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Top-Higgs and Top-pion phenomenology in the Top Triangle Moose model

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We discuss the deconstructed version of a topcolor-assisted technicolor model wherein the mechanism of top quark mass generation is separated from the rest of electroweak symmetry breaking. The minimal deconstructed version of this scenario is a “triangle moose” model, where the top quark gets its mass from coupling to a top-Higgs field, while the gauge boson masses are generated from a Higgsless sector. The spectrum of the model includes scalar (top-Higgs) and pseudoscalar (top-pion) states. In this paper, we study the properties of these particles, discuss their production mechanisms and decay modes, and suggest how best to search for them at the LHC.

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I. INTRODUCTION

Higgsless models, as the name implies, break the electroweak symmetry without invoking a fundamental scalar particle. Inspired by the idea that one could maintain perturbative unitarity in extra-dimensional models through heavy vector resonance exchanges in lieu of a Higgs [1–3], Higgsless models were initially introduced in an extra-dimensional context as $SU(2) \times SU(2) \times U(1)$ gauge theories living in a slice of AdS_5 , with symmetry breaking codified in the boundary condition of the gauge fields [4–10]. It emerged that the low energy dynamics of these extra-dimensional models can be understood in terms of a collection of 4-D theories, using the principle of “deconstruction” [11, 12]. Essentially, this involves latticizing the extra dimension, associating a 4-D gauge group with each lattice point and connecting them to one another by means of nonlinear sigma models. The five dimensional gauge field is now spread in this theory as four dimensional gauge fields residing at each lattice point, and the fifth scalar component residing as the eaten pion in the sigma fields. The picture that emerges is called a “Moose” diagram [13]. The AdS/CFT correspondence suggests that these models can be understood to be dual to strongly coupled technicolor models. The key features of these models [14–22] are the following:

- Spin-1 resonances created by the techni-dynamics are described as massive gauge bosons, following the Hidden-Local-Symmetry setup originally used for QCD [23–27]. The mass of the resonances is roughly $\tilde{g}F$, where F is around the weak scale and \tilde{g} is a large coupling. Interactions of the resonances with other gauge bosons and fermions are calculated as a series in $1/\tilde{g} \ll 1$.
- Standard model (SM) fermions reside primarily on the exterior sites – the sites approximately corresponding to $SU(2)_w$ and $U(1)_Y$ gauge groups. Fermions become massive through mixing with massive, vector-like fermions located on the interior, ‘hidden’ sites.
- Precision electroweak parameters (S,T,U), perennially a thorn in the side of dynamical electroweak breaking models [28], are accommodated by delicately spreading the SM fermions between sites. By adjusting the fermion distribution across sites to match the gauge boson distribution, S, T, U can all be reduced to acceptable levels. This is identical to the solution used in extra-dimensional Higgsless models, where the spreading of a fermion among sites becomes a continuous distribution, or profile, in the extra dimension [5]. This adjustment is called “ideal delocalization” [17].

The most economical deconstructed Higgsless model constructed along these lines (a “three site” model) was presented in [29], and had, in addition to the SM spectrum, one heavy partner for each fermion and the W and Z bosons. Though providing an excellent ground for studying the low-energy properties of Higgsless models, the mass of the heavy Dirac partners of the SM fermions in this model was constrained to lie at ~ 2 TeV, because of the tension between obtaining the correct value for the top-quark mass and keeping $\Delta\rho$ within experimental bounds. To alleviate this constraint, an extension of the three site model was constructed [30], whose goal was to separate the top-quark mass generation from the rest of electroweak symmetry breaking (EWSB), thus relaxing the aforementioned constraint.

This idea of treating the top-quark mass as arising from a separate dynamics is not new - in fact, Topcolor-assisted technicolor models [31–36] employ precisely this idea. Topcolor-assisted technicolor is a scenario of dynamical electroweak symmetry breaking in which the strong dynamics is partitioned into two different sectors. One sector, the technicolor sector, is responsible for the bulk of electroweak symmetry breaking and is therefore characterized by a scale $F \sim v$, where $v = 246$ GeV is the EWSB scale. Consequently, technicolor dynamics is responsible for the majority of gauge boson masses and, more indirectly, light fermion masses. The second strong sector, the topcolor sector, only communicates directly with the top quark. Its sole purpose is to generate a large mass for the top quark. In generating a top quark mass, this second sector also breaks the electroweak symmetry. If the characteristic scale of the topcolor sector is low, $f \ll F$, it plays only a minor role in electroweak breaking, but can still generate a sufficiently large top quark mass given a strong enough top-topcolor coupling. At low-energies, the top-color dynamics is summarized by the existence of a new dynamical top-Higgs which couples preferentially to the top-quark. The introduction of the top-Higgs field serves to alleviate the tension between obtaining the correct top quark mass and keeping $\Delta\rho$ small that exists in Higgsless models by separating the top quark mass generation from the rest of electroweak symmetry breaking. An important consequence of this separation is that the model permits heavy Dirac partners for the fermions that are potentially light enough to be seen at the LHC. Thus, the combination of two symmetry-breaking mechanisms can achieve both dynamical electroweak breaking and a realistic top quark mass.

