Gauge coupling unification in heterotic string models with gauge mediated supersymmetry breaking

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Abstract

We calculate the weak scale MSSM spectrum starting from a heterotic string theory compactified on an anisotropic orbifold. Supersymmetry breaking is mediated by vector-like exotics that arise naturally in heterotic string theories. The messengers that mediate SUSY breaking come in incomplete GUT multiplets and give rise to non-universal gaugino masses at the GUT scale. Models with non-universal gaugino masses at the GUT scale have the attractive feature of allowing for precision gauge coupling unification at the GUT scale with negligible contributions from threshold corrections near the unification scale. The unique features of the MSSM spectrum are light gluinos and also large mass differences between the lightest and the next-to-lightest neutralinos and charginos which could lead to interesting signatures at the colliders.

1 Introduction

Grand unification of the fundamental forces is a very appealing idea. It was noticed in as early as 1974, that when the couplings of the three fundamental interactions are run to high energies, they seem to meet at a point [1],[2]. Supersymmetry is required for precise unification [3], without which there is a discrepancy of about 12 σ [4]. With supersymmetry, assuming universal scalar and gaugino masses at the GUT scale, precision electroweak data requires [5], for the strong coupling constant to match experiments, that α_3 be about $3-4\%$ smaller than α_1 and α_2 at the GUT scale. This conflict between the coupling constants at the GUT scale can be eased by including 'threshold corrections' from extra states around the GUT scale.

In grand unified theories, the Higgs fields have to respect the GUT symmetry and thus existence of Higgs doublets also implies the existence of Higgs triplets. In order to avoid rapid proton decay the triplets necessarily have mass greater than the GUT scale which introduces some unpleasantness into these SUSY GUT theories. In addition, complicated symmetry breaking potentials are required to break the GUT symmetry. Theories with extra dimensions have gained popularity in this respect, since they can eliminate some of the problems with 4D SUSY GUTs. In theories with extra dimensions, connection is made to the low-energy world, by compactifying the extra dimensions. The choice of boundary conditions then can lead to natural and simple solutions to the problems hindering SUSY GUTs. The threshold corrections required to match precision electroweak data can come from massive states around the GUT scale and from Kaluza-Klein states living between the compactification scale of the extra dimensions, M_C and the cut-off scale, M_*; in string theory M_* is the string scale, M_S.

Recent searches for the MSSM from heterotic string theory have yielded interesting results. Orbifold compactifications of the $E_8 \times E_8$ heterotic string theory have been shown to yield realistic models that include the gauge group and the matter content of the MSSM [12, 13]. In addition, the models also have vector-like exotics with Standard Model charges that obtain mass in the supersymmetric limit. They may couple to the SUSY breaking field, and mediate supersymmetry breaking. The mechanism of supersymmetry breaking plays a very important role in understanding the low-energy spectrum, and in this case the possibility of *Gauge Mediated Supersymmetry breaking*

(GMSB) [9]. In GMSB, the gauginos receive mass at one-loop as:

$$M_i \sim \frac{\alpha_i}{4\pi}\frac{\langle F\rangle}{\langle M_\phi\rangle} \tag{1}$$

where, ϕ is the messenger field with mass, M_ϕ, and $\langle F\rangle$ is the SUSY breaking VEV. Thus, a heavier messenger (in this case, the exotics) corresponds to a lighter gaugino.

It was shown earlier in [6] that light(of the order $10^9 - 10^{13}$ GeV) vector-like exotic states were required for gauge coupling unification, assuming the standard scenario with universal gaugino masses at the GUT scale and threshold corrections of about -3%. Solutions to gauge coupling unification were constrained by the bounds on proton decay. It was also assumed that all the vector-like exotics obtain mass at the same scale. The exotics come in incomplete GUT multiplets and hence, in general could obtain mass at different scales. In this work, we generalize the solutions, allowing the exotics that carry SU(3) and SU(2) charges to obtain mass at different scales. We build a *consistent* MSSM spectrum at the weak scale with these exotic messengers. We find that this generalization increases the number of solutions satisfying gauge coupling unification. In addition, the weak scale spectrum now allows unification with moderate or even zero threshold corrections at the GUT scale. The low energy spectrum in such a case has light gluinos which should be detected at the Tevatron and/or LHC.

2 Gauge Mediated Supersymmetry Breaking

Gauge Mediated SUSY Breaking(GMSB) [18] models have chiral supermultiplets called *messenger* fields that mediate supersymmetry breaking. The messenger fields carry SU(3) × SU(2) × U(1) charges and hence couple to the matter fields of the MSSM through the usual SU(3) × SU(2) × U(1) gauge interactions. The messenger fields are very massive at some scale, denoted by M_{mess}. Sources of flavor violation near the messenger scale are given by (4+d)-dimension operators that are suppressed by $\frac{1}{(M_{mess})^d}$. Hence, the major source of flavor violation is due to Yukawa couplings, similar to the Standard Model(SM). This suppression of flavor changing neutral currents (FCNC) is the most attractive feature of GMSB.

In string theories where the extra-dimensions are compactified on an orbifold, there exist extra vector-like non-standard model particles, usually called "exotics". These exotics need to be heavy in order for them to decouple from the low-energy theory. The exotics carry charges under the SM gauge group, and hence are perfect candidates to mediate supersymmetry breaking via gauge mediation. We build a *consistent* MSSM spectrum at the weak scale with these exotic messengers. By consistent, we mean that if we start at the highest scale in the model and run the coupling constants and soft SUSY breaking parameters all the way down to the weak scale, integrating out heavy states during this running, we must end up with the coupling constants that match the experimental values at the weak scale. Since this requires knowledge of the spectrum of exotics and Kaluza-Klein modes, as well as the MSSM spectrum, we perform our analysis in two steps:

