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Pulsar Timing Sensitivity to Very-Low-Frequency Gravitational Waves

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Abstract

At nanohertz frequencies gravitational waves (GWs) cause variations in time-of-arrival of pulsar signals potentially measurable via precision timing observations. Here we compute very-low-frequency GW sensitivity constrained by instrumental, propagation, and other noises fundamentally limiting pulsar timing observations. Reaching expected GW signal strengths will require estimation and removal of $\simeq 99\%$ of time-of-arrival fluctuations caused by typical interstellar plasma turbulence and a reduction of white rms timing noise to ~ 100 nsec or less. If these were achieved, single-pulsar signal-to-noise ratio (SNR) = 1 sensitivity is then limited by the best current terrestrial time standards at $h_{rms} \sim 2 \times 10^{-16} [f/(1 \text{ cycle/year})]^{-1/2}$ for $f < 3 \times 10^{-8}$ Hz, where f is Fourier frequency and a bandwidth of 1cycle/(10 years) is assumed. This sensitivity envelope may be optimistic in that it assumes negligible intrinsic pulsar rotational noise, perfect time transfer from time standard to observatory, and stable pulse profiles. Nonetheless it can be compared to predicted signal levels for a broadband astrophysical GW background from supermassive black hole binaries. Such a background is comparable to time-keeping-noise only for frequencies lower than about 1 cycle/(10 years), indicating that reliable detections will require substantial improvements in signal-to-noise ratio through pulsar array signal processing.

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The pulsar timing GW detector uses the earth and a distant pulsar as electromagnetically-tracked separated test masses. The pulsar emission serves as a clock, in the idealized case producing perfectly periodic radio pulses transmitted to the earth. These are “timed” by cross correlation of the received pulses against a template of the pulsed waveform. Time-of-arrival residuals, $R(t)$, are produced after correcting for known effects. Signals and noises enter the observed time series via transfer functions [1]. A GW of characteristic strain amplitude h incident on the earth-pulsar system produces variations of order h in the time series of relative dimensionless frequency fluctuations, $y(t)$, of the pulsar signal [2] $y(t) = ((1 - \mu)/2)[\Psi(t - T_1(1 + \mu)) - \Psi(t)]$. Here $\mu = \mathbf{k} \cdot \mathbf{n}$, \mathbf{k} is a unit vector parallel to the GW propagation direction, \mathbf{n} is a unit vector from the earth to the pulsar, $\mathbf{h}(t)$ is the GW strain tensor, $\Psi(t) = (\mathbf{n} \cdot \mathbf{h} \cdot \mathbf{n})/(1 - \mu^2)$, and T_1 is the one-way light travel time between the pulsar and earth. The fractional frequency time series is the derivative [3] of the observed time-of-arrival residuals, $R(t)$: $y(t) = dR(t)/dt$. The pulsar timing technique has been used to bound GW signal strengths, e.g. [4–9].

To assess instrumental and other noises currently and fundamentally limiting detections, we compute here the sensitivity of pulsar GW observations. Sensitivity is conventionally expressed as the sky- and polarization-averaged sinusoidal signal strength necessary to achieve a given signal-to-noise ratio in a given bandwidth, e. g. [2, 10]. Explicitly, we compute the signal strength required to produce $\text{SNR} = 1$ in bandwidth B : $[S_{yn}(f)B]^{1/2}/(\text{rms signal response})$, where $S_{yn}(f)$ is the spectrum of noise and the rms signal response in general also depends on Fourier frequency.

