Forward hadronic scattering at 7 TeV: An update on predictions for the LHC

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Forward hadronic scattering at 7 TeV: predictions for the LHC; an update.

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The LHC has successfully run for a long period at half energy, 7 TeV. In this note, we update earlier full-energy Large Hadron Collider (LHC) forward hadronic scattering predictions [1], giving new predictions, including errors, for the pp total and inelastic cross sections, the ρ-value, the nuclear slope parameter B, $d\sigma_{\rm el}/dt$, and the large gap survival probability at the current 7 TeV energy.

I. INTRODUCTION

The LHC has run at 7 TeV (half-energy) for an extended period of time and large amounts of data have been collected. Five years ago, we made hadronic forward scattering predictions for the full energy (14 Tev) Large Hadron Collider; for details see the review article by M. Block [1]. Recently, we have had inquiries from LHC experimental groups for 7 TeV predictions. The purpose of this note is to gather together in one convenient location an update to the 2006 publication, in which we furnish comparisons between new 7 TeV and (already published) 14 TeV results, including errors in the 7 TeV predictions due to model uncertainties, as well as presenting new calculations for pp elastic scattering, $d\sigma/dt$, at 7 TeV. We have combined two separate models to make these predictions, the first being the analyticity-constrained analytic amplitude model of Block and Halzen [2] that saturates the Froissart bound [3] and the second being the "Aspen Model", a revised version of the eikonal model of Block, Gregores, Halzen and Pancheri [5] that now incorporates analyticity constraints. We purposely keep explanations very brief; for complete details, see Ref. [1].

II. PREDICTIONS AT 7 TEV

A. The analytic amplitude model

We make the most accurate predictions of the forward pp scattering properties,

$$\sigma_{\rm tot} \equiv \frac{4\pi}{p}{\rm Im}f(\theta_L=0) \tag{1}$$

$$\rho \equiv \frac{{\rm Re}f(\theta_L=0)}{{\rm Im}f(\theta_L=0)}, \tag{2}$$

using the analyticity-constrained analytic amplitude model of Block and Halzen [2] that saturates the Froissart bound [3]. By saturation of the Froissart bound, we mean that the total cross section defined in Eq. (1) rises as $\ln^2 s$, where s is the square of the cms energy. In Eq. (1) and Eq. (2), $f(\theta_L)$ is the pp laboratory scattering amplitude as a function of θ_L, the laboratory scattering angle and p is the laboratory momentum. In Fig. 1, the solid line is the total pp cross section as a function of the cms energy, $\sqrt{s}$. Our use of analyticity constraints—employing new Finite Energy Sum Rules (FESR) [4]—allows us to use *very accurate low energy cross section measurements to act as an anchor* that accurately fixes our high energy cross section predictions.

At 7 TeV, we find that $\sigma_{\rm tot} = 95.4 \pm 1.1$ mb. The same set of parameters predict $\sigma_{\rm tot} = 107.3 \pm 1.2$ mb at 14 TeV [1]. Further, at 7 TeV we predict that $\rho_{pp} = 0.135 \pm 0.001$.

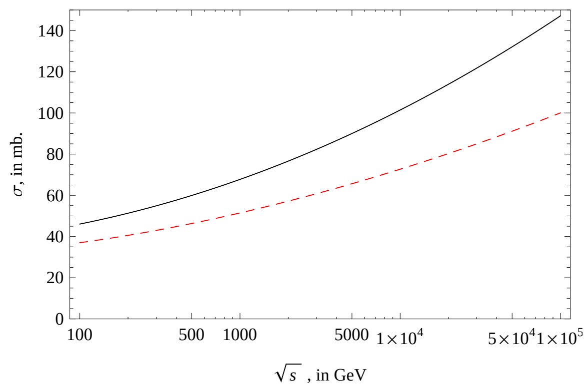

FIG. 1: The total pp cross section, σ_{tot}, and the inelastic cross section σ_{inel}, in mb, vs. $\sqrt{s}$, the cms energy in GeV. The solid (black) curve is the total cross section and the dashed (red) curve is the inelastic cross section.