Because electroweak symmetry is effectively broken twice in this scenario, there are two sets of Goldstone bosons in the theory. One triplet of these Goldstones is eaten to become the longitudinal modes of the W^\pm/Z^0 , while the other triplet remains in the spectrum. This remaining triplet, typically called the top-pions, and a singlet partner, the top-Higgs, are the focus of this paper.

The top-pions and top-Higgs couple preferentially to the third generation of quarks, which makes them interesting for a number of reasons. First, the interactions of the top quark are the least constrained of all fermions, so new dynamics coupling preferentially to the top quark is not phenomenologically excluded. Second, the gluon fusion mechanism involves a top quark loop and is an efficient method for singly producing top-Higgses and neutral top-pions at the LHC. In fact, the strong top-quark–topcolor interaction, manifest in a top Yukawa of order $\sim \text{few}$, significantly enhances the coupling of top-Higgs and top-pions relative to a SM Higgs of equal mass. Such a large cross section leads to exciting LHC signals which may be discoverable in the initial low-energy, low-luminosity run.

Our goal in this paper is to lay the foundation for phenomenological studies of the top-pions and top-Higgs at the LHC. We begin in Sec. II by setting out the relevant details of the Top Triangle Moose model. In Sec. III we identify the physical top-pion states and summarize their couplings to other particles. Section IV contains the bulk of our phenomenological results. After identifying the existing experimental constraints on the top-pions and top-Higgs, we study their decay branching ratios, direct production cross sections in pp collisions, and production in decays of the heavy vector-like top quark partners or the heavy gauge bosons in the model. We identify cases where the LHC has the clear ability to discover the new states and others where good potential for discovery exists and further detailed study is warranted. We summarize our findings and discuss their implications in Sec. V.

II. THE MODEL

The Top Triangle Moose model [30] is shown in moose notation in Fig. 1. The circles represent global $SU(2)$ symmetry groups; the full $SU(2)$ at sites 0 and 1 are gauged with gauge couplings g and \tilde{g} , respectively, while the τ^3 generator of the global $SU(2)$ at site 2 is gauged with $U(1)$ gauge coupling g' . The lines represent spin-zero link fields which transform as a fundamental (anti-fundamental) representation of the group at the tail (head) of the link. Σ_{01} and Σ_{12} are nonlinear sigma model fields, while Φ (the top-Higgs doublet) is a linear sigma model field.

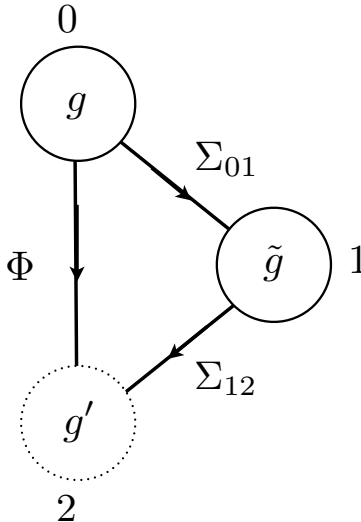


FIG. 1. The gauge structure of the model in Moose notation. g and g' are approximately the SM $SU(2)$ and hypercharge gauge couplings while \tilde{g} represents the ‘bulk’ gauge coupling. The left (right) handed light fermions are mostly localized at site 0 (2) while their heavy counterparts are mostly at site 1. The links connecting sites 0 and 1 and sites 1 and 2 are nonlinear sigma model fields while the one connecting sites 0 and 2 is a linear sigma field. Site 2 is dotted to indicate that only the τ_3 component is gauged.

