Spin filter in deeply virtual Compton scattering amplitudes

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Spin Filter in DVCS amplitudes

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Whether the kinematics includes the hard transverse photon momenta or not makes a dramatic difference in computing deeply virtual Compton scattering (DVCS) in terms of the widely used reduced operators that define generalized parton distributions (GPDs). Our tree-level complete DVCS amplitude including the lepton current plays the role of spin filter to analyze such kinematic dependence on the contribution of longitudinally polarized virtual photon as well as the conservation of angular momentum.

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For some time already, it has been realized that in non-forward kinematics, e.g. deeply virtual Compton scattering (DVCS), the scattering amplitudes, and thus cross sections, can be expressed in terms of objects, generalized parton distributions (GPDs), which complement the knowledge encoded in parton distribution functions [1–3]. This idea has inspired many authors, whose work has been summarized in several important review papers [4–6].

The paramount feature of the treatment of deep inelastic scattering (DIS) and DVCS is factorization, i.e., writing the full scattering amplitude as a convolution of a hard-scattering amplitude to be calculated in perturbation theory, and a soft part embodying the hadronic structure. The use of a hard photon that is far off-shell, say $-q^2 = Q^2 \gg$ any relevant soft mass scale, enables factorization theorems [7] with the identification of the hard scattering amplitude. Light-front dynamics (LFD) (see e.g. Ref. [8]) can be invoked to further analyze the physics, as it has the advantage that vacuum diagrams are either rigorously absent or suppressed. In the context of single-photon physics (e.g. hadron form factors), it means that in a reference frame where the momentum of the photon q^μ has vanishing plus component [9]: $q^+ \equiv (q^0 + q^3)/\sqrt{2} = 0$, it cannot create partons, as their momenta must have positive plus-components and these components are conserved in LFD. This simplification facilitates the partonic interpretation of amplitudes [10]. In two-photon physics such as DVCS, however, both photons cannot have vanishing plus components simultaneously and thus further investigation is called for to analyze the choice of a preferred kinematics in which the amplitudes are calculated and the link between the theoretical quantities, GPDs, and the cross sections can be established.

This paper is devoted to the issue of kinematics in computing the DVCS amplitude in terms of widely used reduced operators that define GPDs. For example, the operator γ^+ is not invariant under the transformation from the $\vec{q}_\perp = 0$ frame to the $q^+ = 0$ frame. In effect, the choice of reference frame matters in computing the DVCS amplitude in terms of GPDs. We discuss this in the simplest possible setting, namely DVCS on a structure-less spin-1/2 particle. Although this might seem to preclude the discussion of the GPD formalism, we shall argue that important lessons can be learnt from the anlysis of this "bare bone" structure on top of which the GPDs are formulated.

Before we get into the discussion of the GPD formalism, we first report our benchmark calculation of the complete full DVCS amplitude for the scattering of a massless lepton ℓ off a point-like fermion f of mass m. In the final state, we find the scattered lepton ℓ', the fermion f' with momentum k' and a (real) photon γ', viz $\ell \to \ell' + \gamma^*$, $\gamma^* + f \to \gamma' + f'$. ('Complete' means that the amplitude includes the leptonic part and 'full' means that no approximations are made in the calculation of the hadronic amplitude.) The complete amplitude at tree level can be written as

$$\mathcal{M} = \sum_h \mathcal{L}(\{\lambda', \lambda\}h) \frac{1}{q^2} \mathcal{H}(\{s', s\}\{h', h\}), \qquad (1)$$

where the quantities λ', λ, h', h, s', and s are the helicities of the outgoing and incoming leptons, outgoing and incoming photons, and the rescattered and target fermions, respectively. Leaving out inessential factors, we may write

$$\begin{aligned} \mathcal{L}(\{\lambda', \lambda\}h) &= \bar{u}(\ell'; \lambda') \not{\epsilon}^*(q; h) u(\ell; \lambda), \\ \mathcal{H}(\{s', s\}\{h', h\}) &= \bar{u}(k'; s')(\mathcal{O}_s + \mathcal{O}_u) u(k; s), \end{aligned} \qquad (2)$$

where the s- and u-channel operators of the intermediate fermion are given by

$$\begin{aligned} \mathcal{O}_s &= \frac{\not{\epsilon}^*(q'; h')(\not{k} + \not{q} + m)\not{\epsilon}(q; h)}{(k+q)^2 - m^2}, \\ \mathcal{O}_u &= \frac{\not{\epsilon}(q; h)(\not{k} - \not{q}' + m)\not{\epsilon}^*(q'; h')}{(k-q')^2 - m^2}. \end{aligned} \qquad (3)$$

