Symmetries and strings in field theory and gravity

Tom Banks and Nathan Seiberg

Phys. Rev. D **83**, 084019 — Published 12 April 2011

DOI: 10.1103/PhysRevD.83.084019

Symmetries and Strings in Field Theory and Gravity

Tom Banks[1,2,3] *and Nathan Seiberg*[1]

[1]School of Natural Sciences
Institute for Advanced Study
Einstein Drive, Princeton, NJ 08540

[2]New High Energy Theory Center
Department of Physics
Rutgers University
Piscataway, NJ 08854

[3]Santa Cruz Institute for Particle Physics
University of California
Santa Cruz, CA 95064

We discuss aspects of global and gauged symmetries in quantum field theory and quantum gravity, focusing on discrete gauge symmetries. An effective Lagrangian description of $\mathbb{Z}_p$ gauge theories shows that they are associated with an emergent $\mathbb{Z}_p$ one-form (Kalb-Ramond) gauge symmetry. This understanding leads us to uncover new observables and new phenomena in nonlinear σ-models. It also allows us to expand on Polchinski's classification of cosmic strings. We argue that in models of quantum gravity, there are no global symmetries, all continuous gauge symmetries are compact, and all charges allowed by Dirac quantization are present in the spectrum. These conjectures are not new, but we present them from a streamlined and unified perspective. Finally, our discussion about string charges and symmetries leads to a more physical and more complete understanding of recently found consistency conditions of supergravity.

1. Introduction and conclusions

This work addresses several seemingly unrelated topics. We will comment on some subtleties in gauge theories and in particular in discrete (e.g. $\mathbb{Z}_p$) gauge theories. We will explore the classifications of strings in four dimensions. We will discuss some conjectures about symmetries in quantum gravity, and we will present some constraints on consistent supergravity theories. Even though these topics seem to be distinct we will see that they are related. Our improved understanding of the field theory of discrete gauge symmetries, is crucial to a proper analysis of gravitational theories.

A key focus of our work is a discussion of three conjectures about quantum gravity[1]. Some of them are widely accepted as "folk theorems." Others are known to some experts (see e.g. [1-3] and in particular [4] and the earlier reference [5]). We present new, streamlined arguments for them, and explore their inter-relation and their implications for other issues. The conjectures are:

1. There are no global symmetries. Here we slightly extend the known "no global symmetries" dictum to include also discrete symmetry groups and higher brane charges. In particular, we argue that all stable branes are associated with gauge symmetries. For example, stable strings in four dimensions are associated with a continuous or discrete one-form gauge symmetry (a.k.a. Kalb-Ramond (KR) gauge symmetry)[2].
2. All continuous gauge groups, including Kalb-Ramond gauge symmetries, are compact. This excludes for example, the possibility of a gauge group being $\mathbb{R}$ rather than $U(1)$.
3. The Completeness Hypothesis: The spectrum of electric and magnetic charges forms a complete set consistent with the Dirac quantization condition. By "complete set" we mean that every allowed charge is present in the spectrum.

Clearly, these conjectures are satisfied in all known examples of string theory, i.e. in perturbative string theory, Matrix Theory, and for strings in an asymptotically AdS background.

---

[1] We will concentrate on models with exactly four non-compact dimensions, which are asymptotically flat, but we expect that some version of our conjectures is true more generally.

[2] Our conventions for higher form gauge symmetries and gauge fields follows that of branes. A q-brane has a $q+1$-dimensional world-volume. It couples to a $q+1$-form gauge field, which has a q-form gauge symmetry. In this terminology ordinary gauge symmetry is a 0-form gauge symmetry and a global symmetry is a "-1-form gauge symmetry." We will use this terminology both for continuous and for discrete gauge symmetries.

Our arguments in favor of the three conjectures show that the distinction between observable operators and dynamical particles, which pervades quantum field theory, disappears in models of quantum gravity. In particular, Wilson and 't Hooft loops *must* correspond to propagating particles, which might be stable charged black holes.

Since our arguments thread together a number of themes and results, which are at first sight quite unrelated, we end this introduction with a reader's guide to the sections and their relationships.

Section 2: Here we review properties of gauge theories, many of which are well known. We emphasize the analysis of these theories in terms of their deep infrared (IR) behavior. We give a universal IR Lagrangian for $\mathbb{Z}_p$ gauge theories in four dimensions. This involves a two-form and a one-form gauge potential, whose holonomies are valued in $U(1)$. The integer p enters in the Lagrangian. In sections 2 and 3 we will strive to find universal properties of the long distance quantum field theory, as well as a description of operators in the IR theory, which parameterize possible objects in *any* UV completion of the theory. In some more complete effective field theory at a higher scale, we may find some of these objects as dynamical (perhaps solitonic) particles or strings. The others remain as observables (Wilson lines *etc.*). In section 4, we will argue that in models including gravity, all the observables represent dynamical objects.

We also study a $\mathbb{Z}_p$ gauge theory coupled to σ-models, like the $SO(3)$ model, with non-trivial two-cycles in their target space. Here we use our improved understanding of the $\mathbb{Z}_p$ gauge theories to shed new light on the constructions of [6]. The strings of these models might be a macroscopic manifestation of the strings of a $\mathbb{Z}_p$ gauge theory. We point out the existence of point, line and surface operators, which describe the violation of topological symmetries of σ-models in a universal manner independent of the UV completion of the model. We describe applications of these operators to violation of the string conservation law of the σ-models referred to above. The latter operators are analogs of magnetic monopoles for Nielsen-Olesen strings. In Appendix B we note the application of these operators to baryon number violation and the existence of confining strings, in chiral Lagrangians derived from QCD like gauge theories. Appendix C outlines how all of these questions arise in lattice models.

Section 3: Here we review and refine Polchinski's classification of cosmic strings [7]. Our general conclusion is that stable strings are always coupled to a KR gauge field, which might be a $\mathbb{Z}_p$ gauge field of the type described in section 2. In the latter case p strings can end at a point. We also mention the possibility of discrete non-Abelian strings. If the

KR gauge group is continuous, the corresponding 2-form gauge field is massless and is dual to a scalar. Here we argue that the theory should be supersymmetric in order to avoid a potential for that scalar. Furthermore, in order to render the string tension finite, that scalar should be part of a moduli space of vacua and the latter should have a singularity around which the scalar winds. In addition, the boundary conditions should set the moduli to that singular value. Otherwise, in the presence of gravity the strings cannot be viewed as excitations of the model. If one tries to create a large loop of string in such a model, there will always be a radius above which one forms a black hole instead[3].

Section 4: Here we formulate the three interlocking conjectures about symmetries in models of quantum gravity we mentioned above. The arguments for these conjectures are all based on black hole physics. They are also valid in all *known* consistent models of quantum gravity.

We argue that the covariant entropy bound (CEB) leads to an elegant and simple way to formulate the principle of quantum gravity that forbids global continuous symmetries. The same argument shows that all continuous gauge groups are compact. We use the CEB and other arguments from black hole physics to argue for the completeness hypotheses and to show that all global cosmic string charges are violated as well. Stable cosmic strings must be coupled to a gauge field, which might be discrete. In section 3 we use these observations to refine Polchinski's classification of cosmic strings.

Section 5: We return to the origins of this paper [8,9,6] and explore the consequences of all of these results for $\mathcal{N}=1$ supergravity in four dimensions. These papers argued that certain rigid supersymmetric theories cannot be coupled to supergravity in a straightforward way. In particular, problems arise either when the theory includes Fayet-Iliopoulos (FI) terms or when the target space topology has a non-exact Kähler form. We will give a more physical interpretation of these results by emphasizing the role of a certain 2-form string current[4] in the supersymmetry multiplet. Such nontrivial string currents imply the

[3] One should be careful about applying many of our conclusions, including this one, to phenomenology. In many cases, the instabilities we describe are exponential in the ratio of the cosmic string tension scale and the Planck scale.

[4] More generally, q-branes can carry a conserved q-brane charge which are characterized by the associated $q+1$-form conserved current. It is convenient to dualize it to a closed $d-q-1$-form J. We define a finite "charge" by integrating

$$Q(\Sigma_{d-q-1}) = \int_{\Sigma_{d-q-1}} J \quad , \tag{1.1}$$

existence of cosmic strings. Hence our general discussion of cosmic strings gives a new perspective on the conclusions of [8,9,6] and extends them.

Throughout most of this paper, we will neglect the gravitational back-reaction of cosmic strings. In all the models we consider, that back-reaction is a simple deficit angle much smaller than 2π, and this certainly appears justified.

2. Comments on gauge symmetries, observables and dynamical branes in quantum field theory

One theme of our paper is the description of quantum field theory and quantum gravity models using their long distance behavior. In this section we will make some general comments (many of them are well known) about the long distance properties of gauge theories.

2.1. Continuous Abelian gauge theories

A simple question, which often causes confusion, that one can ask about an Abelian gauge theory in continuum quantum field theory, is whether the gauge group is $U(1)$ or $\mathbb{R}$. It has long been known to *cognoscenti* that this question can be answered by specifying the list of allowed observables in the model. We can have Wilson loops

$$W_A(\Sigma_1, n_A) = \exp\left(i n_A \oint_{\Sigma_1} A \right) \quad . \tag{2.1}$$

In a $U(1)$ gauge theory only $n_A \in \mathbb{Z}$ are allowed, while in an $\mathbb{R}$ gauge theory every $n_A \in \mathbb{R}$ is allowed. Correspondingly, in a $U(1)$ gauge theory we can have 't Hooft operators

$$T_A(\Sigma_1, m_A) = \exp\left(i m_A \oint_{\Sigma_1} V \right) \tag{2.2}$$

where V is the dual photon and $m_A \in \mathbb{Z}$. Alternatively, we can define $T_A(\Sigma_1, m_A)$ by removing a tubular neighborhood of Σ_1 which is locally $\mathbb{R} \times \mathbb{S}^2$ and impose $\int_{\mathbb{S}^2} F = 2\pi m_A$.

Equivalently, we can characterize the compactness of the gauge group, by specifying the allowed fluxes of F and $*F$ through various cycles. The allowed fluxes and Wilson/'t

---

where Σ_{d-q-1} is a closed $d-q-1$ subspace. Note that this definition of the charge coincides with the standard definition for $q=0$ where Σ_{d-q-1} is all of space. Shifting the current by an exact $d-q-1$-form does not affect its conservation and does not change the charges (1.1). This is known as an improvement transformation.

Hooft lines are restricted by the Dirac-quantization condition, but in quantum field theory nothing requires us to include *all* charges consistent with the Dirac condition as dynamical objects in the theory. Some of the Wilson and/or 't Hooft lines may just be non-dynamical probes of the theory.

The relation between dynamical particles and probes can be usefully thought of as an infinite mass limit. If we take the mass of some charged field to infinity, we can find the effect on low energy fields due to virtual particles of large mass by the usual Wilsonian methods. However, one can analyze more general states, containing one or more large mass particles, by using the particle path expansion of the functional integral over the massive field [10]. For the case of Abelian gauge theories in a perturbative phase, the particle path expansion involves Wilson loops for heavy electric charges and 't Hooft loops for magnetic charges[5].

We end this subsection with a comment about non-compact gauge groups. One simple way to guarantee that one is talking about a non-compact gauge group is to insist that the model has two relatively irrational charges, say 1 and $\sqrt{2}$. We would like to point out that any gauge invariant Lagrangian for this pair of charges also has a global Abelian symmetry under which just the field of charge $\sqrt{2}$ rotates. If we accept that there are no global conserved charges in quantum gravity, then such a model cannot be coupled to gravity. In section 4 we will see that global charges and irrational charges are both ruled out by the same argument about black hole physics. Of course, in field theory with gauge group $\mathbb{R}$ we include Wilson loops of irrational charge in the list of observables, even if all dynamical charged fields have only integer charges. Alternatively we can insist that we do not allow magnetic flux when we study the theory on $\mathbb{S}^2 \times \mathbb{R}^2$. In quantum gravity, we will see that the *completeness hypothesis* does not allow us to make such choices. $\mathbb{R}$ gauge theories would have to allow irrational electric charges, and this would lead to a forbidden global quantum number, as above. In other words, $\mathbb{R}$ *gauge fields do not exist in the quantum theory of gravity.* All continuous gauge groups are compact.

---

[5] In perturbation theory, only mutually local dyons can be light simultaneously, and by convention we call those electric charges.

2.2. The low energy description of $\mathbb{Z}_p$ gauge theories and an emergent $\mathbb{Z}_p$ one-form gauge symmetry

The four dimensional BF-theory with the Lagrangian [11]

$$\frac{ip}{2\pi} B \wedge dA \tag{2.3}$$

is known to be a simple topological field theory which does not have any local degrees of freedom. Following reference [12] we now show that this theory is equivalent to another topological field theory – a $\mathbb{Z}_p$ gauge theory.

