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Vector Multiplet in Three Dimensions with Non-Polynomial Interactions

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Abstract

We present $f(\varphi)(F_{\mu\nu})^2$-type non-canonical and non-polynomial interactions for an $N = 1$ vector multiplet in three dimensions. We couple a Yang-Mills multiplet $(A_\mu{}^I, \lambda^I)$ to a scalar multiplet (φ, χ), where φ appears in the arbitrary scalar function $f(\varphi)$ in the coupling $\alpha f(\varphi)(F_{\mu\nu}{}^I)^2$. Supersymmetric Chern-Simons terms for the vector multiplet and a potential term for the scalar multiplet can be also added. We first give the component lagrangian, and we give its superspace re-formulation. We also give exact solutions for the vector and scalar fields in the Abelian case with a *finite* total energy.

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1. Introduction

There seem to be limited types of higher-order consistent couplings of global Yang-Mills (YM) multiplets. The most well-known example is the Dirac-Born-Infeld (DBI) interaction [1]. General supersymmetric higher-order interactions in four dimensions (4D) in terms of Abelian field strengths were considered, and its causal structures were studied [2]. The investigation of deformations from BPS solutions to vector fields with slowly-varying field strengths, showed that the DBI action is a unique deformation at least in the Abelian case [3].

On the other hand, it is well known in 4D that the vector multiplet can have non-canonical and non-polynomial couplings such as $f_{\alpha\beta}(z) F_{\mu\nu}{}^{\alpha} F^{\mu\nu\beta}$ for a general YM multiplet. Here $f_{\alpha\beta}(z)$ with two symmetric adjoint indices is an arbitrary non-polynomial function of the complex scalar field z in the chiral multiplet $(z, z^*, \chi_{\rm L}, \chi_{\rm R})$ [4]. In other words, in 4D a vector multiplet allows the presence of an arbitrary non-polynomial function of scalars in front of its kinetic terms. These couplings arise in the context of local supersymmetry or supergravity. Similar non-polynomial interaction terms have been found also with local supersymmetry, for example, in 5D [5][6] or in 9D [7][8].

These non-polynomial couplings in $D \ge 5$ exist with *local* supersymmetry, and in the 4D case, they also exist even with *global* symmetry [4]. However, it is a non-trivial question, if such non-polynomial couplings exist in 3D with *global* supersymmetry, and what features such couplings possess.

In this brief report, we seek such non-polynomial couplings in 3D, with $N = 1$ global supersymmetry. We also add supersymmetric Chern-Simons terms for a vector multiplet, as well as a potential for a scalar multiplet. As an important application, we present exact solutions when the function $h(\varphi)$ in front of the vector kinetic term $h(\varphi)(F_{\mu\nu}{}^{I})^2$ is an exponential function. It turns out that the total energy is finite, in contrast to the usual static electric field in 3D giving a divergent total energy.

We first give our result in components, and next re-formulate it in terms of superfields [9][10]. The former formulation projects more practically transparent results with component fields. On the other hand, superspace formulation has its own advantage of compact expressions of all terms. The combination of these two formulations provides a solid ground of the validity of our total system.

Note that couplings such as $\varphi(F_{\mu\nu}{}^I)^2$ are *not* renormalizable, because the scalar field φ has the physical dimension $m^{1/2}$ in 3D. However, we do not exclude these couplings in this brief report, for the same reason non-renormalizable DBI actions [1] are not excluded.

2. The Lagrangian

The multiplets we deal with are the YM multiplet $(A_\mu{}^I, \lambda^I)$ and the scalar multiplet (φ, χ) with global $N = 1$ supersymmetry. The indices $I,~J,~\cdots = 1,~2,~\cdots,~\dim G$ are for the adjoint representation of an arbitrary Lie group G with positive definite metric. Therefore, the YM multiplet has $2 + 2$ degrees of freedom (DOF) up to $\dim G$, while the scalar multiplet has $1 + 1$ DOF. The spinors λ and χ are both Majorana spinors.

