\mathcal{P}_I$ have been studied in Ref. [8, 10, 11].

It is perhaps worthwhile to compare the present approach based on non–Abelian flavor symmetries to those based on Abelian flavor symmetries. The latter can also lead to predictive scenarios for quark and lepton mixings with "texture zeros" in the fermion mass matrices [12, 13]. While for two families of fermions, such symmetric matrices with nearest neighbor interactions work well, the three family generalization is inconsistent with data [13]. Furthermore, these models by themselves do not solve the SUSY flavor problem. Our approach here based on Q_6 symmetry is similar in spirit to the texture zero models in that certain entries in the fermion mass matrices are zero, and thus the model has a precise prediction for one of the quark mixing parameters. The non–Abelian symmetry also enables us to

solve the SUSY flavor problem, due to the degeneracy of the first two families implied by the Q_6 symmetry. After Q_6 symmetry breaking, flavor changing operators can be generated, but as shown in Ref. [8,10,11], these are all sufficiently suppressed. Higher dimensional operators are suppressed by the Planck scale and have negligible effects.

With Abelian flavor symmetries, one has the prospect for explaining the mass hierarchy via the Froggatt–Nielsen mechanism [14]. This involves the use non–renormalizable operators for light family mass generation, or equivalently, the introduction of new fermionic degrees of freedom that generate such operators. While such an approach can lead to a qualitative understanding of the mass and mixing hierarchies, because of uncertainties involved with order one Yukawa couplings, *quantitative* explanation of the observed fermion spectrum is difficult to achieve. In the Q_6 model presented here, we aim for a quantitative prediction for the quark mixing parameter, which can be used to confirm or rule out the model. As such, we do not explain the hierarchies in the Yukawa couplings, but we rather accommodate them. No new fermions beyond those of the MSSM are employed, unlike the Froggatt–Nielsen models, at the price of having an extended Higgs sector.

In our model, the effective theory below a TeV has the MSSM spectrum plus two additional Higgsino doublets arising from the extended Higgs sector. The new spin zero Higgs bosons will have to be heavier than a few TeV, in order to be compatible with flavor changing processes, and thus are unlikely to be discovered at the LHC. On the other hand, the two Higgsino doublets beyond the MSSM are predicted to be light (with masses of order 100 GeV), in order to solve the SUSY CP problem. Spontaneous breaking of CP implies that the MSSM soft SUSY breaking parameters are real, which solves bulk of the strong CP problem. Since the VEVs of the Higgs fields are complex, after symmetry breaking, there is residual CP violation, which is however small if the Higgsinos are light [11].

The plan of the paper is as follows. In Sec. 2 we present the supersymmetric Q_6 model. In Sec. 3 we analyze the predictions of model $\mathcal{P}_I$. In Sec. 4 we provide the new model $\mathcal{P}_{II}$ and analyze its predictions for the quark mixing angles and CP violation. Sec. 5 discusses a simple extension of model $\mathcal{P}_{II}$ to the neutrino and charged lepton sector and the resulting predictions. In Sec. 6 we have our concluding remarks.

2 CP invariant SUSY Q_6 model

2.1 Q_6 group theory and the Yukawa sector of the model

We work within the context of supersymmetric standard model, with a non–Abelian flavor symmetry Q_6 acting on the three families of quarks, leptons and their superpartners. The group theory of Q_6 is discussed in detail in Ref. [8]. We briefly recall its salient features relevant for model building. Q_6 is a binary dihedral group, a subgroup of $SU(2)$, of order 12. It has the presentation

$$\{A, B; A^6 = E, B^2 = A^3, B^{-1}AB = A^{-1}\} \,. \tag{2.1}$$

The irreducible representations of Q_6 fall into $2, 2', 1, 1', 1'', 1'''$, where the 2 is complex–valued but pseudoreal, while the $2'$ is real valued. The $\{1,\ 1',\ 1'',\ 1'''\}$ singlets form a Z_4 subgroup with the 1 and $1'$ being real and the $1''$ and $1'''$ being complex conjugates of each other. The group multiplication rules are given as

$$1' \times 1' = 1,\ 1'' \times 1'' = 1',\ 1''' \times 1''' = 1',\ 1'' \times 1''' = 1,\ 1' \times 1''' = 1'',\ 1' \times 1'' = 1''' \tag{2.2}$$

$$2 \times 1' = 2,\ 2 \times 1'' = 2',\ 2 \times 1''' = 2',\ 2' \times 1' = 2',\ 2' \times 1'' = 2,\ 2' \times 1''' = 2 \tag{2.3}$$

$$2 \times 2 = 1 + 1' + 2',\ 2' \times 2' = 1 + 1' + 2',\ 2 \times 2' = 1'' + 1''' + 2 \tag{2.4}$$

The Clebsch–Gordon coefficients for these multiplication can be found in Ref. [8].

In Table 1 we list the Q_6 assignment of the quark, lepton and Higgs chiral supermultiplets in our model,[4] where Q, Q_3, L, L_3 stand for the $SU(2)_L$ quark and lepton fields, and H^u, H_3^u, H^d, H_3^d are the Higgs doublets. The $SU(2)_L$ singlet supermultiplets for quarks, charged leptons and neutrinos are denoted by $u^c, u_3^c, d^c, d_3^c, e^c, e_3^c$ and ν^c, ν_3^c. Three pairs of Higgs doublets are introduced in order to generate fermion masses directly in the presence of Q_6 symmetry using renromalizable couplings. The singlet field T_3 is needed to generate the Majorana mass for ν_3^c. The other singlet scalar fields are needed to achieve spontaneous breaking of Q_6 symmetry as well as CP symmetry without giving rise to pseudo–Nambu–Goldstone bosons. This point will be clarified in the next subsection. Table 1 also shows a flavor universal Z_4 symmetry, the purpose of which is to realize an unbroken interchange symmetry in the scalar sector even after spontaneous symmetry breaking. Such an interchange symmetry, for which we have two solutions, enables us to predict one quark mixing parameter.

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[4]Essentially the same model can be realized with any Q_{2N} if N is odd and a multiple of 3.

Table 1: Particle content of the Q_6 model along with their transformation under $Q_6 \times Z_4$.

	$\{Q, L\}$	$\{Q_3, L_3\}$	$\{u^c, d^c, \nu^c, e^c\}$	$\{u_3^c, d_3^c, \nu_3^c, e_3^c\}$	$H^{u,d}$	$H_3^{u,d}$	S	S_3	T	T_3	U
Q_6	2	$1'$	$2'$	$1'''$	$2'$	$1'''$	2	1	$2'$	$1'$	1
Z_4	$-i$	$-i$	$+$	$+$	i	i	$-$	$-$	$+$	$+$	$+$

The most general Yukawa superpotential involving the quark and lepton fields invariant under the $Q_6 \times Z_4$ symmetry, assuming matter parity in the usual way, is:

$$
\begin{aligned}
W_{\text{Yukawa}} &= \{a_u Q_3 u_3^c H_3^u + b_u (Q * H^u) u_3^c + b_u' Q_3 (H^u * u^c) + c_u (Q \star u^c) H_3^u + u \to d\} \\
&+ \{a_\ell L_3 e_3^c H_3^d + b_e (L * H^d) e_3^c + b_e' L_3 (H^d * e^c) + c_e (L \star e^c) H_3^d + e \to \nu\} \\
&+ \frac{M_1}{2} \nu^c \cdot \nu^c + \frac{a_{\nu^c}}{2} \nu_3^c \nu_3^c T_3 ,
\end{aligned}
\tag{2.5}
$$

where we have defined

$$
x \cdot y = x_1 y_1 + x_2 y_2 , \ \ x * y = x_1 y_2 + x_2 y_1 , \ \ x \star y = x_1 y_2 - x_2 y_1 .
\tag{2.6}
$$

We have used the explicit basis for Q_6 given in Ref. [8] and the notation $u^c \equiv (-u_1^c, \ u_2^c)$ etc, for the right–handed Q_6 doublet fermion fields. Note that the Z_4 symmetry plays no role in the construction of Eq. (2.5).