B. The"Aspen" Model: an eikonal model for pp scattering

The "Aspen" model uses an unconventional definition of the eikonal $\chi(b,s)$ in impact parameter space b, so that

$$\sigma_{\text{tot}}(s) = 2\int \left[1 - e^{-\chi_I(b,s)}\cos\left(\chi_R(b,s)\right)\right] d^2\vec{b}, \tag{3}$$

$$\rho(s) = \frac{\int e^{-\chi_I(b,s)}\sin(\chi_R(b,s))\, d^2\vec{b}}{\int \left[1 - e^{-\chi_I(b,s)}\cos(\chi_R(b,s))\right] d^2\vec{b}} \quad , \tag{4}$$

$$B(s) = \frac{1}{2}\frac{\int |e^{-\chi_I(b,s)+i\chi_R(b,s)} - 1| b^2\, d^2\vec{b}}{\int |e^{-\chi_I(b,s)+i\chi_R(b,s)} - 1|\, d^2\vec{b}}, \tag{5}$$

$$\frac{d\sigma_{\text{el}}}{dt} = \pi\left|\int J_0(qb)\left[e^{-\chi_I(b,s)+i\chi_R(b,s)} - 1\right] b\, db\right|^2, \tag{6}$$

$$\sigma_{\text{el}}(s) = \int \left|e^{-\chi_I(b,s)+i\chi_R(b,s)} - 1\right|^2 d^2\vec{b}, \tag{7}$$

$$\sigma_{\text{inel}}(s) \equiv \sigma_{\text{tot}}(s) - \sigma_{\text{el}}(s) = \int \left(1 - e^{-2\chi_I(b,s)}\right) d^2\vec{b}, \tag{8}$$

where $\sigma_{\text{inel}}(s)$ is the total inelastic cross section. The even eikonal profile function χ^{even}, which is the only surviving term at the high energies considered here, receives contributions from quark-quark, quark-gluon and gluon-gluon interactions, and can be written in the factorized form

$$\begin{aligned} \chi^{even}(s,b) &= \chi_{qq}(s,b) + \chi_{qg}(s,b) + \chi_{gg}(s,b) \\ &= i\left[\sigma_{qq}(s)W(b;\mu_{qq}) + \sigma_{qg}(s)W(b;\sqrt{\mu_{qq}\mu_{gg}}) + \sigma_{gg}(s)W(b;\mu_{gg})\right] , \end{aligned} \tag{9}$$

where σ_{ij} is the cross sections of the colliding partons, and $W(b;\mu)$ is the overlap function in impact parameter space, parameterized as the Fourier transform of a dipole form factor. The parameters μ_{qq} and μ_{gg} are masses which describe the "area" occupied by the quarks and gluons, respectively, in the colliding protons. In this model hadrons asymptotically evolve into black disks of partons. For details of the parameterization of the model, see Ref. [1].

From Eq. (8) and Eq. (3), we calculate the *ratio* $r(s) = \sigma_{\text{inel}}(s)/\sigma_{\text{tot}}(s)$, because most errors due to parameter uncertainties cancel in the ratio. We then multiply $r(s)$ by the (more accurate) total cross section using Eq. (1) (the analytic amplitude model) to obtain the inelastic cross section shown in Fig. 1, as the dashed (red) curve. At 7 TeV, we find $\sigma_{\text{inel}} = 69.0 \pm 1.3$ mb.

Further, from, Eq. (5) we find that the nuclear slope parameter B, the logarithmic derivative of the elastic cross section (as a function of squared momentum transfer t) with respect to t, at $t = 0$ is given by $B = 18.28 \pm 0.12$ (GeV/c)$^{-2}$ at 7 TeV.

At 7 TeV, using Eq. (6), we plot the differential elastic scattering cross section $d\sigma_{\text{el}}/dt$, in mb/(GeV/c)^2, against $|t|$, in (Gev/c)^2 as the solid (black) curve in Fig. 2. Also shown is the approximation, $\frac{d\sigma}{dt}|_{t=0}e^{-B|t|}$, valid for small $|t|$, which is the dashed (red) curve. The agreement is striking for small t.