We start with our action $I \equiv \int d^3x\, \mathcal{L}$ with the lagrangian[3)]

$$
\begin{aligned}
\mathcal{L} = &- \tfrac{1}{4}(F_{\mu\nu}{}^I)^2 + \tfrac{1}{2}(\overline{\lambda}{}^I \not{D} \lambda^I) + \tfrac{1}{2}(\overline{\chi} \not{\partial} \chi) - \tfrac{1}{2}(\partial_\mu \varphi)^2 \\
&+ \tfrac{1}{4} m \epsilon^{\mu\nu\rho} \left(F_{\mu\nu}{}^I A_\rho{}^I - \tfrac{1}{3} g f^{IJK} A_\mu{}^I A_\nu{}^J A_\rho{}^K \right) + \tfrac{1}{2} m (\overline{\lambda}{}^I \lambda^I) \\
&+ \alpha f(\varphi) (F_{\mu\nu}{}^I)^2 - 2\alpha f(\varphi) (\overline{\lambda}{}^I \not{D} \lambda^I) \\
&+ \alpha f'(\varphi) (\overline{\chi} \gamma^{\mu\nu} \lambda^I) F_{\mu\nu}{}^I - \tfrac{1}{2} \alpha f''(\varphi) (\overline{\chi}\chi)(\overline{\lambda}{}^I \lambda^I) - \tfrac{1}{2} \alpha^2 [f'(\varphi)]^2 (\overline{\lambda}{}^I \lambda^I)^2 \\
&- \tfrac{1}{2} [\, W'(\varphi)\,]^2 + \tfrac{1}{2} W''(\varphi)(\overline{\chi}\chi) + \alpha W'(\varphi) f'(\varphi) (\overline{\lambda}{}^I \lambda^I) ~,
\end{aligned}
\tag{2.1}
$$

where $f(\varphi)$ is an arbitrary continuous and differentiable scalar function of φ. The *primes* on $f(\varphi)$ are for the differentiations by φ, *e.g.,* $f'(\varphi) \equiv d\, f(\varphi)/d\,\varphi$. Similarly, $W(\varphi)$ is an arbitrary differentiable function of φ, corresponding to the superpotential $W(\Phi)$ in superspace. Again, *primes* on the W's imply derivatives by φ. The α is a non-zero real constant that parametrizes the new couplings starting with $\alpha f(\varphi)(F_{\mu\nu}{}^I)^2$, while the m is another constant for the supersymmetric Chern-Simons terms. The field strength and the covariant derivative are defined as usual by

$$
F_{\mu\nu}{}^I \equiv + \partial_\mu A_\nu{}^I - \partial_\nu A_\mu{}^I + g f^{IJK} A_\mu{}^J A_\nu{}^K ~, \qquad D_\mu \lambda^I \equiv + \partial_\mu \lambda^I + g f^{IJK} A_\mu{}^J \lambda^K ~, \tag{2.2}
$$

where g is a gauge coupling constant.

[3)] We are using the metric $(\eta_{\mu\nu}) = \text{diag.}\ (-,+,+)$, where the indices $\mu,~\nu,~\cdots = 0,~1,~2$ are for the $D = 2 + 1$ dimensions.

Our action I is invariant under $N = 1$ supersymmetry

$$\delta_Q A_\mu{}^I = + (\overline{\epsilon}\gamma_\mu \lambda^I) \quad , \tag{2.3a}$$

$$\delta_Q \lambda^I = + \tfrac{1}{2}(\gamma^{\mu\nu}\epsilon) F_{\mu\nu}{}^I \quad , \tag{2.3b}$$

$$\delta_Q \varphi = + (\overline{\epsilon}\chi) \quad , \tag{2.3c}$$

$$\delta_Q \chi = - (\gamma^\mu \epsilon)\partial_\mu \varphi - \alpha f'(\varphi)\, \epsilon\, (\overline{\lambda}{}^I \lambda^I) + W'(\varphi)\epsilon \quad . \tag{2.3d}$$