Step #1 We concentrate on a class of models based on SU(6) gauge-Higgs unification in 5D [12, 13]. Starting from heterotic string theory compactified on an anisotropic orbifold, $T^6/\mathbb{Z}_6$-II and applying the 'Phenomenological Priors' - Inequivalent models with the SM gauge group, 3 SM families, Higges, and non-anomalous $U(1)_Y \subset SU(5)$; the authors end up with 15 models consisting of low energy spectrum that is similar to that of MSSM. In addition, the spectrum consists of heavy vector-like exotics that decouple from the low-energy theory. Gauge coupling unification was studied [6] in 2 out of the above 15 models [13] - "Model 1A" and "Model 2". The matter content of both these models are very similar and is summarized in Table 3. For the gauge couplings to unify in the heterotic orbifold theory, it was noted in Ref.[6] that there had to be at least $\vec{n} = (n_3, n_2, (n_1, n_1'))$ 'light' exotics at some intermediate scale M_{EX}, below the 4D unification(GUT) scale. This scale, M_{EX} was determined by matching the Renormalization Group Equations(RGE) from the two theories - heterotic orbifold model and the 4D MSSM at some low energy scale μ, where both the theories predict the same running for the couplings. In the 4D MSSM, the gauge couplings unify at the GUT scale with some threshold corrections from new physics near the GUT scale whereas, on the heterotic side, the gauge couplings unify at the string scale, M_S(See Fig.3). The analysis in Ref.[6] was done in the context of a minimal scenario where all the light exotics obtained mass at the same scale and to accommodate precision electroweak data, a -3 % threshold correction was assumed at the GUT scale.

The exotics come in incomplete SU(5) GUT multiplets and in general could obtain mass at different scales. We therefore relax the previous assumption that the light exotics obtain mass at the same scale and allow those that carry SU(3) and SU(2) charges to obtain mass at different scales. This leads to non-universal gaugino masses at the GUT scale, as a consequence of which, the threshold corrections required to match precision data at this scale is no longer of order -3%. The GUT scale threshold correction is a priori a free parameter which depends on the spectrum of states near the GUT scale. However, it's value needs to be fixed by evaluating the 2-loop RGE running from the string scale to the weak scale and including one loop threshold corrections at both the weak and the GUT scales, self-consistently. This is done in the next step. The new intermediate scales, M_{EX3} and M_{EX2} are determined self-consistently using the RGEs. The details of the calculation of the light exotics mass spectrum is given in Appendix A.

Step #2 Once the exotic masses are determined, the soft SUSY breaking terms are calculated at this scale, i.e. the messenger scale [9]. We then use SOFTSUSY [11] to run them down to the weak scale.[1] SOFTSUSY uses the 2-loop RG running to determine the weak scale MSSM spectrum and the 1-loop weak scale threshold corrections. The MSSM parameters are then run back again to the GUT scale and the GUT scale threshold corrections are calculated, i.e. the (output) values fixed by SOFTSUSY (see Eq. (7)). We compare this value of the GUT threshold corrections with the (input) value determined independently by the exotic mass spectrum (see Eq. (11)). We vary the arbitrary parameters of the orbifold string theory and save only those cases where the input (determined by the exotic mass spectrum) and output (required by the low energy MSSM spectrum) threshold corrections match. We also only keep cases consistent with the bound on the proton lifetime and a lower bound on the Higgs mass.

[1]Note, SOFTSUSY runs from the presumed 4D GUT scale to the weak scale. In our case the soft masses are only determined at the messenger scale. The error made by matching the 5D theory to the 4D theory at the messenger scale is however, small. Using the fact that up to 1-loop, the ratio M_i/α_i = constant, we calculate the soft-masses at the GUT scale where they get small 2-loop corrections. As shown in Appendix. B, the corrections to the gaugino masses are found to be less than 1% and can be neglected. A detailed discussion of the soft SUSY breaking masses is in Section 2.1.

2.1 Soft Masses

The exotics of the orbifold theory mediate supersymmetry breaking by acting as messengers of gauge mediation. Due to the gauge interactions of the messengers, soft terms are generated at the messenger scale. We assume that the gravity-mediated SUSY breaking contributions to gaugino masses are much smaller than the gauge-mediated contribution. There can be anomaly mediated contributions to the soft masses, proportional to the gravitino mass, $m_{3/2}$, or dilaton contributions proportional to F_S/M_{Pl}.[2] We allow for a large gravitino mass of the order of a few TeV. However, if the ratio, $\frac{F}{M_{EX}} \gg m_{3/2}$, we can ignore the gravitino contribution to the gaugino masses. At one loop, the gauginos masses are given by:

$$M_i = b_i^{EX3}\ \frac{\alpha_i}{4\pi}\frac{F}{M_{EX3}} + b_i^{EX2}\ \frac{\alpha_i}{4\pi}\frac{F}{M_{EX2}} \tag{2}$$

where F is the SUSY breaking VEV, which at this point is chosen to be arbitrary.

The scalars obtain mass at two-loops, and the dominant contribution to their mass is from the gravitino. In addition, in string models, it is natural to have an anomalous $U(1)_X$ gauge interaction. Such interactions can add an additional Fayet-Iliopoulos D-term to the scalar potential. In such cases, the scalar masses can receive a contribution from the D-term that is of the same order as the gauge mediation contribution. This was discussed in [21] where the contribution to scalar masses was modeled by a term:

$$\delta m_{\phi_i}^2 = d\ Q_i^X M_2^2 \tag{3}$$

with, d, an arbitrary parameter and, Q_i^X, the $U(1)_X$ charge of the field ϕ_i. For the matter fields this charge is taken to be +1, and for the Higgs fields, it is set equal to -2; i.e. $U(1)_X$ is the U(1) in $SO(10)$ commuting with $SU(5)$. M_2 represents the wino mass calculated earlier in Eq. (2). With contributions from the gravitino, gauge mediation, and the D-term, the scalar masses are

[2] We assume the SUSY breaking dilaton VEV, F_S, is negligibly small. Kahler and complex structure moduli (denoted generically by T) contribute to gaugino masses via one loop corrections to the gauge kinetic function with scale set by F_T/M_{Pl}. F is then assumed to be a linear combination of geometric moduli and chiral matter moduli.

given by:

$$m_{\phi_i}^2 = m_{3/2}^2 + 2\left(b_3^{EX3}\ \frac{\alpha_3}{4\pi}\frac{F}{M_{EX3}}\right)^2 C_3(i) + 2\left(b_2^{EX2}\ \frac{\alpha_2}{4\pi}\frac{F}{M_{EX2}}\right)^2 C_2(i)$$
$$+2\left(\frac{\alpha_3}{4\pi}\left(b_1^{EX3}\ \frac{F}{M_{EX3}} + b_1^{EX2}\ \frac{F}{M_{EX2}}\right)\right)^2 C_1(i) + dQ_i^X M_2^2 \quad (4)$$

where, C_is represent the quadratic Casimir invariants [10]. The low energy spectrum is now computed for different values of $\vec{n}$, ϵ_3, F, $m_{3/2}$, and d. Table 1 shows the GUT scale parameters for four sample points.

