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The vacuum revealed: the final state of vacuum instabilities in compact stars

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Quantum fields in compact stars can be amplified due to a semiclassical instability. This generic feature of scalar fields coupled to curvature may affect the birth and the equilibrium structure of relativistic stars. We point out that the semiclassical instability has a classical counterpart, which occurs exactly in the same region of the parameter space. For negative values of the coupling parameter the instability is equivalent to the well-known "spontaneous scalarization" effect: the plausible end-state of the instability is a static, asymptotically flat equilibrium configuration with nonzero expectation value for the quantum fields, which is compatible with experiments in the weak-field regime and energetically favored over stellar solutions in general relativity. For positive values of the coupling parameter the new configurations are energetically disfavored, and the end-point of the instability remains an open and interesting issue. The vacuum instability may provide a natural mechanism to produce spontaneous scalarization, leading to new experimental opportunities to probe the nature of vacuum energy via astrophysical observations of compact stars.

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One of the great achievements of quantum field theory in curved spacetime is Hawking's semiclassical prediction of black hole evaporation. This process is made possible by the special nature of the vacuum in quantum field theory, but it is extremely feeble by any astrophysical standards. It was recently shown that the vacuum in strongly curved spacetimes might play an important role even in the absence of horizons [1, 2], so that the formation and evolution of relativistic stars could be affected by semiclassical effects.

In particular, Lima, Matsas and Vanzella (henceforth LMV) [1] showed that the vacuum expectation value of nonminimally coupled scalar fields can grow exponentially in relativistic stars. An understanding of the final state of the LMV instability is important: if this final state supports nonvanishing scalar fields, the semiclassical amplification of vacuum energy could have cosmological and astrophysical implications.

In this paper we show that, for certain values of the coupling parameter, the most likely end-state of the instability is an asymptotically flat stellar solution with nonvanishing scalar field, which is compatible with gravitational experiments in the weak-field regime. We also point out that these new solutions correspond to the well-known "spontaneous scalarization" phenomenon [3, 4], and that they are *energetically favored* over stellar solutions in general relativity (GR). Our findings support the relevance of vacuum amplification scenarios. Vacuum amplification should be investigated carefully when devising strong-field tests of GR, for example in the context of gravitational collapse and gravitational-wave emission.

I. SETUP: STATIC SOLUTIONS

We look for static, spherically symmetric equilibrium solutions of the field equations admitting a nonzero scalar field with metric

$$ds^2 = -f(r)dt^2+(1-2m(r)/r)^{-1}\,dr^2+r^2d\theta^2+r^2\sin^2\theta\,d\phi^2\,.$$

Following LMV [1], we study a nonminimally coupled scalar field in the presence of a perfect-fluid star in GR. We consider the action

$$S=\frac{1}{16\pi G}\int d^4x\sqrt{-g}R+\int d^4x\sqrt{-g}\,\mathcal{L}_m\,, \qquad (1)$$

where $\mathcal{L}_m=\mathcal{L}_{\rm scalar}+\mathcal{L}_{\rm perfect\,fluid}$. The Lagrangian for a scalar field with conformal coupling ξ is given by

$$\mathcal{L}_{\rm scalar}=-\xi\,R\,\Phi^2-g^{\mu\nu}\Phi_{,\mu}\Phi_{,\nu}-\mu^2\Phi^2\,, \qquad (2)$$

where μ denotes the scalar field mass. Here we set $\mu=0$; we will consider the massive case in a follow-up paper. For $\xi=1/6$, $\mu=0$ the action is invariant under conformal transformations ($g_{\mu\nu}\to\Omega^2 g_{\mu\nu}\,,\Phi\to\Omega^{-1}\Phi$). For $\xi=0$, $\mu=0$ one recovers the usual minimally coupled massless scalar. The above Lagrangian corresponds to a viable theory of gravity, passing all weak-field tests [5, 6].

