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The four jet production at LHC and Tevatron in QCD.

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We demonstrate that in the back to back kinematics the production of four jets in the collision of two partons is suppressed in the leading log approximation of pQCD, compared to the hard processes involving the collision of four partons. We derive the basic equation for four jet production in QCD in terms of the convolution of generalized two parton distributions of colliding hadrons in the momentum space representation. Our derivation leads to geometrical approach in the impact parameter space close to that suggested within the parton model and used before to describe the four-jet production. We develop the independent parton approximation to the light-cone wave function of the proton. Comparison with the CDF and D0 data shows that the independent parton approximation to the light cone wave function of the proton is insufficient to explain the data. We argue that the data indicate the presence of significant multi-parton correlations in the light cone wave functions of colliding protons.

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In spite of an extensive theoretical and experimental work, various aspects of the high energy hadronic collisions at Tevatron and LHC are still poorly understood. This is especially true for the multijet production which is of a paramount importance for the understanding of pQCD dynamics at high energy colliders, and for the search of new particles. The topic of multiparton interactions is now one of the focal points of studying pQCD and building an adequate basis for modeling the final states at the LHC. In particular, description of multiparton interactions requires treatment of a significant disbalance of the momenta of the jets (presence of the Sudakov form factors). In this paper we summarize the first steps of the program to address these issues. Among original results of the paper are the derivation of the formulae in the leading logarithmic approximation for production of 4 jets. Our key finding is that it is possible to isolate the kinematics where the leading twist processes $2 \rightarrow 4$ are not enhanced. This result will allow one to improve the reliability of the Tevatron studies of the four jet production in the multiparton kinematics and point out directions for the corresponding analysis at the LHC.

Another critical issue is the formulation of the problem in terms of the generalized two parton distributions in the momentum space representation and introduction of the mean field approximation for this object. This new formulation is well suited for the more detailed studies which are now under way. In addition it establishes a link with the original formulation in the coordinate space [1–10], and resolves an issue of the value of the strength of the double interaction within this approximation. Previously there was a question whether a conclusion of ref. [4], that the observed rate is a factor of two larger than the theoretical prediction, can be due to uncertainties re-

lated to many Fourier transforms required to convert the HERA data to the experimental number. A new formulation, though mathematically equivalent, has completely resolved this issue. This poses serious constraints on the Monte Carlo models of pp scattering at collider energies which are not satisfied by many of the current models.

These issues are of broad interest, both theoretical and experimental.

The standard approach to the multijet production is the QCD improved parton model. It is based on the assumption that the cross section of a hard hadron–hadron interaction is calculable in terms of the convolution of parton distributions within colliding hadrons with the cross section of a hard two-parton collision. An application of this approach to the processes with production of four jets implies that all jets in the event are produced in a hard collision of *two* initial state partons.

The recent data due to CDF and D0 collaborations [11–13] do not contradict to the dominance of this mechanism in the well defined part of the phase space. At the same time these data provide the evidence that there exists a kinematical domain where a more complicated mechanism becomes important, namely the double hard interaction of two partons in one hadron with two partons in the second hadron.

Within the parton model picture, the four jets produced this way should pair into two groups such that the transverse momenta of two jets in each pair compensate each other. In what follows we refer to this kinematics as *back-to-back dijet production*. We consider the dijets for the case

$$\delta_{13}^2 \equiv (\vec{j}_{1t} + \vec{j}_{3t})^2 \ll j_{1t}^2 \simeq j_{3t}^2, \quad \delta_{24}^2 \ll j_{2t}^2 \simeq j_{4t}^2, \quad (1)$$

where δ is the total transverse momentum of the dijet and j_{it} the transverse momentum of an individual jet (see

Fig. 1). The hardness condition $\delta^2 \gg R^{-2}$ is implied, with R the characteristic hadron size (non-perturbative scale). The events with disbalances $\delta^2 \leq R^{-2}$, give a small contribution both to total and differential cross sections, since they are suppressed by the Sudakov form factors. For a detailed discussion of this issue see review [15].