Table 1: The GUT scale parameters for four different cases. g_{string} is discussed in Appendix A.3.2. Dimensionful quantities in units of GeV unless specified.

Observable	**Case 1**	**Case 2**	**Case 3**	**Case 4**
$\vec{n}$	(4,2,(2,1))	(4,2,(2,1))	(4,2,(1,1))	(4,2,(2,0))
$m_{3/2}$	4 TeV	10 TeV	10 TeV	4 TeV
d	0	5	5	1
g_{string}	0.99412	0.99604	0.8233	0.8588
M_S	6.04×10^{17}	6.05×10^{17}	7.39×10^{17}	7.27×10^{17}
M_C	1.2×10^{16}	1.2×10^{16}	3.2×10^{16}	2.8×10^{16}
M_{EX3}	5.03×10^{13}	1.10×10^{14}	1.07×10^{14}	5.05×10^{13}
M_{EX2}	1.69×10^{13}	8.54×10^{13}	5.35×10^{13}	8.87×10^{12}
M_{GUT}	2.5×10^{16}	2.0×10^{16}	3.25×10^{16}	1.75×10^{16}
ϵ_3	-2.5 %	0 %	-2.5 %	-0.5 %
F	$1.0 \times 10^{18}\text{GeV}^2$	$1.0 \times 10^{18}\text{GeV}^2$	$1.0 \times 10^{18}\text{GeV}^2$	$1.0 \times 10^{18}\text{GeV}^2$
$M_3(M_{GUT})$	257.296	155.269	120.882	260.894
$M_2(M_{GUT})$	392.844	600.865	119.793	747.307
$M_1(M_{GUT})$	124.900	128.947	39.666	260.894

3 Features of the Spectrum

The MSSM spectrum is calculated using SOFTSUSY [11]. For the four cases shown in Table 1, the spectrum from SOFTSUSY is shown in Table. 2.

Non-universal gaugino masses: The split exotics give rise to non-universal gaugino masses at the GUT scale, as is clear from Eq.(2). As a result of this

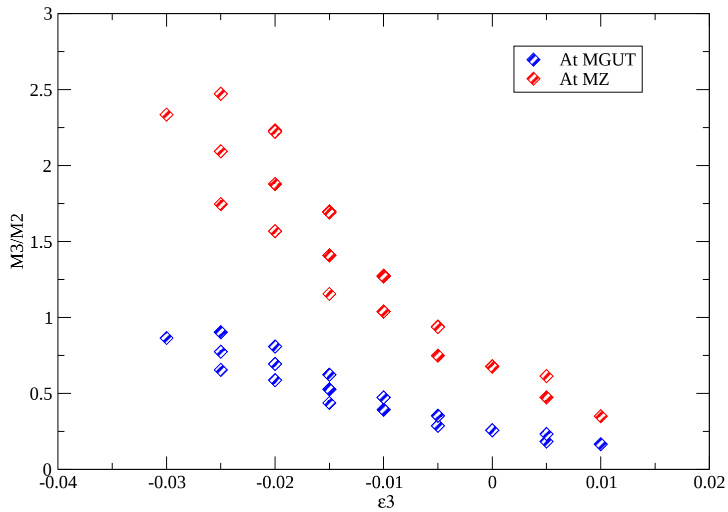

Figure 1: The scatter plot of consistent points for the case, $\vec{n} = (4, 2, (2, 1))$. The gravitino mass, $m_{3/2}$ was taken to be 4 TeV. The running of the couplings depend only on the ratio M_3/M_2 since the scalars are very heavy. We find an anti-correlation between this ratio and the value of ϵ_3. Precision unification favors $M_3/M_2 \sim 0.3$ at the GUT scale.

non-universality, the GUT scale threshold corrections required to match the precision electroweak data need not necessarily be of order -3 % . In fact, we notice that it is possible to obtain precision unification when $M_3 < M_2$. This requirement was observed in [7] in a variety of SUSY breaking scenarios including a Higgs-messenger mixing model where SUSY is broken via gauge mediation. The weak scale MSSM spectrum now has a light gluino. Case 2 in Table 2 illustrates this feature of the gluino being the second lightest sparticle after the neutralino. Figure 1 demonstrates the correlation between the GUT scale threshold corrections ϵ_3 and the ratio of gaugino masses at the weak scale and the GUT scale. Note, the scalar masses are heavy and degenerate. As a result, they do not introduce differential running of the coupling constants.

SUSY breaking scale, gravitino mass, and D-Term: The low energy spectrum depends on the following parameters that are chosen arbitrarily - the SUSY breaking scale F, the gravitino mass $m_{3/2}$ and the d parameter in the D-term. Although the doublet and the triplet exotics could couple to

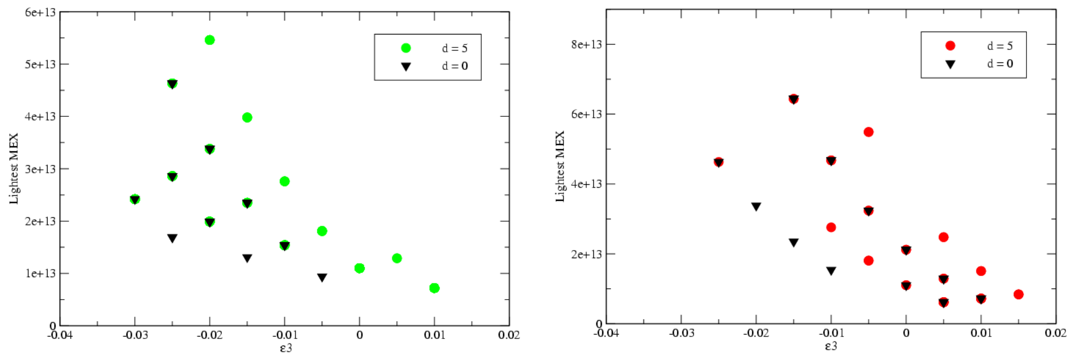

Figure 2: The plot shows the consistent points with varying $m_{3/2}$ and d. The plot on the left is for $m_{3/2} = 4$ TeV and the one on the right is for $m_{3/2} = 10$ TeV.