We take the following three kinematics for the momenta of the incoming and outgoing particles in the hadronic amplitude:

(1) δ-Kinematics ($q^+ \to 0$ as $\delta \to 0$)

$$\begin{aligned}
q^\mu &= \left(\delta p^+, Q, 0, \frac{Q^2}{2(\zeta+\delta)p^+} + \frac{\zeta m^2}{2x(x-\zeta)p^+}\right), \\
q'^\mu &= \left((\zeta+\delta)p^+, Q, 0, \frac{Q^2}{2(\zeta+\delta)p^+}\right), \\
k^\mu &= \left(xp^+, 0, 0, \frac{m^2}{2xp^+}\right), \\
k'^\mu &= \left((x-\zeta)p^+, 0, 0, \frac{m^2}{2(x-\zeta)p^+}\right), \qquad (4)
\end{aligned}$$

(2) $q'^+ = 0$ Kinematics (effectively, '1+1' dim.)

$$\begin{aligned}
q^\mu &= \left(-\zeta p^+, 0, 0, \frac{Q^2}{2\zeta p^+}\right), \\
q'^\mu &= \left(0, 0, 0, \frac{Q^2}{2\zeta p^+} - \frac{\zeta m^2}{2x(x-\zeta)p^+}\right). \qquad (5)
\end{aligned}$$

The momenta k^μ and k'^μ are the same as in case (1).

(3) Nonvanishing q^+ and q'^+ Kinematics (with $m = 0$)

$$\begin{aligned}
q^\mu &= \left(-\frac{\zeta}{2}p^+, \frac{Q}{\sqrt{2}}, 0, \frac{Q^2}{2\zeta p^+}\right), \\
q'^\mu &= \left(\frac{\zeta}{2}p^+, \frac{Q}{\sqrt{2}}, 0, \frac{Q^2}{2\zeta p^+}\right). \qquad (6)
\end{aligned}$$

The momenta k^μ and k'^μ are the same as in case (1) if the limit $m \to 0$ is taken.

These kinematics correspond to the hard-scattering part of a DVCS amplitude where the fermions are the quarks and p^+ is the plus-component of the momentum of the parent hadron target. We use the Kogut-Soper spinors [11] normalized to $2m$ and the polarization vectors

$$\begin{aligned}
\epsilon(q;\pm 1) &= \frac{1}{\sqrt{2}}\left(0, \mp 1, -i, \mp\frac{q_x \pm i q_y}{q^+}\right), \\
\epsilon(q;0) &= \frac{1}{\sqrt{q^2}}\left(q^+, q_x, q_y, \frac{q_\perp^2 - q^2}{2q^+}\right), \qquad (7)
\end{aligned}$$

that correspond to the LF gauge $A^+ = 0$.

All of these three kinematics yield identical kinematical invariants such as $s = \frac{x-\zeta}{\zeta}Q^2$ and $u = -\frac{x}{\zeta}Q^2$ in the DVCS limit as $\delta \to 0$ and $m \to 0$. To make sure of the consistency in limiting procedures, we have checked explicitly that the two limits, $\delta \to 0$ and $Q \to \infty$, commute with each other. As we discuss below, each of the above three kinematics has its own merit of consideration.

In the $\delta \to 0$ limit, the δ-kinematics coincides with the well-known $q^+ = 0$ frame [12] frequently cited in the discussion of the GPD formalism. Noticing that taking $q^+ = 0$ will lead to singular polarization vectors in the LF gauge $A^+ = 0$ (see e.g. Eq. (7)), we proceed with care: q^+ is set to δp^+ and all amplitudes are expanded in powers of δ, taking the limit $\delta \to 0$ at the very end of the calculation of the complete, physical amplitude.