Importantly, in this kinematical region the hard scattering of four partons from the wave functions of the colliding hadrons remains the dominant source for four-jet production even when the pQCD parton multiplication phenomena are taken into account.

The reason for that is the following. When the two partons from each hadron emerge from the *initial state parton cascades* and then engage into double hard scattering, the resulting differential distribution of the final state jets lacks the double back-to-back enhancement factor $d\sigma \propto \delta_{13}^{-2} \delta_{24}^{-2}$ which is there in the case of two independent hard scatterings [14]. For the two-parton scattering, the characteristic perturbative enhancement $d\sigma \propto \delta^{-2}$ results from a coherent enhancement of the amplitude due to integration over a large transverse disk, $\rho^2 \sim \delta^{-2} \gg j_t^{-2}$. The two partons that originate from a perturbative splitting form a relatively compact system in the impact parameter space, so that the double hard interaction of such pairs produces only a single perturbative enhancement factor, $(\vec{\delta}_{13} + \vec{\delta}_{24})^{-2}$, which does not favor the back-to-back dijet kinematics (1). The distribution of four jets so produced is much more isotropic and can be suppressed by choosing proper kinematical cuts.

So, the aim of this letter is to consider the four-jet production in the hard collisions of *four* initial state partons. We show that the cross section of back-to-back dijet production is calculable in terms of new nonperturbative objects — the Generalized Two Parton Distributions ($_2$ GPD). The properties of the $_2$ GPD can be rigorously studied within QCD. In particular, we report here the derivation of the geometric picture for multiple parton collisions in the impact parameter space.

In the kinematical domain (1) the direct calculation of the light cone Feynman diagrams (momenta of the partons in the initial and final states are shown in Fig. 1) using the separation of hard and soft scales shows that the four \rightarrow four cross section for the collisions of hadrons "a" and "b" has the form:

$$\sigma_4(x_1, x_2, x_3, x_4) = \int \frac{d^2 \vec{\Delta}}{(2\pi)^2} D_a(x_1, x_2, p_1^2, p_2^2, \vec{\Delta}) \times D_b(x_3, x_4, p_1^2, p_2^2, -\vec{\Delta}) \times \frac{d\sigma^{13}}{d\hat{t}_1} \frac{d\sigma^{24}}{d\hat{t}_2} d\hat{t}_1 d\hat{t}_2. \quad (2)$$

Here $D_\alpha(x_1, x_2, p_1^2, p_2^2, \vec{\Delta})$ are the new $_2$ GPDs for hadrons "a" and "b" defined below. (In the following we will consider the case of pp collisions and omit the subscripts a and b . Summing over collisions of various types of partons is implied. In practice however we will keep

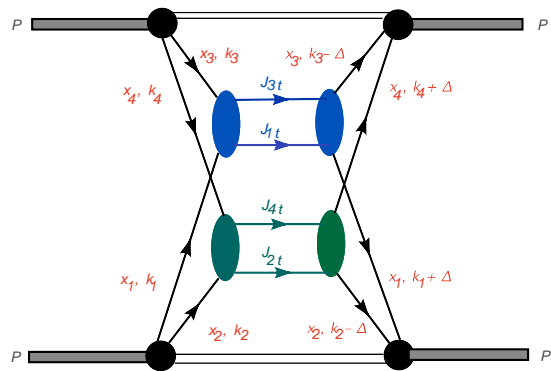


FIG. 1: Kinematics of double hard collision - momenta of the colliding partons in in and out states

hard scattering of gluons only since it gives the dominant contribution.). Remember that the light cone fractions x_i are actually fixed by the final jet parameters and the energy momentum constraints.

With account of the radiative pQCD effects, in full analogy with the "DDT formula" for two-body collisions, the differential distribution (2) acquires Sudakov form factors [15, 16] depending on the logarithms of the large ratios of scales, j_t^2/δ^2 , and the $_2$ GPDs become scale dependent: $p_1^2 \sim \delta_{13}^2$, $p_2^2 \sim \delta_{24}^2$. It should be mentioned that the structure of the final formula depends on what one actually measures in the experiment — energetic single particles with large transverse momenta in the final state or "jets" — and on how the jets are precisely defined. A more detailed account of the pQCD effects will be given in a future publication [14].