The kinetic energy terms of the link fields corresponding to these charge assignments are:

$$\mathcal{L}_{gauge} = \frac{F^2}{4} \text{Tr}[(D_\mu \Sigma_{01})^\dagger D^\mu \Sigma_{01}] + \frac{F^2}{4} \text{Tr}[(D_\mu \Sigma_{12})^\dagger D^\mu \Sigma_{12}] + (D_\mu \Phi)^\dagger D^\mu \Phi, \quad (1)$$

where the covariant derivatives are:

$$\begin{aligned} D_\mu \Sigma_{01} &= \partial_\mu \Sigma_{01} + ig W_{0\mu} \Sigma_{01} - i\tilde{g} \Sigma_{01} W_{1\mu}, \\ D_\mu \Sigma_{12} &= \partial_\mu \Sigma_{12} + i\tilde{g} W_{1\mu} \Sigma_{12} - ig' \Sigma_{12} \tau^3 B_\mu, \\ D_\mu \Phi &= \partial_\mu \Phi + ig W_{0\mu} \Phi - \frac{ig'}{2} B_\mu \Phi. \end{aligned} \quad (2)$$

Here the gauge fields are represented by the matrices $W_{0\mu} = W_{0\mu}^a \tau^a$ and $W_{1\mu} = W_{1\mu}^a \tau^a$, where $\tau^a = \sigma^a/2$ are the generators of SU(2). The nonlinear sigma model fields Σ_{01} and Σ_{12} are 2×2 special unitary matrix fields. To mimic the symmetry breaking caused by underlying technicolor and topcolor dynamics, we assume all link fields develop vacuum expectation values (vevs):

$$\langle \Sigma_{01} \rangle = \langle \Sigma_{12} \rangle = \mathbf{1}_{2 \times 2}, \quad \langle \Phi \rangle = \begin{pmatrix} f/\sqrt{2} \\ 0 \end{pmatrix}. \quad (3)$$

In order to obtain the correct amplitude for muon decay, we parameterize the vevs in terms of a new parameter ω ,

$$F = \sqrt{2} v \cos \omega, \quad f = v \sin \omega, \quad (4)$$

where $v = 246$ GeV is the weak scale. As a consequence of the vacuum expectation values, the gauge symmetry is broken all the way down to electromagnetism and we are left with massive gauge bosons (analogous to techni-resonances), top-pions and a top-Higgs. To keep track of how the degrees of freedom are partitioned after we impose the symmetry breaking, we expand Σ_{01} , Σ_{12} and Φ around their vevs. The coset degrees of freedom in the bi-fundamental link fields Σ_{01} and Σ_{12} can be described by nonlinear sigma fields:

$$\Sigma_{01} = \exp \left(\frac{2i\pi_0^a \tau^a}{F} \right), \quad \Sigma_{12} = \exp \left(\frac{2i\pi_1^a \tau^a}{F} \right), \quad (5)$$

while the degrees of freedom in Φ fill out a linear representation,

$$\Phi = \begin{pmatrix} (f + H_t + i\pi_t^0)/\sqrt{2} \\ i\pi_t^- \end{pmatrix}. \quad (6)$$

The gauge-kinetic terms in Eq. (1) yield mass matrices for the charged and neutral gauge bosons. The photon remains massless and is given by the exact expression

$$A_\mu = \frac{e}{g} W_{0\mu}^3 + \frac{e}{\tilde{g}} W_{1\mu}^3 + \frac{e}{g'} B_\mu, \quad (7)$$

where e is the electromagnetic coupling. Normalizing the photon eigenvector, we get the relation between the coupling constants:

$$\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{\tilde{g}^2} + \frac{1}{g'^2}. \quad (8)$$

This invites us to conveniently parametrize the gauge couplings in terms of e by

$$g = \frac{e}{\sin \theta \cos \phi} = \frac{g_0}{\cos \phi}, \quad \tilde{g} = \frac{e}{\sin \theta \sin \phi} = \frac{g_0}{\sin \phi}, \quad g' = \frac{e}{\cos \theta}. \quad (9)$$

We will take $\tilde{g} \gg g$, which implies that $\sin \phi \equiv x$ is a small parameter. The result of the diagonalization of the gauge boson matrices perturbatively in x is summarized in Appendix A.

Counting the number of degrees of freedom, we see that there are six scalar degrees of freedom on the technicolor side (Σ_{01}, Σ_{12}) and four on the topcolor side (Φ). Six of these will be eaten to form the longitudinal components of the W^\pm , Z^0 , W'^\pm , and Z'^0 . This leaves one isospin triplet of scalars and the top-Higgs H_t as physical states in the spectrum. While the interactions in Eq. (1) are sufficient to give mass to the gauge bosons, the top-pions and top-Higgs remain massless at tree level. Quantum corrections will give the top-pions a mass, however this loop-level mass is far too small to be consistent with experimental constraints. To generate phenomenologically acceptable masses for the top-pions and top-Higgs, we add two additional interactions:

$$\mathcal{L}_M = -\lambda \text{Tr} \left(M^\dagger M - \frac{f^2}{2} \right)^2 - \kappa f^2 \text{Tr} \left| M - \frac{f}{\sqrt{2}} \Sigma_{01} \Sigma_{12} \right|^2, \quad (10)$$

where the first of these interactions arises from topcolor interactions, and the second from ETC-like interactions [37]. Here λ and κ are two new parameters, f is the same vacuum expectation value appearing in Eq. (4), and M is the Φ field expressed as a matrix, schematically given by $M = (\tilde{\Phi}, \Phi)$ with $\tilde{\Phi} = -i\sigma_2\Phi^*$:

$$M = \begin{pmatrix} i\pi_t^+ & (f + H_t + i\pi_t^0)/\sqrt{2} \\ (f + H_t - i\pi_t^0)/\sqrt{2} & i\pi_t^- \end{pmatrix}, \quad (11)$$

where $\pi_t^+ = (\pi_t^-)^*$. The first term in Eq. (10) depends only on the modulus of M , and therefore contributes only to the mass of the top-Higgs. The second term gives mass to both the top-Higgs and the physical (uneaten) combination of pion fields, as we will show shortly. Because these masses depend on two parameters, λ and κ , we can treat the mass of the top-Higgs and the common mass of the uneaten top-pions as two independent parameters. In addition to generating masses, the potential in Eq. (10) also induces interactions between the top-Higgs and top-pions which can be important.

Finally, we note that the mass terms for the light fermions arise from Yukawa couplings of the fermionic fields with the nonlinear sigma fields

$$\mathcal{L} = M_D \left[\epsilon_L \bar{\psi}_{L0} \Sigma_{01} \psi_{R1} + \bar{\psi}_{R1} \psi_{L1} + \bar{\psi}_{L1} \Sigma_{12} \begin{pmatrix} \epsilon_{uR} & 0 \\ 0 & \epsilon_{dR} \end{pmatrix} \begin{pmatrix} u_{R2} \\ d_{R2} \end{pmatrix} \right]. \quad (12)$$

We have denoted the Dirac mass that sets the scale of the heavy fermion masses as M_D . Here, ϵ_L is a parameter that describes the degree of delocalization of the left handed fermions and is flavor universal. All the flavor violation for the light fermions is encoded in the last term; the delocalization parameters for the right handed fermions, ϵ_{fR} , can be adjusted to realize the masses and mixings of the up and down type fermions. The mass of the top quark arises from similar terms with a unique left-handed delocalization parameter ϵ_{tL} and also from a unique Lagrangian term reflecting the coupling of the top-Higgs to the top quark:

$$\mathcal{L}_{top} = -\lambda_t \bar{\psi}_{L0} \Phi t_R + h.c. \quad (13)$$

Details of the fermion masses and mass eigenstates are given in Appendix A.

III. PHYSICAL TOP-PIONS AND THEIR COUPLINGS

The next step towards understanding top-pion phenomenology is to identify the combination of degrees of freedom which make up the physical (uneaten) top-pions. While the top-Higgs H_t remains a mass eigenstate, the pions π_0^a , π_1^a and π_t^a mix. We can identify the physical top-pions as the linear combination of states that cannot be gauged away. We do this by isolating the Goldstone boson states that participate in interactions of the form $V_\mu \partial^\mu \phi$ in the Lagrangian.

We start by expanding the nonlinear sigma fields to first order in π/F ,

$$\Sigma_{01} = 1 + \frac{2i\pi_0^a \tau^a}{F} + \mathcal{O}\left(\frac{\pi^2}{F^2}\right), \quad (14)$$

$$\Sigma_{12} = 1 + \frac{2i\pi_1^a \tau^a}{F} + \mathcal{O}\left(\frac{\pi^2}{F^2}\right). \quad (15)$$

Plugging this in Eq. (1)¹, we can read off the various interaction terms. The complete details are given in Appendix B. Here, we concentrate on the gauge-Goldstone mixing terms:

$$\mathcal{L}_{\text{mixing}} = \frac{g}{2} W_0^{a\mu} \partial_\mu [F\pi_0^a + f\pi_t^a] + \frac{\tilde{g}}{2} W_1^{a\mu} \partial_\mu [F\pi_1^a - F\pi_0^a] - \frac{g'}{2} B_2^\mu \partial_\mu [F\pi_1^3 + f\pi_t^3]. \quad (16)$$

Note that the pion combination in the third term can be written as a linear combination of those appearing in the first two terms:

$$F\pi_1^3 + f\pi_t^3 = [F\pi_0^3 + f\pi_t^3] + [F\pi_1^3 - F\pi_0^3]. \quad (17)$$

¹Here and in the Appendices, the subscripts appearing in the fields will refer to the “site” numbers and the superscripts will be reserved for $SU(2)$ indices.