2.2 The Higgs sector

In order to break the Q_6 symmetry spontaneously while avoiding pseudo–Nambu–Goldstone bosons one needs to introduce SM singlet Higgs fields. The minimal such set will involve a 2, $2'$, $1'$ and two 1's of Q_6. These are listed in Table 1. The SM singlet S's are needed to mix the Q_6 doublets $H^{u,d}$ with the Q_6 singlets $H_3^{u,d}$. Without the Q_6 doublet T there will be an accidental $O(2)$ symmetry in the Higgs potential. The $O(2)$ symmetry is violated by the cubic coupling of T. The field T_3 is introduced for the Majorana mass for ν_3^c, and the Q_6 singlet U is introduced to generate a spontaneous CP violation and also to enable the spontaneous breaking of $Q_6 \times Z_4$ within the SM singlet sector. Thus the SM singlet Higgs sector employed appears to be the minimal set consistent with the demands we wish to meet.

The most general Higgs superpotential involving the Higgs fields of Table 1 invariant under the $Q_6 \times Z_4$ symmetry along with the usual matter parity (with all the Higgs fields being

even) has the form

$$W_{\text{Higgs}} = W_U + W_{ST} + W_H , \qquad (2.7)$$

where

$$W_U = \mu_U U^2 + \lambda U^3 + \left(\lambda_1 S_3^2 + \lambda_2 T_3^2 + \lambda_3 T \cdot T\right) U, \qquad (2.8)$$

$$W_{ST} = \mu_{S_3} S_3^2 + \mu_T T \cdot T + \mu_{T_3} T_3^2 + \lambda_3' T \cdot (T \otimes T)$$
$$+ \lambda_1'[-2S_2 S_1 T_1 + (S_1^2 - S_2^2)T_2] + \lambda_2' S \cdot ST_3 , \qquad (2.9)$$

$$W_H = \lambda_1'' H_3^u(H^d * S) + \lambda_2'' (H^u * S)H_3^d + \lambda_3'' (H^u \cdot H^d)S_3 \qquad (2.10)$$

with the notation

$$A \cdot (B \otimes C) = A_1(-B_1 C_1 + B_2 C_2) + A_2(B_1 C_2 + B_2 C_1) . \qquad (2.11)$$

Thus $T \cdot (T \otimes T) = 3T_1 T_2^2 - T_1^3$. The Z_4 symmetry has restricted the form of Eqs. (2.8)-(2.10); without the Z_4, the following couplings would be allowed:

$$W'_{\text{Higgs}} = (-H_1^u H_1^d + H_2^u H_2^d)T_1 + (H_1^u H_2^d + H_2^u H_1^d)T_2 . \qquad (2.12)$$

We wish to avoid these terms, since in their absence we can define an unbroken discrete symmetry, as discussed below.

The Higgs potential contains F terms derived from Eqs. (2.8)- (2.10), D terms associated with $SU(2)_L \times U(1)_Y$ breaking, and the following soft SUSY breaking Lagrangian[5]

$$\begin{aligned}
\mathcal{L}_{\text{soft}} = \ & m_U^2|U|^2 + m_S^2(|S_1|^2 + |S_2|^2) + m_{S_3}^2|S_3|^2 + m_T^2(|T_1|^2 + |T_2|^2) + m_{T_3}^2|T_3|^2 \\
& + \ m_{H_3^u}^2|H_3^u|^2 + m_{H_3^d}^2|H_3^d|^2 + m_{H^u}^2(|H_1^u|^2 + |H_2^u|^2) + m_{H^d}^2(|H_1^d|^2 + |H_2^d|^2) \\
& + \ \{ \ B_U U^2 + B_{S_3} S_3^2 + B_T T \cdot T + B_{T_3} T_3^2 \\
& + \ [A U^2 + A_1 S_3^2 + A_2 T_3^2 + A_3 (T \cdot T)] \ U + A_3' T \cdot (T \otimes T) \\
& + \ A_1'[-2S_2 S_1 T_1 + (S_1^2 - S_2^2)T_2] + A_2' S \cdot ST_3 \\
& + \ A_1'' H_3^u(H^d * S) + A_2'' (H^u * S)H_3^d + A_3'' (H^u \cdot H^d)S_3 + h.c.\} \ , \qquad (2.13)
\end{aligned}$$

where the $\cdot$ and $*$ products are defined in (2.6). We assume CP invariance, which implies that all the Yukawa couplings and the parameters in the Higgs potential are real. The Higgs potential would then admit two interesting minima which leave two separate discrete symmetries $\mathcal{P}_I$ or $\mathcal{P}_{II}$ unbroken. We analyze these two ground states in the next two sections.

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[5]We have used the same symbol for the scalar components as the superfields.

3 Ground state with unbroken interchange symmetry $\mathcal{P}_I$

The following symmetry $\mathcal{P}_I$ is respected by the $Q_6 \times Z_4$ invariant Higgs superpotentials Eqs. (2.8)-(2.10), and (2.13) and the D terms:

$$H_1^u \leftrightarrow H_2^u, \ H_1^d \leftrightarrow H_2^d, \ S_1 \leftrightarrow S_2, \ T_2 \to -T_2,$$
$$H_3^u \to H_3^u, \ H_3^d \to H_3^d, \ S_3 \to S_3 \ , T_1 \to T_1, \ T_3 \to T_3, \ U \to U. \quad (3.14)$$

The VEVs of the various Higgs fields can be consistently chosen such that this symmetry remains unbroken:

$$\left\langle H_1^{u,d} \right\rangle = \left\langle H_2^{u,d} \right\rangle = v_1^{u,d} e^{i\phi_+^{u,d}}, \ \left\langle H_3^{u,d} \right\rangle = v_3^{u,d} e^{i\phi_3^{u,d}}, \langle S_1 \rangle = \langle S_2 \rangle = v_S e^{i\phi_S},$$
$$\langle T_1 \rangle = v_T e^{i\phi_T}, \ \langle T_2 \rangle = 0, \ \langle S_3 \rangle = v_{S_3} e^{i\phi_{S_3}}, \ \langle T_3 \rangle = v_{T_3} e^{i\phi_{T_3}}, \ \langle U \rangle = v_U e^{i\phi_U} \ . (3.15)$$

In Eq. (3.15), we have explicitly displayed the complex phases. It should be noted that this symmetry $\mathcal{P}_I$ is an accidental symmetry of the Higgs potential, and is not respected by the full theory. For example, the Yukawa sector explicitly breaks this symmetry. Nevertheless, the existence of $\mathcal{P}_I$ enables us to choose a ground state given as in Eq. (3.15) consistently.

We have explicitly verified that the minimum of Eq. (3.15) is indeed a local minimum, and that spontaneous breaking of $Q_6 \times Z_4$ and CP symmetries occurs without generating pseudo–Nambu–Goldstone bosons. The scalar spectrum of our model is in fact arrived at by meeting these requirements.