For brevity we will not write explicitly the virtuality scales of the $_2$ GPD and will use the form: $D(x_1, x_2, \vec{\Delta})$. Note that these distributions depend on the new transverse vector $\vec{\Delta}$ that is equal to the difference of the momenta of partons from the wave function of the colliding hadron in the amplitude and the amplitude conjugated. Such dependence arises because the difference of parton transverse momenta within the parton pair is not conserved. The integration limits in x_i, \hat{t} are subject to standard limits determined by experimental kinematic cuts.

Within the parton model approximation the cross section has the form:

$$\sigma_4 = \sigma_1 \sigma_2 / \pi R_{\text{int}}^2, \quad (3)$$

where σ_1 and σ_2 are the cross sections of two independent hard binary parton interactions. The factor πR_{int}^2 which in principle depends on x_i characterizes the transverse area occupied by the partons participating in the hard collision. (In the experimental [11, 12] and some of the theoretical papers this factor was denoted as an effective cross section. Our Eq. 4 below shows that such wording is not satisfactory since πR_{int}^2 does not have the meaning of the interaction cross section.) The data [11–13] indicate that πR_{int}^2 is practically constant in the kinematical range studied at the Tevatron.

Eq. 2 leads to the general model independent expression for

$$\frac{1}{\pi R_{\text{int}}^2} = \int \frac{d^2 \vec{\Delta}}{(2\pi)^2} \frac{D(x_1, x_2, -\vec{\Delta}) D(x_3, x_4, \vec{\Delta})}{D(x_1) D(x_2) D(x_3) D(x_4)}, \quad (4)$$

in terms of the ${}_2$ GPDs. Here $D(x_i)$ are the corresponding structure functions. Note that hereafter we do not write dependence of σ_4 and R_{int}^2 on the light cone fractions x_i explicitly.

The ${}_2$ GPDs are expressed through the light cone wave functions of the colliding hadrons as follows. Suppose that in a four \rightarrow four process the two partons in the nucleon in the initial state wave function have the transverse momenta \vec{k}_1, \vec{k}_2 . Then in the conjugated wave function they will have the momenta $\vec{k}_1 + \vec{\Delta}, \vec{k}_2 - \vec{\Delta}$. This is because only sum of parton transverse momenta but not the difference is conserved.

The relevant ${}_2$ GPDs are:

$$\begin{aligned} D(x_1, x_2, p_1^2, p_2^2, \vec{\Delta}) &= \sum_{n=3}^{\infty} \int \frac{d^2 k_1}{(2\pi)^2} \frac{d^2 k_2}{(2\pi)^2} \theta(p_1^2 - k_1^2) \\ &\times \theta(p_2^2 - k_2^2) \int \prod_{i \neq 1,2} \frac{d^2 k_i}{(2\pi)^2} \int_0^1 \prod_{i \neq 1,2} dx_i \\ &\times (\psi_n(x_1, \vec{k}_1, x_2, \vec{k}_2, \dots, \vec{k}_i, x_i \dots)) \\ &\times \psi_n^+(x_1, \vec{k}_1 + \vec{\Delta}, x_2, \vec{k}_2 - \vec{\Delta}, x_3, \vec{k}_3, \dots) + h.c.) \\ &\times (2\pi)^3 \delta\left(\sum_{i=1}^{i=n} x_i - 1\right) \delta\left(\sum_{i=1}^{i=n} \vec{k}_i\right). \end{aligned} \quad (5)$$

Note that this distribution is diagonal in the space of all partons except the two partons involved in the collision. Here ψ is the parton wave function normalized to one in a usual way. An appropriate summation over color and Lorentz indices is implied. In the case of kinematics $1 \gg x_1 \geq x_2$ we expect only distributions without the spin flip to be important.