Vertex	Strength
$A_\mu \Pi_t^+ \Pi_t^-$	$e (p_{\Pi^+} - p_{\Pi^-})_\mu$
$Z_\mu \Pi_t^+ \Pi_t^-$	$\frac{g_0}{\cos \theta} \left[\left(\frac{1}{2} - \sin^2 \theta \right) + \frac{x^2}{16} \sec^2 \theta (2 + \cos 2\theta \sec^2 \omega) \right] (p_{\Pi^-} - p_{\Pi^+})_\mu$
$Z'_\mu \Pi_t^+ \Pi_t^-$	$\frac{g_0}{2x} \left[\sin^2 \omega - \frac{x^2}{16} (7 + \cos 2\omega) \sec^2 \theta \right] (p_{\Pi^-} - p_{\Pi^+})_\mu$
$W_\mu^+ \Pi_t^0 \Pi_t^-$	$-\frac{g_0}{2} \left[1 + \frac{x^2}{8} (2 + \cos 2\omega) \sec^2 \omega \right] (p_{\Pi^0} - p_{\Pi^-})_\mu$
$W_\mu'^+ \Pi_t^0 \Pi_t^-$	$-\frac{g_0}{2x} \left[\sin^2 \omega - \frac{x^2}{16} (7 + \cos 2\omega) \right] (p_{\Pi^0} - p_{\Pi^-})_\mu$

TABLE I. Couplings of two top-pions to a vector boson. These have been calculated to $\mathcal{O}(x^2)$. Here p_Π is the outgoing momentum of particle Π .

Vertex	Strength
$\Pi_t^- A_\mu W_\nu^+$	0
$\Pi_t^- A_\mu W_\nu'^+$	0
$\Pi_t^- Z_\mu W_\nu^+$	$-\frac{ie^2 x^2}{16} v \sec^3 \theta \tan \omega$
$\Pi_t^- Z_\mu W_\nu'^+$	$-\frac{ig_0^2}{4x} v \sec \theta \sin 2\omega \left[1 + \frac{x^2}{16} (3 + 5 \cos 2\theta) \sec^2 \theta \right]$
$\Pi_t^- Z'_\mu W_\nu^+$	$\frac{ig_0^2}{4x} v \sin 2\omega \left[1 + \frac{x^2}{16} (5 + 3 \cos 2\theta) \sec^2 \theta \right]$
$\Pi_t^- Z'_\mu W_\nu'^+$	$-\frac{ig_0^2}{8} v \tan^2 \theta \left[\sin 2\omega + \frac{x^2}{32} (10 - 2 \cos 2\theta + 3 \cos[2(\theta - \omega)] + 2 \cos 2\omega + 3 \cos[2(\theta + \omega)]) \sec^2 \theta \tan \omega \right]$
$\Pi_t^0 W_\mu^+ W_\nu'^-$	$\frac{ig_0^2}{4x} v \sin 2\omega \left(1 + \frac{x^2}{2} \right)$

TABLE II. Couplings of a top-pion to a pair of gauge bosons. The corresponding Feynman rule is obtained by inserting a $g_{\mu\nu}$ in each coupling. These have been calculated to $\mathcal{O}(x^2)$.

The two eaten triplets of pions span the linear combinations that appear in the first two terms of Eq. (16), leaving the third linear combination as the remaining physical top-pions, which we will denote Π_t^a :

$$\Pi_t^a = -\sin \omega \left(\frac{\pi_0^a + \pi_1^a}{\sqrt{2}} \right) + \cos \omega \pi_t^a, \quad (18)$$

where we have normalized the state properly using the definitions of F and f in Eq. (4).

The physical top-pions can also be identified by expanding the top-Higgs potential given in Eq. (10) and collecting the mass terms. The masses of H_t and Π_t^a are given by,

$$M_H^2 = 2v^2(\kappa + 4\lambda) \sin^2 \omega, \quad M_{\Pi_t}^2 = 2v^2 \kappa \tan^2 \omega, \quad (19)$$

while the other two linear combinations of pions are massless, as true Goldstone bosons should be. Equation (10) also contains trilinear couplings between H_t and two top-pions; we find

$$\begin{aligned} \lambda_{H_t \Pi_t^0 \Pi_t^0} &= \lambda_{H_t \Pi_t^+ \Pi_t^-} = 2v \sin \omega (\kappa \sin^2 \omega \tan^2 \omega + 4\lambda \cos^2 \omega) \\ &= \frac{1}{2v \sin \omega} [(M_H^2 - 2M_{\Pi_t}^2) \cos 2\omega + M_H^2]. \end{aligned} \quad (20)$$

These couplings are important for top-Higgs decays when $M_H > 2M_{\Pi_t}$.