A simple way to establish the relation of (2.3) to a $\mathbb{Z}_p$ gauge theory [12] is to start with the Lagrangian

$$t^2(d\phi - pA) \wedge *(d\phi - pA) + \mathcal{L}(A), \tag{2.4}$$

which describes the Higgsing of a $U(1)$ gauge theory with gauge field A by a Higgs field ϕ. ϕ is subject to the identification $\phi \sim \phi + 2\pi$ such that $\exp(i\phi)$ carries charge p under the gauge group and hence the $U(1)$ gauge symmetry is Higgsed to $\mathbb{Z}_p$. We are interested in the low energy limit of this theory which is obtained in the limit $t \to \infty$. In this limit $A = \frac{1}{p} d\phi$ and therefore the low energy theory does not include local degrees of freedom.

In order to analyze the low energy $\mathbb{Z}_p$ gauge theory in more detail we start with (2.4) and dualize ϕ to derive the Lagrangian

$$\frac{1}{(4\pi)^2 t^2} H \wedge *H + \frac{ip}{2\pi} B \wedge dA + \mathcal{L}(A) \quad , \tag{2.5}$$

where $H = dB$.[6]

If

$$\mathcal{L}(A) = \frac{1}{2e^2} F \wedge *F \quad , \tag{2.7}$$

we can further dualize A to another one-form V with the Lagrangian

$$\frac{1}{(4\pi)^2 t^2} H \wedge *H + \frac{e^2}{8\pi^2} (dV - pB) \wedge *(dV - pB) \quad . \tag{2.8}$$

---

[6] If $\mathcal{L}(A) = 0$ we can shift

$$A \to A + \frac{i}{8\pi p t^2} * H \tag{2.6}$$

and remove the first term in (2.5) showing that the Lagrangian is only $\frac{ip}{2\pi} B \wedge dA$. However, in the presence of observables, this shift does not eliminate the dependence on t.

In this form we interpret the vector field V as a matter field, which Higgses the $U(1)$ gauge symmetry of B down to $\mathbb{Z}_p$.

Now we can take the low energy limit ($t \to \infty$) and if $\mathcal{L}(A)$ depends only on $F = dA$ (i.e. there is no Chern-Simons term or charged matter fields), we end up with the low energy Lagrangian

$$\frac{ip}{2\pi} B \wedge dA \tag{2.9}$$

which is a BF-theory with coefficient p.

It is important that the gauge fields A and B have their standard $U(1)$ gauge transformation laws with standard periodicities. In particular, for every closed two-surface Σ_2 and for every closed three-volume Σ_3

$$\begin{aligned} &\oint_{\Sigma_2} F \in 2\pi\mathbb{Z} \\ &\oint_{\Sigma_3} H \in 2\pi\mathbb{Z} \quad . \end{aligned} \tag{2.10}$$

Note that the equations of motion of (2.3) set $F = H = 0$ and therefore, (2.10) might look meaningless. This is not correct. (2.10) can be interpreted as a rule defining which bundles are included in the path integral, and the allowed gauge transformations on these bundles. When the path integral is used to compute expectation values of line and surface operators, these rules have content because F and H have delta function contributions at the location of the operators.

We conclude that the topological field theory Lagrangian (2.3) can be interpreted as a universal description of a $\mathbb{Z}_p$ gauge theory. In fact, it also shows that an ordinary $\mathbb{Z}_p$ gauge theory must be accompanied by a $\mathbb{Z}_p$ one-form gauge symmetry (recall our conventions of labeling higher form gauge symmetries by the dimension of the gauge parameter). From the perspective of our starting point (2.4) this new one-form gauge symmetry is an *emergent gauge symmetry.* It arises, like most emergent gauge symmetries, out of a duality transformation. Note that both gauge fields are $U(1)$ gauge fields, but as we will see below, the distinct observables are labeled by $\mathbb{Z}_p$.

We point out that this observation is consistent with the result of [13] that the theory with $p = 1$ is trivial.

What are the observables in the theory (2.3)? The local gauge invariant operators are

$$\begin{aligned} &d\phi - pA \sim *H \\ &dV - pB \sim *F \quad . \end{aligned} \tag{2.11}$$

However, the equations of motion in the low energy theory (2.3) show that they vanish; i.e. their long distance correlation functions are exponentially small in the parameter t or other relics of short distance physics.

We can also consider electric (Wilson) operators. For every closed line Σ_1 and for every closed two-surface Σ_2 we study

$$\begin{aligned} W_A(\Sigma_1, n_A) &= \exp\left(i n_A \oint_{\Sigma_1} A\right) \\ W_B(\Sigma_2, n_B) &= \exp\left(i n_B \oint_{\Sigma_2} B\right) \end{aligned} \tag{2.12}$$

with integers n_A and n_B. They are interpreted as the effect of a charge n_A particle with world-line Σ_1 and a string with charge n_B with world-sheet Σ_2.

It easily follows from the equations of motion of (2.3) that every one of these operators induces a holonomy for the other gauge field around it. In particular

$$\left\langle W_A(\Sigma_1, n_A) W_B(\Sigma_2, n_B)\right\rangle \sim \exp\left(2\pi i \frac{n_A n_B \#(\Sigma_1, \Sigma_2)}{p}\right) \quad , \tag{2.13}$$

where $\#(\Sigma_1, \Sigma_2)$ is the linking number of Σ_1 and Σ_2. Therefore only $n_A \mathrm{mod}(p)$ and $n_B \mathrm{mod}(p)$ label distinct operators. Alternatively, the fact that only $n_A \mathrm{mod}(p)$ and $n_B \mathrm{mod}(p)$ are important follows from performing singular gauge transformations. A $U(1)$ gauge transformation on A, which winds around Σ_2 (and is singular on Σ_2) shifts $n_B \to n_B + p$. Similarly, a $U(1)$ gauge transformation on B, which winds around Σ_1 shifts $n_A \to n_A + p$.

In terms of the equivalent $\mathbb{Z}_p$ gauge theory the operator W_A in (2.12) is simply the $\mathbb{Z}_p$ gauge theory Wilson line; i.e. it describes the world-line of a probe particle in the representation labeled by $n_A \mathrm{mod}(p)$ of $\mathbb{Z}_p$. The operator W_B in (2.12) represents the world-sheet of a probe string which is characterized by the $\mathbb{Z}_p$ holonomy around it being $\exp\left(\frac{2\pi i n_B}{p}\right)$. Clearly, this interpretation is consistent with (2.13).

We can also attempt to construct magnetic ('t Hooft) operators. Naively, the operator $\exp(i\phi(P))$ at a point P is an "'t Hooft point operator" for B and $\exp\left(i \oint_{\Sigma_1} V\right)$ is an 't Hooft line operator of A. However, these operators are not gauge invariant under the $U(1)$ gauge symmetries of A and B respectively. In order to make them gauge invariant, we attach p open line electric operators to $\exp(i\phi)$ and consider $\exp\left(i\phi(P) + i\sum_i \int_{l_i} A\right)$, where all the lines l_i end at P, which is gauge invariant. Similarly, we attach p open surface operators to $\exp\left(i\oint_{\Sigma_1} V\right)$ to make it gauge invariant. Special cases of these configurations are the two 't Hooft operators

$$\begin{aligned} T_B(P,m_B) &= \exp\left[im_B\left(\phi(P)+p\int_l A\right)\right] \\ T_A(\Sigma_1,m_A) &= \exp\left[im_A\left(\oint_{\Sigma_1} V+p\int_L B\right)\right] \quad , \end{aligned} \tag{2.14}$$

where the line l ends at the point P and the surface L ends at Σ_1. The other ends of these open line and open surface can be on another magnetic operator or can be taken to infinity. Instead of using ϕ and V in (2.14) we could have cut out of our space an $\mathbb{S}^3$ around P and a tubular neighborhood around Σ_1, which is locally $\mathbb{S}^2 \times \mathbb{R}$, and impose $\int_{\mathbb{S}^3} H = 2\pi m_B$ and $\int_{\mathbb{S}^3} F = 2\pi m_A$. This way the operators (2.14) are expressed only in terms of the variables A and B.

However, these constructions of the operators (2.14) shows that their long distance behavior is in fact trivial. First, note that the charge p line and surface in (2.14) are invisible – they are like Dirac strings. Second, we can always locally gauge $\phi = V = 0$. Equivalently, as we remarked above the equations of motion of the long distance theory (2.3) set $F = H = 0$ and hence we cannot fix any nontrivial integrals of them to be non-vanishing.

We conclude that the only significance of the magnetic operators (2.14) is that a number of electric operators can end at the same point provided their electric charges sum up to zero modulo p. Other than that, the operators (2.14) are trivial in the low energy $\mathbb{Z}_p$ theory.

The Wilson loop operators may be viewed, in a familiar fashion, as the remnants of high mass particles of charge $1 \leq k < p$ in the low energy theory. For example, we could include in the original Higgs model, from which we derived our BF-Lagrangian, fields ψ_k of charge k with a mass of order the Higgs VEV or higher. These particles can be represented using the Schwinger particle path expansion. If k is a factor of p, then $\frac{p}{k}$ of these paths can end at a point, indicating the existence of the gauge invariant operator $\psi_k^{\frac{p}{k}} e^{-i\phi}$ in the underlying theory at the scale of the VEV. Similarly, the fact that p elementary flux tubes of the BF-theory can end at a point and are therefore unstable (in the BF-theory, this just means that their AB-phase is unobservable, so they are indistinguishable from the vacuum), could be attributed to the existence of magnetic monopoles carrying the minimal Dirac unit with respect to the particles with charge $k = 1$. Note however that in an underlying continuum Higgs model, nothing requires these monopoles to exist at finite mass. In the $\mathbb{Z}_p$ lattice gauge theory they do exist, as well as in continuum theories in which the $U(1)$ is embedded in a simple spontaneously broken non-Abelian gauge theory. We will see in section 4 that the completeness hypothesis of quantum gravity, guarantees the existence of these particles below a mass of order Planck scale.

2.3. Example 1: The 4d Abelian Higgs Model

We have seen that the universal low energy description of $\mathbb{Z}_p$ gauge theories, involves a KR gauge field, and that the observables include cosmic string sources. We now turn to two examples of explicit microscopic models in which such strings are present.

The first is the $U(1)$ gauge theory with a charge p scalar. The short distance Lagrangian is

$$\mathcal{L} = |(\partial - ipA)\Phi|^2 - V(|\Phi|) + \frac{1}{2e^2} F \wedge *F \quad . \tag{2.15}$$

We take $V(|\Phi|)$ such that $\langle|\Phi|^2\rangle = t^2 \neq 0$. This vev Higgses the gauge symmetry to $\mathbb{Z}_p$ and the spectrum is gapped. Writing $\Phi = t\exp(i\phi)$ in (2.15) we recover the Lagrangian (2.4). Hence, the low energy description of this theory is given by the universal Lagrangian (2.3).

Given an explicit UV theory we can consider the operators (2.12). First, the theory (2.15), has only charge p elementary quanta. Therefore, the nontrivial Wilson lines $W_A(\Sigma_1, n)$ with $n \neq 0 \mathrm{mod}(p)$ correspond to probe particles rather than dynamical particles. If, however, we add to (2.15) additional massive particles with electric charge one, then all these Wilson lines can be realized by dynamical particles.

It follows from (2.15) that the theory has smooth string configurations. The Higgs field Φ vanishes at the core of the string, where the full $U(1)$ gauge symmetry is restored, and its phase winds around the core. The low energy description of k strings with world-sheet Σ_2 is given by the operator $W_B(\Sigma_2, k)$. These strings can be detected by using the electric Wilson operators $W_A(\Sigma_1, n)$ where Σ_1 circles around the string world-sheet.

Next we consider the magnetic 't Hooft operators (2.14). Because of the Higgsing by Φ, magnetic monopoles are confined – they are attached to strings. More precisely, since Φ has electric charge p, the magnetic flux in our strings is $1/p$ times the fundamental unit of magnetic charge and a charge m monopole is connected to mp strings. This configuration is the one we discussed above when we considered p open surface operators. We see that $\langle T_A(\Sigma_1, m)\rangle$ exhibits an area law with string tension mp times that of the basic strings. In accordance with our general discussion above, this area law sets this operator to zero in the long distance theory.

More interesting is the possibility of adding to our theory dynamical massive magnetic charges. If they carry the fundamental unit of magnetic charge, the number of strings k is not conserved but $k\text{mod}(p)$ is conserved. This fact is consistent with our general discussion above in which a low energy observer was sensitive only to $k\text{mod}(p)$ rather than to k.

2.4. Example 2: Coupling a $\mathbb{Z}_p$ gauge theory to the $SO(3)$ σ-model and topology tearing operators

In our first example above the theory was gapped and the low energy theory was only the $\mathbb{Z}_p$ gauge theory. The universal behavior of its correlation functions were determined by the low energy theory, but additional details depended on the UV theory. In particular, some properties of the strings including their tension depended on the details of the potential at $\Phi \approx 0$ where the $U(1)$ gauge symmetry was restored. We now study a variant of this theory in which the low energy $\mathbb{Z}_p$ theory is coupled to massless matter fields. Here, in addition to various possible massive excitations, some charged particles and strings are constructed out of the low energy matter theory.