Our lagrangian (2.1) is rewritten in terms of a new function $h(\varphi) \equiv \alpha f(\varphi) - 1/4$ as

$$\begin{aligned} \mathcal{L} = & + h(\varphi) \Big[(F_{\mu\nu}{}^I)^2 - 2(\overline{\lambda}{}^I \not{D} \lambda^I) \Big] + \tfrac{1}{2}(\overline{\chi} \not{\partial} \chi) - \tfrac{1}{2}(\partial_\mu \varphi)^2 \\ & + \tfrac{1}{4} m \epsilon^{\mu\nu\rho} \Big(F_{\mu\nu}{}^I A_\rho{}^I - \tfrac{1}{3} g f^{IJK} A_\mu{}^I A_\nu{}^J A_\rho{}^K \Big) + \tfrac{1}{2} m (\overline{\lambda}{}^I \lambda^I) \\ & + h'(\varphi)(\overline{\chi}\gamma^{\mu\nu}\lambda^I) F_{\mu\nu}{}^I - \tfrac{1}{2} h''(\varphi)(\overline{\chi}\chi)(\overline{\lambda}{}^I\lambda^I) - \tfrac{1}{2}[h'(\varphi)]^2 (\overline{\lambda}{}^I\lambda^I)^2 \\ & - \tfrac{1}{2}[\, W'(\varphi)\,]^2 + \tfrac{1}{2} W''(\varphi)(\overline{\chi}\chi) + \alpha W'(\varphi) h'(\varphi)(\overline{\lambda}{}^I \lambda^I) \quad . \end{aligned} \tag{2.4}$$

The expression (2.4) is more compact than (2.1), absorbing the kinetic terms into the general function $h(\varphi)$.

We can choose any continuous differentiable function for $h(\varphi)$. For example, if we choose the dilaton-like coupling $h(\varphi) \equiv -(1/4)\, e^{a\varphi}$, then the lowest order term for the YM kinetic terms becomes canonical, and there is a peculiar global symmetry:

$$\varphi \rightarrow \varphi + 2c \quad , \qquad A_\mu{}^I \rightarrow e^{-ac} A_\mu{}^I \quad , \qquad \lambda^I \rightarrow e^{-ac}\lambda^I \quad , \qquad g \rightarrow e^{+ac} g \quad , \tag{2.5}$$

for an arbitrary constant-shift parameter c.

The invariance $\delta_Q I = 0$ for (2.1) is confirmed by straightforward computation, containing non-Abelian contributions. Due to the scalar function $h(\varphi)$ multiplied in front of the YM and λ-kinetic terms, the partial integrations after the δ_Q-variations of these terms generate terms like $\alpha h'(\varphi)(\overline{\epsilon}\gamma^{\cdot\cdot}\lambda^I) F_{\cdot\cdot}{}^I \partial_\cdot \varphi$. However, these terms are cancelled by the like terms arising in the variation of $h'(\varphi)(\overline{\chi}\gamma^{\mu\nu}\lambda^I)F_{\mu\nu}{}^I$.

3. Superspace Formulation

We have so far relied only on component formulation. We now reconfirm the superinvariance $\delta_Q I = 0$ in superspace in a more transparent manner.

Our total action in superspace with the coordinates $(Z^M) \equiv (x^\mu, \theta^\alpha)$ ($\mu, \nu, \cdots = 0, 1, 2;$ $\alpha, \beta, \cdots = 1, 2$),[4] is given in terms of the conventional superfield $\Gamma_\alpha{}^I(Z)$ and $\Phi(Z)$ respectively for the vector multiplet $(A_\mu{}^I, \lambda_\alpha{}^I)$ and the scalar multiplet $(\varphi, \chi_\alpha, g)$ [9]. We basically follow the notation in [9], such as $\widetilde{\mathcal{L}}(x) = \nabla^2 \widetilde{\mathcal{L}}(z)\Big|$[5], where

$$\begin{aligned} \widetilde{\mathcal{L}}(z) \equiv & + \frac{1}{4} W^{\alpha I} W_\alpha{}^I + \tfrac{1}{2}\Phi D^2 \Phi - \alpha f(\Phi) W^{\alpha I} W_\alpha{}^I + W(\Phi) \\ & + \tfrac{1}{4} m \Big[\Gamma^{\alpha I} W_\alpha{}^I + \tfrac{1}{6} \{\Gamma^\alpha, \Gamma^\beta\}^I D_\alpha \Gamma_\beta{}^I + \tfrac{1}{12} \{\Gamma^\alpha, \Gamma^\beta\}^I \{\Gamma_\alpha, \Gamma_\beta\}^I \Big] \quad . \end{aligned} \tag{3.1}$$