TABLE I: Leptonic amplitudes in kinematics corresponding to Eqs. (4)-(6)

	$\mathcal{L}(\{\lambda',\lambda\}h)$		
$\{\lambda',\lambda\}$ h	Eq. (4)	Eq. (5)	Eq. (6)
$\{\frac{1}{2},\frac{1}{2}\}$ $+1$	$-Q\left(1-\frac{\delta}{4\zeta}+\frac{2\zeta}{\delta}\right)$	0	$2Q$
$\{\frac{1}{2},\frac{1}{2}\}$ -1	$-Q\left(1-\frac{3\delta}{4\zeta}-\frac{2\zeta}{\delta}\right)$	$-2Q$	$-4Q$
$\{\frac{1}{2},\frac{1}{2}\}$ 0	$-i2\sqrt{2}Q\,\frac{\zeta}{\delta}$	0	$4iQ$

TABLE II: Hadronic amplitudes in DVCS in three kinematics given by Eqs. (4)-(6)

		$\mathcal{H}(\{h',h\}\{s',s\})$		
$\{h',h\}$	$\{s',s\}$	Eq. (4)	Eq. (5)	Eq. (6)
$\{+1,+1\}$	$\{\frac{1}{2},\frac{1}{2}\}$	$2\sqrt{\frac{x}{x-\zeta}}\left(1+\frac{\zeta}{\delta}\right)$	$2\sqrt{\frac{x-\zeta}{x}}$	$-2\sqrt{\frac{x}{x-\zeta}}$
$\{+1,+1\}$	$\{-\frac{1}{2},-\frac{1}{2}\}$	$2\sqrt{\frac{x-\zeta}{x}}\left(1+\frac{\zeta}{\delta}\right)$	$2\sqrt{\frac{x}{x-\zeta}}$	$-2\sqrt{\frac{x-\zeta}{x}}$
$\{+1,-1\}$	$\{\frac{1}{2},\frac{1}{2}\}$	$-2\sqrt{\frac{x}{x-\zeta}}\,\frac{\zeta}{\delta}$	0	$4\sqrt{\frac{x}{x-\zeta}}$
$\{+1,-1\}$	$\{-\frac{1}{2},-\frac{1}{2}\}$	$-2\sqrt{\frac{x-\zeta}{x}}\,\frac{\zeta}{\delta}$	0	$4\sqrt{\frac{x-\zeta}{x}}$
$\{+1,\ 0\}$	$\{\frac{1}{2},\frac{1}{2}\}$	$i\sqrt{2}\sqrt{\frac{x}{x-\zeta}}\left(1+\frac{2\zeta}{\delta}-\frac{\delta}{4\zeta}\right)$	0	$-i4\sqrt{\frac{x}{x-\zeta}}$
$\{+1,\ 0\}$	$\{-\frac{1}{2},-\frac{1}{2}\}$	$i\sqrt{2}\sqrt{\frac{x-\zeta}{x}}\left(1+\frac{2\zeta}{\delta}-\frac{\delta}{4\zeta}\right)$	0	$-i4\sqrt{\frac{x-\zeta}{x}}$

The $q'^+ = 0$ kinematics without any transverse component (effectively, '1+1' dimensional) avoids the singularity in the polarization vectors of the real photon and consequently provides a convenient framework of calculation without encountering any singularity. Similarly, the nonvanishing q^+ and q'^+ kinematics also avoids the singularity in the amplitude calculation, while the photons carry the same order of transverse momenta as the ones in the δ-kinematics given by Eq. (4).

The results from these three kinematics are summarized in Tables I, II, and III. A straightforward evaluation of $\mathcal{L}(\{\lambda',\lambda\}h)$ gives the result in Table I, where we have used the corresponding lepton kinematics[1] to Eqs. (4)-(6) and presented the results only up to order δ as well as in the DVCS limit. For the massless leptons helicity is conserved. The amplitudes not shown in Table I can be obtained using the helicity rule

$$\mathcal{L}(\{-\lambda',-\lambda\}\ -h) = (-1)^{\lambda'-\lambda+h}\mathcal{L}(\{\lambda',\lambda\}\ h). \qquad (8)$$

The full hadronic amplitudes are shown in Table II, where we again presented the results only up to order δ. They obey the rule

$$\mathcal{H}(\{-h',-h\}\{-s',-s\}) = (-1)^{h-h'-s+s'}\mathcal{H}(\{h',h\}\{s',s\}). \qquad (9)$$

The complete DVCS amplitude $\mathcal{M}$ in Eq. (1) is shown in Table III. Since all the singular terms of orders δ^{-2} and

[1] The details of lepton kinematics and spinors will be presented somewhere else.