The theory can be recast as a scalar-tensor theory of gravity in the Einstein frame [3] via the transformation

$$g_{\mu\nu}\to(1-8\pi\xi\Phi^2)g_{\mu\nu}\,,\quad d\Phi\to\frac{\sqrt{1-8\pi\xi(1-6\xi)\Phi^2}}{1-8\pi\xi\Phi^2}d\Phi\,.$$

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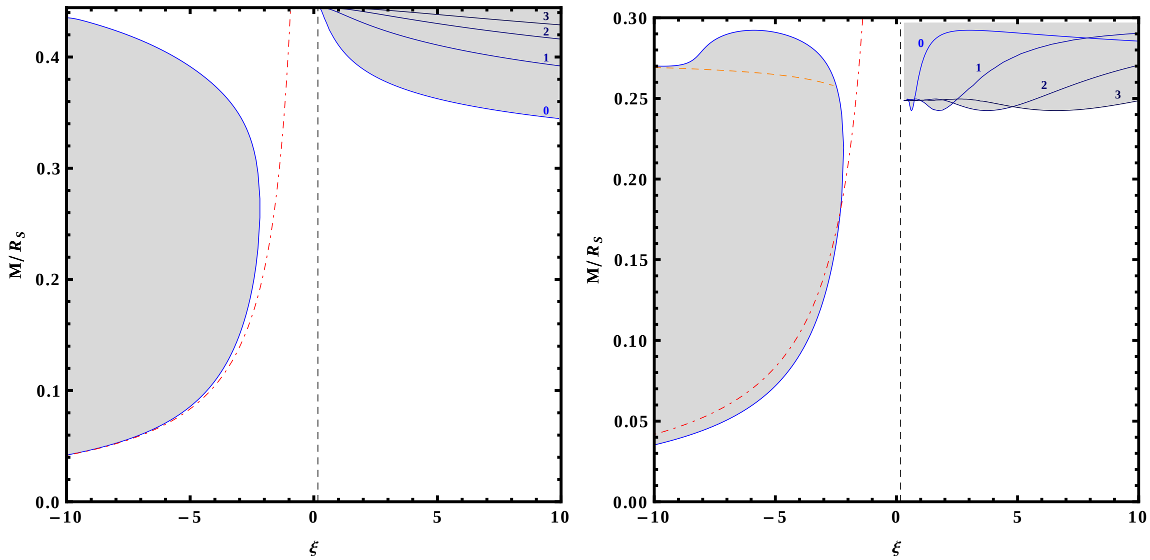

FIG. 1. Left panel: Existence diagram for constant density stars, showing the values of the ratio M/R_s of uniform density compact objects which support a dynamical scalar field. The dash-dotted line (red in the online version) corresponds to the analytical prediction in the Newton ian limit, Eq. (7). The vertical dashed line indicates the conformal-coupling value $\xi = 1/6$. We present this plot in "standard" geometrical units so that the compactness of our models can be easily compared to the maximum compactness for constant-density stars in GR: $M/R_s \leq 4/9 \simeq 0.444$ (Buchdahl's limit). Modulo this trivial unit conversion, the shaded regions in the left panel match *exactly* the shaded regions in LMV's Fig. 1 (see main text). In the upper-right region of the diagram, different critical lines correspond to "excited" equilibrium configurations and the integers refer to the number N of nodes in $\Phi(r)$ (see the main text). Right panel: Same for stars with a polytropic EOS. The dashed line (orange online) through the negative–ξ region marks the configuration with maximum mass (i.e., the radial stability limit) for each ξ.

In this form, both the classical counterpart of the LMV instability and its final state have been thoroughly studied [3, 4, 7].

We can explicitly construct static, asymptotically flat, spherically symmetric solutions to the theory (1) with nonzero scalar field and verify that our solutions match those found in [3, 4, 7], as follows. The equations of motion following from the Lagrangian are (hereafter we set $c = G = 1$)

$$G_{\mu\nu} = 8\pi T_{\mu\nu}\,, \tag{3}$$

$$\nabla_\alpha \nabla^\alpha \Phi = (\mu^2 + \xi R)\Phi\,, \tag{4}$$

where

$$T_{\mu\nu} = \left(2\Phi_{,\mu}\Phi_{,\nu} - g_{\mu\nu}g^{\alpha\beta}\Phi_{,\alpha}\Phi_{,\beta} - \mu^2 g_{\mu\nu}\Phi^2 - 2\xi\Phi^2_{,\mu;\nu} + 2\xi\, g_{\mu\nu}\Phi^2_{,\alpha;\beta}g^{\alpha\beta} + T^{\text{perfect fluid}}_{\mu\nu}\right)\left(1 - 16\pi\,\xi\Phi^2\right)^{-1}\,. \tag{5}$$