We replace the theory (2.15) with a similar theory with two fields $\Phi_{i=1,2}$ with charge p. We can take the potential to constrain

$$|\Phi_1|^2 + |\Phi_2|^2 = t^2 \quad . \tag{2.16}$$

This constraint guarantees that the $U(1)$ gauge symmetry is Higgsed everywhere in field space to $\mathbb{Z}_p$. This theory has been studied in [14-16,6] from various points of view.

Naively, the low energy theory is simply the $SO(3)$ σ-model. However, the unbroken $\mathbb{Z}_p$ gauge symmetry has interesting low energy consequences. In order to derive them we parameterize the two fields $\Phi_{1,2}$ subject to the constraint (2.16) as

$$\begin{aligned} \Phi_1 &= t\frac{ze^{i\phi}}{\sqrt{1+|z|^2}} \\ \Phi_2 &= t\frac{e^{i\phi}}{\sqrt{1+|z|^2}} \quad . \end{aligned} \tag{2.17}$$

Here z parameterizes the $\mathbb{S}^2$ target space and ϕ can be changed by a $U(1)$ gauge transformation. Next, we dualize ϕ as in (2.4)(2.5) and shift A as in (2.6) to find that the σ-model couples to A and B through

$$\frac{i}{2\pi} B \wedge (pF - \omega) \quad . \tag{2.18}$$

where ω is the pull back of the Kähler form of the target space to space-time normalized such that if we wrap the target space once, $\oint \omega \in 2\pi$.

We conclude that the σ-model is coupled to the $\mathbb{Z}_p$ gauge theory in the BF presentation.

The equation of motion of B sets $pF = \omega$. In fact, up to a gauge transformation we can write

$$pA = a \tag{2.19}$$

where a is the pull back of the Kähler connection to space-time satisfying

$$\omega = da \quad . \tag{2.20}$$

Therefore the integral over every closed two-surface Σ_2 in our space time must satisfy [6]

$$\oint_{\Sigma_2} \omega = p \oint_{\Sigma_2} F \in 2\pi p\mathbb{Z} \quad . \tag{2.21}$$

The paper [6] studied two classes of theories with nontrivial constraints on the "instanton number" $\int_{\Sigma_2} \omega$. First, the theory was coupled to a BF-theory, where B played the role of a Lagrange multiplier implementing the constraint on ω. Second, the constrained theory was derived as a $\mathbb{Z}_p$ gauge theory by using charge p fields. Here we see that in fact these two presentations are dual to each other.

To explore the low energy content of our model, we study its allowed operators. These include both "matter" components constructed out of the σ-model field z and $\mathbb{Z}_p$ components constructed out of A and B.

Let us start with the matter operators. First, we have all the obvious local σ-model operators, which are constructed out of functions of z and its derivatives. Another local operator at a point P, $\mathcal{O}_{Hopf}(P)$ is constructed by removing a neighborhood of P from our spacetime and constraining the σ-model variables on its $\mathbb{S}^3$ boundary to have a nontrivial Hopf map. Such an operator is interpreted as creating a nontrivial particle – a "Hopfion", at the point P. We can also construct line operators $\exp\left(in \oint_{\Sigma_1} a\right)$ for integer n. Finally, we can attempt to construct a line operator that represents the creation of m strings. Ordinarily, it is defined by removing a tubular neighborhood of the line whose boundary is locally $\mathbb{R} \times \mathbb{S}^2$ and imposing $\int_{\mathbb{S}^2} \omega = 2\pi m$. However, because of the condition (2.21) this is possible only for m which is a multiple of p. In other words, only operators that create p strings can be constructed out of the σ-model variables.

Next we construct additional operators out of the gauge sector A and B. We start with the Wilson line

$$W_A(\Sigma_1, n_A) = \exp\left(in_A \oint_{\Sigma_1} A\right) = \exp\left(i\frac{n_A}{p}\oint_{\Sigma_1} a\right) \quad . \tag{2.22}$$

For n_A a multiple of p this operator was mentioned above as a "matter operator." The appearance of the fraction $\frac{n_A}{p}$ in (2.22) is interesting. It shows that this is not a standard σ-model operator. If we think of it as a result of a massive particle of charge n_A the fraction means that the field of this particle cannot be described as a section of an integer power of the line bundle over our σ-model target space. It appears like "the p'th root of that line bundle." The interpretation of this fact is obvious. The massive field is not a section on the target space, which is pulled back to spacetime. Instead, it is a section of a bundle on spacetime, whose transition functions depend on the σ-model fields. Since the σ-model configurations are restricted by (2.21), the corresponding massive field is well defined.

The second gauge sector operator is the surface Wilson operator $W_B(\Sigma_2, n_B)$. In the presence of this operator the equation of motion $pF - \omega$ has a delta function on Σ_2. Hence this operator represents the world-sheet of a string with n_B units of flux. These strings can be interpreted either as σ-model strings or as strings constructed out of massive degrees of freedom.

For n_B which is not a multiple of p the world-sheet Σ_2 must be closed. If however, n_B is a multiple of p, these strings can end as in the second equation in (2.14). Their end can be described by the string creation operator made out of the σ-model fields that we discussed above. We conclude that in this case we can use the matter degrees of freedom to construct an analog of the second operator in (2.14). However, since the equation of motion of A still sets $H = 0$ there is no analog of the first operator in (2.14).

Now suppose we add dynamical particles with magnetic charge k. They are characterized by the integral $\int_{\mathbb{S}^2} \omega = 2\pi pk$ around their world line. As above, such particles, are confined – they are attached to strings. The wrapping number of these strings is pk. If such excitations are present, then a string with wrapping number m can decay to a lower winding number while preserving $m \mathrm{mod}(pk)$. In particular, if particles with $k = 1$ are present, then only $m \mathrm{mod}(p)$ is conserved. It is important to note that since $\int_{\mathbb{S}^2} \omega = 2\pi pk$ around a magnetically charged particle, creating or annihilating such a particle has the effect of "tearing the topology." We will refer to these operators and analogous ones below as "topology tearing operators."

As in the Abelian Higgs model, the expectation value of a purely electric loop $\langle W_A(\Sigma_1, n)\rangle$ has a perimeter law. If Σ_1 is large and it winds once around a string with winding number m, $\langle W_A(\Sigma_1, n)\rangle$ depends on m through a phase $\exp(2\pi i n m/p)$. Again, only $m \mathrm{mod}(p)$ is measurable this way. Also, as in the Abelian Higgs model, if there are no magnetically charged particles, an 't Hooft loop of charge m, $T_A(\Sigma_1, m)$, exhibits an area law associated with a string tension proportional to mp. However, dynamical magnetically charged particles with charge k can screen the tension and the area law is determined by $m \mathrm{mod}(k)$. In particular, for $k = 1$ the loop can be totally screened.

In conclusion, we have presented new operators in the standard $SO(3)$ σ-model. In addition, this model can be coupled to a topological field theory. There are no new local degrees of freedom but the set of operators is modified. We have also shown that this theory can be modified at short distance (without breaking its $SO(3)$ global symmetry) by adding electric and magnetic charges. The electrically charged fields do not transform like sections of a line bundle on the target space. The magnetic charges violate conservation of the string current $*\omega$. Equivalently, because of the monopoles the 2-form operator ω is not closed

$$d\omega = p dF \neq 0. \tag{2.23}$$

Hence, the topological conservation law is violated. As we said above, this arises from the fact that creation of magnetically charged particles tears the topology.

The topological classification of the space of maps from space-time to the target space depends on the topology of both domain and range, and violation of the conservation law can come from either. For example, we can, as above, embed the σ-model in a linear model in which the constraint (2.16) is a low energy artifact. This eliminates the topology of the target space, while keeping space-time continuous. Alternatively, we can look at the lattice version of the σ-model, with the constraint imposed at the microscopic level. Here, topology tearing operators and topology tearing excitations can "hide their singularities in the holes between lattice points" and they exist even though the target space topology is intact. We will see that in theories with gravity, black holes act in some respect like both of these mechanisms at once.

Finally, we would like to comment on another aspect of this theory, which is unrelated to our $\mathbb{Z}_p$ discussion. The σ-model has particle like excitations, "Hopfions" associated with nontrivial Hopf maps of space to the target space. Their number is conserved[7] and hence the theory has a *global continuous symmetry.* This symmetry cannot survive when the model is coupled to gravity. The operator $\mathcal{O}_{Hopf}$, when added to the Lagrangian, will describe the breaking of this symmetry. Although formally it is a local operator, we are dealing with a non-renormalizable theory, so it is at best local on the scale of the cutoff t. So we should imagine that the hole which it cuts in space-time is of size $\sim 1/t$. A smooth Hopfion configuration can shrink to the cutoff scale and when, in first order perturbation theory in $\mathcal{O}_{Hopf}$, it encounters the hole, it disappears. Its energy is radiated into topologically trivial fields of the σ-model. If in fact the underlying physics responsible for violation of Hopf number comes from a shorter distance scale, the dimensionless coefficient of $\mathcal{O}_{Hopf}$ in the Lagrangian will be small.

There is a similar description of proton decay, due to *e.g.* unification scale exchanges, in terms of a topology tearing operator in the Skyrme model. We will discuss it in Appendix B.

3. The taxonomy of stable strings

In this section we review and expand on Polchinski's classification of cosmic strings [7]. Polchinski classifies strings according to a long distance observer's ability to detect them:

1. "Local" strings cannot be detected by a long distance observer.
2. "Global" strings are coupled to a massless 2-form gauge field B or equivalently to a massless dual scalar ϕ, which winds around the string.
3. Aharonov-Bohm (AB) strings [17-19] are characterized by a nontrivial discrete holonomy around the string. We can divide this category in two by asking whether or not

[7] We point out that the corresponding conserved current is

$$J_H \sim \sum_{i,j} \left(\overline{\Phi}^i d\Phi_i - \Phi_i d\overline{\Phi}^i \right) d\overline{\Phi}^j d\Phi_j \sim d\phi \wedge \omega \quad , \tag{2.24}$$

where we have used the parametrization (2.17). It is not gauge invariant under the gauge symmetry of A and therefore this current cannot be gauged. (Equivalently, if we express it in terms of the gauge invariant data of the σ-model as $a \wedge \omega$, it is not invariant under the global $SU(2)$.) However, the corresponding charge is gauge invariant and therefore the Hopf number is conserved.

the discrete group is Abelian. We will leave a discussion of the IR theory for discrete non-Abelian groups to future work.

4. Quasi-Aharonov-Bohm strings have such a holonomy but it is embedded in a continuous gauge symmetry.

We would like to make several comments about these classes, emphasizing the perspective of this paper:

1. The term "local strings" originates from the cosmic string literature. From our perspective this terminology is confusing because such strings are characterized by a *global string* charge, which is not gauged. If the corresponding symmetry is continuous, the theory has a conserved string current – a closed but not exact 2-form current J. Following the three quantum gravity conjectures, such global string symmetries cannot be present and, as discussed in [7], such stable strings should not exist in a gravity theory.
2. Again, the term "global strings", which originates from the cosmic string literature is confusing. Such strings are characterized by a *local string* charge. We will discuss them in more detail below.
3. Here the Aharonov-Bohm (AB) phase is associated with a *discrete gauge symmetry*, e.g. $\mathbb{Z}_p$. The discussion of section 2 shows that these are $\mathbb{Z}_p$ strings. They arise from an ordinary $\mathbb{Z}_p$ gauge symmetry and are coupled to an emergent $\mathbb{Z}_p$ one-form gauge symmetry. So the difference between this case and the previous one is that here the one-form gauge symmetry is discrete. As we will discuss in section 4, in the context of a gravity theory p such strings can combine and disappear.
4. Unlike the previous case, here the ordinary gauge symmetry is embedded in a continuous gauge symmetry, e.g. $\mathbb{Z}_p \subset U(1)$. Since the low energy theory does not have a $\mathbb{Z}_p$ gauge symmetry, we do not find the emergent $\mathbb{Z}_p$ one-form gauge symmetry and hence these strings are at best associated with a global charge and should not be present in quantum gravity. In fact, in most situations such strings can decay by emitting the massless $U(1)$ gauge fields.

To summarize, classes 2 and 3 are associated with a long range one-form gauge symmetry, which is continuous and discrete respectively, while cases 1 and 4 are not associated with such a gauge symmetry and hence cannot exist in a consistent model of quantum gravity. We have given a complete IR description of class 3 strings when the discrete AB-phase is Abelian. The non-Abelian case certainly exists, because we know examples with discrete non-Abelian gauge groups in perturbative string compactification (e.g. the $\mathbf{S}_5$ symmetry of the quintic is a case in point). We will leave a discussion of the IR Lagrangian for such theories to future work. For the rest of this paper we restrict the AB-phase to lie in $\mathbb{Z}_p$.