TABLE III: Complete DVCS amplitudes, in three kinematics given by Eqs. (4)-(6)

			$\sum_h \mathcal{L}(\{\lambda'=\lambda\},h)\frac{1}{q^2}\mathcal{H}(\{h',h\}\{s',s\})$		
$\lambda'=\lambda$	h'	$s'=s$	Eq. (4)	Eq. (5)	Eq. (6)
$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{4}{Q}\sqrt{\frac{x}{x-\zeta}}$	0	$\frac{4}{Q}\sqrt{\frac{x}{x-\zeta}}$
$\frac{1}{2}$	1	$-\frac{1}{2}$	$\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}$	0	$\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}$
$-\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}$	0
$-\frac{1}{2}$	1	$-\frac{1}{2}$	0	$-\frac{4}{Q}\sqrt{\frac{x}{x-\zeta}}$	0
$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{4}{Q}\sqrt{\frac{x}{x-\zeta}}$	0
$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}$	0
$-\frac{1}{2}$	-1	$\frac{1}{2}$	$-\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}$	0	$-\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}$
$-\frac{1}{2}$	-1	$-\frac{1}{2}$	$-\frac{4}{Q}\sqrt{\frac{x}{x-\zeta}}$	0	$-\frac{4}{Q}\sqrt{\frac{x}{x-\zeta}}$

δ^{-1} are exactly cancelled out in the complete amplitude, we have taken $\delta = 0$ in Table III. Note in Table III that there is an interchange[2] of the polarization of the final photon in the result of the '1+1' dim. kinematics in comparison with the other kinematics, in which the momenta of photons have transverse components. This is remarkable in view of the LF helicity [13]. One should realize that the LF helicity states are defined for a momentum q' by taking a state at rest with the spin projection along the z direction equal to the desired helicity, then boosting in the z direction to get the desired q'^+, and then doing a LF transverse boost (i.e., $E_1 = K_1+J_2$ [13]) to get the desired transverse momentum $\vec{q'}_\perp$. Whether the kinematics includes the LF transverse boost (E_1) or not makes a dramatic difference in the spin direction because E_1 rotates the spin direction. Thus, the spin direction of the LF helicity state is opposite (or antiparallel) to the direction of the photon momentum when the photon doesn't carry any transverse momentum but moves only in the $-z$ direction as in the case of the outgoing photon given by Eq. (5). When the photon carries the transverse momentum of order Q as in the kinematics given by Eq. (4) or Eq. (6), the spin directions of the LF helicity state and the Jacob-Wick helicity state [14] are related [13] by the Wigner function $d^1_{h',h'}(\tan^{-1}\frac{2m}{Q})$, which becomes unity for the outgoing photon. This illustrates the correspondence between the results of a kinematics with $\vec{q'}_\perp = 0$ and a kinematics with the transverse momentum of order Q: e.g. in Table III, the result of $h' = 1$ in the effective '1+1D' kinematics corresponds to the result of $h' = -1$ in the δ-kinematics or the nonvanishing q^+ and q'^+ kinematics for $\lambda' = \lambda = \frac{1}{2}$ and $s' = s = \frac{1}{2}$. One should note that the conservation of angular momentum is satisfied in the complete full amplitudes for <u>any</u> kinematics. Therefore, we may take the calculation <u>up</u> to now as a benchmark for the discussion of the GPD formalism as we do below. Rewriting the s- and u- channel hadronic amplitudes as

$$\begin{aligned}\bar{u}(k';s')\mathcal{O}_s u(k;s) &= \epsilon_\mu{}^*(q';h')\epsilon_\nu(q;h)T_s{}^{\mu\nu},\\ \bar{u}(k';s')\mathcal{O}_u u(k;s) &= \epsilon_\mu{}^*(q';h')\epsilon_\nu(q;h)T_u{}^{\mu\nu}, \quad (10)\end{aligned}$$