In the ground state $\mathcal{P}_I$, the mass matrices for the up and down quarks take the form:

$$M_{u,d} = \begin{pmatrix} 0 & C_{u,d} & \frac{B_{u,d}}{\sqrt{2}} e^{i\Delta\phi_{u,d}} \\ -C_{u,d} & 0 & \frac{B_{u,d}}{\sqrt{2}} e^{i\Delta\phi_{u,d}} \\ \frac{B'_{u,d}}{\sqrt{2}} e^{i\Delta\phi_{u,d}} & \frac{B'_{u,d}}{\sqrt{2}} e^{i\Delta\phi_{u,d}} & A_{u,d} \end{pmatrix} . \quad (3.16)$$

Here we have defined the following parameters:

$$A_{u,d} = a_{u,d} \, v_3^{u,d}, \ B_{u,d} = \sqrt{2} \, b_{u,d} \, v_1^{u,d}, \ B'_{u,d} = \sqrt{2} \, b'_{u,d} \, v_1^{u,d}, \ C_{u,d} = c_{u,d} \, v_3^{u,d},$$
$$\Delta\phi_{u,d} = \phi_3^{u,d} - \phi_1^{u,d} \ . \quad (3.17)$$

We have ignored irrelevant overall phases of the two mass matrices. CP invariance of the Lagrangian implies that the parameters $(A_{u,d}, \ B_{u,d}, \ B'_{u,d}, \ C_{u,d})$ are all real. In this case,

after a common 45 degree rotation in the (1-2) sector that would set the (1,3) and (3,1) entries of $M_{u,d}$ of Eq. (3.16) to zero without inducing CKM mixing, we can write

$$M_{u,d} = P_{u,d} \hat{M}_{u,d} P_{u,d} \, , \tag{3.18}$$

where $\hat{M}_{u,d}$ are real matrices given as

$$\hat{M}_{u,d} = \begin{pmatrix} 0 & C_{u,d} & 0 \\ -C_{u,d} & 0 & B_{u,d} \\ 0 & B'_{u,d} & A_{u,d} \end{pmatrix} \, , \tag{3.19}$$

and $P_{u,d}$ are diagonal phase matrices given as

$$P_{u,d} = \mathrm{diag.}\{e^{-i\Delta\phi_{u,d}}, \ e^{i\Delta\phi_{u,d}}, \ 1\} \, . \tag{3.20}$$

The CKM matrix is then given by

$$V_{\mathrm{CKM}} = O_u^T P O_d \, , \tag{3.21}$$

where $O_{u,d}$ are the orthogonal matrices that diagonalize $\hat{M}_{u,d}$ via

$$O_{u,d}^T \hat{M}_{u,d} M_{u,d}^T O_{u,d} = \mathrm{diag.}\{m_{u,d}^2, \ m_{c,s}^2, \ m_{t,b}^2\} \, , \tag{3.22}$$

and P is a diagonal phase matrix

$$P = \mathrm{diag.}\{e^{i\phi}, \ e^{-i\phi}, \ 1\} \tag{3.23}$$

with $\phi = \Delta\phi_d - \Delta\phi_u$. Since $\hat{M}_u$ and $\hat{M}_d$ each has four real parameters, once the six quark masses are fixed, O_u and O_d will have one undetermined parameter each. These two parameters and the phase ϕ appearing in the matrix P of Eq. (3.23) will completely fix the three CKM mixing angles and the one CP violating phase. That will lead to one sum rule involving the CKM mixing angles, the CP violating phase, and the quark mass ratios. This prediction was analyzed in Refs. [8, 15] and shown to be fully consistent with data.

Here we update the results of Ref. [8] for the quark mixing parameter prediction. We use the most recent values of the quark masses. Lattice calculations have reduced the errors in the light quark masses, which we adopt for our fits. Furthermore, we compare the model prediction with the global fits provided recently in Refs. [9, 16] assuming specific new physics contributions. The new physics contributions are motivated by certain discrepancies that have been observed in the CKM fits. For example, the CP violation parameter $\bar{\eta}$ determined from ϵ_K differs from that obtained from the decay $B \to J/\psi K_S$ by more than 2 standard

9

deviations. We compare our model fits with the best fit of the standard model, as well as with the best fit for Scenario 1 of Refs. [9, 16]. This scenario is characterized by independent new contributions $\Delta_{d,s}$ to $B_{d,s} - \overline{B}_{d,s}$ mixing amplitude. It turns out that there is room for small ($\sim 25\%$) new contributions to these mixings in our model, arising from gluino–squark box diagrams. The Q_6 assignment of quarks implies that the third family squark is not degenerate with the first two family squarks (which are nearly degenerate). Once the quark mass matrices are diagonalized, there will be small off–diagonal entries in the squark mass matrix, which leads to $B_{d,s} - \overline{B}_{d,s}$ mixings. These diagrams have been evaluated in Ref. [11]. While real, these amplitudes are still in the interesting range for new physics to influence the CKM parameter fits. In Ref. [17] the radiative corrections to these mixing parameters, arising through Higgs boson exchange, have been computed, and have been shown to be complex. Thus, it appears that the Q_6 model admits small deviations in the CKM fits to $B_{d,s} - \overline{B}_{d,s}$ mixings. It should be noted, however, that the prediction of the present model agrees well with the best fit values of the CKM fits, with or without new physics assumed.

Guided by the analytic expressions for the CKM mixing parameters from (3.19) - (3.23), we have done a numerical fit to all quark masses and mixings. An excellent fit is obtained with the following choice of parameters at $\mu = 1$ TeV:

$$
\begin{aligned}
A_u/m_t &= 0.9963, \ B_u/m_t = 0.06051, \ B'_u/m_t = 0.06051, \ C_u/m_t = 1.748 \times 10^{-4}, \\
A_d/m_b &= 0.8895, \ B_d/m_b = 0.04214, \ B'_d/m_b = 0.4554, \ C_d/m_b = -5.043 \times 10^{-3}, \\
\phi &= 0.71875.
\end{aligned} \tag{3.24}
$$

The resulting mass eigenvalues at $\mu = 1$ TeV are:

$$
\begin{aligned}
m_u &= 1.25 \text{ MeV}, \ m_c = 552 \text{ MeV}, \\
m_d &= 2.74 \text{ MeV}, \ m_s = 50.0 \text{ MeV},
\end{aligned} \tag{3.25}
$$

where we have used $m_t = 150.3$ GeV and $m_b = 2.46$ GeV. These values are to be compared with quark masses extrapolated from low energy scale to $\mu = 1$ TeV [18]:

$$
\begin{aligned}
m_u &= 0.85 \sim 1.65 \text{ MeV} , \ m_d = 2.05 \sim 2.90 \text{ MeV} , \\
m_s &= 39.6 \sim 64.4 \text{ MeV} , \ m_c = 502 \sim 570 \text{ MeV} , \\
m_b &= 2.39 \sim 2.53 \text{ GeV} , \ m_t = 148.9 \sim 151.6 \text{ GeV} ,
\end{aligned} \tag{3.26}
$$

where we have updated the result of [18] by using the updated quark masses given in PDG 2011 [19], while neglecting the uncertainties due to the RG running. The input values of Eq.