The GW signal response depends on the angle of arrival of a wave relative to the earth-pulsar line. For GWs from a specific direction the above formula for $y(t)$ can be used directly [7]. We are interested here in signal response averaged over the sky. To get the rms signal response as a function of Fourier frequency, the Fourier transform squared of the GW signal response, above, is averaged over the sky and polarization states to obtain $S_y(f)/S_h(f) = 1/3 - 1/(8\pi^2 f^2 T_1^2) + \sin(4\pi f T_1)/(32 f^3 \pi^3 T_1^3)$, where S_y is the spectrum of fractional frequency fluctuations, S_h is the spectrum of GW strengths, and f is Fourier frequency. In the practical case T_1 is hundreds of years or longer and the duration of pulsar timing observations is \sim decades, so the second and third terms are negligible for $f > 1/(\text{duration of observations})$: $S_y(f)/S_h(f) \approx 1/3$, implying that rms signal response is constant ($\simeq 0.58$) over the accessible frequency band. (This assumes observations suffi-

ciently long compared with the lowest frequency of interest, since GW variability could be absorbed into fits for pulsar spin and position parameters [11, 12], reducing the apparent signal response.)

To compute sensitivity we also need spectra of the noises. Important noise sources include finite signal-to-noise ratio in the raw observations, instability of the local clock against which pulsars are timed (and errors in time transfer if the clock is not located at the observatory), uncertainties in solar system ephemerides (used to correct arrival times at the earth to the barycenter of the solar system), pulsar position uncertainty, intrinsic pulsar rotational stability, stability and accuracy of the average pulse templates used to measure $R(t)$, dispersion measure (DM) variability in the interstellar and interplanetary plasmas, tropospheric scintillation (the wet and dry components of the troposphere cause delay variability), antenna mechanical noise (stability of the phase center of the antenna tracking the pulsar), and station location errors (changes in antenna location due to atmospheric and tidal loading of the crust). Figure (1) shows the GW sensitivity for several of these noise sources individually, as discussed briefly below, with sensitivity computed for bandwidth $B = 1$ cycle/10 years.

The green curve is the station-location-noise limit; S_{yn} was computed from 30 years of absolute value of vector ground displacement using data [13] taken near the NASA/JPL Goldstone CA tracking complex. The derivative theorem for Fourier transforms [14] was used to convert the spectrum of displacement to the spectrum of velocity and hence the spectrum of $y = \Delta v/c$. The black curve is derived from the power spectrum of hourly zenith dry tropospheric pressure fluctuations [15] provided by the National Climate Data Center and taken at the NASA/JPL deep space tracking complex near Madrid, Spain. Pressure was converted to zenith path variation using (path variation in centimeters) = $0.022768 \times$ (surface pressure in millibars), ignoring a factor close to unity which depends on latitude and height. The blue curve is from the power spectrum of zenith wet tropospheric path delay, computed from 1.5 years of data [16] taken at the NASA/JPL Goldstone, California tracking complex. Tropospheric path variation spectra were similarly converted to spectra of y using the derivative theorem. The light blue curve is the measurement (for $f > 0.0001$ Hz) and upper limit (for $10^{-6} - 10^{-4}$ Hz) for antenna mechanical noise fluctuations observed with a 34m tracking antenna at Goldstone [17]; smaller stiffer antennas give lower antenna mechanical noise [18] [19]. The solid black lines are for white timing noise with rms amplitudes of 100

nsec, 1 nsec, and 1 ps in a Fourier band ± 0.5 cycles/day (i.e. one sample per day; current observations are more typically 1 sample per 2 weeks, which would result in curves $\sqrt{14}$ higher). The 100 nsec level (or better) is the current timing goal of leading timing array experiments; three pulsars are being timed to these levels [9]. One picosecond is the absolute best possible timing accuracy one can achieve using millisecond pulsars. Since the pulsar signal itself is an amplitude modulated noise process, it can be said to have “self-noise”. In the absence of all other sources of noise, including timing and antenna noise, the pulsar signal self-noise would still be present. Assuming that the narrowest possible average pulse profile is 10 μ sec, the pulsar signal bandwidth is 1 GHz, and that one can observe the pulsar for 12 hours at a time, the self-noise yields an rms timing accuracy of about 1 ps. The dotted curve shows approximate limits due to the uncertainties in the masses of the planets [20–22], Mercury through Jupiter, affecting knowledge of the solar system barycenter.