we may neglect an inessential fermion mass m to express the tensorial amplitudes $T_s{}^{\mu\nu}$ and $T_u{}^{\mu\nu}$ as

$$\begin{aligned}T_s{}^{\mu\nu} &= \frac{k_\alpha + q_\alpha}{s}\bar{u}(k';s')\gamma^\mu\gamma^\alpha\gamma^\nu u(k;s),\\ T_u{}^{\mu\nu} &= \frac{k_\alpha - q'_\alpha}{u}\bar{u}(k';s')\gamma^\nu\gamma^\alpha\gamma^\mu u(k;s), \quad (11)\end{aligned}$$

respectively. Using the identity

$$\gamma^\mu\gamma^\alpha\gamma^\nu = g^{\mu\alpha}\gamma^\nu + g^{\alpha\nu}\gamma^\mu - g^{\mu\nu}\gamma^\alpha + i\epsilon^{\mu\alpha\nu\beta}\gamma_\beta\gamma_5 \quad (12)$$

and the Sudakov variables $n^\mu(+) = (1,0,0,0)$ and $n^\mu(-) = (0,0,0,1)$, one may expand $T_s{}^{\mu\nu}$ and $T_u{}^{\mu\nu}$ to find the terms proportional to $\bar{u}(k';s')\not{n}(-)u(k;s)$ and $\bar{u}(k';s')\not{n}(-)\gamma_5 u(k;s)$ that correspond to the nucleon GPDs $H(x,\Delta^2,\zeta)$ and $\tilde{H}(x,\Delta^2,\zeta)$ defined e.g. in Ref. [1], respectively (here, $\Delta^2 = (q'-q)^2$). One should note, however, that a special system of coordinates without involving any large transverse momentum (see e.g. Eq. (5)) was chosen in Ref. [1] to compute the scattering amplitude in terms of GPDs [3].

In order to cover the more general kinematics involving large transverse momenta such as given in Eqs. (4) and (6), we may expand q^μ (similarly q'^μ) and k^μ as $q^\mu = q^+n^\mu(+)+q^-n^\mu(-)+q_\perp{}^\mu$ and $k^\mu = k^+n^\mu(+)+k^-n^\mu(-)$ with $q_\perp{}^\mu$ representing the transverse momentum corresponding to q^μ. For $m = 0$, $k^- = 0$ and $T_s{}^{\mu\nu}$ (similarly $T_u{}^{\mu\nu}$) can be expanded as

$$\begin{aligned}T_s{}^{\mu\nu} = \frac{1}{s}\Big[&\big(\{(k^+ + q^+)n^\mu(+) + q^- n^\mu(-) + q_\perp{}^\mu\}n^\nu(+)\\ &+\{(k^+ + q^+)n^\nu(+) + q^- n^\nu(-) + q_\perp{}^\nu\}n^\mu(+) - g^{\mu\nu}q^-\big)\\ &\times\bar{u}(k';s')\not{n}(-)u(k;s)\\ &-i\epsilon^{\mu\nu\alpha\beta}\{(k^+ + q^+)n_\alpha(+) + q^- n_\alpha(-) + q_{\perp\alpha}\}n_\beta(+)\\ &\times\bar{u}(k';s')\not{n}(-)\gamma_5 u(k;s)\Big]. \quad (13)\end{aligned}$$

Since q^- has the highest power of Q among the components of momenta, one may just take the terms proportional to q^- as shown in Ref. [1], i.e.,

$$\begin{aligned}T_s{}^{\mu\nu} = \frac{q^-}{s}\big[&\{n^\mu(-)n^\nu(+) + n^\nu(-)n^\mu(+) - g^{\mu\nu}\}\\ &\times\bar{u}(k';s')\not{n}(-)u(k;s)\\ &-i\epsilon^{\mu\nu\alpha\beta}n_\alpha(-)n_\beta(+)\times\bar{u}(k';s')\not{n}(-)\gamma_5 u(k;s)\big].\end{aligned}$$

[2] We have also confirmed the similar interchange of the helicity amplitudes between the kinematics with and without the transverse momentum of the virtual photon in the case of a form-factor calculation.