(3.24) give also the following output for the CKM parameters:

$$\lambda \;=\; 0.2252, \; A = 0.7962 \;, \; \bar{\rho} = 0.1613 \;, \; \bar{\eta} = 0.4230 \;,$$

$$\sin 2\beta \;=\; 0.8042 \;, \; \alpha = 84.1 \,[\text{deg}] \;, \; \beta = 26.8 \,[\text{deg}] \;, \; \gamma = \; 69.1[\text{deg}] \;, \qquad (3.27)$$

which should be compared with the fit result of the CKMfitter group (scenario I) [9]

$$\lambda \;=\; 0.22542 \pm 0.00077 \;, \; A = 0.801^{+0.024}_{-0.017} \;, \qquad\qquad (3.28)$$

$$\bar{\rho} \;=\; 0.159^{+0.036}_{-0.035} \;, \; \bar{\eta} = 0.438^{+0.019}_{-0.029} \;, \; \sin 2\beta = 0.813^{+0.022}_{-0.068} \;,$$

$$\alpha \;=\; 79^{+22}_{-15} \,[\text{deg}] \;, \; \beta = 27.2^{+1.1}_{-3.1} \,[\text{deg}] \;, \; \gamma = 70.0^{+4.3}_{-4.5} \,[\text{deg}] \;. \qquad (3.29)$$
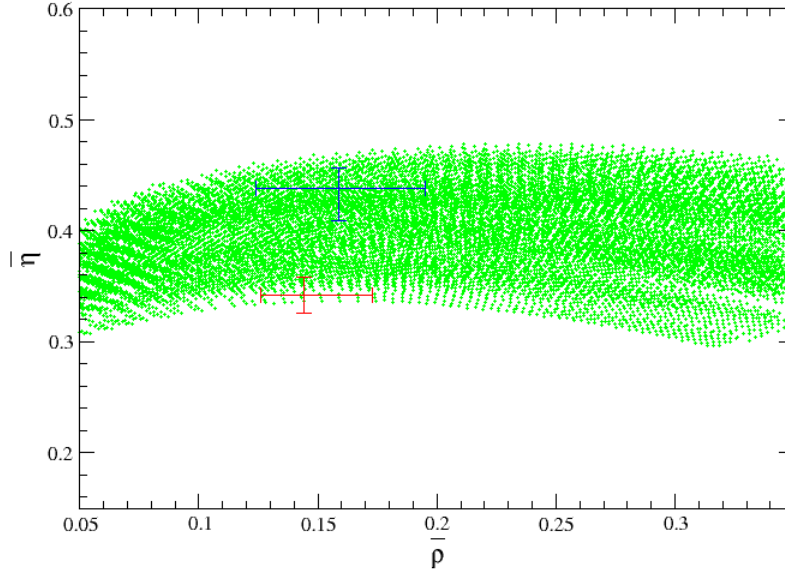

Figure 1: The prediction in the $\bar{\rho} - \bar{\eta}$ plane for the model $\mathcal{P}_I$, where we have used as the input parameters; the quark masses, λ and A given in Eqs. (3.26) and (3.28), respectively. We also have imposed the constraints on the quark mass ratios [19]: $2m_s/(m_u + m_d) = 22 \sim 30$, $m_s/m_d = 17 \sim 22, m_u/m_d = 0.35 \sim 0.60$. The crosses are the CKMfitter group values [9]; blue (scenario I) and red (SM).

Since there are nine model parameters for six quark masses and four CKM mixing parameters, we can make one prediction in a two dimensional plane if we fix eight of the nine model parameters. To fix these eight parameters we use the quark masses, λ and A given in (3.26) and (3.28), respectively. Fig. 1 shows the prediction in the $\bar{\rho} - \bar{\eta}$ plane, and Fig. 2 shows the
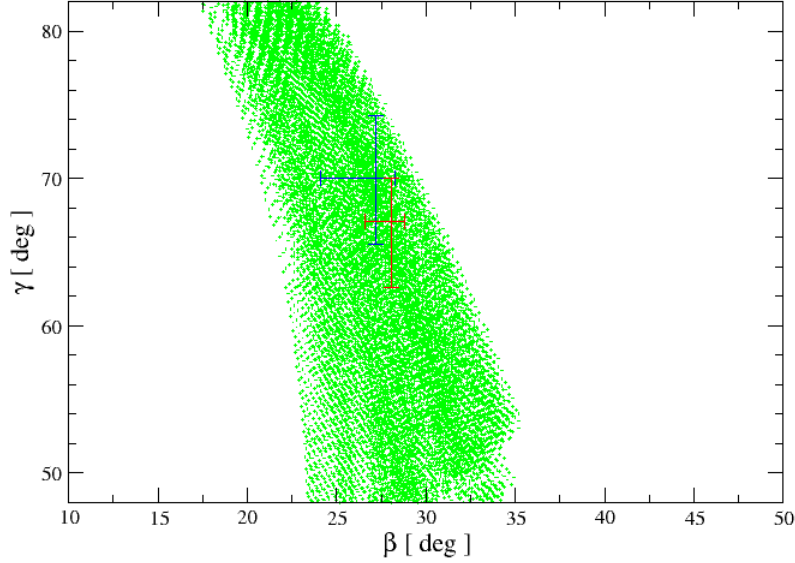
11

Figure 2: The prediction in the $\beta - \gamma$ plane for the model $\mathcal{P}_I$. The input parameters and the constraints are the same as for Fig. 1.

prediction in the $\beta - \gamma$ plane. The CKMfitter group best fit values (3.29) are also indicated in these figures. We see from Eqs. (3.25), Fig. 1 and Fig. 2 that the model $\mathcal{P}_I$ reproduces the quark masses, CKM mixings and the CP violating phase in an excellent way.

4   New ground state with unbroken interchange symmetry $\mathcal{P}_{II}$

The same Higgs potential as derived from Eqs. (2.7), and the soft SUSY breaking Lagrangian (2.13) including the D terms, with all parameters taken to be real so that CP is an exact symmetry, admits a new unbroken interchange symmetry as given below:

$$H_1^u \leftrightarrow H_2^{u*}, \ H_1^d \leftrightarrow H_2^{d*}, \ S_1 \leftrightarrow S_2^*, \ T_2 \to -T_2^*,$$
$$H_3^u \to H_3^{u*}, \ H_3^d \to H_3^{d*}, \ S_3 \to S_3^*, \ T_1 \to T_1^*, \ T_3 \to T_3^*, \ U \to U^*. \tag{4.30}$$

This symmetry $\mathcal{P}_{II}$ enables us to choose a ground state given by

$$\langle H_1^u \rangle = v_1^u e^{-i\phi_u}, \ \ \langle H_2^u \rangle = v_1^u e^{i\phi_u}, \ \ \langle H_1^d \rangle = v_1^d e^{-i\phi_d}, \ \ \langle H_2^d \rangle = v_1^d e^{i\phi_d},$$

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$$\langle H_3^u \rangle = v_3^u, \quad \langle H_3^d \rangle = v_3^d,$$

$$\langle S_1 \rangle = v_S e^{-i\phi_S}, \quad \langle S_2 \rangle = v_S e^{i\phi_S}, \quad \langle S_3 \rangle = v_{S_3}, \tag{4.31}$$

$$\langle T_1 \rangle = v_{T_1}, \quad \langle T_2 \rangle = -iv_{T_2}, \quad \langle T_3 \rangle = v_{T_3}, \quad \langle U \rangle = v_U,$$

where the complex phases are all explicitly displayed. Note that there are only three phases, ϕ_S, ϕ_u and ϕ_d in the VEVs, along with a purely imaginary VEV of T_2.

In the background $\mathcal{P}_{II}$, the fermion mass matrices $M_{u,d}$ following from Eq. (2.5) take the form

$$M_{u,d} = \begin{pmatrix} 0 & C_{u,d} & \frac{B_{u,d}}{\sqrt{2}} e^{-i\phi_{u,d}} \\ -C_{u,d} & 0 & \frac{B_{u,d}}{\sqrt{2}} e^{i\phi_{u,d}} \\ \frac{B'_{u,d}}{\sqrt{2}} e^{-i\phi_{u,d}} & \frac{B'_{u,d}}{\sqrt{2}} e^{i\phi_{u,d}} & A_{u,d} \end{pmatrix} \tag{4.32}$$

with the parameters as defined in Eq. (3.17). CP invariance of the Lagrangian implies that the parameters $\{A_{u,d}, B_{u,d}, B'_{u,d}, C_{u,d}\}$ are all real.