Having worked out the physical top-pion combination, all that remains is to express the interactions in the Lagrangian in terms of mass eigenstates. The top-pion combination is given above, while the gauge boson and fermion mass eigenstates are given in Ref. [30] and are summarized in Appendix A. This conversion is straight-forward, but tedious, so we just summarize the results for the three-point couplings in Tables I–III. We write the couplings in terms of x , $\sin \theta$, and g_0 , with the latter two defined as in Eq. (9). The results in this section are given as an expansion in powers of x and include terms up to order x^2 .

Notice in particular that the couplings of the heavy gauge bosons Z' and W'^\pm to two top-pions are proportional to the large gauge coupling $\tilde{g} = g_0/x$ associated with site 1. The leading term in these couplings is in fact $\tilde{g} \sin^2 \omega/2$, with the two $\sin \omega$ factors reflecting the overlap of the Π_t^a wavefunction with the combination of nonlinear sigma fields $(\pi_0^a + \pi_1^a)/\sqrt{2}$. The couplings of the top-Higgs and top pions to third generation fermions (and their heavy partners)

Vertex	Strength
$H_t W_\mu^+ W_\nu^-$	$\frac{g_0^2}{2} v \sin \omega \left(1 + \frac{3x^2}{4}\right)$
$H_t W_\mu'^+ W_\nu^-$	$-\frac{g_0^2 x}{4} v \sin \omega$
$H_t W_\mu'^+ W_\nu'^-$	$\frac{g_0^2 x^2}{8} v \sin \omega$
$H_t Z_\mu Z_\nu$	$\frac{g_0^2}{4 \cos^2 \theta} v \sin \omega \left[1 + \frac{x^2}{4} (1 + 2 \cos 2\theta) \sec^2 \theta\right]$
$H_t Z_\mu' Z_\nu$	$-\frac{g_0^2 x}{4 \cos^2 \theta} v \sec \theta \cos 2\theta \sin \omega$
$H_t Z_\mu' Z_\nu'$	$\frac{g_0^2 x^2}{16 \cos^2 \theta} v \sec^2 \theta \cos^2 2\theta \sin \omega$
$H_t \Pi_t^- W_\mu^+$	$\frac{g_0}{2} \cos \omega \left(1 + \frac{3x^2}{8}\right) (p_H - p_{\Pi^-})_\mu$
$H_t \Pi_t^- W_\mu'^+$	$-\frac{g_0 x}{4} \cos \omega (p_H - p_{\Pi^-})_\mu$
$H_t \Pi_t^0 Z_\mu$	$-\frac{g_0}{2 \cos \theta} \cos \omega \left[1 + \frac{x^2}{8} (1 + 2 \cos 2\theta) \sec^2 \theta\right] (p_H - p_{\Pi^0})_\mu$
$H_t \Pi_t^0 Z_\mu'$	$\frac{g_0 x}{4 \cos \theta} \cos \omega \sec \theta \cos 2\theta (p_H - p_{\Pi^0})_\mu$

TABLE III. Three-point couplings of the top-Higgs, again calculated to $\mathcal{O}(x^2)$. The Feynman rules for the couplings involving two gauge bosons are obtained by multiplying the coupling strength given here by $ig_{\mu\nu}$.