different SUSY breaking fields, for simplicity we use a single SUSY breaking VEV, F. The gravitino mass $m_{3/2}$, is the dominant contribution to the scalar masses. In order to obtain consistent solutions, we find $m_{3/2} \geq 2$ TeV, otherwise the scalars become non-degenerate at the GUT scale and spoil unification through differential running. At the same time, if $m_{3/2} > 10$ TeV, the assumption that gauge mediation is the dominant contribution to gaugino masses no longer holds and the gravitino corrections to the gaugino masses must be included. The D-term introduces a splitting between the sparticle masses and the Higgs masses, since they carry different charges under the $U_X(1)$. For the two cases of $m_{3/2} = 4$ TeV and 10 TeV, the graph 2 shows the set of consistent points with varying d = 0, 5.

MSSM Spectrum: The MSSM spectrum for the four particular cases (Table 1) is given in Table 2. The $m_{3/2}$ contribution makes the scalars very heavy, with the third family being slightly lighter. The gauginos receive the dominant contributions only from the gauge messengers and are light in comparison with the scalars. The LSP is the lightest neutralino, $\tilde{\chi}_1^0$, which is predominantly "bino-like". The gluino and chargino masses depend on the threshold corrections at the GUT scale, as is seen for the four cases given in Table 2. Most of the points that were found to be consistent with the low energy data have small values of $\tan\beta < 10$. Finally, increasing the d parameter gives a handle on the possible values for $\tan\beta$.

Collider prospects: We have found light gauginos which can be produced at the LHC or possibly the Tevatron. Since the gauginos are lighter than the scalars, they will decay only through off-shell squarks. Gluinos can decay via the process: $\tilde{g} \to q\,\bar{q}\,\tilde{\chi}_i$. The produced $\tilde{\chi}_i$ would then undergo cascade decay until the final product is the LSP and Standard Model leptons. This would give a striking signature of at least 4 jets + missing E_T [16]. The heavier $\tilde{\chi}_i^0$ and $\tilde{\chi}_i^{\pm}$ could decay into their lighter counterparts and leptons that could be the cleanest signature at the LHC. The unique feature of the spectrum is the mass difference between the heavier neutralinos and the LSP - about 150 GeV in one case and close to 500 GeV in the other. This could lead to very high energy leptons and a lot of missing energy making this a very favorable channel at the LHC. Once detected, this would give useful information about the GUT scale threshold corrections.

4 Summary

We have calculated the spectrum of exotics as well the MSSM spectrum starting from a heterotic string theory compactified on an anisotropic orbifold. Allowing the exotics, that come in incomplete GUT multiplets to obtain mass according to their quantum numbers allows for more possible solutions to gauge coupling unification. We find that we can build consistent MSSM spectra starting with such theories with the exotics acting as messengers of SUSY breaking through the gauge mediation mechanism. The gaugino masses in the low energy spectrum depend on the threshold corrections at the GUT scale. They are lighter than the scalars and are within the kinematic reach of the LHC. The unique features of the spectrum are light gluinos and also large mass differences between the lightest and the next-to-lightest neutralinos and charginos which could lead to interesting signatures at the LHC.

Acknowledgments

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Table 2: The MSSM spectrum at the weak scale for the four different cases given in Table 1.

Observable	**Case 1**	**Case 2**	**Case 3**	**Case 4**
$\tan\beta$	7	4	4	6
mu	-206.217	-1932.930	937.044	958.984
m_{h^0}	119.311	117.384	117.791	117.384
m_{H^0}	4039.466	10327.840	10323.870	4129.50
m_{A^0}	4037.954	10323.182	10319.566	4127.62
m_{H^+}	4039.133	10323.750	10320.322	4128.69
$m_{\tilde{g}}$	708.987	455.343	369.786	712.66
$m_{\tilde{\chi}_1^0}$	52.867	60.336	13.608	24.671
$m_{\tilde{\chi}_2^0}$	198.922	548.051	97.766	629.15
$m_{\tilde{\chi}_3^0}$	-221.679	1947.429	-947.105	-962.34
$m_{\tilde{\chi}_4^0}$	360.330	-1949.000	952.206	974.12
$m_{\tilde{\chi}_1^\pm}$	199.225	548.196	98.136	629.87
$m_{\tilde{\chi}_2^\pm}$	351.551	1974.466	967.940	984.45
$m_{\tilde{d}_L} \simeq m_{\tilde{s}_L}$	4035.319	10017.133	9891.84	4195.60
$m_{\tilde{u}_L} \simeq m_{\tilde{c}_L}$	4034.710	10017.005	9891.67	4195.03
$m_{\tilde{b}_1}$	3312.041	8329.932	8180.58	3489.22
$m_{\tilde{t}_1}$	2357.034	6181.548	6034.69	2419.63
$m_{\tilde{e}_L} \simeq m_{\tilde{\mu}_L}$	4023.779	10095.067	9973.61	4185.18
$m_{\tilde{\nu}_{eL}} \simeq m_{\tilde{\nu}_{\mu L}}$	4022.711	10094.474	9973.00	4184.15
$m_{\tilde{\tau}_L}$	3982.090	10068.311	9966.998	4053.27
$m_{\tilde{\nu}_{\tau L}}$	4014.664	10088.009	9966.40	4178.30
$m_{\tilde{d}_R} \simeq m_{\tilde{s}_R}$	4011.295	10009.631	9919.58	4078.18
$m_{\tilde{u}_R} \simeq m_{\tilde{c}_R}$	4009.916	10005.369	9914.576	4076.57
$m_{\tilde{b}_2}$	3994.851	9997.358	9907.23	4065.98
$m_{\tilde{t}_2}$	3314.600	8330.918	8181.58	3491.80
$m_{\tilde{e}_R}$	3998.343	10081.311	9993.46	4065.36
$m_{\tilde{\mu}_R}$	3998.289	10081.268	9993.42	4065.32
$m_{\tilde{\tau}_R}$	4015.735	10088.622	9980.25	4179.34