Model $\mathcal{P}_{II}$, while different from model $\mathcal{P}_I$, is just as predictive in the quark sector as $\mathcal{P}_I$. It is then interesting to see if the quark mixing sum rule of $\mathcal{P}_{II}$ is consistent with data. To address this question we proceed to diagonalize $M_{u,d}$ of Eq. (4.32). The phases in the matrices of Eq. (4.32) can be factorized:

$$M_{u,d} = P_{u,d} M^r_{u,d} P_{u,d} \tag{4.33}$$

where

$$P_{u,d} = \text{diag.}\{e^{i\phi_{u,d}}, \ e^{-i\phi_{u,d}}, \ 1\} \tag{4.34}$$

with $M^r_{u,d}$ given as in Eq. (4.32), but with $\phi_{u,d}$ set to zero. Quark field redefinitions can absorb the phases in $P_{u,d}$, however a phase matrix will then appear in the quark mixing matrix:

$$P = \text{diag.}\{e^{i\phi}, \ e^{-i\phi}, \ 1\}, \tag{4.35}$$

where

$$\phi = \phi_d - \phi_u. \tag{4.36}$$

Now we do a 45 degrees rotation in the (1-2) plane to bring $M^r_{u,d}$ into $\hat{M}_{u,d}$ as given in Eq. (3.19), but this will generate a non-trivial quark mixing matrix given by

$$K = \begin{pmatrix} \cos\phi & i\sin\phi & 0 \\ i\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{4.37}$$

The CKM mixing matrix is then obtained as

$$V_{\text{CKM}} = O_u^T K O_d, \tag{4.38}$$

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where $O_{u,d}$ diagonalize the matrices of Eq. (3.19) as specified in Eq. (3.22).

Using the approximate analytic expressions for the CKM mixing parameters, we have done a numerical fit to all quark masses and mixings within this model. An excellent fit is obtained with the following choice of parameters at $\mu = 1$ TeV:

$$
\begin{aligned}
A_u/m_t &= 0.01389, \ B_u/m_t = -0.003282, \ B'_u/m_t = 0.9999, \ C_u/m_t = 1.381 \times 10^{-3}, \\
A_d/m_b &= 0.9020, \ B_d/m_b = 0.04512, \ B'_d/m_b = 0.4297, \ C_d/m_b = 4.554 \times 10^{-3}, \\
\phi &= 0.1038.
\end{aligned}
\tag{4.39}
$$

The resulting mass eigenvalues at $\mu = 1$ TeV are:

$$
\begin{aligned}
m_u &= 1.12 \text{ MeV}, \ m_c = 535 \text{ MeV}, \\
m_d &= 2.27 \text{ MeV}, \ m_s = 50.0 \text{ MeV},
\end{aligned}
\tag{4.40}
$$

where we have used $m_t = 150.3$ GeV and $m_b = 2.46$ GeV as in the case of $\mathcal{P}_I$. These values are to be compared with quark masses given in Eq. (3.26). The input values of Eq. (4.39) give the output for the CKM parameters:

$$
\begin{aligned}
\lambda &= 0.2254, , \ A = 0.7987 \ , \ \bar{\rho} = 0.1575 \ , \ \bar{\eta} = 0.4231 \ , \\
\sin 2\beta &= 0.8021 \ , \ \alpha = 83.7 \, [\text{deg}] \ , \ \beta = 26.7 \, [\text{deg}] \ , \ \gamma = 69.9 [\text{deg}] \ ,
\end{aligned}
\tag{4.41}
$$

which should be compared with the fit result of the CKMfitter group (3.29).

Fig. 3 shows the prediction in the $\bar{\rho} - \bar{\eta}$ plane, and Fig. 4 shows the prediction in the $\beta - \gamma$ plane for model $\mathcal{P}_{II}$. The CKMfitter group best fit values (3.29) as well as the SM best fit values are indicated in these plots. As in the case of $\mathcal{P}_I$, we see from Eqs. (4.40), Fig. 3 and Fig. 4 that the model $\mathcal{P}_{II}$ also reproduces the quark masses, CKM mixings and the CP violating phase in an excellent way.

5 Predictive scenario for neutrino mixing

The lepton sector of model $\mathcal{P}_I$ with the Q_6 assignment given in Table 1 has been studied in Ref. [8], and therefore we will not discuss it further here. It is interesting to see if there are any constraints on neutrino oscillation parameters for model $\mathcal{P}_{II}$. Here we explore an alternative possibility of the Q_6 assignment for the leptons, which is given in Table 2.

In this new scenario, the leptonic part of the superpotential (2.5) becomes

$$
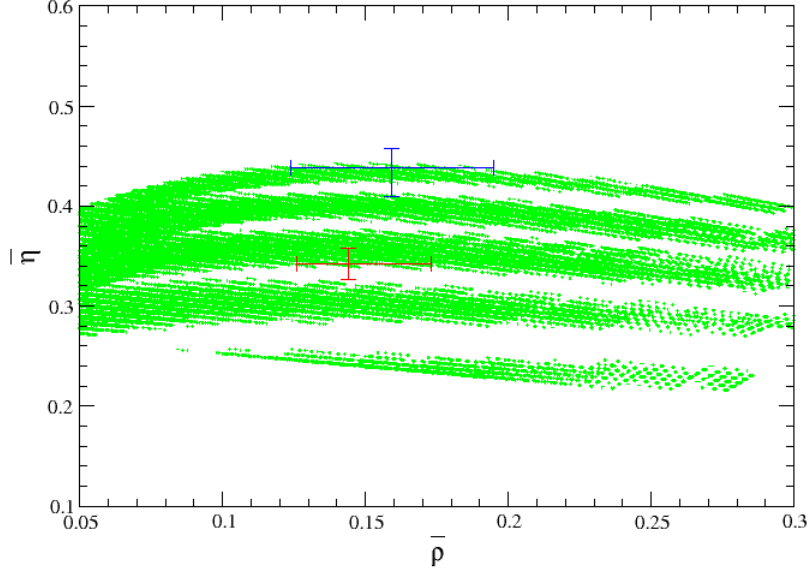W_{\text{Yukawa}}\ell = b_e(L \cdot H^d)e_3^c + b'_e L_3(H^d \cdot e^c) + c_e(L \otimes e^c) \cdot H^d
$$

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Figure 3: The prediction in the $\bar{\rho}-\bar{\eta}$ plane for the model $\mathcal{P}_{II}$, where we have used as the input parameters; the quark masses, λ and A given in Eqs. (3.26) and (3.28), respectively. We also have imposed the constraints on the quark mass ratios [19]: $2m_s/(m_u + m_d) = 22 \sim 30$, $m_s/m_d = 17 \sim 22$, $m_u/m_d = 0.35 \sim 0.60$. The crosses are the CKMfitter group values [9]; blue (scenario I) and red (SM).

$$
\begin{aligned}
&+ \quad a_\nu L_3 \nu_3^c H_3^u + b_\nu' L_3 (H^u \cdot \nu^c) + c_\nu (L \otimes \nu^c) \cdot H^u \\
&+ \quad \frac{M_1}{2} \nu^c \cdot \nu^c + \frac{a_{\nu^c}}{2} \nu_3^c \nu_3^c T_3 \ ,
\end{aligned}
\tag{5.42}
$$

where the $\cdot$ and $\otimes$ products are defined in (2.6) and (2.11), respectively.