[3] In Ref. [2], $q = q' - \zeta p$ was taken although the physical momenta instead of the Sudakov variables were used. It was explicitly stated in Ref. [2] that writing $q = q' - \zeta p$ is equivalent to using the Sudakov decomposition in a situation when there is no transverse momentum.

Although this is correct in the frame of reference chosen in Ref. [1], one should note that Eq. (14) cannot provide the full result of the hadronic amplitude in the kinematics involving large transverse momenta such as Eq. (4) and Eq. (6), because the polarization vectors ${\epsilon_\mu}^*(q';h')$ and $\epsilon_\nu(q;h)$ in Eq. (10) amplify the contributions neglected in the tensorial amplitude ${T_s}^{\mu\nu}$ (similarly ${T_u}^{\mu\nu}$) given by Eq. (14). For example, the coefficient of $\bar{u}(k';s')\not{n}(-)u(k;s)$ in the s-channel hadronic amplitude $\bar{u}(k';s')\mathcal{O}_s u(k;s)$ is given by the following four terms:

$$\begin{aligned}\tfrac{1}{s}\ &\big[\, 2(k^+ + q^+)\epsilon^{*-}(q';h')\epsilon^-(q;h) \\ &+ \epsilon^{*-}(q';h')\ q_\perp\cdot\epsilon_\perp(q;h) + \epsilon^-(q;h)\ q_\perp\cdot{\epsilon_\perp}^*(q';h') \\ &- q^-{\epsilon_\perp}^*(q';h')\cdot\epsilon_\perp(q;h)\,\big]\,. \end{aligned} \tag{14}$$

Since all of the above four terms have the same powers of Q, one cannot just take the last term proportional to q^- but must keep all terms together. In other words, the factorization such as $\left(\frac{1}{x-\zeta}+\frac{1}{x}\right)\{n^\mu(-)n^\nu(+)+n^\nu(-)n^\mu(+)-g^{\mu\nu}\}$ for the coefficient of $\bar{u}(k';s')\not{n}(-)u(k;s)$ in the tensorial amplitude ${T_s}^{\mu\nu}+{T_u}^{\mu\nu}$ cannot hold in general because the polarization vectors ${\epsilon_\mu}^*(q';h')$ and $\epsilon_\nu(q;h)$ can amplify the terms neglected in the tensorial amplitude unless a special system of coordinates is chosen to avoid the large transverse momenta of initial and final photons such as given by Eq. (5). This is the main point of this paper. In the following, we demonstrate this point explicitly, presenting the consequence of taking the reduced amplitude that keeps only the terms proportional to q^- in the tensorial amplitude as done in the formulation of GPDs. Unless the kinematics is chosen properly to avoid the large transverse momenta of initial and final photons, we find that the reduced amplitude does not agree with the full amplitude but yield the wrong result, not even satisfying the conservation of angular momentum.

Since the $q^+=0$ frame is used [12] in the GPD formalism, we utilize the δ-kinematics for our demonstration. We perform an expansion in the hard momentum scale Q, which allows us to define reduced hadronic amplitudes. In the expansion, it is important to retain terms of orders $\delta^{-1}, \ldots \delta^2$ as well as orders $Q^{-1}, \ldots Q^2$, as it turns out that not only are the order δ^{-1}-terms cancelled by order δ terms in the convolution of $\mathcal{L}$ and $\mathcal{H}$, but also that the order Q^{-1}-contribution of the longitudinally polarized virtual photon gives a finite contribution in leading order. (We have checked that the two limits, $\delta\to 0$ and $Q\to\infty$, commute.)

The reduced hadronic operators used in the formulation of GPDs are defined as the limits $Q\to\infty$ of the operators given in Eq. (3) and found to be, as expected:

$$\begin{aligned}\mathcal{O}_s|_{\rm Red} &= \frac{\not{\epsilon}^*(q';h')\gamma^+\not{\epsilon}(q;h)}{2p^+}\,\frac{1}{x-\zeta}, \\ \mathcal{O}_u|_{\rm Red} &= \frac{\not{\epsilon}(q;h)\gamma^+\not{\epsilon}^*(q';h')}{2p^+}\,\frac{1}{x}. \end{aligned} \tag{15}$$