A Renormalization Group Equations

A.1 4D SUSY GUT Theory

The grand unification (GUT) scale, M_{GUT} is defined by SOFTSUSY [11] as the point where

$$\begin{aligned} \alpha_1(M_{GUT}) &= \alpha_{GUT} = \alpha_2(M_{GUT}) \\ \alpha_3(M_{GUT}) &= \alpha_{GUT}(1+\epsilon_3). \end{aligned} \tag{5}$$

Precision electroweak data requires, in the standard supersymmetry breaking scenarios, with universal gaugino and scalar masses at the GUT scale:

$$\epsilon_3 = \frac{\alpha_3 - \alpha_{GUT}}{\alpha_{GUT}} \simeq -0.03. \tag{6}$$

Thus the 1-loop renormalization group equations for SUSY GUT from the weak scale to the GUT scale, with 2-loop threshold corrections near the GUT scale are given (in the vicinity of the GUT scale) by:

$$\alpha_i^{-1}(\mu) = \alpha_{GUT}^{-1} + \frac{b_i}{2\pi} log \frac{M_{GUT}}{\mu} - \Delta\delta_{i3} \tag{7}$$

where i = 3,2,1, represent $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ respectively. The $b_i s$ are the β-function coefficients, $b_i = (-3, 1, \frac{33}{5})$ for the MSSM and δ_{i3} is the threshold corrections to α_3,

$$\Delta = \alpha_{GUT}^{-1} \frac{\epsilon_3}{(1+\epsilon_3)}. \tag{8}$$

It should be noted that when we work in the non-standard scenarios such as models with non-universal gaugino masses at the GUT scale, the value of ϵ_3 need not be -3 %.

A.2 Orbifold GUT Theory

The RGEs for the orbifolded model can be arrived at by taking into account all the particles in the spectrum. The highest scale in this theory is the string scale, M_S, above which there is one unified grand unified coupling constant, α_{string}. In the heterotic framework, the unified coupling constant is related to Newton's constant, G_N, by the relation:

$$G_N = \frac{1}{8}\, \alpha_{\text{string}}\, \alpha' \tag{9}$$

where, $\alpha' = \frac{1}{M_S^2}$, and from the observed value of $G_N = \frac{1}{M_{PL}^2}$; $M_{PL} \sim 1.2 \times 10^{19}$ GeV. Thus we have,

$$\alpha_{string} = 8\frac{M_S^2}{M_{PL}^2} \tag{10}$$

The three gauge couplings renormalize independently below the string scale. New states enter the theory at the intermediate scales:

- M_{EX3} - Mass scale of the triplet exotics.
- M_{EX2} - Mass scale of the doublet exotics.

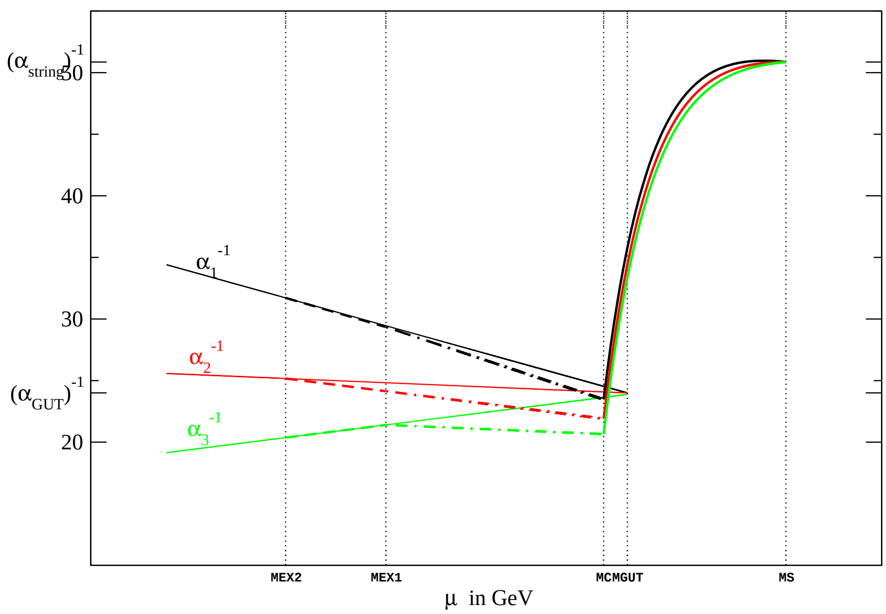

Figure 3: The figure shows the evolution of couplings for one particular model $\vec{n}$ = (4,2,(2,1)). The 4D GUT scale is $M_{GUT} = 2 \times 10^{16}$ GeV. The doublet exotics enter at $M_{EX2} = 1.29 \times 10^{13}$ GeV; the triplets enter at $M_{EX3} = 1.115 \times 10^{14}$ GeV. At $M_C = 1.2 \times 10^{16}$ GeV, the Kaluza-Klein states of the MSSM particles enter and couplings run with a power law to unify at the string scale, M_S =6.06 $\times 10^{17}$ GeV.

In general each of the exotics could obtain different mass. We generalize from the previous work [6], where it was assumed that all the exotics obtain mass at the same scale. This generalization is motivated by the fact that the exotics do not come in complete GUT multiplets. Hence the possibility of a splitting between the doublets and triplets is well motivated. The running of the couplings begin to alter at these scales, as shown in Fig. 3. Taking into account all particles in the spectrum below the string scale: MSSM particles, exotic states and the Kaluza-Klein modes from the compactification, the renormalization group equation for the orbifold GUT theory becomes:

$$\begin{aligned}\alpha_i^{-1}(\mu) = \alpha_{string}^{-1} &+ \frac{b_i^{MSSM}}{2\pi} log\frac{M_s}{\mu} + \frac{b_i^{EX3}}{2\pi} log\frac{M_s}{M_{EX3}} + \frac{b_i^{EX2}}{2\pi} log\frac{M_s}{M_{EX2}} \\ &- \frac{1}{4\pi}(b_i^{++} + b_i^{--}) log\frac{M_s}{M_c} + \frac{b^G}{2\pi}(\frac{M_s}{M_c} - 1). \end{aligned} \tag{11}$$