The Majorana mass matrix for the right–handed neutrinos is given by

$$
M_{\nu^c} = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_1 & 0 \\ 0 & 0 & M_3 \end{pmatrix} \ ,
\tag{5.43}
$$

where $M_3 = a_{\nu^c}\, v_{T_3}$. Note that M_1 and M_3 are both real. The Dirac neutrino and charged lepton mass matrices are:

$$
M_{\nu^D} = \begin{pmatrix} -C_\nu e^{i\phi_u} & C_\nu e^{-i\phi_u} & 0 \\ C_\nu e^{-i\phi_u} & C_\nu e^{i\phi_u} & 0 \\ B_\nu' e^{i\phi_u} & B_\nu' e^{-i\phi_u} & A_\nu \end{pmatrix} \ , \ M_\ell = \begin{pmatrix} -C_\ell e^{i\phi_d} & C_\ell e^{-i\phi_d} & B_\ell e^{i\phi_d} \\ C_\ell e^{-i\phi_d} & C_\ell e^{i\phi_d} & B_\ell e^{-i\phi_d} \\ B_\ell' e^{i\phi_d} & B_\ell' e^{-i\phi_d} & 0 \end{pmatrix} \ .
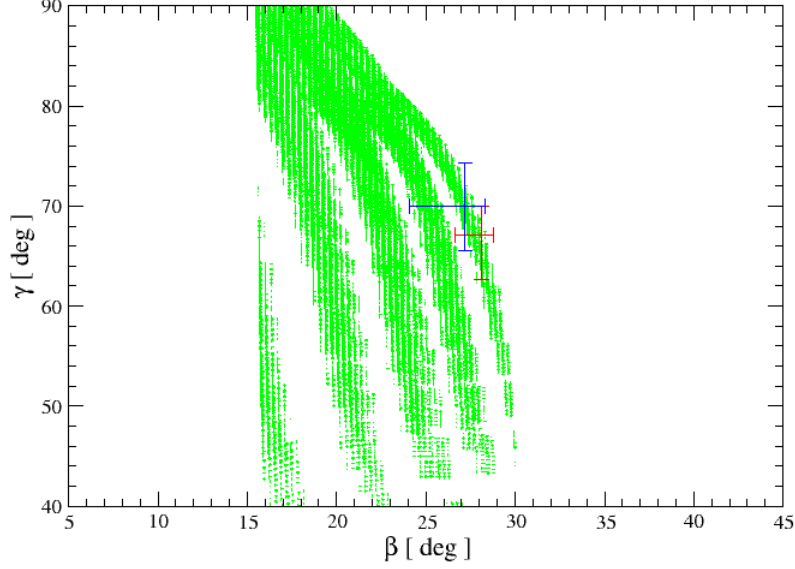\tag{5.44}
$$

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Figure 4: The prediction in the $\beta - \gamma$ plane for the model $\mathcal{P}_{II}$. The input parameters and the constraints are the same as for Fig. 3.

Table 2: An alternative $Q_6 \times Z_4$ assignment for the leptons.

	L	$\{e^c, \nu^c\}$	L_3	e_3^c	ν_3^c
Q_6	$2'$	$2'$	1	1	$1''$
Z_4	$-i$	$+$	$-i$	$+$	$+$

The light neutrino Majorana mass matrix is found (by the seesaw formula) to be

$$M_\nu^{\text{light}} = m_0 \begin{pmatrix} 2\rho_2^2 \cos(2\phi_u) & 0 & -2i\rho_2\rho_4 \sin(2\phi_u) \\ 0 & 2\rho_2^2 \cos(2\phi_u) & 2\rho_2\rho_4 \\ -2i\rho_2\rho_4 \sin(2\phi_u) & 2\rho_2\rho_4 & -\rho_3^2 + 2\rho_4^2 \cos(2\phi_u) \end{pmatrix} , \qquad (5.45)$$

where

$$\rho_2^2 = (C_\nu)^2/M_1 , \quad \rho_3^2 = -(A_\nu)^2/M_3 , \quad \rho_4^2 = (B_\nu')^2/M_1 . \qquad (5.46)$$

We have assumed that M_1 is positive, while M_3 is negative. When $\phi_u = 0$, the neutrino mass matrix is exactly the same as the matrix discussed in [20], and yields only a tiny

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$U_{e3} \sim m_e/m_\mu \sim 10^{-3}$, where U_{e3} is the $(e,3)$ element of the Maki-Nakagawa-Sakata (MNS) mixing matrix U_{MNS}. It was also shown there that the mass matrix (5.45) with $\phi_u = 0$ can yield consistent neutrino masses and mixing only if M_3 is negative, and the mass spectrum is inverted. This conclusion also applies to the present case with non-vanishing ϕ_u. For non-zero ϕ_u, we obtain $U_{e3} \sim \sin 2\phi_u$, which can be small or large. We vary $|U_{e3}|$ in its entire range allowed by experiments and correlate its value with other observables.
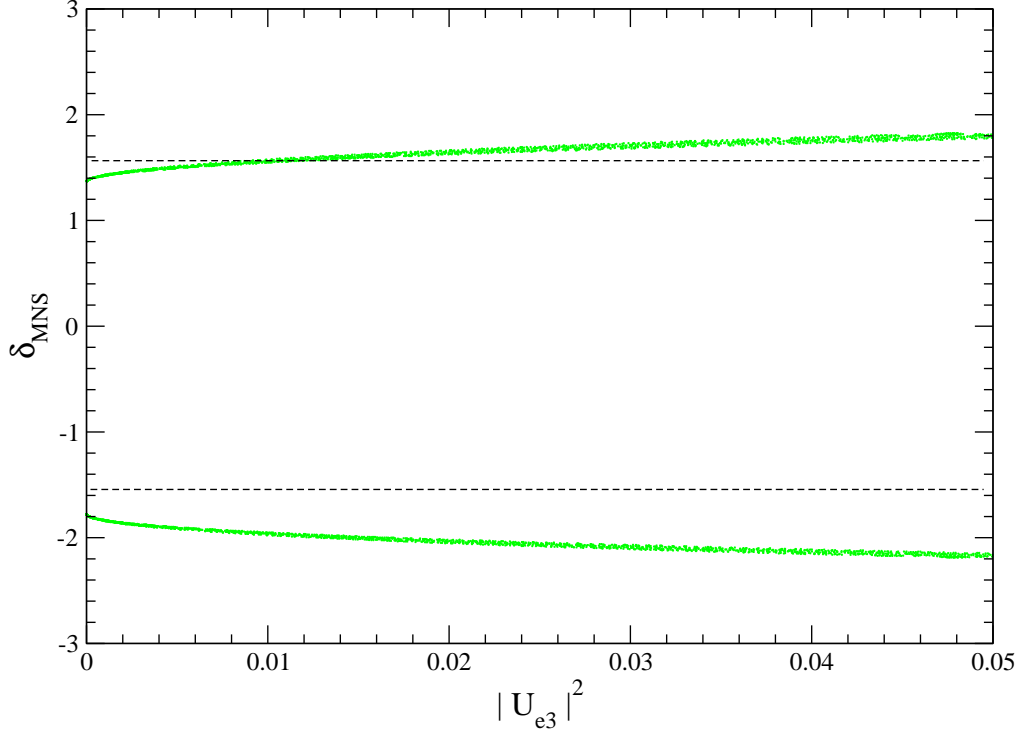

Figure 5: The prediction in the $|U_{e3}|^2 - \delta_{\mathrm{MNS}}$ plane for the model $\mathcal{P}_{II}$ with the Q_6 assignment of the leptons given in Table 2, where we have used the parameters given in (5.52), and $\phi_d = \phi_u + 0.1038$. The dashed vertical line is the maximal CP violation.