Vertex	Strength
$H_t t_L \bar{t}_R + h.c.$	$\frac{\lambda_t}{\sqrt{2}} \left[-1 + \frac{(1+a^2)(x^2+2\epsilon_{tR}^2)+4\sqrt{2}ax\epsilon_{tR}}{4(a^2-1)^2} \right]$
$H_t T_L \bar{T}_R + h.c.$	$-\frac{\lambda_t}{\sqrt{2}} \left[\frac{a(x^2+2\epsilon_{tR}^2)+\sqrt{2}(1+a^2)x\epsilon_{tR}}{2(a^2-1)^2} \right]$
$H_t t_L \bar{T}_R + h.c.$	$\lambda_t \left[\frac{ax+\sqrt{2}\epsilon_{tR}}{2(a^2-1)} \right]$
$H_t t_R \bar{T}_L + h.c.$	$\lambda_t \left[\frac{x+\sqrt{2}a\epsilon_{tR}}{2(a^2-1)} \right]$
$\Pi_t^0 \bar{t}_L t_R - h.c.$	$\frac{i\lambda_t}{\sqrt{2}} \left[-\cos \omega + \frac{(a^2+\cos 2\omega)\sec \omega (x^2+2\epsilon_{tR}^2)+\frac{2\sqrt{2}}{a}x\epsilon_{tR}[2a^2\cos \omega+(a^2-1)\sin \omega \tan \omega]}{4\sqrt{2}(a^2-1)^2} \right]$
$\Pi_t^0 \bar{T}_L T_R - h.c.$	$\frac{i\lambda_t}{4\sqrt{2}} \left[\frac{2\sqrt{2}x\epsilon_{tR}(a^2+\cos 2\omega)+(2a^2\cos \omega+(a^2-1)\sin \omega \tan \omega)(x^2+2\epsilon_{tR}^2)}{(a^2-1)^2} \right]$
$\Pi_t^0 \bar{t}_L T_R - h.c.$	$\frac{i\lambda_t}{(a^2-1)} \left[\frac{x\sec \omega (-1+3a^2+(1+a^2)\cos 2\omega)}{8a} + \frac{\epsilon_{tR}\cos \omega}{\sqrt{2}} \right]$
$\Pi_t^0 \bar{T}_L t_R - h.c.$	$\frac{i\lambda_t}{(a^2-1)} \left[\frac{x\cos \omega}{2} + \frac{\epsilon_{tR}\sec \omega (-1+3a^2+(1+a^2)\cos 2\omega)}{4\sqrt{2}a} \right]$
$\Pi_t^- t_R \bar{b}_L - h.c.$	$i\lambda_t \left[\cos \omega - \frac{x^2(a^4+(a^4-2a^2+2)\cos 2\omega)\cos \omega}{8(a^2-1)^2} - \frac{x\epsilon_{tR}\sec \omega (-2+5a^2-a^4+(a^4-a^2+2)\cos 2\omega)\cos \omega}{4\sqrt{2}a(a^2-1)^2} - \frac{\epsilon_{tR}^2\cos \omega}{2(a^2-1)^2} \right]$
$\Pi_t^- T_R \bar{B}_L - h.c.$	$\frac{i\lambda_t\sec \omega}{(a^2-1)} \left[-\frac{x^2(-1+3a^2+(1+a^2)\cos 2\omega)}{8a} - \frac{x\epsilon_{tR}(1+3\cos 2\omega)}{4\sqrt{2}a} + \frac{\sin^2 \omega \epsilon_{tR}^2}{2a} \right]$
$\Pi_t^- T_R \bar{b}_L - h.c.$	$-\frac{i\lambda_t}{(a^2-1)} \left[\frac{x(-1+3a^2+(1+a^2)\cos 2\omega)\sec \omega}{4\sqrt{2}a} + \epsilon_{tR}\cos \omega \right]$
$\Pi_t^- t_R \bar{B}_L - h.c.$	$i\lambda_t \left[\frac{x\cos \omega}{\sqrt{2}} - \frac{\epsilon_{tR}\sin \omega \tan \omega}{2a} \right]$

TABLE IV. The couplings of the top-Higgs and top pions to third generation fermions and their heavy partners, calculated to $\mathcal{O}(x^2, \epsilon_{tR}^2)$.

can be likewise computed, by plugging in the mass eigenstates into the top quark mass term, Eq. (13), with Φ is given by Eq. (6)). The results are shown in Table IV, written in terms of the parameter $a = v \sin \omega / \sqrt{2} M_D$.

We have also worked out the four-point interactions. While these are less important phenomenologically, we list the mass-basis couplings in Appendix C for completeness.

IV. TOP-HIGGS AND TOP-PION PHENOMENOLOGY

We are now prepared to investigate the phenomenology of the new states related to the top quark: the top-Higgs (H_t), the top-pions (Π_t), and the heavy vector fermion partner of the top quark (T). First, we will show how existing Tevatron data can be applied to place limits on the top triangle moose model. Essentially, rescaling to take altered coupling values into account allows limits derived for other models to be transformed into limits on our model's top-pions and top-Higgs. Next, we study top-Higgs and top-pion production at LHC. As indicated in Figure (2) below, the new scalars can be produced either directly, through gluon fusion via a top loop, or indirectly, via decays of the heavy T quarks.