Let us stress that it follows from the above formulae that in the impact parameter space these GPDs have a probabilistic interpretation. In particular they are positively definite in the impact parameter space, cf. Eq. 11. Note that in the same way one can introduce the N-particle GPD, G_N , which can be probed in the production of N pairs of jets. In this case the first N arguments k_i in Eq. 5 are shifted by $\vec{\Delta}_i$ subject to the constraint $\sum_i \vec{\Delta}_i = 0$. So the cross section is proportional to

$$\begin{aligned} \sigma_{2N} &\propto \int \prod_{i=1}^{i=N} \frac{d\vec{\Delta}_i}{(2\pi)^2} D_a(\vec{\Delta}_1, \dots, \vec{\Delta}_N) \\ &\times D_b(\vec{\Delta}_1, \dots, \vec{\Delta}_N) \delta\left(\sum_{i=1}^{i=N} \vec{\Delta}_i\right). \end{aligned} \quad (6)$$

These GPDs can be easily rewritten in the form of the matrix elements of the operator product. For example:

$$\begin{aligned} D(\Delta) &= \langle N | \int d^4 x_1 d^4 x_2 d^4 x_3 \\ &\times G_{i+}^a(x_1) G_{j+}^b(x_2) G_{i+}^a(x_3) G_{j+}^b(x_4) \\ &\times \exp(ip_1^+(x_1 - x_3)^- + ip_2^+(x_2 - x_4)^- \\ &+ i\vec{\Delta}_t(\vec{x}_4 - \vec{x}_3)_t) | N \rangle, \end{aligned} \quad (7)$$

calculated at the virtualities p_1^2, p_2^2 at fixed $\vec{\Delta}$. Here we gave an example for the most relevant case of gluons without a flip in color and spin spaces. In general a number of distributions can be written, depending on different contractions of transverse Lorentz indices and color indices. The classification of the relevant distributions is the same as the classification of the quasipartonic operators in ref. [18]. Note that the presence of the transverse external parameter $\vec{\Delta}$ does not change the classification, since the corresponding new structures will be strongly suppressed at high energies. We wrote the operator expression in the light cone gauge. In an arbitrary gauge we shall need Wilson lines $W(C)$ connecting points with contracted color indices.

In the approximation of uncorrelated partons it follows from Eq. 5 that

$$D(x_1, x_2, p_1^2, p_2^2, \vec{\Delta}) = G(x_1, p_1^2, \vec{\Delta}) G(x_2, p_2^2, \vec{\Delta}), \quad (8)$$

where $G(x, \vec{\Delta})$ are conventional one-particle GPDs. These GPDs can be approximated as $G_N(x, Q^2, \vec{\Delta}) = G_N(x, Q^2) F_{2g}(\Delta)$, where $F_{2g}(\Delta)$ is the two-gluon form factor of the nucleon extracted from hard exclusive vector meson production (we suppress here the dependence of F_{2g} on x) [19] and $G_N(x, Q^2)$ conventional parton distribution of a nucleon. (Here Q^2 is the virtuality due to the radiation, cf. discussion after Eq. 2.) Thus :

$$\frac{1}{\pi R_{\text{int}}^2} = \int \frac{d^2 \Delta}{(2\pi)^2} F_{2g}^4(\Delta) = \frac{m_g^2}{28\pi}. \quad (9)$$

Here at the last step we used the dipole fit $F_{2g}(\Delta) = 1/(\Delta^2/m_g^2 + 1)^2$ to the two-gluon form factor ($m_g^2(x \sim 0.03, Q^2 = 3\text{GeV}^2) \approx 1.1\text{GeV}^2$). Using the transverse gluon radius of the nucleon we obtain

$$R_{\text{int}}^2 = 7/2r_g^2, \quad r_g^2/4 = dF_{2g}(t)/dt_{t=0}. \quad (10)$$

This result coincides with the one for the area πR_{int}^2 obtained earlier in [4] using the geometric picture in the impact parameter space. That derivation involved taking Fourier transform of the two-gluon form factor and calculating a rather complicated six-dimensional integral which could potentially lead to large numerical uncertainties. The form of Eq. 10 clearly indicates that numerical uncertainties are small.