We now turn to a more detailed discussion of the second class in which the strings are coupled to a massless scalar ϕ, which winds around the string. This massless ϕ leads to two problems:

1. As emphasized in [7], such a configuration of a winding ϕ is possible only if ϕ does not have a potential. Typically, this is natural only if the theory has a global shift symmetry of ϕ. The lack of global symmetries in gravity makes this possibility unnatural.
2. Even if a scalar ϕ without a potential is present, its kinetic term $(\partial\phi)^2$ makes the tension of the string infinite.

These two problems can be solved naturally, if our theory and our vacuum are supersymmetric. In that case it is natural to have no potential for ϕ without a shift symmetry. ϕ and its supersymmetric scalar partner lie on a supersymmetric moduli space.

In the context of supersymmetry the problem of infinite tension can also be ameliorated. ϕ is typically the real part of a massless complex field τ without a potential. More generally, we should view τ as a complex coordinate on the moduli space. For simplicity, consider a common example of a one dimensional moduli space where the metric on that space is determined by the kinetic term

$$g_{\tau\overline{\tau}}\partial\tau\partial\overline{\tau} = \frac{1}{(\mathrm{Im}\tau)^2}\partial\tau\partial\overline{\tau} \quad . \tag{3.1}$$

Because of the singularity as $\mathrm{Im}\tau \to \infty$, if $\mathrm{Im}\tau \to \infty$ near the boundary of space-time, the string tension can be finite.[8]

[8] Using polar coordinates in two dimensions, the most general solution of the equations of motion, which winds around infinity n times is $\tau = -\frac{n\phi}{2\pi} + i\frac{n}{4\pi C}\left((br)^C - (br)^{-C}\right)$, where b and C are integration constants. The action of this solution suffers from an IR divergence as $r \to \infty$. It is removed by taking the $C \to 0$ limit in which the solution becomes $\tau = i\frac{n}{2\pi}\log(bz)$ where $z = re^{i\phi}$. Note that this solution becomes singular at $z = 1/b$. In standard string applications this singularity is is removed by a duality transformations. We thank J. Distler for a useful discussion about this point.

One can generalize this construction to multi-dimensional moduli spaces. Finite tension strings will exist for every non-trivial circle *near the boundary of moduli space*, if the metric has the appropriate asymptotic behavior there. To our knowledge, this is the case for all the boundaries of moduli space that have been explored in string theory. Note that it does not matter whether the circle can be unwound in the interior of moduli space. The existence of a string is characterized by the winding of ϕ at spatial infinity, but the finite tension condition forces the fields to the boundary of moduli space at spatial infinity. This is analogous to the finite action condition on four dimensional gauge theory, which leads us to characterize gauge fields in terms of maps of the sphere at infinity into the gauge group.

This understanding leads us to an important point: the behavior of these finite tension strings is such that they cannot be viewed as excitations of a model at a generic point in moduli space. Generic models are characterized by generic values of the moduli at infinity and such boundary conditions do not accommodate finite tension strings.

At this point we would like to make a few comments:

1. For generic values of the moduli we can consider a circular loop of one of these strings, of radius R. Far from the string the moduli must first approach the boundary of moduli space, and then return to their fixed value at infinity. As $R \to \infty$, the fields must return to the fixed value at infinity over a larger region in space and hence the energy of this configuration must grow more rapidly than R. (Of course, this statement is equivalent to our statement above that such strings have infinite tension unless the asymptotic value of the moduli is infinite.) Consequently, for large enough R this configuration collapses into a black hole, and there are no large loops of cosmic string. For example, in the simple moduli space with asymptotic metric (3.1), the energy grows like $R \ln R$ and the maximal scale of the string loop is of order $\mu^{-1} e^{\frac{m_P^2}{\mu^2}}$, where μ^2 is the string tension.
2. Special cases of this discussion of cosmic strings with a $\mathbb{Z}$ valued quantum number in four asymptotically flat dimensions are fundamental Type II or heterotic $E_8 \times E_8$ superstrings, in compactifications that preserve minimal super-Poincare invariance in four dimensions. The infinite tension in these cases is of order g_s^2 as is the back reaction on the metric. For this reason this effect is often ignored.
3. We would like to point out an important fact, which follows from the analysis of [8], and to which we will return in section 5. The target space of every supersymmetric

field theory is a Kähler manifold. If its Kähler form is not exact, it cannot be coupled to linearized minimal supergravity. The only way to do that is through the introduction of an additional chiral superfield ρ. In the linearized approximation the Kähler potential K is replaced by $K + \rho + \overline{\rho}$. The real field Im$\rho$ plays the role of ϕ above – it is dual to a 2-form that couples to the string current. However, taking into account the gravitational corrections to the metric on the target space, this modification of K has the effect of ruining the topology of the target space. The strings are no longer stable, even classically.

4. At this point it is worth mentioning the gravitational back-reaction of the cosmic string, which is a deficit angle at infinity, and has been neglected in all of our discussions so far. This by itself means that there is a sense in which the string is not an excitation in asymptotically flat space, but if the deficit angle is small this is clearly sort of academic. One can make arbitrarily large closed loops of such strings without forming black holes. In particular, if the tension of $\mathbb{Z}_p$ strings is small compared to the Planck scale, then macroscopic strings are observable.
5. We caution again that the phenomenological import of our statements is unclear. Many of the instabilities we describe may have exponentially small probabilities. For example, our general arguments only imply the existence of magnetic monopoles of mass $\sim M_P$. In that case the Schwinger pair production probability per unit string length for these monopoles to destabilize an infinite cosmic string is $\exp\left(-\frac{M_P^2}{f^2}\right)$, where f is of the order of the energy scale of the magnetic flux line in some low energy Higgs field. Similarly, the collapse of a long but finite string for generic values of the moduli to a black hole takes place only for exponentially large R. Therefore, in many cases, these instabilities will have no phenomenological relevance.

Our conclusion then is that in models of quantum gravity in four asymptotically flat dimensions, with finite values of the moduli at infinity, there is only one kind of stable cosmic string, and it is stable modulo some integer p.[9] In models that have a non-compact moduli space, the existence of strings of finite tension depends on the behavior of the metric on target space at infinity and on the "fundamental group of the boundary." All other types of cosmic strings suffer from instabilities. In the next section we will provide arguments for the claims about quantum gravity, which we have used to come to these conclusions.

[9] There is a generalization to strings whose AB-phases lie in a more complicated finite group.

4. Symmetries in quantum gravity

In this section we will discuss the following three conjectures about symmetries in quantum gravity:

1. No global symmetries – all the symmetries including brane symmetries and discrete symmetries are gauged.
2. All continuous gauge symmetries are compact.
3. The entire set of allowed states in the lattice of charges is populated.

Before getting into a detailed discussion of these conjectures, we would like to point out that some special cases of them can be proven or at least be argued for quite convincingly, but others are more speculative. Furthermore, these conjectures are not logically independent. Assuming a subset of them, or some special cases, we can prove the others.

As an example of the interrelation between the conjectures, consider a gauge symmetry associated with a q-form conserved current. Then the corresponding field strength is a $d-q+1$ form F. If the gauge symmetry is noncompact, it does not have magnetic charges and F is closed and leads to a conserved current. Using the first conjecture, it must be gauged, i.e. be coupled to a $q-1$ gauge field B through the Chern-Simons coupling $B \wedge F$. As in section 2, this has the effect of Higgsing the two gauge symmetries. If such a coupling is not present, we conclude that the gauge group must be compact and that the theory includes dynamical magnetic monopoles, so that F is not closed. A similar example was discussed at the end of section 2.1. There we argued that if a gauge group is $\mathbb{R}$ and two relatively irrational charges are present, the theory must have a global symmetry, thus violating the first conjecture.

4.1. No global continuous symmetries

The idea that any theory of quantum gravity cannot have global symmetries has a long history and is often referred to as a "folk-theorem." Below we will discuss the rationale behind this idea. But before doing that we would like to make some comments:

1. The simplest form of this conjecture applies to continuous symmetries, which are associated with point particles in space-time dimensions four or larger. The usual argument for this case involves black hole physics.
2. The lack of such ordinary global continuous symmetries is known to be satisfied in all controlled constructions of quantum gravity. It was shown in [20] to be satisfied in perturbative string theory – global symmetries on the string world-sheet lead to gauge

symmetries in spacetime, and there is no way to have global symmetries in spacetime. The situation for strings in AdS background [21] is similar – global symmetries in the boundary theory are associated with gauge symmetries in the bulk and there is no way to have global symmetries in the bulk. Similar comments apply to the matrix model of [22].

3. Assuming that no such global symmetries exist, we can easily extend the result to all continuous symmetries associated with branes of codimension larger than two. Such branes can be compactified on circles to ordinary point particles in lower dimensions. The absence of global symmetries for these particles leads to the absence of global symmetries of the higher dimensional branes.
4. We will be particularly interested in string currents in four dimensions. These situations are not covered by the discussion above. Yet, we will argue in section 4.4 that such currents should also be coupled to gauge fields. One special string current will be discussed in section 5. This current appears in the supersymmetry current algebra. Hence, in the context of supergravity it must be gauged, independent of our more general conjectures.

We begin by reviewing the argument that models of quantum gravity cannot have global continuous symmetries. The lightest particle transforming under the group, is in a representation $\mathbf{r}$ and has mass m. We can make multi-particle states of an arbitrary representation $\mathbf{R} \subset \mathbf{r} \otimes \mathbf{r} \otimes \mathbf{r}$... and collide those particles to make a black hole. As long as the representation is not correlated with a long range gauge field, this will be a Schwarzschild black hole, and its macroscopic fields will be independent of $\mathbf{R}$. Hawking's calculation shows that this state decays to a remnant with mass of order XM_P, without emitting its global charge. Here XL_P is the value of the Schwarzschild radius for which Hawking's calculation becomes unreliable, . The value of X for which the Hawking calculation breaks down is hard to estimate but it is surely no more than an order of magnitude or so.

If m is non-zero, the remnant state is absolutely stable, since any combination of particles carrying such a large representation of the global group is heavier than the remnant. Even if $m = 0$, as would be the case for the global $SU(8)$ symmetry of $N = 8$ SUGRA, the lifetime of the state must go to infinity with the number of particles in $\mathbf{r}$, which are needed to make a black hole in $\mathbf{R}$. Indeed, the only way to emit an infinite number of massless particles with finite energy is through bremsstrahlung processes in which the decay products accelerate through re-scattering. Neither the remnant state, nor any of its decay products carry charge under any low energy gauge field apart from gravity itself. Gravitons are of course neutral under the global symmetry, so gravitational bremsstrahlung cannot carry away global charge. Thus, even if $m = 0$ we get an infinite number of arbitrarily long lived remnants, whose external geometry is that of Schwarzschild with $R_S = XL_P$.

The Covariant Entropy Bound (CEB) [23,24] is a conjectured bound on the entropy contained in any causal diamond in Lorentzian geometry. The original statement of the bound does not specify which density matrix is (implicitly) referred to, but it would be completely meaningless if it did not count entropy associated with verifiably different states that could form the same black hole geometry. Causal diamonds whose past boundary is a portion of the remnant black hole horizon[10] have a holographic screen whose area is that of the horizon. Thus, the CEB implies that the entropy of the remnant object is less than a finite number of order πX^2, which contradicts the existence of an infinite number of remnant states. Other arguments have been given for the absence of infinitely degenerate remnants, but all of them make *some* assumption about the most general possible theory of quantum gravity. For example, it has been argued that quantum loops of an infinite number of degenerate remnants are inconsistent. All such arguments contain *some* hidden assumptions. We view the CEB as the most elegant and general criterion for characterizing the differences between gravitational and non-gravitational quantum theories.

The CEB also enables us to get a soft bound on the maximal size of a *finite* global symmetry group. Let Σ be the sum of the dimensions of irreducible unitary representations of such a group. Since each state, in each representation, would be another possible state for the remnant, we get a bound

$$\Sigma < e^{\pi X^2} \quad . \tag{4.1}$$

The bound is soft, both because we do not know the value of X, and because for X of order one we expect corrections to the Bekenstein-Hawking formula.

Actually, as we said above and as we will discuss in section 4.2, it is our prejudice/conjecture that there are no global discrete symmetries in gravitational theories.

[10] The black hole remnant might be only meta-stable in the massless case and purists might object to using its interior geometry. We can instead talk about the causal diamond of an observer orbiting the black hole in the last stable orbit, over one period of rotation. This would affect only the quantitative estimate of the bound on the order of finite global symmetry groups, which we present below.