The $J=0$ fixed pole contribution in Eq. (15) for pointlike scattering has been discussed in Ref. [15] along with the universality of this contribution in two-photon processes. These reduced propagators contain the nilpotent Dirac matrix γ^+ only, which kills the singular parts of the polarization vectors, namely $\epsilon^-(q;h)\gamma^+$. This is the reason for disregarding the singularities in the polarization vectors in $q^+=0$ kinematics, as the reduced hadronic amplitude does not 'see' it. However, the leptonic part $\mathcal{L}$ of the complete amplitude is also singular. Consequently, the complete amplitude calculated with the reduced hadronic part and taking into account the transverse polarizations only, is wrong, even in the limit $Q\to\infty$. Contrary to the expectation[12], the contribution from the longitudinal polarization of the virtual photon is not suppressed by a factor $1/Q$ compared to the contribution from the transverse polarizations. Not only $\epsilon^-(q;h=0)$ is singular but also $q_\perp\cdot\epsilon_\perp(q;h=0)$ contributes in the same leading order of Q as the $h=\pm 1$ contributions do.

Table IV clearly shows that the reduced amplitudes and the full ones disagree. We have checked that the same disagreement occurs in the nonvanishing q^+ and q'^+ kinematics given by Eq. (6), although for the kinematics without any transverse component, e.g. Eq. (5), the reduced amplitudes and the full ones do agree. Upon convoluting the leptonic and hadronic amplitudes to obtain the complete ones, we find that the singular $1/\delta$-terms cancel in δ-kinematics, but the full and reduced hadronic amplitudes do not produce the same complete ones. Moreover, if the contribution of the longitudinal polarization of the virtual photon is neglected, i.e., if its propagator is reduced too, the singular parts do not cancel out. As such, the contribution of the longitudinal polarization should not be neglected when the photons carry transverse momenta of order Q. The inclusion of the longitudinal polarization can also be found in the previous work on the virtual Compton scattering, e.g. Ref.[16], specifically in the center of momentum frame.

TABLE IV: Complete amplitudes in δ-kinematics for $\lambda'=\lambda=\frac{1}{2}$ and $s'=s=\frac{1}{2}$

h	$\mathcal{L}\frac{1}{q^2}\mathcal{H}_{\rm Full}$ for $h'=1$	$\mathcal{L}\frac{1}{q^2}\mathcal{H}_{\rm Red}$ for $h'=1$
$+1$	$\frac{1}{Q}\sqrt{\frac{x}{x-\zeta}}\left(\frac{4\zeta^2}{\delta^2}+\frac{6\zeta}{\delta}+\frac{3}{2}-\frac{\delta}{4\zeta}\right)$	$\frac{2}{Q}\sqrt{\frac{x-\zeta}{x}}\left(\frac{2\zeta}{\delta}+1-\frac{\delta}{4\zeta}\right)$
0	$\frac{1}{Q}\sqrt{\frac{x}{x-\zeta}}\left(\frac{-8\zeta^2}{\delta^2}-\frac{4\zeta}{\delta}+1-\frac{\delta}{2\zeta}\right)$	$\frac{2}{Q}\sqrt{\frac{x-\zeta}{x}}\left(-\frac{2\zeta}{\delta}-1+\frac{\delta}{4\zeta}\right)$
-1	$\frac{1}{Q}\sqrt{\frac{x}{x-\zeta}}\left(\frac{4\zeta^2}{\delta^2}-\frac{2\zeta}{\delta}+\frac{3}{2}-\frac{5\delta}{4\zeta}\right)$	0
$\sum_h$	$\frac{1}{Q}\sqrt{\frac{x}{x-\zeta}}\left(4-\frac{2\delta}{\zeta}\right)$	0
h	$\mathcal{L}\frac{1}{q^2}\mathcal{H}_{\rm Full}$ for $h'=-1$	$\mathcal{L}\frac{1}{q^2}\mathcal{H}_{\rm Red}$ for $h'=-1$
$+1$	$\frac{1}{Q}\sqrt{\frac{x-\zeta}{x}}\left(-\frac{4\zeta^2}{\delta^2}-\frac{2\zeta}{\delta}+\frac{1}{2}-\frac{\delta}{4\zeta}\right)$	0
0	$\frac{1}{Q}\sqrt{\frac{x-\zeta}{x}}\left(\frac{8\zeta^2}{\delta^2}+\frac{4\zeta}{\delta}-1+\frac{\delta}{2\zeta}\right)$	$\frac{2}{Q}\sqrt{\frac{x}{x-\zeta}}\left(\frac{2\zeta}{\delta}+1-\frac{\delta}{4\zeta}\right)$
-1	$\frac{1}{Q}\sqrt{\frac{x-\zeta}{x}}\left(-\frac{4\zeta^2}{\delta^2}-\frac{2\zeta}{\delta}+\frac{1}{2}-\frac{\delta}{4\zeta}\right)$	$\frac{2}{Q}\sqrt{\frac{x}{x-\zeta}}\left(-\frac{2\zeta}{\delta}+1-\frac{3\delta}{4\zeta}\right)$
$\sum_h$	0	$\frac{1}{Q}\sqrt{\frac{x}{x-\zeta}}\left(4-\frac{2\delta}{\zeta}\right)$