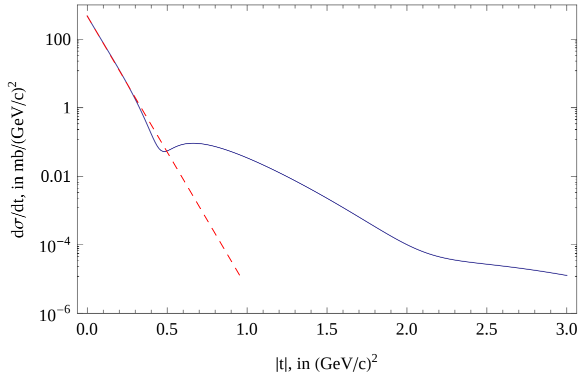

FIG. 2: The pp differential elastic scattering cross section , $d\sigma_{\rm el}/dt$, in mb/(GeV/c)^2, vs. $|t|$, in (GeV/c)^2 is the solid (black) curve. The dashed (red) curve is the small $|t|$ approximation, $\frac{d\sigma}{dt}|_{t=0}e^{-B|t|}$.

C. Rapidity gap survival probabilities

As shown in Ref. [1], the survival probability $<|S|>^2$ of *any* large rapidity gap is given by

$$<|S|^2>=\int W(b\,;\mu_{\rm qq})\,e^{-2\chi_{\rm I}(s,b)}d^2\,\vec{b}, \tag{10}$$

which is the differential probability density in impact parameter space b for *no* subsequent interaction (the exponential suppression factor) multiplied by the quark probability distribution in b space from Eq. (9)), which is then integrated over b. It should be emphasized that Eq. (10) is the probability of *survival* of a large rapidity gap and *not* the probability for the production and survival of large rapidity gaps, which is the quantity observed experimentally. The energy dependence of the survival probability $<|S|^2>$ is through the energy dependence of $\chi_{\rm I}$, the imaginary portion of the eikonal given in Eq. (9). A plot of $<|S|^2>$ as a function of $\sqrt{s}$, the cms energy in GeV, is given in Fig. 3. At 7 TeV, we find the gap survival probability to be $<|S|^2>=15.5\pm 0.05$ %.

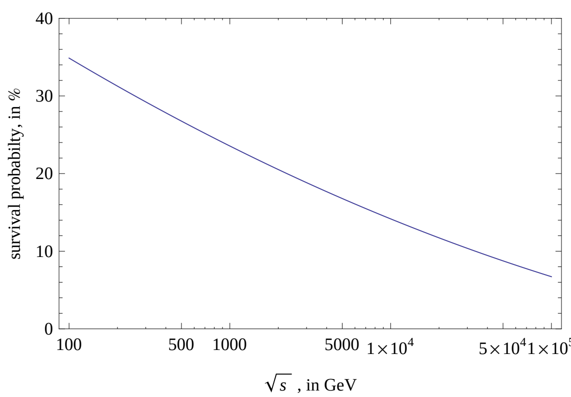

FIG. 3: $<|S|^2>$, the survival probability of large rapidity gaps in pp collisions, in %, vs. $\sqrt{s}$, the cms energy in GeV.

III. SUMMARY

We summarize our pp forward scattering parameters for the LHC in Table I, comparing the 7 and 14 TeV values. The 14 TeV values are taken from Ref.[1].

TABLE I: Values of forward scattering parameters for the LHC, at 7 and 14 TeV.

$\sqrt{s}$ (TeV)	$\sigma_{\rm tot}$ mb	$\sigma_{\rm inel}$ mb	ρ	B $({\rm GeV/c})^{-2}$	$<\|S\|^2>$ %
7	95.4 ± 1.1	69.0 ± 1.3	0.135 ± 0.001	18.28 ± 0.12	15.5 ± 0.05
14	107.3 ± 1.2	76.3 ± 1.4	0.132 ± 0.001	19.39 ± 0.13	12.6 ± 0.06

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