It was emphasized in [4] that the experiments on four-jet production report a smaller value of πR_{int}^2 as compared to the one obtained above in the independent particle approximation (though the issue of how well the contribution of the $2 \rightarrow 4$ processes was subtracted still remains, cf. discussion in beginning of the paper). It is at least a factor of two smaller — that is a four-jet cross section is a factor of two larger — than Eq. 10 gives. (The GPDs for sea quarks appear to decrease with Δ somewhat faster, resulting in a smaller $1/\pi R_{\text{int}}^2$, see discussion in [17].)

It follows from Eq. 4 that the value of R_{int}^2 is determined by the range of integration over Δ . Hence the characteristic Δ in the integral measures the effective distance between the parton pairs (which in principle may differ for different flavor combinations). According to the above evaluation within the independent parton approximation the integral for $1/R_{\text{int}}^2$ is dominated by small $\Delta^2 \sim 0.1 m_g^2$. The contribution of large Δ is suppressed by the two-gluon form factor of a nucleon. This reasoning indicates the important role of inter-parton correlations. In other words, the integral over Δ is effectively cut off by a scale of the nonperturbative correlations. Such correlations naturally arise in nonperturbative QCD regime in a number of nucleon models, such as constituent quark model (gluon cloud around constituent quark) [4], or string model (gluon structure of string) [20]. The detailed analysis of the additional correlations due to the hard– soft interplay will be reported elsewhere [14].

Let us now show that results obtained in the paper lead to the geometric picture in the impact parameter space mentioned above [1–10].

The first step is to make transformation into coordinate space i.e., to make Fourier transform from variables k_i in Eq. 5 to coordinates ρ_i . Performing integration over k_i we obtain that transverse coordinates of partons in the amplitude and the amplitude conjugated are equal $\rho_i = \rho_f$. In the calculation we use the fact that upper limit of integration over k_t^2 is very large compared with the inverse hadron size. Next step is to perform integration over Δ which produces $\delta(\vec{\rho}_1 - \vec{\rho}_2 - \vec{\rho}_3 + \vec{\rho}_4) = \int d^2 B \delta(\vec{\rho}_1 - \vec{\rho}_3 - \vec{B}) \delta(\vec{\rho}_2 - \vec{\rho}_4 - \vec{B})$.

The delta functions express the fact that within the accuracy $1/p_t$ where p_t is the hard scale, the interactions of partons from different nucleons occur at the same point. \vec{B} is the relative impact parameter of two nucleons.

The expression for the cross section in the impact parameter space has the form which corresponds to the ge-

ometry of Fig.2

$$\begin{aligned} \sigma_4 &= \int d^2 B d^2 \rho_1 d^2 \rho_2 d^2 \rho_3 d^2 \rho_4 D(x_1, x_2, \vec{\rho}_1, \vec{\rho}_2) \\ &\times D(x_3, x_4, \vec{\rho}_3, \vec{\rho}_4) \delta(\vec{\rho}_1 - (\vec{B} + \vec{\rho}_3)) \delta(\vec{\rho}_2 - (\vec{B} + \vec{\rho}_4)) = \\ &= \int d^2 B d^2 \rho_1 d^2 \rho_2 D(x_1, x_2, \vec{\rho}_1, \vec{\rho}_2) \\ &\times D(x_3, x_4, -\vec{B} + \vec{\rho}_1, -\vec{B} + \vec{\rho}_2). \end{aligned} \quad (11)$$

Here the ${}_2$ GPD in the impact parameter space representation is given by

$$\begin{aligned} D(x_1, x_2, \vec{\rho}_1, \vec{\rho}_2) &= \\ &= \sum_{n=3}^{n=\infty} \int \prod_{i \geq 3}^{i=n} [dx_i d^2 \rho_i] \psi_n(x_1, \vec{\rho}_1, x_2, \vec{\rho}_2, \dots, x_i, \vec{\rho}_i, \dots) \\ &\times \psi_n^+(x_1, \vec{\rho}_1, x_2, \vec{\rho}_2, \dots, x_i, \vec{\rho}_i, \dots) \delta\left(\sum_{i=1}^{i=n} x_i \vec{\rho}_i\right). \end{aligned} \quad (12)$$