We see in Table IV that summing the complete amplitudes over h gives the same result for the full and the reduced amplitudes, *but for the interchange of the polarization of the final photon*. As this polarization is an observable, we observe that the reduced amplitude gives the wrong amplitude. Clearly the tree-level hard amplitude plays the role of a spin filter. Using the reduced amplitudes means using a spin filter that provides an erroneous connection between the data and the GPD. Using it only for the spin-averaged data would not do, as in DVCS the GPD amplitude is added to the Bethe-Heitler amplitude with its own spin structure, so using the reduced amplitudes would mean to obtain the wrong interference terms in the expression for the cross section. Unless the full amplitude is used, the spin filter is artificial as the complete amplitude depends on the choice of the reference frame.

Since GPDs are carried on top of our bare bone spin filter, the reference frame where the experimental data are taken to extract GPDs should be transformed into the frame where the spin filter provides the correct result. Likewise, theoretical predictions based on the dominance of the handbag diagrams should be analyzed in the special system of coordinates without involving large transverse momenta as given by Eq. (5). We realize that our concern discussed in this work doesn't apply to the bulk of the GPD discussion[17–19] which refers to the kinematics where the transverse momentum of the virtual photon is not of order Q but small or zero (e.g. to the center-of-mass of virtual photon and target hadron, or to the kinematics given by Eq. (5)). We stress, however, that for a correct analysis of the experimental data and/or theoretical predictions based on the handbag dominance, one must transform the experimentally measured and/or theoretically predicted quantities to the corresponding ones in the reference frame where the transverse momenta of the photons are small compared to Q. Once the handbag diagrams are reduced to a triangle diagram, the relevant kinematic variable becomes $\Delta = q - q'$ instead of the two independent variables q and q' and the GPD parametrization made in the level of triangle diagram would not be affected by our findings. On the other hand, the handbag diagram calculations formulated in $q^+ = 0$ frame [12] should be reanalyzed in $q_\perp = 0$ frame to extract GPDs and likewise any future DVCS computations based on the box diagrams to extract GPDs in QCD or in any effective model field theories should be analyzed ultimately in the frame where there is no hard transverse photon momenta.

Based on these straightforward tree-level calculations of DVCS amplitudes, we conclude:

(i) The formulation of GPDs corresponding to the tensorial amplitude given by Eq. (14) is not general enough to cover the kinematics with large transverse momenta such as given by Eqs. (4) and (6) but requires the transformation of experimental data and/or theoretical predictions to the corresponding quantities in the special system of coordinates without involving large transverse momenta as given by Eq. (5).

(ii) In kinematics where the transverse components of the momenta are of order Q the full hadronic amplitudes and the reduced ones do not agree, even in the limit $Q \to \infty$, which means that the calculations of the DVCS amplitudes using the GPD cannot be trusted in this kinematics. It is crucial to realize that the contribution of the longitudinally polarized virtual photon is not down by one order in Q but even plays the role of cancelling the singular parts.

(iii) The singularities we have found are in no way connected to the strong-interaction part, but entirely due to the minus components of the photon-polarization vectors, meaning that a calculation beyond tree level will encounter the same singularities.

We have found [20] the same singularities to occur in real Compton scattering using the same kinematics. There they turn out to be of equal magnitude but opposite sign in the s- and u-channel amplitudes and thus cancel out, as expected.

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