We want to point out that this set of arguments also rules out Abelian gauge fields with particles of relatively irrational charge. Suppose for example that there are particles of charge 1 and $\sqrt{2}$. Then we can make Reissner-Nordstrom black holes of charge

$$q = n_1 + \sqrt{2} n_2 \quad , \tag{4.2}$$

where the n_i are any integers. Thus, there are an infinite number of black holes of charge $q < \epsilon$, for any ϵ. Again, Hawking radiation will allow all of these to decay down to a remnant of Schwarzschild radius XL_P, but an infinite number of these states are indistinguishable, violating the CEB. This argument complements that given in section 2.1, that the theory has a global $U(1)$ symmetry, which counts the number of particles of charge $\sqrt{2}$. The argument for compactness of continuous symmetry groups and the fact that they must be gauged, are really one and the same.

4.2. Are discrete symmetries gauged?

In perturbative string theory there is an enormous amount of evidence that all discrete symmetries are gauge symmetries. For example, the $SL(2, \mathbb{Z})$ duality symmetry of ten dimensional Type IIB string theory is, from the M-theory point of view, just part of the eleven-dimensional reparametrization group. Similarly, the T-duality of Heterotic strings compactified on a circle, is a gauge symmetry [25], because it is part of a continuous $SU(2)$ gauge symmetry that is restored at the self dual radius. Some of the torsion part of the K-theory group of D-brane systems can be viewed as part of the gauge group of a space-filling brane anti-brane system out of which we construct lower dimensional D-branes [26-30]. For example, the $\mathbb{Z}_2$ of the stable non-BPS particle of Type I string theory is the center of the $Spin(32)/\mathbb{Z}_2$ gauge group of the branes. And so it goes for all known discrete symmetries.

We anticipate a general argument for the fact that discrete symmetries are gauged, based solely on simple principles of any theory of quantum gravity, but we have not found one as compelling as those for continuous symmetries. The best we could do is to find the argument around (4.1), which leads to a weak bound on the size of possible discrete groups.

A first step in this direction might be to establish that a $\mathbb{Z}_p$ symmetry is gauged if there is a finite tension cosmic string, around which particles pick up a $\mathbb{Z}_p$ phase. The existence of such a string would enable us to measure the $\mathbb{Z}_p$ charge of a black hole, proving that the $\mathbb{Z}_p$ charge is conserved and measurable from arbitrarily far away. There is clearly much more to be understood about this.

4.3. The Completeness hypothesis and black holes

On the basis of perturbative string theory examples, Polchinski has conjectured that consistent models of quantum gravity always contain a complete set of electric and magnetic objects consistent with the Dirac quantization condition [4]. This is certainly not the case in ordinary quantum field theory. There we can often take the masses of monopoles or charged particles to infinity, leaving behind an incomplete but consistent spectrum.

In a theory of gravity, this procedure will fail. When the mass of the particle becomes larger than the Planck mass, gravitational back-reaction cannot be neglected and the state becomes a black hole carrying the electric or magnetic charge of the erstwhile particle. This black hole will decay by Hawking radiation down to the extremal black hole for that charge. Thus, there is no way to tune parameters to eliminate charges.

Still, we might speculate about the possibility of models in which certain allowed states were "never there." In quantum field theory, there are two ways of specifying the global structure of a gauge group. We can either specify the allowed set of observable including line and surface operators, or we can specify the allowed fluxes of the field strength and its dual on geometries containing a non-trivial cycles. The line operators require renormalization by a factor

$$e^{c\int\sqrt{-g}}\quad , \tag{4.3}$$

and therefore interact with the gravitational field like a point particle. Thus, an allowed line operator will produce a charged black hole with the "missing" charge, even if the field theorist has refused to include a low energy field carrying that charge in the model.

Furthermore, the black hole geometry has an $\mathbb{S}^2$, which is topologically non-trivial, and so the field theorist will be forced to allow black holes with every flux that went into the definition of the gauge group.

Without a rigorous definition of the most general possible theory of quantum gravity, we cannot call these remarks a proof, but they constitute a very strong argument that the completeness hypothesis is valid in any such theory. As a consequence of the completeness hypothesis, our argument about relatively irrational charges becomes the stronger statement that *all continuous gauge groups are compact.* The completeness hypothesis eliminates the possibility of a non-compact gauge group (no monopoles), which "just happens" to have only quantized electric charge states.

Our argument that gauge groups must be compact applies only to continuous groups. We know that Type IIB string theory has an infinite $SL(2,\mathbb{Z})$ gauge symmetry. We also have not found an argument that bounds the value of p in a discrete $\mathbb{Z}_p$ gauge theory. Thus, we cannot rule out the $p \to \infty$ limit. We note however that $SL(2, \mathbb{Z})$ is always Higgsed to a finite subgroup by a choice of the moduli fields at infinity. Furthermore, known string compactifications to asymptotically flat space always seem to have finite discrete gauge groups, acting on the Hilbert space of excitations. Thus, it may be that more refined arguments than those we have presented could rule out infinite discrete gauge groups that leave the boundary conditions at infinity invariant.

It is quite likely that eventually the completeness hypothesis would be derived from general string theory considerations without relying on the existence of black holes. In fact, there are known simple solvable examples of string theory in one space or in one space and one time dimensions in which this question can be addressed. Clearly, in such degenerate situations there are no black hole solutions. So one could ask whether all allowed charges are present. The relevant charges are the RR charges of the type 0 theory. The authors of [31-35] have shown that all the RR charges of these theories have a manifestation in the matrix models. Furthermore, the allowed charges are quantized and *all* of them are populated.

4.4. Black holes and the stability of strings

In this subsection we focus on strings and follow the point of view of [7], which emphasizes measurements, which can be performed far from the string. We restate it as follows from the three conjectures: every stable string must be coupled to a KR gauge field. Its gauge group can be either $U(1)$ or $\mathbb{Z}_p$.[11] In the first case the total number of strings is conserved while in the latter case it is conserved only modulo p. Let us see how these ideas are realized in two examples of strings: Nielsen-Olesen strings of the Abelian Higgs model (section 2.3) and σ-model strings (section 2.4).

The completeness hypothesis shows us that in quantum gravity Nielsen-Olesen strings of low energy Higgs models are at best unbreakable modulo p, for some integer p. The value of p is determined by the charge of the Higgs field which in turn determines the unbroken gauge symmetry. The string can be detected by the AB-phase around it. The completeness assumption guarantees that the system always has dynamical particles of electric charge one, which can detect the $\mathbb{Z}_p$ phase. Furthermore, while in field theory the

[11] In this discussion we ignore the interesting situation which arises when strings are associated with a non-Abelian AB-phase.

system might not have magnetic monopoles of the lowest allowed charge, the completeness hypothesis now tells us that in gravity theories such monopoles must be present. p strings can end on such a monopole. Hence, not only can't we detect p strings using an AB-phase, but such a configuration of p strings is unstable. In terms of the $U(1)$ field strength we have

$$dF = J_m \neq 0 \quad , \tag{4.4}$$

where J_m is the monopole current, so the local string current is not conserved.

As we have explained in sections 2 and 3, one way to think about such a situation is in terms of a continuum $\mathbb{Z}_p$ gauge theory, which is a pair consisting of a 2-form gauge field B and a one-form A which are both valued in $U(1)$. The action is

$$\frac{ip}{2\pi}\int B \wedge F \quad . \tag{4.5}$$

Next we turn to σ-model strings. One way to generate these σ-models is by using a Higgs system and then the strings are similar to those we have just discussed. Alternatively, we can use a description which is intrinsic to the σ-model variables. The strings are maps of a plane or a two-cycle in space into a non-trivial two-cycle in the target space. Now consider a black hole space-time. The black hole horizon is a two sphere and we can consider maps which take this two sphere into the non-trivial two cycle in the target space. These can end a string configuration without any violation of continuity. Colloquially, we can say that "the other end of the string has fallen down the black hole." We can make a particularly compelling case for the existence of such string ends if the model contains a $U(1)$ gauge field. We can study the space-time of an extremal charged black hole, which has an infinitely long throat, ending in an AdS_2 horizon. It would appear to violate locality to claim that one could not thread a σ-model string through the throat. If we now drop charged particles into the extremal hole, we are left with the Schwarzschild string ender, when the dust of Hawking radiation has cleared.

As we explained in section 2.4, if the σ-model is coupled to a $\mathbb{Z}_p$ gauge theory through (2.18), then there can be massive particles which are not sections of a line bundle over the target space – they appear like "fractional powers of that line bundle." The completeness assumption guarantees that such dynamical particles exist. Such particles can detect the $\mathbb{Z}_p$ AB-phase around the string. If such a string can end, this phase would change discontinuously as we pass the string end. Thus, string ends can exist only if they are not detectable by this phase. In fact, the completeness assumption guarantees that such string ends must be present – each is an end of p strings. Again, we can say: "the number of strings, which can penetrate the black hole horizon is a multiple of p."

We have seen that by coupling the σ-model to a $\mathbb{Z}_p$ gauge theory, we can arrange for $\mathbb{Z}_p$ AB-phases in the interaction of probe particles with the σ-model strings. We conclude that in a model of quantum gravity, these strings, like those of the Higgs model, have no integer valued quantum number, but might be AB-strings for some finite group.

The violation of string current conservation by string ends is analogous both to its violation in lattice σ-models, where topology change can happen in the holes between lattice points, and in models where the target space topology is a low energy artifact. From the external observer's point of view, the black hole horizon is a hole in space, on which we are free to specify boundary conditions with non-trivial topology. From the internal point of view, the Hamiltonian is time dependent and singular, and all low energy field theoretic restrictions must be abandoned. The same is true for an external observer supported very close to the horizon, which experiences Hawking radiation of extremely high temperature.

Finally, we should add a word of caution. Throughout this subsection we have ignored the back reaction of the gravitational field. An infinitely long string in four dimensions creates a deficit angle at infinity and as such some of the analysis above is not necessarily justified. If the string tension is parametrically smaller than the Planck scale, it is reasonable to ignore this back reaction. It is not clear to us how to think about it for strings with Planck scale tensions.

5. Relation to Supergravity

So far supergravity has not played a role in our discussion. As we will now see, the discussion above allows us to streamline and to reinterpret some recent results about constraints on supergravity [8,9,6] and to further extend them. Throughout this section we will limit ourselves to $\mathcal{N} = 1$ supersymmetry in four dimensions. The extension to other cases is straightforward.

An important distinction in [8,9,6] is between supersymmetric theories containing a continuous parameter, which can be dialed to make gravity arbitrarily weakly coupled, and theories which are intrinsically gravitational, in the sense that all scales are discrete multiples of the Planck scale. We will devote the next two subsections to these two cases and will find that all of the problems exposed in [8,9,6] can be phrased in terms of the existence of string currents. When all such currents are gauged or their conservation laws violated, following the strictures of section 4, we find rather different outcomes in the two classes of models. In models with continuous parameters, the duals of the KR gauge fields are new chiral multiplets. These make all FI-terms field dependent, and remove all non-contractible two cycles from the low energy target space. In models with quantized FI-terms or periods, the KR gauge symmetry is discrete, and the low energy theory includes our universal $\mathbb{Z}_p$ Lagrangian.

5.1. Linearized SUGRA

Following [9] we start with constraints on linearized supergravity. Here one starts with a well defined rigid supersymmetric theory and studies its supersymmetry current multiplet. Then, this current is coupled to gauge fields – the supergravity multiplet. This is a standard and well known procedure (see e.g. the text books [36,37]). Depending on the super-current multiplet three different approaches can be taken:

1. Old minimal supergravity [38-40] uses the Ferrara-Zumino (FZ) multiplet [41]

$$\begin{aligned}&\overline{D}^{\dot\alpha}\mathcal{J}_{\alpha\dot\alpha}=D_\alpha X\\&\overline{D}_{\dot\alpha}X=0\quad .\end{aligned}\tag{5.1}$$

   This multiplet exists in all Lagrangian field theories, which have no FI-terms and for which the Kähler form of their target space is exact.

2. New minimal supergravity [42,43] is based on the R-multiplet (see e.g. section 7 of [36])

$$\begin{aligned}&\overline{D}^{\dot\alpha}\mathcal{R}_{\alpha\dot\alpha}=\chi_\alpha\\&\overline{D}_{\dot\alpha}\chi_\alpha=\overline{D}_{\dot\alpha}\overline{\chi}^{\dot\alpha}-D^\alpha\chi_\alpha=0\quad .\end{aligned}\tag{5.2}$$

   This multiplet exists in all field theories that have a global $U(1)_R$ symmetry, even in the presence of FI-terms or a target space with a nontrivial Kähler form. The conserved $U(1)_R$ current $j^{(R)}$ is the bottom component of $\mathcal{R}_{\alpha\dot\alpha}$.

3. 16/16 supergravity [44-46] is based on a larger multiplet [47,9], called the S-multiplet,

$$\begin{aligned}&\overline{D}^{\dot\alpha}\mathcal{S}_{\alpha\dot\alpha}=D_\alpha X+\chi_\alpha\\&\overline{D}_{\dot\alpha}X=0\\&\overline{D}_{\dot\alpha}\chi_\alpha=\overline{D}_{\dot\alpha}\overline{\chi}^{\dot\alpha}-D^\alpha\chi_\alpha=0\quad .\end{aligned}\tag{5.3}$$

This multiplet exists in all known Lagrangian field theories. It is particularly important in theories without a global $U(1)_R$ symmetry, which have FI-terms or a target space with a nontrivial Kähler form.