Even though the basic notation is similar to [9], since we are using the 3D metric $(-, +, +)$, we have slight difference reflected in the gamma matrices, such as

$$\{D_\alpha, D_\beta\} = +\partial_{\alpha\beta} \equiv +(\gamma^\mu)_{\alpha\beta}\, \partial_\mu \quad , \qquad (D^2)^2 = -\tfrac{1}{2} \partial^{\alpha\beta} \partial_{\alpha\beta} = +\partial_\mu^2 \quad . \tag{3.2}$$

Using the universal rules [9], such as

$$\widetilde{\mathcal{L}}(x) = \nabla^2 \widetilde{\mathcal{L}}(z)\Big| \quad , \qquad \nabla_{(\alpha} W_{\beta)}{}^I = + F_{\alpha\beta}{}^I \equiv + \tfrac{1}{2} (\gamma^{\mu\nu})_{\alpha\beta} F_{\mu\nu} \quad , \qquad \nabla^2 W_\alpha{}^I = \nabla_\alpha{}^\beta W_\beta{}^I \quad , \tag{3.3}$$

we get the 3D lagrangian $\widetilde{\mathcal{L}}(x) = \nabla^2 \widetilde{\mathcal{L}}(z)\Big|$ now with the auxiliary field g:

$$\begin{aligned} \widetilde{\mathcal{L}}(x) = & - \tfrac{1}{4} (F_{\mu\nu}{}^I)^2 + \tfrac{1}{2} (\overline{\lambda}{}^I \not{D} \lambda^I) + \tfrac{1}{2} (\overline{\chi} \not{\partial} \chi) - \tfrac{1}{2} (\partial_\mu \varphi)^2 \\ & + \tfrac{1}{4} m \epsilon^{\mu\nu\rho} \Big(F_{\mu\nu}{}^I A_\rho{}^I - \tfrac{1}{3} g f^{IJK} A_\mu{}^I A_\nu{}^J A_\rho{}^K \Big) + \tfrac{1}{2} m (\overline{\lambda}{}^I \lambda^I) \\ & + \alpha f(\varphi) (F_{\mu\nu}{}^I)^2 - 2\alpha f(\varphi) (\overline{\lambda}{}^I \not{D} \lambda^I) \\ & + \alpha f'(\varphi) (\overline{\chi} \gamma^{\mu\nu} \lambda^I) F_{\mu\nu}{}^I - \tfrac{1}{2} \alpha f''(\varphi) (\overline{\chi}\chi) (\overline{\lambda}{}^I \lambda^I) - \tfrac{1}{2} \alpha^2 [f'(\varphi)]^2 (\overline{\lambda}{}^I \lambda^I)^2 \\ & - \tfrac{1}{2} [\, W'(\varphi)\,]^2 + \tfrac{1}{2} W''(\varphi) (\overline{\chi}\chi) + \alpha W'(\varphi) f'(\varphi) (\overline{\lambda}{}^I \lambda^I) \\ & + \tfrac{1}{2} \Big[g - \alpha f'(\varphi) (\overline{\lambda}{}^I \lambda^I) + W'(\varphi) \Big]^2 \quad , \end{aligned} \tag{3.4}$$

where the last complete-square term is for the elimination of the auxiliary field g. After its elimination, (3.4) coincides with (2.1).

The important point here is that the invariance of the action for (3.4) is valid *without* any higher-order terms in α. Our component lagrangian (2.1) was originally obtained by ignoring the $\mathcal{O}(\alpha^3)$ terms. However, it turned out to be valid to all orders in α, when re-formulated in superspace.

[4] We use the indices $\mu, \nu, \cdots$ for bosonic coordinate indices, in order to comply with the component notation.

[5] We use $\widetilde{\mathcal{L}}(x)$ for the superspace-based 3D lagrangian, to be distinguished from (2.1).