We consider perfect-fluid, spherically symmetric stars with energy density $\rho(r)$ and pressure $P(r)$ such that $T^{\mu\nu}_{\text{perfect fluid}} = (\rho + P)\,u^\mu\,u^\nu + g^{\mu\nu}P$, where the fluid four-velocity $u^\mu = (1/\sqrt{f}, 0, 0, 0)$. We specify some equation of state (EOS) $P = P(\rho)$ and we impose regularity conditions at the center of the star, i.e.

$$m(0) = 0\,, \quad \rho(0) = \rho_c\,, \quad \Phi(0) = \Phi_c\,, \quad \Phi'(0) = 0\,. \tag{6}$$

We also require continuity at the stellar radius R_s, defined by the condition $P(R_s) = 0$. We focus on two different stellar models: (i) the constant density stars ($\rho =$ const) studied by LMV, and (ii) the polytropic model $\rho = nm_b + Kn_0m_b(\Gamma-1)^{-1}(n/n_0)^\Gamma$, $P = Kn_0m_b(n/n_0)^\Gamma$, with $\Gamma = 2.34, K = 0.0195, m_b = 1.66\times10^{-24}$ g and $n_0 = 0.1\,\text{fm}^{-3}$ (this is the model that was considered in Ref. [3] in the context of spontaneous scalarization). We have checked that nuclear-physics based EOS models would yield qualitatively similar results.

Relativistic stellar configurations in GR correspond to $\Phi_c = 0$, so that $\Phi = 0$ everywhere. For each central density ρ_c, we used a shooting method to search for nonzero values of Φ_c such that $\Phi(r) \to 0$ as $r \to \infty$.

For constant-density configurations, we find solutions with nonzero scalar field in the shaded regions of the $(\xi\,, M/R_s)$ diagram shown in the left panel of Fig. 1. The right panel of Fig. 1 shows that the qualitative features of the existence diagram are the same for a polytropic EOS. These diagrams effectively reproduce previous results obtained many years ago in the context of spontaneous scalarization (see e.g. [4]). It is remarkable that static solutions exist in the *same* region where the LMV instability operates (cf. Fig. 1 in LMV). This provides strong evidence that these solutions (when they are stable) represent a plausible final state of the instability.

In fact, the exact overlap between our own Fig. 1 and Fig. 1 of LMV can be proved analytically. Focus for simplicity on constant-density stars and massless scalar fields (but our reasoning applies in general). The critical lines in Fig. 1 of LMV represent the curves where marginally stable modes exist. These modes are zero-frequency solutions of Eq. (4) in LMV, where the potential is given by their Eq. (6). On the other hand, the critical lines in our Fig. 1 represent the boundary of regions where spherically symmetric, static solutions with nontrivial scalar field profiles cease to exist. These are solutions of the Einstein-Klein-Gordon equations with $\Phi = 0$. As $\Phi \to 0$ the spacetime becomes arbitrarily close to that of a constant-density star and the Klein-Gordon equation reduces to Eq. (4) in LMV, with potential given by their Eq. (6). Furthermore, the same boundary conditions apply in both cases. Thus, the critical lines are obtained from the *very same* equations and they are indeed coincident, not just similar.

Quite interestingly, the LMV instability threshold can be found *analytically* in the Newtonian limit $M/R_s \ll 1$ (i.e., in the bottom left corner of Fig. 1). The instability line defines the existence of static solutions with a small but nonvanishing massless scalar field. The relevant equation in this limit is $\Psi'' - 8\pi\xi(\rho - 3P)\Psi = 0$, where a prime denotes a derivative with respect to r, and we use the ansatz for the scalar field $\Phi = (\Psi/r)e^{-i\omega t}$ (i.e., we consider an s-wave). Assuming that ρ is constant in the stellar interior (this assumption holds exactly for uniform-density stars and is a good approximation for most EOSs), a regular solution at the origin and at infinity that is also continuous at R_s corresponds to

$$24M\xi = -\pi^2 R_s \,. \tag{7}$$

As shown in Fig. 1, this prediction is in very good agreement with the LMV results[1].