The first term in the above equation is the tree level boundary condition from the heterotic string theory as given in Eq. (10). The next three terms include contributions to the β-function coefficients from the from the MSSM and exotic brane states and the zero-KK modes of the states living in the bulk.

$$b_i^X = b_i^{X,++} + b_i^{X,\text{brane}} \qquad \text{where } X = MSSM,\ EX3,\ EX2. \tag{12}$$

For the MSSM, $b_i = (3, -1, 33/5)$. The infinite sum over the KK modes of the MSSM can be evaluated to give the last term in Eq. (11) [14]. States that are not localized on the "branes" are free to propagate in the higher-dimensional "bulk". In this work, we have considered that only the minimum amount of matter lives in the bulk - the MSSM third family $\bar{b}$ and L. In principle we could allow two more pairs of chiral multiplets, $\mathbf{6} + \mathbf{6^c}$ in the bulk that would correspond to "bulk" exotics. The contribution from the massive KK modes of the exotics that live in the bulk would also include an infinite sum over the KK modes. The case of bulk exotics was analyzed in [6] and the spectrum of exotics was calculated. For the purposes of the current work, we shall assume that the exotics live only on the brane. The current treatment can be extended similarly for the bulk exotics case also, without any significant change in the low energy phenomenology.

The brane-localized exotics' β-function coefficients depend on the exotic matter content of the theory, see Table 3. The exotic matter content is defined in terms of the parameters n_i with:

$$\begin{aligned} n_3 \times \left[(\mathbf{3}, 1)_{1/3,*} + (\overline{\mathbf{3}}, 1)_{-1/3,*}\right] + n_2 \times [(1, \mathbf{2})_{0,*} + (1, \mathbf{2})_{0,*}] + \\ n_1 \times [(1, 1)_{1,*} + (1, 1)_{-1,*}] \end{aligned} \tag{13}$$

The coefficients

$$b^{EX3} = (n_3, 0, \frac{n_3 + 3n_1}{10}) \qquad b^{EX2} = (0, n_2, \frac{3n_1'}{10})$$

(given in table 4) can be calculated for each different value of the parameters $\overrightarrow{n} = (n_3, n_2, (n_1, n_1'))$. In the above expression, n_1 of the exotics with just U(1) hypercharge obtain the same mass as the triplets and n_1' of them obtain the same mass as the doublets. We check for gauge coupling unification by comparing the following quantities (i) $1/\alpha_3 - 1/\alpha_2$, (ii) $1/\alpha_2 - 1/\alpha_1$, (iii) α_3 from the 4D SUSY GUT theory and the orbifold GUT theory at the scale μ. We scan over all possible, n_i, and check for gauge coupling unification.

Table 3: Exotic matter content in Models 1A/B and 2 from [13]. Listed are the quantum numbers of the states under the MSSM and hidden sector gauge groups, with the hypercharge denoted in the subscript. The brane localized exotic matter in Model 1 is a subset of that in Model 2.

Model	Hidden Sector		Exotic Matter Irrep	Name
1 A/B	$SU(4) \times SU(2)$	brane	$2 \times \left[(\mathbf{3}, 1; 1, 1)_{1/3,2/3} + (\overline{\mathbf{3}}, 1; 1, 1)_{-1/3,-2/3}\right]$	$v + \bar{v}$
		exotics	$4 \times [(1, \mathbf{2}; 1, 1)_{0,*} + (1, \mathbf{2}; 1, 1)_{0,*}]$	$m + m$
			$1 \times [(1, \mathbf{2}; 1, \mathbf{2})_{0,0} + (1, \mathbf{2}; 1, \mathbf{2})_{0,0}]$	$y + y$
			$2 \times \left[(1, 1; \mathbf{4}, 1)_{1,1} + (1, 1; \overline{\mathbf{4}}, 1)_{-1,-1}\right]$	$f^+ + \bar{f}^-$
			$14 \times [(1, 1; 1, 1)_{1,*} + (1, 1; 1, 1)_{-1,*}]$	$s^+ + s^-$
		bulk	$6 \times \left[(\mathbf{3}, 1; 1, 1)_{-2/3,-2/3} + (\overline{\mathbf{3}}, 1; 1, 1)_{2/3,2/3}\right]$	$\delta + \bar{\delta}$
		exotics	$1 \times \left[(\mathbf{3}, 1; 1, 1)_{-2/3,-1/3} + (\overline{\mathbf{3}}, 1; 1, 1)_{2/3,1/3}\right]$	$d + \bar{d}$
			$1 \times [(1, \mathbf{2}; 1, 1)_{-1,-1} + (1, \mathbf{2}; 1, 1)_{1,1}]$	$\ell + \bar{\ell}$
2	$SO(8) \times SU(2)$	brane	$4 \times \left[(\mathbf{3}, 1; 1, 1)_{1/3,*} + (\overline{\mathbf{3}}, 1; 1, 1)_{-1/3,*}\right]$	$v + \bar{v}$
		exotics	$2 \times [(1, \mathbf{2}; 1, 1)_{0,*} + (1, \mathbf{2}; 1, 1)_{0,*}]$	$m + m$
			$1 \times [(1, \mathbf{2}; 1, \mathbf{2})_{0,0} + (1, \mathbf{2}; 1, \mathbf{2})_{0,0}]$	$y + y$
			$2 \times [(1, 1; 1, \mathbf{2})_{1,1} + (1, 1; 1, \mathbf{2})_{-1,-1}]$	$x^+ + x^-$
			$20 \times [(1, 1; 1, 1)_{1,*} + (1, 1; 1, 1)_{-1,*}]$	$s^+ + s^-$
		bulk	$3 \times \left[(\mathbf{3}, 1; 1, 1)_{-2/3-2/3} + (\overline{\mathbf{3}}, 1; 1, 1)_{2/3,2/3}\right]$	$\delta + \bar{\delta}$
		exotics	$1 \times \left[(\mathbf{3}, 1; 1, 1)_{-2/3,2/3} + (\overline{\mathbf{3}}, 1; 1, 1)_{2/3,-2/3}\right]$	$d + \bar{d}$
			$1 \times [(1, \mathbf{2}; 1, 1)_{-1,-1} + (1, \mathbf{2}; 1, 1)_{1,1}]$	$\ell + \bar{\ell}$
			$3 \times [(1, \mathbf{2}; 1, 1)_{-1,0} + (1, \mathbf{2}; 1, 1)_{1,0}]$	$\phi + \bar{\phi}$