where the delta function expresses the center of mass constraint $\sum_{i=1}^{i=n} x_i \vec{\rho}_i = 0$. This is analogous to the case of single parton GPDs, see [21]. The functions $\psi(x_1, \vec{\rho}_1, x_2, \vec{\rho}_2, \dots)$ are just the Fourier transforms in the impact parameter space of the light cone wave functions and are given by

$$\begin{aligned} \psi_n(x_1, \vec{\rho}_1, x_2, \vec{\rho}_2, \dots) &= \int \prod_{i=1}^{i=n} \frac{d^2 k_i}{(2\pi)^2} \exp(i \sum_{i=1}^{i=n} \vec{k}_i \vec{\rho}_i) \\ &\times \psi_n(x_1, \vec{k}_1, x_2, \vec{k}_2, \dots) (2\pi)^2 \delta\left(\sum \vec{k}_i\right). \end{aligned} \quad (13)$$

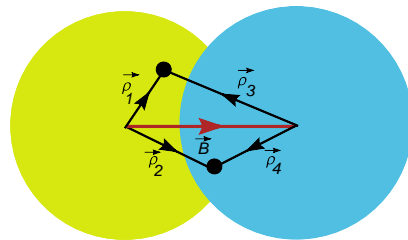


FIG. 2: Geometry of two hard collisions in impact parameter picture.

Thus the ${}_2$ GPD based description of the *four* \rightarrow *four* processes is equivalent to the representation for the cross section corresponding to the simple geometrical picture, but instead of a triple integral we now have an integral over one momentum Δ . The ${}_2$ GPD defined in Eq. 5 is useful for calculation of many different processes. At the same time the knowledge of the full double GPD is

necessary for complete description of events with a double jet trigger since the pedestal strongly depends on the impact parameter \vec{B} [4].

Let us stress that this picture is a natural generalization of the correspondence between momentum representation and geometric picture for a conventional case of two \rightarrow two collisions. Indeed in this case it is easy to see that the cross section in the momentum representation

$$\sigma_2 = \int f(x_1, p^2) f(x_2, p^2) \frac{d\sigma^h}{d\hat{t}} d\hat{t} \quad (14)$$

has a simple geometric representation

$$\sigma_2 = \int d^2\rho_1 d^2B f(x_1, \vec{\rho}_1, p^2) f(x_2, \vec{B} - \vec{\rho}_1, p^2) \frac{d\sigma^h}{d\hat{t}} d\hat{t}, \quad (15)$$

where $f(x, \vec{\rho}, p^2) = \psi^+(x, \vec{\rho}, p^2) \psi(x, \vec{\rho}, p^2)$ and $\psi(x, \vec{\rho}, p^2)$ is the Fourier transform of the light cone wave function defined above.

Let us now summarize our results. We have argued that there exists the kinematical domain where the four \rightarrow four hard parton collisions form the dominant mechanism of four-jet production. In this region we calculated the cross section, see Eqs. 2-4 and found that it can be expressed through new two parton GPDs (see Eq. 5), expressed through the light cone wave functions of the colliding hadrons. These ${}_2$ GPDs depend on a transverse vector $\vec{\Delta}$ that measures the transverse distance within the parton pairs. (Equivalent expressions for these GPDs can be easily given in terms of the operator products.) In the impact parameter space we derived the widely used intuitive geometric picture. We argued that the observed enhancement of a four-jet cross section indicates the presence of short range two parton correlations in the nucleon parton wave function, as determined by the range of integral over Δ . The contribution of perturbative correlations in the appropriate kinematic domain is suppressed. The detailed study of the interplay of the contribution of hard/soft correlations will be reported elsewhere [14].

It was argued recently [22] that the cross section can be expressed in terms of two parton distribution functions. Our analysis indicates that a more detailed treatment of the QCD evolution effects is necessary. We found that it is necessary to introduce the new 2-particle ${}_2$ GPDs which depend on additional parameter Δ . The parameter Δ expresses the fact that the difference of the transverse components of the parton momenta is not conserved and therefore different in $|in\rangle$ and $\langle out|$ states in the double hard collisions.

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