We make the matrix (5.45) real by redefining $\nu_1 = i\nu_1'$. The resulting mass matrix $\hat{M}_\nu^{\mathrm{light}}$ can be diagonalized by an orthogonal matrix $\mathcal{O}_\nu$ as $\mathcal{O}_\nu^T \hat{M}_\nu^{\mathrm{light}} \mathcal{O}_\nu$. As for the charged lepton mass matrix M_ℓ, we can obtain hierarchical masses, e.g., $m_e \sim B_\ell'$, $m_\mu \sim C_\ell$, $m_\tau \sim B_\ell$. Keeping this in mind we rotate M_ℓ according to
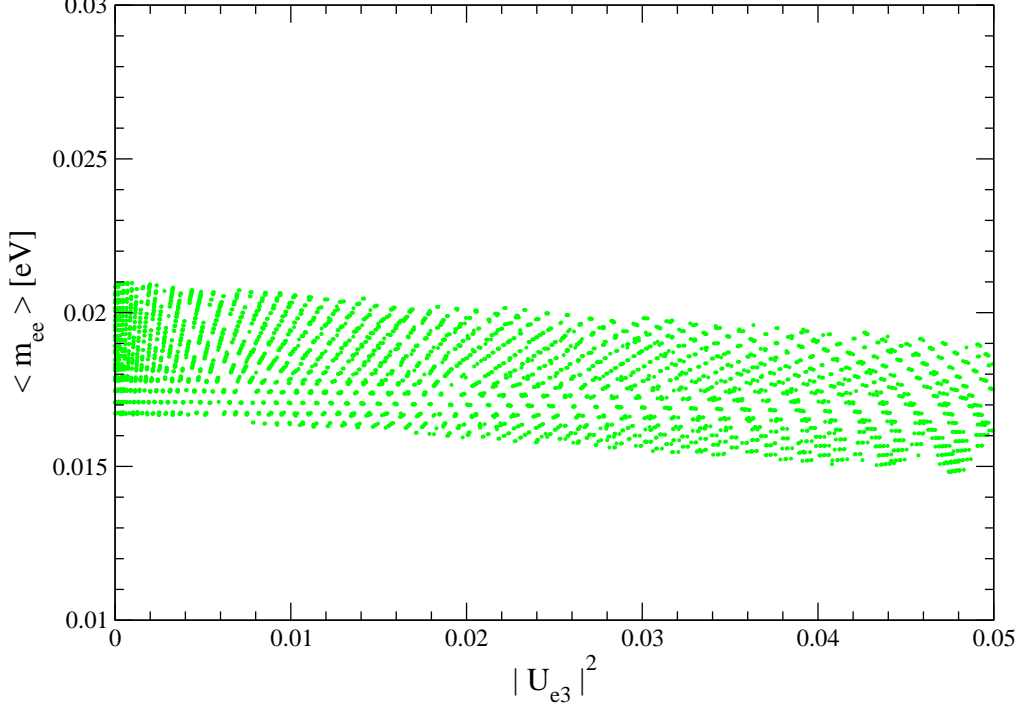
$$\hat{M}_\ell = P_L M_\ell P_R , \qquad (5.47)$$

Figure 6: The prediction in the $|U_{e3}|^2 - <m_{ee}>$ plane for the same input parameters as Fig. 5.

where

$$P_L = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\phi_d} & -e^{i\phi_d} & 0 \\ -ie^{-i\phi_d} & -ie^{i\phi_d} & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix} , \quad P_R = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\phi_d} & e^{-i\phi_d} & 0 \\ -e^{i\phi_d} & e^{i\phi_d} & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix} . \quad (5.48)$$

Then we consider $\hat{M}_\ell \hat{M}_\ell^\dagger$ in the limit $B'_\ell \to 0$, (i.e. $m_e \to 0$), and find

$$\hat{M}_\ell \hat{M}_\ell^\dagger = \begin{pmatrix} C_\ell^2(3 - \cos(4\phi_d)) & C_\ell^2 \sin(4\phi_d) & 0 \\ C_\ell^2 \sin(4\phi_d) & 2B_\ell^2 + C_\ell^2(1 + \cos(4\phi_d) & 0 \\ 0 & 0 & 0 \end{pmatrix} . \quad (5.49)$$

The eigenvalues in the limit are

$$m_e^2 = 0 , \quad m_\mu^2 \simeq C_\ell^2(3 - \cos(4\phi_d)) , \quad m_\tau^2 \simeq 2B_\ell^2 + C_\ell^2(1 + \cos(4\phi_d)) , \quad (5.50)$$

and the (inverse) diagonalizing orthogonal matrix $(\mathcal{O}_\ell^T \hat{M}_\ell \hat{M}_\ell^\dagger \mathcal{O}_\ell)$ is found to be

$$\mathcal{O}_\ell^T \simeq \begin{pmatrix} 0 & 0 & 1 \\ 1 & -\frac{C_\ell^2}{2B_\ell^2} \sin(4\phi_d) & 0 \\ \frac{C_\ell^2}{2B_\ell^2} \sin(4\phi_d) & 1 & 0 \end{pmatrix} . \quad (5.51)$$
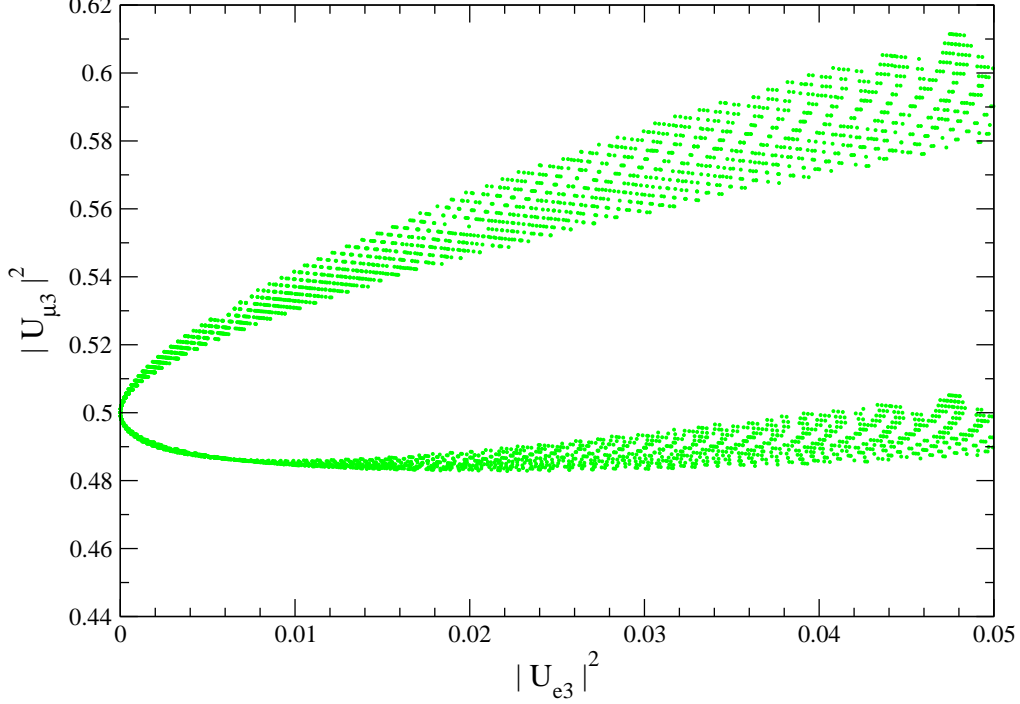
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Figure 7: $|U_{\mu 3}|^2$ against $|U_{e3}|^2$ for the same input parameters as Fig. 5.