A.3 Spectrum of Exotics and Constraints

We look for models with gauge coupling unification allowing the triplets and the doublets to obtain mass at different scales: M_{EX3}, and M_{EX2}. The

irrep	b_3	b_2	b_Y
$[(\mathbf{3},1)_{1/3,*} + (\bar{\mathbf{3}},1)_{-1/3,*}]$	1	0	1/10
$[(1,\mathbf{2})_{0,*} + (1,\mathbf{2})_{0,*}]$	0	1	0
$[(1,1)_{1,*} + (1,1)_{-1,*}]$	0	0	3/10

Table 4: Values of β-function coefficients for the brane-localized exotics.

relevant quantities in the RGEs are given by:

$$
\begin{aligned}
\text{MSSM } \beta\text{-function coefficients} \qquad & b^{MSSM} = (-3, 1, \frac{33}{5}) \\
\text{MSSM bulk states } \beta\text{-function coefficients} \qquad & b^{++} = (-7, -3, \frac{13}{5}) \\
& b^{--} = (5, 1, \frac{1}{5}) \\
b^G \equiv \sum_{P=\pm, P'=\pm} b_i^{MSSM,PP'}; \qquad & b^G = -4 \\
\text{Planck Scale} \qquad & M_{PL} \; = \; 1.22 \times 10^{19} \text{ GeV}
\end{aligned}
$$

M_C is allowed to vary between a minimum of 7×10^{15} GeV, the approximate bound determined in [6] and higher. We find the maximum number of solutions for $M_C = 1.2 \times 10^{16}$ GeV. All further results shown will be from the data set with this value of M_C. This also seems to be a good choice of M_C since it is only an order of magnitude smaller than the string scale. Since we are deviating from the standard scenario of universal gaugino masses at the GUT scale, we need to consider a larger range of ϵ_3. In fact, we assume that the GUT threshold corrections, in order to agree with the precision electroweak data, can be anywhere between $\epsilon_3 \sim$ -4% and +2%. In comparison with the earlier analysis, [6], we find a larger number of cases that satisfy gauge coupling unification. Introducing the splitting of the doublets and triplets gives more freedom in fixing the compactification scale, and hence a larger number of solutions fall within the proton lifetime bound (See eq. 14).

A.3.1 Proton Decay

The bound on the proton lifetime is an important constraint on the solutions, since the models discussed here are SU(6) GUTS in 5D. At the scale,

$M_X \sim M_C$, the grand unified gauge group is broken and the GUT gauge **X** boson gets mass at this scale. It can mediate proton decay via dimension 6 operators. In a 4D effective theory formalism, the decay rate $\Gamma(p \to e^+ + \pi^0)$ can be calculated in terms of g_{GUT}, M_S, M_C, and M_X. In Appendix B of Ref. [6] they obtain

$$\tau(p \to e^+\pi^0) = 5.21 \times 10^{40} \left(\frac{M_C}{M_S}\right)^4 \text{years.} \tag{14}$$

When calculating the spectrum of exotics satisfying gauge coupling unification, we only keep cases consistent with the experimental bounds on the proton lifetime. The strongest experimental bound comes from Super-Kamiokande in Japan that searches for $p \to\ e^+ + \pi^0$ signatures. The current bound is [8]

$$\tau(p \to e^+\pi^0) > 1 \times 10^{34} \text{years.} \tag{15}$$

A.3.2 String Coupling Constant

We have assumed that the $E_8 \times E_8$ model considered in this paper is in the weakly coupled regime. In [17] a simple formula relating the string coupling constant, g_{string}, to other relevant quantities of the model was obtained

$$g^2_{string} = \alpha_{GUT} \frac{M_S}{M_C}. \tag{16}$$

To be in the perturbative regime, we need $g_{string} \leq 1$. In the heterotic framework, we also have a relation between $\alpha_{GUT}(M_S) \equiv \alpha_{string}$, i.e. the unified coupling constant at the string scale, given in Eq. 10:

$$\alpha^{-1}_{GUT} = \frac{1}{8} \frac{M^2_{PL}}{M^2_S}. \tag{17}$$

This condition then imposes an additional constraint on the possible consistent solutions. The value of g_{string} for the four cases discussed earlier can be found in Table. 1.

B 2-loop corrections to soft masses

The 2-loop renormalization group equations for the MSSM parameters have been studied extensively in literature [19]. The RGEs for the couplings and gaugino masses at 2-loop are given by:

$$\frac{dM_i}{dt} = \frac{2g_i^2}{16\pi^2} b_i^{(1)} M_i + \frac{2g_i^2}{(16\pi^2)^2} \sum_{j=1}^{3} b_{ij}^{(2)} g_j^2 (M_i + M_j) \tag{18}$$

$$\frac{dg_i}{dt} = \frac{g_i^3}{16\pi^2} b_i^{(1)} + \frac{g_i^3}{(16\pi^2)^2} \sum_{j=1}^{3} b_{ij}^{(2)} g_j^2 \tag{19}$$

where, $\frac{g_i^2}{4\pi} = \alpha_i$. The 1-loop and 2-loop β- function coefficients for the MSSM are:

$$b_i^{(1)} = (-33/5, -1, 3) \qquad b_{ij}^{(2)} = \begin{pmatrix} 199/100 & 27/20 & 22/5 \\ 9/20 & 25/4 & 6 \\ 11/20 & 9/4 & 7/2 \end{pmatrix} \tag{20}$$

At 1-loop the gaugino masses and the couplings obey the relation:

$$\frac{M_i}{\alpha_i} = constant \tag{21}$$

At 2-loops this equation gets small corrections. Solving the above two equations at 2-loops, we find:

$$\frac{M_i}{\alpha_i} = \sum_{j=1}^{3} b_{ij}^{(2)} \alpha_j M_j \tag{22}$$

Approximating the right-hand side by the 1-loop result, and using the values of the β-function coefficients, we find that the 2-loop corrections to Eq. (21) are less than 1% [20].