II. MASS AND BINDING ENERGY

The orbits of bodies far away from the star depend on the star's gravitational mass M, shown as a function of the central baryonic density $\rho_c = m_b n(0)$ in the left panel of Fig. 2. In GR, maxima of this curve correspond to marginally stable equilibrium configurations, and all solutions after the first maximum are unstable to radial perturbations in the polytropic case (see e.g. [8]). The baryonic mass of the configuration,

$$\bar{m} = m_b \int d^3x \sqrt{-g} u^0 n(r) \,, \tag{8}$$

corresponds to the energy that the system would have if all baryons were dispersed to infinity. The normalized binding energy $E_b/M = \bar{m}/M - 1$ is plotted in the inset of the left panel in Fig. 2. For bound (not necessarily stable) configurations, $E_b > 0$.

A. Negative ξ

When $\xi \lesssim -2$, in the shaded region of Fig. 1 we found only a single solution coupled to a nontrivial scalar field. As shown in the left panel of Fig. 2, the critical central density for these solutions is roughly the same as in GR, but they have larger maximum mass than their GR counterparts ($\approx 10\%$ larger for $\xi \lesssim -5$). Similar arguments to GR indicate that models after the first maximum in $M(\rho_c)$ (lying above the dashed line in the negative–ξ region of the right panel of Fig. 1) are unstable.

The inset of the left panel in Fig. 2 shows that similar deviations occur also for the binding energy. For a given $\bar{m}$, the binding energy is *higher* than in GR, so these configurations are energetically favored over stellar solutions in GR. As shown in the right panel of Fig. 2, for a fixed EOS the scalar field can sensibly modify the mass-radius relation. These modifications depend on ξ, but they can be of order 10% or more, and as such they could be observable. Present neutron star observations give constraints on the mass-radius relationship (e.g. [10, 11]), and electromagnetic observations of binaries containing X-ray pulsars could in principle constrain the binding energy as well [12, 13]. A recent high-mass neutron star observation ([14]) also rules out many EOS models in GR. If astrophysical measurements are not compatible with any realistic EOS in pure GR, we may be able to constrain the coupling constant ξ and to probe the occurrence of vacuum instabilities in astrophysical environments.

B. Positive ξ

For $\xi > 1/6$ the situation is more complicated. Depending on the compactness M/R_s and on the coupling constant ξ, we have found a hierarchy of new solutions, which can be labeled by the number N of radial nodes of the scalar field. In general, the larger N, the lower the binding energy, so high-N solutions can be thought of as "excitations" of the (energetically favored) ground-state configuration. Critical curves bounding the shaded exis-

[1] The basic features of the instability in compact stars were understood by Ford (who studied unstable scalar fields as a possible mechanism to damp the effective value of the cosmological constant) as early as 1987 [9]; see also [4].

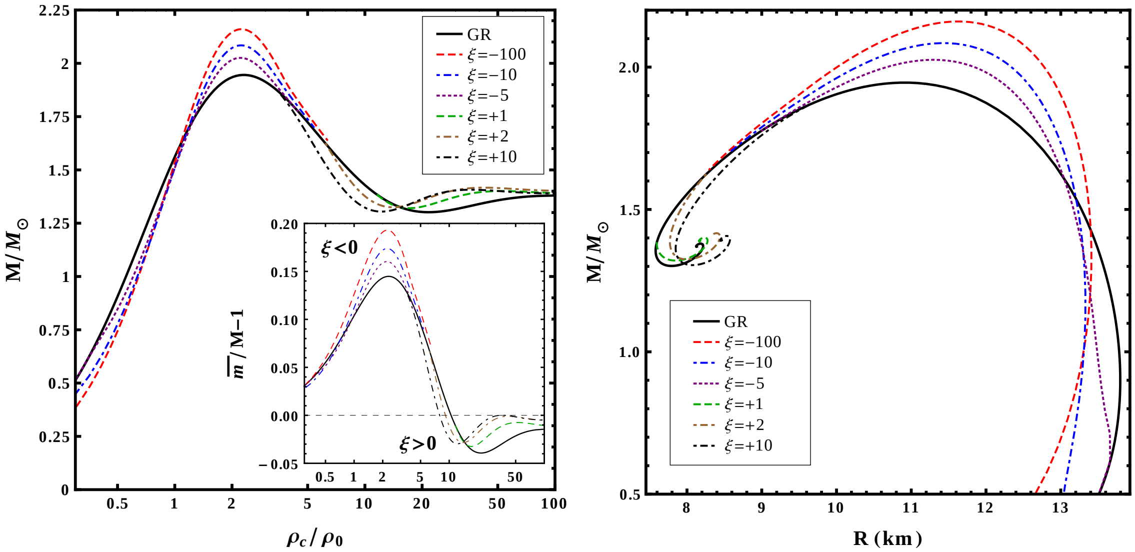

FIG. 2. Left: Gravitational mass as a function of the central baryonic density ρ_c/ρ_0, where $\rho_0 = 8 \times 10^{14}$ g/cm^3 is a typical central density for neutron stars. The inset shows the (normalized) binding energy as a function of ρ_c/ρ_0. Right: Gravitational mass as a function of the radius for different values of the coupling.