Radiowave propagation effects affect precision pulsar timing [11]. The dashed lines are noise from representative interstellar medium dispersion measure (DM) variations [23, 24] (assumed Kolmogorov spectrum with $C_n^2 = 0.001 \text{ m}^{-20/3}$, propagation distance $z = 1 \text{ kpc}$, radio frequency = 1 GHz, transverse velocity $v = 100 \text{ km/sec}$), $S_{yn}(f) = \pi^{-1/6} 2^{-2/3} v^{5/3} \lambda^4 c^{-2} z C_n^2 r_e^2 [\Gamma[4/3]/\Gamma[11/6]] f^{-2/3}$, with the indicated levels of calibration, i.e. 99% calibration means only 1% of the DM fluctuation rms noise remains in the measurement. (If sufficiently stable nearby pulsars – i.e., having smaller integrated interstellar medium turbulence levels – could be used then smaller percentage corrections would be required to reach the indicated line in Figure (1)). The effect of solar wind plasma turbulence is non-negligible [25] but its dispersive character should allow it to be calibrated in addition to the interstellar plasma. Also potentially important are angle-of-arrival variations, particularly if low radio frequency (less than several hundred megahertz) data are used for multi-frequency propagation corrections [11].

Pulsars are timed against terrestrial clocks. Recent stability measurements of linear ion trap time standards [26] give $S_{yn}(f) \simeq 4 \times 10^{-31} (f/1\text{Hz})^{-1}$, measured in the approximate band $10^{-7} - 10^{-6} \text{ Hz}$. We assume this spectrum continues to be valid to lower frequencies; in a 1 cycle/(10 year) bandwidth this noise gives GW sensitivity shown as the dot-dash line in Figure (1). The dotted curve in the lower right, for comparison, is the low-frequency segment of the Laser Interferometer Space Antenna (LISA) mission’s predicted sensitivity curve ([27], 5- σ -in-one-year-integration).

Not included in Figure (1) are intrinsic pulsar rotational instability noise [12, 28] (variable by pulsar and substantial for some), errors due to time transfer from the frequency standard to the observatory [29], pulsar position uncertainty [12] and its apparent variability [11], errors in pulse templates, and errors due to radio frequency dependence and temporal instability of pulse profiles. So the upper envelope sensitivity in Figure (1) is in this sense optimistic.

One application of our sensitivity analysis is the detectability of an astrophysical GW background from incoherently radiating supermassive black hole binaries. Such a background is predicted to produce signal strengths [9, 30–33] in the range $h(f) = [fS_h(f)]^{1/2} \sim (1\text{--to--}10) \times 10^{-16} [f/(1 \text{ cycle/year})]^{-2/3}$. For comparison with our sensitivity curve (SNR = 1 in a fixed $B = 1 \text{ cycle}/(10 \text{ years})$ bandwidth) we convert to $h_{rms}(f) = [BS_h(f)]^{1/2} \sim (3\text{--to--}30) \times 10^{-17} [f/(1 \text{ cycle/year})]^{-7/6}$. These GW strengths are comparable to the SNR = 1 sensitivity limited at low frequencies by time-standard noise only for $f \sim 1 \text{ cycle}/(10 \text{ years})$ or lower (Figure (1)). (Within a broadband astrophysical background there may be some sources strong enough to be detected individually, i.e. detectable GWs coming from specific directions. Figure (1) shows how strong an individual source would have to be for detection above the timekeeping noise limit. When simultaneously timing several pulsars, the directional property of the timing response to gravitational radiation from a single source can be used in the same way as for ground-based networks of broadband GW detectors to improve GW SNR, e.g. [34] and references therein.) Since GW SNR > 5 is conventionally taken as detection threshold, Figure (1) indicates that substantial GW SNR improvements will be required of pulsar timing array signal processing for reliable detections.