Additional multiplets and the relations between them were discussed in [47-49].

Each of these multiplets includes the energy momentum tensor $T_{\mu\nu}$ (6 independent bosonic operators) and the supersymmetry current $S_{\mu\alpha}$ (12 operators) as well as additional operators. Some of these operators can be interpreted as brane currents for branes of various dimensions.[12]

In section 4 we argued that all such brane currents must be coupled to gauge fields. In fact, for brane currents that are in the supersymmetry multiplet this conclusion follows from imposing supersymmetry – since $T_{\mu\nu}$ and $S_{\mu\alpha}$ are coupled to gauge fields, their superpartners must be as well.

There are a few important remarks that we should make about the brane currents that appear in the super-current multiplets:

1. The FZ-multiplet (5.1) includes the conserved, complex 2-brane current dx where x is the lowest component of X in (5.1) (recall that our conventions for currents (1.1) are dual to the more standard ones). For example, in Wess-Zumino models $x = 4W - \frac{1}{3}\overline{D}^2 K$, where W is the superpotential and K is the Kähler potential. The tension of domain walls (2-branes) is constrained by this current [50]. In supergravity this current must be gauged. The corresponding gauge field is a 3-form $A^{(3)}$. If x is well defined, the supergravity Lagrangian depends only on its field strength $F^{(4)} = dA^{(3)}$, which couples to x. Integrating out $F^{(4)}$ eventually leads to the famous $-|W|^2$ in the potential. This interpretation of the $|W|^2$ term as arising from integrating out the gauge field of 2-brane currents is similar to the relation between the Romans mass in ten dimensional IIA supergravity and the gauge field of 8-branes.
2. The R-multiplet (5.2) includes a conserved 2-form string current Z in the θ component of χ_α. For example, in Wess-Zumino models $\chi_\alpha = \overline{D}^2 D_\alpha K$ where K is the Kähler potential and the string current $Z = \omega$ is the pull back of the Kähler form to spacetime. In theories with an FI-term $\chi_\alpha = -4\xi W_\alpha$ where ξ is the FI-term and W_α is the

[12] It is common to refer to them as "brane charges" or "central charges." These terms are misleading. First, when these "charges" correspond to extended branes (not to 0-branes), the corresponding charge is infinite and only the charge per unit brane volume is meaningful. Therefore, we prefer to discuss the corresponding currents and the charges (1.1), which are finite. Second, these charges are not central – they do not commute with the Lorentz group.

superfield including the field strength F. The corresponding string current is $Z = \xi F$. Note that if this gauge field is $U(1)$ rather than $\mathbb{R}$ and it is coupled to dynamical magnetic monopoles, then its field strength F is not closed and therefore the defining equation (5.2) is not satisfied.[13]

3. The S-multiplet (5.3) includes both a string current and a 2-brane current.

The interpretation of the brane currents in the various supersymmetry multiplets leads to a new perspective on some of the issues raised in [8,9]. The problem with theories with nonzero FI-terms or with a non-exact Kähler form is that their supersymmetry current algebra includes string currents that cannot be improved to zero (see the discussion around (1.1)). Therefore, the FZ-multiplet does not exist.

If the rigid theory has a global $U(1)_R$ symmetry, it has an R-multiplet, which includes such a string current Z and therefore such theories can be coupled to supergravity using the new-minimal formalism. Even though both the $U(1)_R$ global current $j^{(R)}$ and the string current Z of the rigid theory are gauged, the resulting supergravity theory has both a global $U(1)_R$ symmetry and a conserved global string current. More explicitly, the supergravity multiplet includes a one-form gauge field $A^{(R)}$ for the global $U(1)_R$ symmetry of the rigid theory, and a 2-form gauge field B for the string current of the rigid theory. These two gauge fields couple through

$$B \wedge dA^{(R)} \quad . \tag{5.4}$$

Therefore, the gauged $U(1)_R$ current is a linear combination of the original the $U(1)_R$ current of the rigid theory $j^{(R)}$ and dB. Similarly, the gauged string current is a linear combination of the original string current Z and the field strength $F^{(R)} = dA^{(R)}$. Therefore, the supergravity theory has global conserved currents which can be taken to be $j^{(R)}$ and Z (or $F^{(R)}$ and $H = dB$). Note that the gauge fields $A^{(R)}$ and B do not correspond to massless propagating degrees of freedom and can be integrated out. The resulting on-shell theory still has global conserved currents. If we accept the arguments in section 4, we conclude that such rigid theories cannot arise in consistent models of quantum gravity [8] (see also [51,52]).

Even though the FZ-multiplet does not exist in these cases one can still use the old-minimal formalism. In fact, it is well known that these two formalisms are dual to each

---

[13] If such a theory indeed exists, one might need to use the more complicated multiplets discussed in [47-49].

other [53] (see however [54]) and therefore we can use either one. As is common in the old minimal formalism, one uses compensator fields. In the problematic situations the FZ-multiplet is either not gauge invariant or not globally well defined. This is taken care of by using compensator fields, which are also non-gauge invariant or fail to be globally well defined. This is possible only when the theory has a global $U(1)_R$ symmetry and ultimately rests on the fact that the R-multiplet exists.

This understanding naturally leads us to examine rigid theories without a global $U(1)_R$ symmetry but with nontrivial string currents. Here we must use the S-multiplet and the corresponding 16/16 supergravity. As pointed out in [46] for pure 16/16 supergravity and in [9] for the case with matter, this theory has an alternate interpretation. It can be interpreted as standard minimal supergravity theory in which the rigid matter system is enlarged by adding to it a chiral superfield τ. The real part of the scalar in τ is dual to the 2-form gauge field B, which couples to the string current. Note that unlike the situation in the new-minimal supergravity, which we discussed above, here B or its dual scalar correspond to a massless propagating particle. With this interpretation of the theory the additional chiral superfield τ removes the problem with the coupling to supergravity [9]. A simple way to understand it is to note that with τ the string current can be improved to zero; i.e. the 2-form current is exact. In theories with an FI-term, that term becomes field dependent [55]; more precisely, the FI-term can be absorbed by shifting τ such that the theory does not really have an FI-term. Similarly, in theories with nontrivial cycles, the cycles become contractible in the higher dimensional moduli space that includes τ [9].

So far we viewed the additional field τ as infinitesimal. However its global properties can be important. Consider a $U(1)$ gauge theory with an FI-term which is eliminated through the coupling to τ. The complex field $\exp(2\pi i\tau)$ transforms linearly under the $U(1)$ gauge symmetry, but we have not yet discussed its charge. It can be an arbitrary integer p. In that case, the nonzero value of $\exp(2\pi i\tau)$ Higgses the $U(1)$ gauge symmetry down to $\mathbb{Z}_p$ and the system can have $\mathbb{Z}_p$ strings. For example, this situation with field dependent FI-terms arises in string constructions with anomalous $U(1)$ [55]. The anomaly is canceled through a Green-Schwarz term $B \wedge F$, where F is the $U(1)$ field strength and B is dual to $\mathrm{Re}\tau$. The coefficient of this Green-Schwarz term is proportional to the sum of the $U(1)$ charges in the problem and thus determines the value of the integer p.

An interesting situation arises when the rigid theory has several $U(1)$ gauge fields V^i with transformation parameters Λ^i and FI-terms ξ_i. The chiral superfield τ is added to the rigid theory so that under gauge transformations it transforms as

$$\tau \to \tau + i\Lambda_i\xi^i \quad . \tag{5.5}$$

When some of the gauge groups are compact, i.e. $\Lambda_i \sim \Lambda_i + 2\pi i$, the global structure of τ becomes important. In this case if ξ_i are generic, the gauge transformation rule (5.5) is incompatible with any periodicity of τ. Similarly, we can consider a rigid theory with a complicated target space with several two-cycles. The addition of τ through $K \to K + i(\tau - \overline{\tau})$ [9] might be incompatible with all the necessary Kähler transformations associated with all the two-cycles[14].

One way to address this question is to constrain the maps from space-time to the target space. In the gauge theory case we could simply declare that all the gauge groups are $\mathbb{R}$ rather than $U(1)$. And in the σ-model case we can follow section 2 and [6] and restrict the maps such that the wrapping of various cycles is compatible with the existence of τ. This amounts to adding an additional discrete gauge symmetry to the system. This modification of the rigid theory by adding τ and an appropriate modification of its global structure makes the coupling to supergravity possible. However, the discussion in section 4 points us in another direction.

The issue with the global behavior of τ arises when the addition of τ to the rigid theory does not eliminate all its string currents. The addition of the discrete gauge symmetry in the previous paragraph, which restricts the maps from spacetime to the target space, induces a global $U(1)$ symmetry which shifts τ by an arbitrary real constant. Although this is perfectly consistent with perturbative supergravity, it violates the conjectures of section 4.

Therefore, the only way to weakly couple such systems to supergravity, while respecting the three conjectures of section 4, is the following. For generic ξ_i and for generic cycles, we need to add a separate τ_i field for every gauge group and for every two-cycle. In other words, all FI-terms must be field dependent and all the Kähler moduli must be massless dynamical fields.[15]

---

[14] We thank N. Nekrasov for a useful discussion about this point.

[15] This observation extends the condition about massless moduli in [9] to many additional moduli. It has the consequence that in a supersymmetric string construction with a continuously variable ratio between low energy scales and the Planck scale, many of the moduli cannot be stabilized in a manner consistent with SUSY.

5.2. Intrinsically gravitational theories

In the previous subsections we studied a quantum field theory weakly coupled to supergravity, with a continuously variable coupling parameter. Assuming the conjectures of section 4., we concluded that all FI-terms are field dependent (which means they can be removed), and no two-cycles are present in the target space.

Here we consider situations in which the separation between the rigid field theory and gravity is not possible. In particular, we are interested in theories with FI-terms or with target spaces that have nontrivial cycles of the order of the Planck scale.

A careful analysis of the component Lagrangian shows that FI-terms must be quantized[16] [6] (see also [56])

$$\xi = 2NM_P^2 \qquad , \qquad N \in \mathbb{Z}\ . \tag{5.6}$$

Similarly, if the target space includes nontrivial two-cycles, their periods must be rational. For example, if the target space includes a $\mathbb{CP}^1$ with the metric

$$ds^2 = f_\pi^2 \frac{d\Phi d\overline{\Phi}}{(1+|\Phi|^2)^2} \quad , \tag{5.7}$$

f_π must satisfy

$$f_\pi^2 = \frac{2N}{p} M_P^2 \qquad , \qquad p, N \in \mathbb{Z} \tag{5.8}$$

and the integer p is related to a discrete $\mathbb{Z}_p$ gauged R-symmetry. (The case $p = 1$ was studied in [57].)

The quantization conditions (5.6)(5.8) were derived in [6] by focusing on the gravitino and its transformation laws. These are gauge transformations in the case with FI-terms, and Kähler and $\mathbb{Z}_p$ transformations in the case with two-cycles. Here we will re-derive and interpret these results from the perspective of our discussion – using the underlying string currents and the coupling to a 2-form gauge field.

The new-minimal formalism is particularly useful if one wants to track the remaining discrete gauge symmetries. Let us start with a model with an R-symmetry such that it is straightforward to use this formalism. As explained around (5.4), even though the R-symmetry is gauged, the theory still has a global R-symmetry whose current can be taken to be $j^{(R)}$. Next we try to add to the Lagrangian terms that explicitly break this symmetry. However, since a linear combination of $j^{(R)}$ and $H = dB$ is gauged, we must preserve that

[16] We use notation $\frac{1}{G_N} = M_{Planck}^2 = \frac{8\pi}{\kappa^2} = 8\pi M_P^2$; i.e. M_P is the reduced Planck mass.

linear combination. In other words, the conservation of H must also be violated. This can be done only if the one-form gauge symmetry of B is compact. Then, as explained in section 2, there exist local operators $\mathcal{O}_N$ around which $\int_{\mathbb{S}^3} H = 2\pi N \neq 0$ ($N \in \mathbb{Z}$). These operators carry charge N under the global symmetry. Using $\mathcal{O}_N$ and R-breaking operators constructed out of matter fields, we can explicitly violate the conservation of both $H = dB$ and $j^{(R)}$, while preserving their gauged linear combination.

Similarly the theory has two conserved string currents, Z and $F^{(R)} = dA^{(R)}$. One linear combination of them is gauged. We can preserve that linear combination, while violating the conservation of Z and $F^{(R)}$, by adding to our system particles magnetically charged under $A^{(R)}$, and having suitable properties under Z.

The upshot of all this is that if the gauge symmetries of $A^{(R)}$ and of B are compact, and the appropriate magnetic objects are present, both the global R-symmetry and the conservation of the global string current can be violated. Of course, since these symmetries are compact, their corresponding charges are quantized. This leads to the quantization conditions (5.6)(5.8).