References

[1] H. Georgi, S. L. Glashow, "Unity of All Elementary Particle Forces," Phys. Rev. Lett. **32**, 438-441 (1974).

[2] H. Georgi, "The State of the Art - Gauge Theories. (Talk)," AIP Conf. Proc. **23**, 575-582 (1975).

H. Fritzsch, P. Minkowski, "Unified Interactions of Leptons and Hadrons," Annals Phys. **93**, 193-266 (1975).

[3] S. Dimopoulos, S. Raby, and F. Wilczek, "Supersymmetry and the scale of unification," *Phys. Rev.* **D24** (1981) 1681–1683.

S. Dimopoulos and H. Georgi, "Softly Broken Supersymmetry and SU(5)," *Nucl. Phys.* **B193** (1981) 150.

L. E. Ibanez and G. G. Ross, "Low-Energy Predictions in Supersymmetric Grand Unified Theories," *Phys. Lett.* **B105** (1981) 439.

N. Sakai, "Naturalness in Supersymmetric Guts," *Zeit. Phys.* **C11** (1981) 153.

M. B. Einhorn and D. R. T. Jones, "The Weak Mixing Angle and Unification Mass in Supersymmetric SU(5)," *Nucl. Phys.* **B196** (1982) 475.

W. J. Marciano and G. Senjanović, "Predictions of Supersymmetric Grand Unified Theories," *Phys. Rev.* **D25** (1982) 3092.

[4] C. Amsler *et al.* [Particle Data Group Collaboration] , Phys. Lett. **B667**, 1 (2008).

[5] D.I. Kazakov, Lectures given at the European School on High Energy Physics, Aug.-Sept. 2000, Caramulo, Portugal [hep-ph/0012288v2].

V. Lucas and S. Raby, "GUT Scale Threshold Corrections in a Complete Supersymmetric SO(10) Model: $\alpha_s(M_Z)$ vs. Proton Lifetime," *Phys. Rev.* **D54**, 2261 (1996) [arXiv:hep-ph/9601303];

T. Blazek, M. Carena, S. Raby and C. E. M. Wagner, "A global chi**2 analysis of electroweak data in SO(10) SUSY GUTs," *Phys. Rev.* **D56**, 6919 (1997) [arXiv:hep-ph/9611217];

G. Altarelli, F. Feruglio and I. Masina, "From minimal to realistic supersymmetric SU(5) grand unification," *JHEP* **0011**, 040 (2000) [arXiv:hep-ph/0007254].

R. Dermíšek, A. Mafi and S. Raby, *Phys. Rev.* **D63**, 035001 (2001);

K.S. Babu, J.C. Pati and F. Wilczek, *Nucl. Phys.* **B566**, 33 (2000).

M. L. Alciati, F. Feruglio, Y. Lin and A. Varagnolo, "Proton lifetime from SU(5) unification in extra dimensions," *JHEP* **0503**, 054 (2005) [arXiv:hep-ph/0501086].

[6] B. Dundee, S. Raby, A. Wingerter, Compactifications," Phys. Rev. **D78**, 066006 (2008). [arXiv:0805.4186 [hep-th]].

[7] S. Raby, M. Ratz and K. Schmidt-Hoberg, Phys. Lett. B **687**, 342 (2010) [arXiv:0911.4249 [hep-ph]].

[8] Makoto Miura [Super-Kamiokande Collaboration], "Search for Nucleon Decays in Super-Kamiokande" ICHEP 2010.

[9] G. F. Giudice and R. Rattazzi, Phys. Rept. **322**, 419 (1999) [arXiv:hep-ph/9801271].

[10] S. P. Martin, *"A Supersymmetry Primer"*, arXiv:hep-ph/9709356.

[11] B. C. Allanach, Comput. Phys. Commun. **143**, 305 (2002) [arXiv:hep-ph/0104145].

[12] W. Buchmuller, K. Hamaguchi, O. Lebedev and M. Ratz, Phys. Rev. Lett. **96**, 121602 (2006) [arXiv:hep-ph/0511035].

W. Buchmuller, K. Hamaguchi, O. Lebedev and M. Ratz, Nucl. Phys. B **785**, 149 (2007) [arXiv:hep-th/0606187].

O. Lebedev, H. P. Nilles, S. Raby, S. Ramos-Sanchez, M. Ratz, P. K. S. Vaudrevange and A. Wingerter, Phys. Rev. Lett. **98**, 181602 (2007) [arXiv:hep-th/0611203].

[13] O. Lebedev, H. P. Nilles, S. Raby, S. Ramos-Sanchez, M. Ratz, P. K. S. Vaudrevange and A. Wingerter, Phys. Lett. B **645**, 88 (2007) [arXiv:hep-th/0611095].

[14] K. R. Dienes, E. Dudas and T. Gherghetta, Nucl. Phys. B **537**, 47 (1999) [arXiv:hep-ph/9806292].

[15] S. Raby and K. Tobe, Nucl. Phys. B **539**, 3 (1999) [arXiv:hep-ph/9807281].

[16] J. D. Wells, arXiv:hep-ph/0306127.

[17] B. Dundee and S. Raby, arXiv:0808.0992 [hep-th].

[18] M. Dine and W. Fischler, Phys. Lett. B **110**, 227 (1982).

C. R. Nappi and B. A. Ovrut, Phys. Lett. B **113**, 175 (1982).

L. Alvarez-Gaume, M. Claudson and M. B. Wise, Nucl. Phys. B **207**, 96 (1982).

[19] S. P. Martin and M. T. Vaughn, Phys. Rev. D **50**, 2282 (1994) [Erratum-ibid. D **78**, 039903 (2008)] [arXiv:hep-ph/9311340].

Y. Yamada, interactions: Supergraph method," Phys. Rev. **D50**, 3537-3545 (1994). [hep-ph/9401241].

I. Jack, D. R. T. Jones, Phys. Lett. **B333**, 372-379 (1994). [hep-ph/9405233].

[20] Y. Yamada, In *Tsukuba 1993, Proceedings, Particle physics* 352-358.

[21] S. Raby and K. Tobe, Phys. Lett. B **437**, 337 (1998) [arXiv:hep-ph/9805317].