tence region of these solutions are plotted in the upper-right corner of the two panels of Fig. 1.

The inset of Fig. 2 shows that ground-state configurations actually have a *negative* binding energy for $\rho_c > \rho_{\rm crit} \sim 6 \times 10^{15}$ g/cm^3. When $\rho_c < \rho_{\rm crit}$, we only found solutions with binding energy *lower* than in GR, so GR solutions are energetically favored in this region of parameter space. The fact that negative-ξ solutions are more stable than positive-ξ solutions is consistent with the interpretation of ξ as an "effective gravitational constant" proposed by van der Bij and Gleiser [15] in their study of boson stars with nonminimal coupling.

It is reasonable to conjecture that the top-right region of Fig. 1 does not correspond to stable "stars" with non-vanishing scalar field. The final state of the instability in this case is an interesting topic for numerical simulations [7] (see also [16] and references therein). Note that no-hair theorems for positive ξ (in particular $\xi \geq 1/2$) have been proven years ago [17, 18] and supported by numerical searches [19]. This may imply that either stars with $\xi > 1/6$ evolve to a black hole solution in pure GR, or that they shed enough mass to leave the forbidden region and become a star in pure GR again.

III. EXTENSIONS

The LMV mechanism, like other strong-field effects in scalar-tensor theories [3, 20], relies on a specific coupling of the scalar field with matter. We found that an instability is *not* present for "minimally-coupled" Weyl fermions or Maxwell fields, independently of the EOS[2].

We have not explicitly shown that the equilibrium configurations reported here arise as a result of nonlinear time evolutions of GR solutions with a "seed" scalar field; however, strong indications that this should be the case come from numerical studies by Novak in the context of spontaneous scalarization [7]. It would be interesting to perform similar simulations for $\xi > 1/6$. Other possible extensions of our work concern the investigation of slowly and rapidly rotating stellar models and of their oscillation frequencies (see e.g. [24]).

IV. CONCLUSIONS

We have reconsidered a generic class of theories where a scalar field is nonminimally coupled to the Ricci scalar, that were recently shown to give rise to a semiclassical instability. We have pointed out an interesting relation between the semiclassical instability and the spon-

[2] The electromagnetic case can be treated as described in Ref. [21]. This leads to an equation of the form $d^2\Psi/dr_*^2 + \left(\omega^2 - V\right)\Psi = 0$ for both polarization states, where we defined $dr/dr_* = \sqrt{f(1-2m/r)}$, and $V = fl(l+1)/r^2$. Massless neutrino fields yield a similar equation with potential $V = (2r^2)^{-1}[2kf(r)\left(k + \sqrt{1-2m/r}\right) - kr\sqrt{1-2m/r}f']$, where k is a separation constant [22]. The potential for Maxwell perturbations is positive-definite, and therefore no instabilities can arise. For neutrinos we were not able to prove stability in general, but we did verify that the two specific models for the EOS discussed in this paper lead to positive-definite potentials.

taneous scalarization effect in classical scalar-tensor theories. For certain values of the coupling parameter the scalar field can leave observable imprints on the equilibrium properties of relativistic stars. Our main finding is that the LMV instability may provide a "natural" seed mechanism to produce spontaneous scalarization, reinforcing the relevance of previous studies of compact stars in scalar-tensor or $f(R)$ theories of gravitation (see e.g. [3, 20, 25]).

We stress that corrections to GR due to scalar fields are a *generic* feature of a large class of unification theories. Our work suggests that strong-field modifications to GR compatible with weak-field tests may be astrophysically viable, with potentially observable consequences in the structure of compact stars. It will be important to explore the implications of vacuum amplification mechanisms for tests of strong-field gravity in compact objects (see e.g. [26] and references therein).

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