4. Exact Solutions with Finite Energy

As another interesting application, we give exact solutions for the purely bosonic part of our $N = 1$ system. These solutions contain certain non-polynomial functions of a scalar, so that they are similar to dilaton black hole solutions in lower dimensions [11]. The difference of our system is that we have only *global* $N = 1$ supersymmetry without gravity. In 4D, exact solutions for similar systems have been studied for the dilaton couplings and confinements [12]. We use the methodology in [12] for solving our 3D field equations.

For our purpose, we consider only the Abelian case, and omit the Chern-Simons terms and scalar potential terms. The relevant lagrangian terms are

$$\mathcal{L}_{A,\varphi} = +h(\varphi)(F_{\mu\nu})^2 - \tfrac{1}{2}(\partial_\mu\varphi)^2 \quad , \tag{4.1}$$

yielding the A_μ and φ-field equations

$$\partial_\nu \left[h(\varphi)F^{\mu\nu} \right] \doteq 0 \quad , \tag{4.2a}$$

$$\partial_\mu^2\varphi + h'(\varphi)(F_{\mu\nu})^2 \doteq 0 \quad . \tag{4.2b}$$

We next restrict the function $h(\varphi)$ to be

$$h(\varphi) = -\tfrac{1}{4}e^{a\varphi} \quad , \tag{4.3}$$

where a is a real constant. Following the method in [12], we set up the ansätze

$$F_{0i} = E_i(\vec{r}) = E(r)\frac{x_i}{r} \quad , \qquad F_{ij} = 0 \quad , \qquad \varphi = \varphi(r) \quad , \tag{4.4}$$

where the indices $i,\, j,\, \cdots = 1,\, 2$ are only for spatial 2D in the 3D coordinate $(x^1, x^2, x^0) = (x, y, t)$, while (r, ϕ) are the polar coordinates for (x, y). The field equations in (4.2) under these ansätze are

$$\vec{\nabla} \cdot \left[e^{a\varphi}\vec{E}(\vec{r}) \right] \doteq 0 \quad , \tag{4.5a}$$

$$\frac{d^2\varphi}{dr^2} + \frac{1}{r}\frac{d\varphi}{dr} + \frac{1}{2}a\, e^{a\varphi}\left[\vec{E}(\vec{r})\right]^2 \doteq 0 \quad . \tag{4.5b}$$

Here all the vectorial symbols are for 2D spatial directions. Since the vector $\vec{E}(r)$ is only the function of the radial coordinate r, one solution to (4.5a) is simply

$$\vec{E}(\vec{r}) = \frac{k}{r}e^{-a\varphi}\,\hat{r} \quad , \tag{4.6}$$

where k is a real constant, and $\hat{r}$ is the unit vector in the r-direction. We now change the variable from r to $\xi \equiv \ln r$, so that (4.5b) is now

$$\frac{d^2\varphi}{d\xi^2} \doteq -\frac{ak^2}{2}e^{-a\varphi} \quad . \tag{4.7}$$

Multiplying both sides by $d\varphi/d\xi$, we get the first integration:

$$\frac{d\varphi}{d\xi} = \pm|k|\sqrt{e^{-a\varphi}+C} \quad , \tag{4.8}$$

where C is a real constant.

We expect that $d\varphi/d\xi \to 0$ as $\varphi \to \infty$, so that $C = 0$ in (4.8) is appropriate. In this case, (4.8) is further integrated to be

$$\varphi = \frac{2}{a}\ln\left[1 \pm \frac{a|k|}{2}\ln\left(\frac{r}{r_0}\right)\right] \quad . \tag{4.9}$$

When $a \to 0$, the standard logarithmic solution of the type $\varphi \approx \ln(r/r_0)$ is recovered from (4.9).

The radius r_0 can be regarded as a 'regulator' for the $r = 0$ singularity, such that the range of r is chosen to be $r_0 \le r < \infty$. After all the computation has been done, we take $r_0 \to 0$. As will be seen in (4.13) below, this is also consistent with the value of $a > 0$ (or $a < 0$) for upper (or lower) sign in (4.9).