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FIGURES

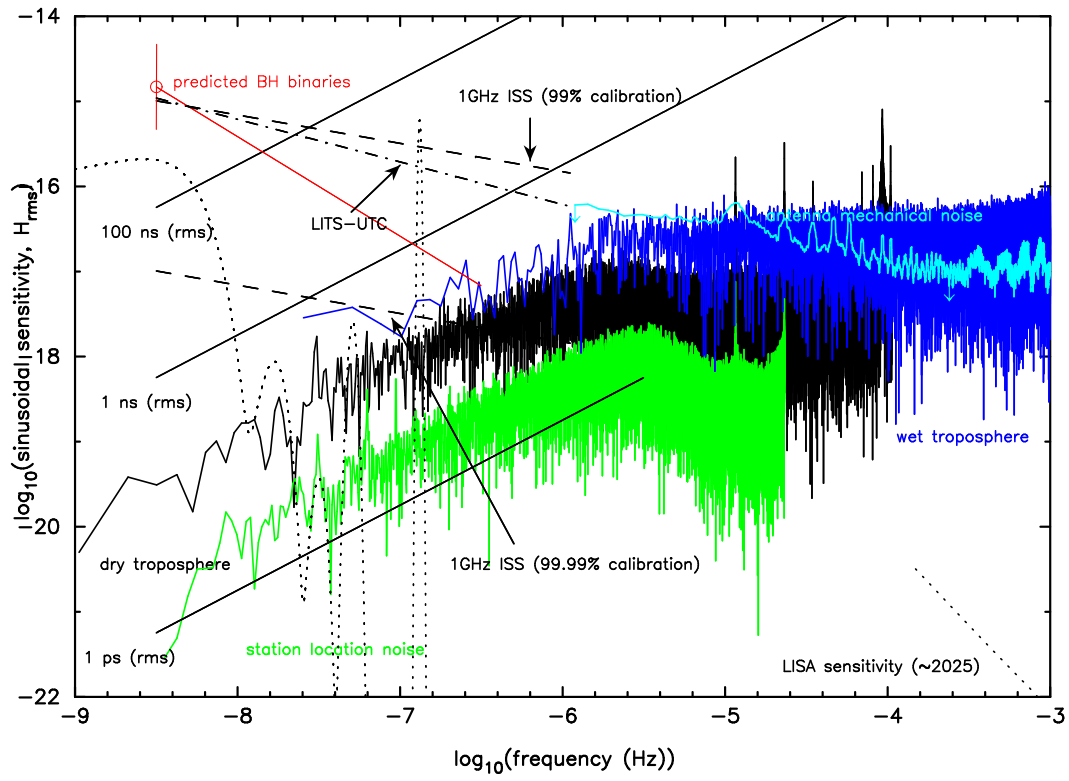


FIG. 1. Gravitational wave sensitivity expressed as strain amplitude required for $\text{SNR} = 1$ in a 1 cycle per 10 years bandwidth, as a function of Fourier frequency. Sensitivity limited by various noises are indicated: green is station location noise [13]; black and blue are respectively due to fluctuations in the zenith dry [15] and wet [16] troposphere; light blue is due to antenna mechanical noise for a 34m beam-waveguide station [10, 17]; dashed lines are for dispersion measure variations in the interstellar medium [23, 24] for 1 GHz observations after 99% and 99.99% calibration; solid black lines are for white timing noise with rms amplitudes of 100 ns, 1ns, and 1ps in a Fourier band ± 0.5 cycles/day; dotted line is an approximate limit due to uncertainties in the masses of the planets [20, 21]; dot-dashed line is sensitivity limited by a linear ion trap time standard [26]. Also shown for reference is the low-frequency sensitivity expected for the LISA detector [27]. Red line and vertical bar shows the dependence and range of predicted signal strengths from an ensemble of supermassive black-hole binaries [30–33]