The same conclusion can be reached using the old-minimal formalism. Here the point is that the compensators are not gauge invariant and not Kähler invariant. Normally, the compensator fields appear only in front of the exponential of the Kähler potential and in front of the superpotential. However, if the quantization conditions (5.6)(5.8) are satisfied, we can add to the Lagrangian terms with other dependence on the compensators such that they explicitly break the global R-symmetry.

In conclusion, the problem with FI-terms and with nontrivial two-cycles can be traced back to the existence of strings in the rigid model that we are trying to couple to SUGRA. With FI-terms the quantization condition (5.6) guarantees that magnetic monopoles can be added to the system such that the strings are unstable. With nontrivial two-cycles the condition (5.8) guarantees that one can couple the strings to a $\mathbb{Z}_p$ gauge theory and violate the string current conservation.

Acknowledgments

We thank N. Arkani-Hamed, O. Aharony, M. Dine, T. Dumitrescu, J. Distler, M. Douglas, D. Freed, Z. Komargodski, J. Maldacena, G. Moore, N. Nekrasov, J. Polchinski, M. Rocek, S. Shenker, A. Strominger, and E. Witten for useful discussions. The work of TB was supported in part by DOE grant DE-FG03-92ER40689 and in part by the Ambrose Monell Foundation at the Institute for Advanced Study. The work of NS is partially supported by DOE grant DE-FG02-90ER40542. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the funding agencies.

Appendix A. Generalizations of the Lagrangian description of the $\mathbb{Z}_p$ gauge theories

It is easy to extend the BF-theory to $\mathcal{N}=1$ supersymmetry. The gauge field A belongs to a vector superfield $\mathcal{V}$ with the gauge symmetry

$$\mathcal{V} \to \mathcal{V} + \Lambda + \Lambda^\dagger \quad . \tag{A.1}$$

The 2-form B belongs to a chiral spinor superfield $\mathcal{B}_\alpha$ (satisfying $\overline{D}_{\dot\alpha}\mathcal{B}_\alpha = 0$) with the gauge symmetry

$$\mathcal{B}_\alpha \to \mathcal{B}_\alpha + \overline{D}^2 D_\alpha L \tag{A.2}$$

for arbitrary real L. The gauge invariant field strength $H = dB$ is embedded in the real linear superfield[17] $\mathcal{H} = D^\alpha \mathcal{B}_\alpha + h.c.$ which is invariant under (A.2).

Then the BF-Lagrangian (2.9) is included in

$$\frac{p}{2\pi}\int d^4\theta \mathcal{H}\mathcal{V} = \frac{p}{2\pi}\int d^2\theta \mathcal{B}_\alpha W^\alpha + h.c. \tag{A.3}$$

which is invariant under the two gauge symmetries (A.1)(A.2).

Our discussion of the BF-theory has an obvious generalization to an arbitrary q-form gauge field A in d dimensions [12]. The $\mathbb{Z}_p$ gauge theory is described at low energies by two $U(1)$ gauge fields: a q-form A and a $d-q-1$-form B with the Lagrangian

$$\frac{ip}{2\pi} B\wedge dA \quad . \tag{A.4}$$

Note that this theory is invariant under $q \to d-q-1$ which exchanges A and B.

A particularly interesting special case is $q=1$ in three dimensions, where both A and B are one-forms. In this case we find a $U(1)\times U(1)$ Lagrangian description of the three dimensional $\mathbb{Z}_p$, level k, Chern-Simons gauge theory of [58,59] (see also the recent discussions [60,61])

$$\frac{ip}{2\pi} B\wedge dA + \frac{ik}{4\pi} A\wedge dA \quad . \tag{A.5}$$

As in (2.12), the theory has p^2 line operators

$$\exp\left(i n_A \oint_{\Sigma_1} A + i n_B \oint_{\Sigma_1} B \right) \tag{A.6}$$

labeled by $n_A, n_B = 0, 1, ...p$.

---

[17] A real linear superfield $\mathcal{H}$ is characterized by $\mathcal{H} = \mathcal{H}^\dagger$ and $D^2\mathcal{H} = 0$ (and using the reality also $\overline{D}^2\mathcal{H} = 0$).

Appendix B. Topology changing operators in the chiral Lagrangian

In this brief appendix, we note that our method of describing operators that violate topological symmetries in terms of boundary conditions, can easily be extended to other σ-models.

Consider for example the $SU(n)$ chiral model. The dynamical field is a group element $\Sigma \in SU(n)$ and the model has an $SU(n)\times SU(n)$ symmetry. We compactify space to $\mathbb{S}^3$ and then the states of the system are labeled by the integer quantum number $\pi_3(SU(n)) = \mathbb{Z}$ and the corresponding current is

$$B \sim \mathrm{tr}\ (\Sigma^\dagger d\Sigma)^3 \quad , \tag{B.1}$$

where the cubic product is a wedge product. As is well known, in the context of QCD, this current should be interpreted as the baryon current, and the corresponding solitons are baryons [62-64].

We can define a local operator $\mathcal{O}_k$ that creates k baryons. We remove a point from spacetime and impose that Σ on its surrounding $\mathbb{S}^3$ is associated with k wrappings in $\pi_3(SU(n)) = \mathbb{Z}$. $\mathcal{O}_{k=1}$ is an interpolating field for a single baryon. We can add $\mathcal{O}_k$ to the Lagrangian to describe the effects of UV physics that violates baryon number but preserves a $\mathbb{Z}_k$ subgroup of it.

Another example of the general phenomena we have explored is the observation that the chiral Lagrangian for an $O(n)$ gauge theory coupled to n chiral fermions in the vector representation, has $\mathbb{Z}_2$ strings arising from $\pi_2(SU(n)/O(n)) = \mathbb{Z}_2$ [64]. We can construct a string creation operator by excising a one dimensional line from space-time and imposing boundary conditions around it. Locally the line is surrounded by $\mathbb{R} \times \mathbb{S}^2$ and we impose that on the $\mathbb{S}^2$ the fields are associated with the non-trivial homotopy class.

Appendix C. Lattice gauge theories

There are higher form $\mathbb{Z}_p$ invariant gauge theories on any d dimensional cubic lattice. In fact, our discussion generalizes rather simply to any finite Abelian group and any d dimensional simplicial complex. A q-form $\mathbb{Z}_p$ gauge theory is an assignment of an integer $L \mathrm{mod} p$ to each $q+1$ dimensional hypercube on the lattice. (Recall our convention labeling higher form gauge symmetry by the dimension of the gauge parameter.) For $q = -1, 0$ the hypercube is a point or a link. To simplify the notation, in this appendix we will restrict attention to $q=-1$ and $d=4$, which is generally called the case with *global* $\mathbb{Z}_p$ symmetry. Our considerations are easily generalized to arbitrary q and d. In particular, our conclusion that, in the symmetric phase, the global $\mathbb{Z}_p$ symmetry is realized as the global limit of an IR gauge symmetry is true for general d.

The lattice partition function is the sum over all $L(x)$ of[18]

$$e^{\sum_{x,\mu} S[d_\mu L(x)]} \quad , \tag{C.1}$$

where the sum in the exponent is over all elementary links. d_μ is the lattice finite difference operator in the direction labeled by μ. Correlation functions are expectation values of local operators of the form

$$e^{\frac{2\pi i}{p} n_k L(x)} \quad , \quad n_k = 0, ..., p-1 \quad . \tag{C.2}$$

(For higher values of q the observables are gauge invariant products of elements like (C.2).) $\mathbb{Z}_p$ invariance would imply that only correlation functions with $\sum n_k = 0 \mathrm{mod} p$ are non-vanishing, but there are values of the parameters where the $\mathbb{Z}_p$ symmetry is spontaneously broken[19].

Next, we apply ordinary electric magnetic duality to this system and replace the variables $L(x)$ on the sites with plaquette variables. We define $e^{\widetilde{S}[z]}$ to be the $\mathbb{Z}_p$ Fourier transform of $e^{S[z]}$. We use the same letter for the arguments of the two functions because the group $\mathbb{Z}_p$ is self-dual. Using lattice integration by parts one easily shows that the partition function is given by

$$\sum_{K_\mu(x)} e^{\sum_{x,\mu} \widetilde{S}[K_\mu(x)]} \prod_x \delta[d_\mu K_\mu(x)] \quad . \tag{C.3}$$

The $\mathbb{Z}_p$ valued vector K_μ must be conserved. The δ function in this equation is a $\mathbb{Z}_p$ valued Kronecker δ .

For correlation functions, conservation of K_μ fails on those points where there are local operators. We can solve the conservation constraint by writing

$$K_\mu = \epsilon^{\mu\nu\rho\sigma} d_\nu L_{\rho\sigma} + \Delta \quad . \tag{C.4}$$

[18] We limit ourselves to nearest neighbor interactions. More complicated local actions which couple more points are possible.

[19] If we were dealing with $q \geq 0$, this language would be slightly misleading, but conventional. The local symmetry is never spontaneously broken. This phase is called the Higgs phase for $q \geq 0$.

The $\mathbb{Z}_p$ valued 2-form $L_{\mu\nu}$ really lives on the dual lattice, but we use the Levi-Civita symbol to assign it to a lattice plaquette. Δ is a sum of Dirac string contributions, emanating from each point on the original lattice where conservation failed. The correlation functions are invariant under $L_{\mu\nu} \to L_{\mu\nu} + d_\mu L_\nu - d_\nu L_\mu$. The Dirac strings must either go to infinity, or connect points with opposite charge. Any choice of strings satisfying these conditions is equivalent to any other choice, because the difference between them defines a K_μ which is conserved, and can be written in terms of $L_{\mu\nu}$.[20] We see that a theory with global $\mathbb{Z}_p$ symmetry is dual to a theory with a one-form $\mathbb{Z}_p$ gauge symmetry. More generally, a theory with a k-form gauge symmetry is dual to one with $(d-4-k)$-form gauge symmetry. Correlation functions of the original fields involve expressions with non-local Dirac strings, when they are written in terms of the dual fields.

The action S depends on p parameters, and in some extreme parameter ranges it is large. We can then approximate the partition sum by finding the configuration that minimizes S and considering a dilute gas of small localized fluctuations around it. This is actually the first term in a convergent expansion and defines a weak coupling phase of the lattice theory. Similarly, there is a different range of parameters in which $\widetilde{S}$ is large, and we have a strong coupling phase. Both phases have a mass gap finite in lattice units. The IR physics is completely topological and is described by a BF-Lagrangian. For the weak coupling, spontaneously broken phase, the IR Lagrangian has a 0-form potential and a 3-form potential. The latter couples to the domain wall excitations of the spontaneously broken phase. In the strong coupling, symmetric, phase the IR Lagrangian has a 2-form potential B and a 1-form potential. The Dirac strings that attach to the charged local operators, are realized as Wilson lines of the 1-form potential. They can be shifted at will, as long as there are no sources coupling to the 2-form.

We conclude that in the symmetric phase, the global symmetry of the original model is realized as the global transformation that is the limit of a $\mathbb{Z}_p$ 0-form (Maxwell) gauge symmetry, coupled by the topological Lagrangian to a 1-form (Kalb-Ramond) gauge potential. If we take the continuum limit, the tension of the co-dimension 2 objects that couple to $B_{\mu\nu}$ goes to infinity, so we can always ignore the fact that the global symmetry is the global part of a gauge group.

We want to emphasize that we are *not* saying that every $\mathbb{Z}_p$ one form gauge symmetry has a global symmetry associated with it. In QFT, exact global symmetries are not

[20] On simplicial complexes with more complicated topology we would have to include variables describing closed but inexact forms.

emergent – at best there can be accidental approximate global symmetries. However gauge symmetries are often emergent. We have shown that a particular class of UV regulators, the lattice, associates a one form $\mathbb{Z}_p$ gauge symmetry to every system with a global $\mathbb{Z}_p$ symmetry. In the IR limit of the symmetric phase, there is an emergent 0-form gauge symmetry, and the original global symmetry becomes the global limit of these gauge transformations in the IR. We can run this argument backward only if we have the full UV complete 1-form lattice gauge theory.

It is illuminating to restate this using our continuum formalism. We start with a continuum $\mathbb{Z}_p$ gauge theory, $\mathcal{L} = \frac{ip}{2\pi} B \wedge dA$ and would like to introduce local operators. We do that by adding to the system charged fields $\chi(x)$ and study the gauge invariant operators

$$\chi(x) e^{i \int_x^\infty A} \quad . \tag{C.5}$$

These are not invariant under gauge transformations that approach a constant at infinity. Therefore, the operators (C.5) can be interpreted as carrying charge under a global symmetry.