The regulator feature in (4.9) both at $r \to 0$, and at $r \to \infty$, as is clear from the solution for $\vec{E}(r)$ *via* (4.6):

$$\vec{E}(r) = \frac{k}{r\left[1 \pm \frac{a|k|}{2}\ln\left(\frac{r}{r_0}\right)\right]^2}\hat{r} \quad , \tag{4.10}$$

because the usual $1/r$-type damping at $r \to \infty$ becomes faster like $1/[r(\ln r)^2]$ by the extra factor $[1 \pm (a|k|/2)\ln(r/r_0)]^2 \approx (\ln r)^2$ in the denominator.

The non-singular feature of (4.9) can be also seen in the total energy. The dynamical energy-momentum tensor is computed *via* the gravitational couplings as

$$T_\mu{}^\nu = -\delta_\mu{}^\nu h(\varphi)(F_{\rho\sigma})^2 + 4h(\varphi)F_{\mu\rho}F^{\nu\rho} + \tfrac{1}{2}\delta_\mu{}^\nu(\partial_\rho\varphi)^2 - \eta^{\nu\rho}(\partial_\mu\varphi)(\partial_\rho\varphi) \quad . \tag{4.11}$$

The 00-component of (4.11) gives the energy density

$$\rho(r) = T_0{}^0 = \frac{k^2}{r^2\left[1 \pm \frac{a|k|}{2}\ln\left(\frac{r}{r_0}\right)\right]^2} \quad , \tag{4.12}$$

so that the total energy $\mathcal{E}$ is

$$\mathcal{E} = \int d^2\vec{x}\,\rho(r) = \lim_{r_0\to 0}\int_{r_0}^{\infty} dr\,2\pi r\rho(r) = +2\pi k^2 \lim_{r_0\to 0}\int_{r_0}^{\infty}\frac{dr}{r\left[1\pm\frac{a|k|}{2}\ln\left(\frac{r}{r_0}\right)\right]^2}$$

$$= \mp\frac{4\pi|k|}{a}\lim_{r_0\to 0}\left[\frac{1}{1\pm\frac{a|k|}{2}\ln\left(\frac{r}{r_0}\right)}\right]_{r_0}^{\infty} = \pm 4\pi\,\frac{|k|}{a} \quad . \tag{4.13}$$

For the reason already mentioned, we have performed the spatial integration from r_0 to ∞, avoiding the singularity at $r = 0$. Note that there is no singularity at $r_0 \to 0$.

For the total energy $\mathcal{E}$ to be positive, as physically meaningful solutions, we have to choose the upper (or lower) sign in (4.13), when a is positive (or negative).

Note that in the conventional case of $a = 0$ with the usual $E(r) = k/r \approx 1/r$, the total energy corresponding to (4.13) is divergent at $r \to \infty$. In this sense, the $h(\varphi)$-interaction provides a 'regulators' for the singularities both at $r \to \infty$ and $r \to 0$.

5. Concluding Remarks

In this brief report, we have established non-polynomial interactions of the type $f(\varphi)(F_{\mu\nu}{}^I)^2$ for an $N = 1$ vector multiplet in 3D, where φ is the scalar in the scalar multiplet (φ, χ). We have no problem with the non-Abelian generalization, such as supersymmetric Chern-Simons terms, and a potential term for the scalar multiplet. We have performed the invariance confirmation $\delta_Q I = 0$ both in component field and superspace formulations.

As an important application, we have a set of exact solutions for the Abelian case, when $h(\varphi)$ is an exponential function, while the Chern-Simons terms and scalar multiplet potential terms are absent. They seem to be physically meaningful solutions with a finite total energy. In particular, we see that the $h(\varphi)$-function plays a role of regulator both for $r \to 0$ and $r \to \infty$.

In principle, we can consider the generalization of our couplings to extended supersymmetries [13]. However, non-Abelian cases will be much more non-trivial. The main obstruction against higher N is that scalar fields are generally *non*-singlets. Therefore, it seems very difficult for them to appear in a scalar function such as $f(\varphi)$ in non-Abelian cases.

Needless to say, the non-polynomial function $f(\varphi)$ or $h(\varphi)$ implies the existence of *infinitely many* coupling constants in the system. The special example of exact solutions presented above form only a small set of more non-trivial features of supersymmetric vector and scalar systems in 3D yet to be explored.

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