Since the relative phase $\phi = \phi_d - \phi_u$ is fixed in the quark sector, there are seven independent parameters in the lepton sector. We use [21]:

$$m_e = 0.511 \text{ MeV} , \ m_\mu = 105.7 \text{ MeV} , \ m_\tau = 1.777 \text{ GeV} , \ |U_{e2}|^2 = 0.318 \ ^{+0.019}_{-0.016}$$

$$\Delta m_{13}^2 = (2.40 \ ^{+0.12}_{-0.11}) \times 10^{-3} \text{ eV}^2 , \ \Delta m_{21}^2 = (7.59 \ ^{+0.23}_{-0.18}) \times 10^{-5} \text{ eV}^2 \qquad (5.52)$$

as input parameters. The MNS neutrino mixing matrix is then given by

$$U_{\text{MNS}} = \mathcal{O}_\ell^T P_L P_\nu \mathcal{O}_\nu \times \text{diag.}\{ 1 , i , 1 \} , \qquad (5.53)$$

where the last phase factor multiplied with U_{MNS} is the Majorana phase, and $P_\nu = \text{diag.}\{ i , 1 , 1 \}$, which was introduced to make the matrix (5.45) real. In the lepton sector we have only one free phase ϕ_u, which controls U_{e3}. In the following calculations we use $\phi_d = \phi_u + 0.1038$ (see (4.39).

Fig. 5 shows the Dirac phase δ_{MNS} (in the convention of Ref. [19]) against $|U_{e3}|^2$. We see that the model predicts nearly maximal CP violation. This can be understood as follows. Consider the limit $m_e, \phi_u, \phi_d \to 0$. In this limit, only P_ν contributes to δ_{MNS}, and the first element of P_ν, $e^{i\pi/2}$, appears as the Dirac phase.

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It is possible to predict the effective neutrino mass $< m_{ee} >= |m_{\nu_1} U_{e1}^2 + m_{\nu_2} U_{e2}^2 + m_{\nu_3} U_{e3}^2|$ for neutrinoless double beta decay as a function of $|U_{e3}|$. Note that the first row of $\mathcal{O}_\ell^T P_L P_\nu$ is diag.$\{ 0 , 0 , 1 \}$ in the $m_e \to 0$ limit. Since $\mathcal{O}_\nu$ is real, the first and third elements of the first row of U_{MNS} are real, while the second element is purely imaginary. Therefore,

$$< m_{ee} > \simeq |m_{\nu_1} \cos^2 \theta_{\text{sol}} - m_{\nu_2} \sin^2 \theta_{\text{sol}}| \simeq m_{\nu_2} \cos 2\theta_{\text{sol}} \simeq 0.4 \; m_{\nu_2}. \tag{5.54}$$

In Fig. 6 we plot the prediction in the $|U_{e3}|^2 - < m_{ee} >$ plane, which verifies the rough estimate above. The main contribution to $|U_{\mu3}|$ comes from $\mathcal{O}_\ell$. In the limit $m_e, \phi_u \to 0$, it is exactly $1/\sqrt{2}$, so the maximal mixing. The deviation from the maximal mixing has terms proportional to m_e/m_μ and to $\sin 2\phi_u$. In Fig. 7 we plot $|U_{\mu3}|^2$ against $|U_{e3}|^2$, verifying our expectation. Note that the entire range of $|U_{e3}|$ allowed by experiments currently is also allowed by atmospheric neutrino oscillations. But once the $|U_{e3}|$ is measured, the model will make precise prediction for $|U_{\mu3}|$ which can be scrutinized with improved precision experiments.

6 Conclusions

The Q_6 model of flavor is constructed to solve the SUSY flavor problem of the supersymmetric standard model. It also yields an interesting prediction for the quark mixing parameters, which compares very well with experimental data. An unbroken interchange symmetry plays an important role in obtaining the quark mixing parameter prediction. In this paper we have updated this prediction, and compared it with the best fit values within the standard model as well as with new physics contributions assumed in $B_{d,s} - \overline{B}_{d,s}$ mixing amplitudes. The model prediction is in very good agreement with the data.

A major observation of the present paper is the existence of a new minimum that violates CP symmetry spontaneously, but leaves a new interchange symmetry unbroken. In this minimum, there is again a prediction for quark mixing parameters. We have analyzed this prediction and found that it fits data (within the CKM model and with new physics included) rather well. We have extended this symmetry to the leptonic sector, and have found various correlations between neutrino oscillation parameters.

We conclude with several comments on the new solution found.

(1) The SUSY flavor problem is solved in the new ground state $\mathcal{P}_{II}$ in the same way it is solved in $\mathcal{P}_I$. Q_6 invariance requires the first two family squarks and sleptons to be degenerate in mass, which provides the needed SUSY GIM mechanism. Since after Q_6 breaking the Q_6

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doublet and singlet quark states mix, there is residual flavor violation mediated by the SUSY particles, but such FCNC processes are well within experimental limits.

(2) The SUSY CP problem is solved in the model by virtue of spontaneous CP violation. The fundamental parameters in the Lagrangian are all real, complex phases develop only spontaneously via the VEVs of $H^{u,d}$ and S, T, U fields. This implies that the soft SUSY breaking parameters such as the gluino mass are all real, which alleviates bulk of the SUSY phase problem. The trilinear SUSY breaking A–terms are not proportional to the corresponding Yukawa couplings, however the phases in these A–terms, since they arise spontaneously, will align with the phases in the fermion mass matrices. Thus the A–terms do not generate CP violation. There is CP violation arising from the μ–terms, but as suggested in Ref. [11], if the Higgsino masses are parametrically smaller than the squark and slepton masses, this CP violation is not excessive. We also note that in the new minimum $\mathcal{P}_{II}$, the spontaneously induced phase that is necessary for KM CP violation is rather small, ~ 0.1038. One can then assume an approximate CP symmetry for the entire Lagrangian, where all the phases remain small, of this order. This will further suppress the SUSY phase effects.

(3) The new interchange symmetry $\mathcal{P}_{II}$ might appear to be CP transformation, but it actually is not. If it were CP transformation, when extended to the fermion Yukawa sector, that would make the parameters $c_{u,d,\ell,\nu}$ in Eq. (2.5) purely imaginary. CP violation will then disappear from the CKM matrix, as it should, since this symmetry remains unbroken. The symmetry $\mathcal{P}_{II}$ is an accidental symmetry of the Higgs potential, and is not respected by the Yukawa couplings, just as it was for the interchange symmetry $\mathcal{P}_I$. This state leads to a new sum rule involving the quark masses and CKM mixing parameters, which is found to be in good agreement with data. Extension of the model to the neutrino sector, by changing the Q_6 assignment of the leptons, can lead to a predictive scenario. In this version we find that neutrino mass hierarchy is inverted with nearly maximal CP violation along with nearly maximal mixing of atmospheric neutrinos. Thus the model lends itself to experimental scrutiny in the near future.

(4) The question of whether it is possible to obtain a large CP violation in the $B_s^0 - \bar{B}_s^0$ mixing for the case of $\mathcal{P}_{II}$, as in the case of $\mathcal{P}_I$ [17], remains to be studied. To distinguish two ground states of the same model, precise measurements of the CKM parameters [22] and neutrino oscillation parameters [23] as well as precise determination of the quark masses [24] are indispensable.

Acknowledgments

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