Conversely, starting with a linearly realized global $\mathbb{Z}_p$, acting on a field $\psi(x)$, we can write $\psi(x) = \sqrt{\psi^* \psi} e^{i \int_x^\infty V}$, where V is a one form and the integral is taken along some contour. Adding to the action $e^{\frac{ip}{2\pi} \int B \wedge dV}$ we see that B acts as a Lagrange multiplier setting $dV = 0$ and therefore the expression for $\psi(x)$ is independent of local changes in the contour. We have thus introduced a fake gauge invariance, and in this formalism the global symmetry is realized as the global part of the gauge group – the group of gauge transformations that act as constants at infinity, modulo those which act as the identify at infinity.

These observations may be relevant to our speculations about discrete gauge symmetries in section 4.2. If gravitational effects force the co-dimension 2 brane tension to be finite, we would be able to prove that all discrete symmetries are gauged.

For many forms of the action S, appropriate values of d, and large enough p, there is another phase of the system, which illustrates some of the principles we have expounded in the text. This is well known in the literature [65,66], so we will only sketch the results. Start from the dual form of the $\mathbb{Z}_p$ theory and use the coset construction of $\mathbb{Z}_p$ in terms of $\mathbb{Z}$ to write this as a $\mathbb{Z}$ valued gauge theory with matter fields of charge p. Then use the Poisson summation formula to write the sum over $\mathbb{Z}$ valued fields in terms of an integral and another sum. For $q = 0$ and $d = 4$, the result is a lattice version of QED, coupled to electric and magnetic charges. The electric charges are p times the fundamental unit of charge, while the monopoles have the minimal Dirac quantum. There is often an intermediate Coulomb phase, where the IR physics is that of a free photon, and the electric and magnetic particles have mass of order the inverse lattice spacing. The weak and strong coupling phases can be viewed as transitions to states where either electric or magnetic charges are condensed. The charges supply the relevant ends of strings in the weak and strong coupling phases. The Coulomb phase has no stable strings and is an illustration of the instability of quasi-Aharonov-Bohm strings [7].

References

[1] D. Garfinkle, G. T. Horowitz and A. Strominger, "Charged black holes in string theory," Phys. Rev. D **43**, 3140 (1991) [Erratum-ibid. D **45**, 3888 (1992)] [Phys. Rev. D **45**, 3888 (1992)].

[2] G. T. Horowitz and A. Strominger, Nucl. Phys. B **360**, 197 (1991).

[3] T. Hartman, K. Murata, T. Nishioka and A. Strominger, "CFT Duals for Extreme Black Holes," JHEP **0904**, 019 (2009) [arXiv:0811.4393 [hep-th]].

[4] J. Polchinski, "Monopoles, duality, and string theory," Int. J. Mod. Phys. A **19S1**, 145 (2004) [arXiv:hep-th/0304042].

[5] C. W. Misner and J. A. Wheeler, "Classical physics as geometry: Gravitation, electromagnetism, unquantized Annals Phys. **2**, 525 (1957).

[6] N. Seiberg, "Modifying the Sum Over Topological Sectors and Constraints on arXiv:1005.0002 [hep-th].

[7] J. Polchinski, "Open heterotic strings," JHEP **0609**, 082 (2006) [arXiv:hep-th/0510033].

[8] Z. Komargodski and N. Seiberg, "Comments on the Fayet-Iliopoulos Term in Field Theory and Supergravity," JHEP **0906**, 007 (2009) [arXiv:0904.1159 [hep-th]].

[9] Z. Komargodski and N. Seiberg, "Comments on Supercurrent Multiplets, Supersymmetric Field Theories and Supergravity," arXiv:1002.2228 [hep-th].

[10] J. S. Schwinger, "On gauge invariance and vacuum polarization," Phys. Rev. **82**, 664 (1951).

[11] G. T. Horowitz, "Exactly Soluble Diffeomorphism Invariant Theories," Commun. Math. Phys. **125**, 417 (1989).

[12] J. M. Maldacena, G. W. Moore and N. Seiberg, "D-brane charges in five-brane backgrounds," JHEP **0110**, 005 (2001) [arXiv:hep-th/0108152].

[13] E. Witten, "SL(2,Z) action on three-dimensional conformal field theories with Abelian symmetry," arXiv:hep-th/0307041.

[14] T. Pantev and E. Sharpe, "GLSM's for gerbes (and other toric stacks)," Adv. Theor. Math. Phys. **10**, 77 (2006) [arXiv:hep-th/0502053].

[15] T. Pantev and E. Sharpe, "Notes on gauging noneffective group actions," arXiv:hep-th/0502027.

[16] A. Caldararu, J. Distler, S. Hellerman, T. Pantev and E. Sharpe, "Non-birational twisted derived equivalences in abelian GLSMs," arXiv:0709.3855 [hep-th].

[17] M. G. Alford and F. Wilczek, "Aharonov-Bohm Interaction of Cosmic Strings with Matter," Phys. Rev. Lett. **62**, 1071 (1989).

[18] L. M. Krauss and F. Wilczek, "Discrete Gauge Symmetry in Continuum Theories," Phys. Rev. Lett. **62**, 1221 (1989).

[19] J. Preskill and L. M. Krauss, "Local Discrete Symmetry And Quantum Mechanical Hair," Nucl. Phys. B **341**, 50 (1990).

[20] T. Banks and L. J. Dixon, "Constraints on String Vacua with Space-Time Supersymmetry," Nucl. Phys. B **307**, 93 (1988).

[21] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, "Large N field theories, string theory and gravity," Phys. Rept. **323**, 183 (2000) [arXiv:hep-th/9905111].

[22] T. Banks, W. Fischler, S. H. Shenker and L. Susskind, "M theory as a matrix model: A conjecture," Phys. Rev. D **55**, 5112 (1997) [arXiv:hep-th/9610043].

[23] W. Fischler and L. Susskind, "Holography and cosmology," arXiv:hep-th/9806039.

[24] R. Bousso, "A Covariant Entropy Conjecture," JHEP **9907**, 004 (1999) [arXiv:hep-th/9905177].

[25] M. Dine, P. Y. Huet and N. Seiberg, "Large and Small Radius in String Theory," Nucl. Phys. B **322**, 301 (1989).

[26] R. Minasian and G. W. Moore, "K-theory and Ramond-Ramond charge," JHEP **9711**, 002 (1997) [arXiv:hep-th/9710230].

[27] A. Sen, "Stable non-BPS states in string theory," JHEP **9806**, 007 (1998) [arXiv:hep-th/9803194].

[28] A. Sen, "Stable non-BPS bound states of BPS D-branes," JHEP **9808**, 010 (1998) [arXiv:hep-th/9805019].

[29] A. Sen, "Tachyon condensation on the brane antibrane system," JHEP **9808**, 012 (1998) [arXiv:hep-th/9805170].

[30] E. Witten, "D-branes and K-theory," JHEP **9812**, 019 (1998) [arXiv:hep-th/9810188].

[31] M. R. Douglas, I. R. Klebanov, D. Kutasov, J. M. Maldacena, E. J. Martinec and N. Seiberg, "A new hat for the c = 1 matrix model," arXiv:hep-th/0307195.

[32] I. R. Klebanov, J. M. Maldacena and N. Seiberg, "Unitary and complex matrix models as 1-d type 0 strings," Commun. Math. Phys. **252**, 275 (2004) [arXiv:hep-th/0309168].

[33] N. Seiberg and D. Shih, "Branes, rings and matrix models in minimal (super)string theory," JHEP **0402**, 021 (2004) [arXiv:hep-th/0312170].

[34] N. Seiberg and D. Shih, "Flux vacua and branes of the minimal superstring," JHEP **0501**, 055 (2005) [arXiv:hep-th/0412315].

[35] J. M. Maldacena and N. Seiberg, "Flux-vacua in two dimensional string theory," JHEP **0509**, 077 (2005) [arXiv:hep-th/0506141].

[36] S. J. Gates, M. T. Grisaru, M. Rocek and W. Siegel, "Superspace, or one thousand and one lessons in supersymmetry," Front. Phys. **58**, 1 (1983) [arXiv:hep-th/0108200].

[37] S. Weinberg, "The quantum theory of fields. Vol. 3: Supersymmetry," *Cambridge, UK: Univ. Pr. (2000) 419 p*

[38] K. S. Stelle and P. C. West, "Minimal Auxiliary Fields For Supergravity," Phys. Lett. B **74**, 330 (1978).

[39] S. Ferrara and P. van Nieuwenhuizen, "The Auxiliary Fields Of Supergravity," Phys. Lett. B **74**, 333 (1978).

[40] E. S. Fradkin and M. A. Vasiliev, "S Matrix For Theories That Admit Closure Of The Algebra With The Aid Of Auxiliary Fields: The Auxiliary Fields In Supergravity," Lett. Nuovo Cim. **22**, 651 (1978).

[41] S. Ferrara and B. Zumino, "Transformation Properties Of The Supercurrent," Nucl. Phys. B **87**, 207 (1975).

[42] V. P. Akulov, D. V. Volkov and V. A. Soroka, "On The General Covariant Theory Of Calibrating Poles In Superspace," Theor. Math. Phys. **31**, 285 (1977) [Teor. Mat. Fiz. **31**, 12 (1977)].

[43] M. F. Sohnius and P. C. West, "An Alternative Minimal Off-Shell Version Of N=1 Supergravity," Phys. Lett. B **105**, 353 (1981).

[44] G. Girardi, R. Grimm, M. Muller and J. Wess, "Antisymmetric Tensor Gauge Potential In Curved Superspace And A (16+16) Supergravity Multiplet," Phys. Lett. B **147**, 81 (1984).

[45] W. Lang, J. Louis and B. A. Ovrut, "(16+16) Supergravity Coupled To Matter: The Low-Energy Limit Of The Superstring," Phys. Lett. B **158**, 40 (1985).

[46] W. Siegel, "16/16 Supergravity," Class. Quant. Grav. **3**, L47 (1986).

[47] M. Magro, I. Sachs and S. Wolf, "Superfield Noether procedure," Annals Phys. **298**, 123 (2002) [arXiv:hep-th/0110131].

[48] S. M. Kuzenko, "Variant supercurrent multiplets," arXiv:1002.4932 [hep-th].

[49] S. M. Kuzenko, "Variant supercurrents and Noether procedure," arXiv:1008.1877 [hep-th].

[50] G. R. Dvali and M. A. Shifman, "Domain walls in strongly coupled theories," Phys. Lett. B **396**, 64 (1997) [Erratum-ibid. B **407**, 452 (1997)] [arXiv:hep-th/9612128].

[51] S. M. Kuzenko, "The Fayet-Iliopoulos term and nonlinear self-duality," arXiv:0911.5190 [hep-th].

[52] K. R. Dienes and B. Thomas, "On the Inconsistency of Fayet-Iliopoulos Terms in Supergravity Theories," arXiv:0911.0677 [hep-th].

[53] S. Ferrara, L. Girardello, T. Kugo and A. Van Proeyen, "Relation Between Different Auxiliary Field Formulations Of N=1 Supergravity Coupled To Matter," Nucl. Phys. B **223**, 191 (1983).

[54] N. D. Lambert and G. W. Moore, "Distinguishing off-shell supergravities with on-shell physics," Phys. Rev. D **72**, 085018 (2005) [arXiv:hep-th/0507018].

[55] M. Dine, N. Seiberg and E. Witten, "Fayet-Iliopoulos Terms in String Theory," Nucl. Phys. B **289**, 589 (1987).

[56] J. Distler and E. Sharpe, "Quantization of Fayet-Iliopoulos Parameters in Supergravity," arXiv:1008.0419 [hep-th].

[57] E. Witten and J. Bagger, "Quantization Of Newton's Constant In Certain Supergravity Theories," Phys. Lett. B **115**, 202 (1982).

[58] R. Dijkgraaf and E. Witten, "Topological Gauge Theories And Group Cohomology," Commun. Math. Phys. **129**, 393 (1990).

[59] D. S. Freed and F. Quinn, "Chern-Simons theory with finite gauge group," Commun. Math. Phys. **156**, 435 (1993) [arXiv:hep-th/9111004].

[60] D. Belov and G. W. Moore, "Classification of abelian spin Chern-Simons theories," arXiv:hep-th/0505235.

[61] A. Kapustin and N. Saulina, "Topological boundary conditions in abelian Chern-Simons theory," arXiv:1008.0654 [hep-th].

[62] T. H. R. Skyrme, "A Nonlinear field theory," Proc. Roy. Soc. Lond. A **260**, 127 (1961).

[63] D. Finkelstein and J. Rubinstein, "Connection between spin, statistics, and kinks," J. Math. Phys. **9**, 1762 (1968).

[64] E. Witten, "Current Algebra, Baryons, And Quark Confinement," Nucl. Phys. B **223**, 433 (1983).

[65] S. Elitzur, R. B. Pearson and J. Shigemitsu, "The Phase Structure Of Discrete Abelian Spin And Gauge Systems," Phys. Rev. D **19**, 3698 (1979).

[66] A. Ukawa, P. Windey and A. H. Guth, "Dual Variables For Lattice Gauge Theories And The Phase Structure Of Z(N) Systems," Phys. Rev. D **21**, 1013 (1980).