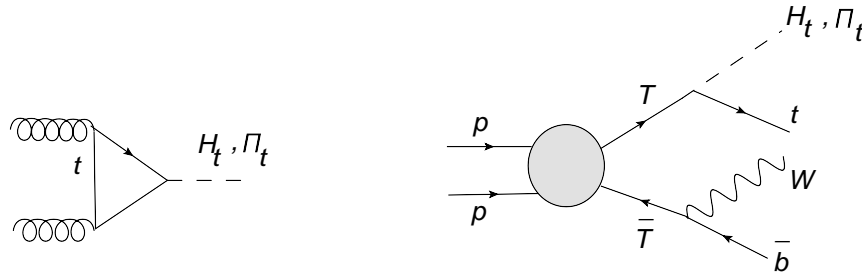


FIG. 2. Feynman diagrams for the single production of the top-Higgs or top pion. Direct production proceeds via a top loop (left). Indirect production occurs as a decay product of heavy T quarks (right), where the pair production of the T quarks proceeds via gluon fusion and quark annihilation.

The multiple production modes will make it possible to confirm that the scalar one has discovered is, in fact, the H_t of this model, rather than some other scalar state. We examine the branching ratios of the H_t , Π_t , and T , in order to identify production channels at LHC that are likely to lead to discovery of these new particles. Then, we discuss how already-planned searches at LHC can be repurposed or extended to yield information about the individual states and the relationships between them.

A. Current constraints on parameter values

Before starting our phenomenological analysis, let us briefly recall some of the limits on the different parameters of our model. First, within the gauge sector there is the mass of the W' boson, $M_{W'}$, and the ratio of gauge couplings x . Second, there are parameters related to fermion masses: the quantities ϵ_L , ϵ_{fR} , ϵ_{tL} , ϵ_{tR} , and M_D . In addition, from the top-Higgs sector, we have λ_t , $\sin\omega$ and the masses of the top-pion and the top-Higgs, M_{Π_t} and M_{H_t} .

As discussed in [29], the value of the W' mass is constrained to lie above 380 GeV by the LEP II measurement of the triple gauge boson vertex and to lie below 1.2 TeV by the need to maintain perturbative unitarity in $W_L W_L$ scattering². We will use the illustrative $M_{W'} = 500$ GeV in our calculations (except where noted otherwise) both for definiteness and because the value of the parameter ϵ_L is then derivable from $M_{W'}$ (and x) via ideal delocalization, as shown below.

In principle, the values of the various ϵ_{fR} are proportional to the masses of the light fermions; since we will be working in the limit $m_f \rightarrow 0$ for fermions other than the top quark, we will set $\epsilon_{fR} = 0$. Similarly, since the top quark mass depends very little on ϵ_{tR} we will set $\epsilon_{tR} = 0$ as well for simplicity. In this limit M_D corresponds to the (degenerate) masses of the heavy fermionic partners of the light ordinary fermions, and is closely related (as shown in Appendix A) to the mass of the heavy partner of the top quark. We will set M_D to the illustrative value $M_D = 400$ GeV in calculations not depending strongly on the precise value, and will otherwise show how results vary with M_D .

Within the top-pion sector we will set M_{H_t} and M_{Π_t} to the illustrative values $M_{H_t} = 250$ GeV and $M_{\Pi_t} = 200$ GeV when the precise value is not critical and will otherwise show how results vary with the values of these masses. Likewise, we will allow $\sin\omega$ to vary to show how various physical quantities depend on it; when the dependence is not critical, we will tend to use the illustrative value $\sin\omega = 0.5$.

²Strictly speaking, the upper bound on $M_{W'}$ in this model should be slightly different than the one in the three site model, because the formula for $M_{W'}$ is slightly different. However, to the extent that $f \ll F$ ($\sin\omega \ll 1$), this may be neglected.

The remaining parameters, x and the top Yukawa coupling λ_t , are now derived from the quantities above via:

$$\begin{aligned} x &= \sqrt{2} \epsilon_L = \frac{2 \cos \omega M_W}{M_{W'}} \\ \lambda_t &= \frac{\sqrt{2} m_t}{v \sin \omega} \left[\frac{M_D^2 (\epsilon_L^2 + 1) - m_t^2}{M_D^2 - m_t^2} \right], \end{aligned} \quad (21)$$

where m_t is the physical top quark mass and in the last expression we have set $\epsilon_{tR} = 0$. The relationship between ϵ_L and x is imposed by ideal delocalization; similarly, as discussed below in subsection IV.A.3, flavor constraints tend to force $\epsilon_{tL} \simeq \epsilon_L$, so the value of this last parameter is set as well.

1. Tevatron limits from Higgs searches

The Tevatron experiments analyzed the channel $gg \rightarrow H \rightarrow WW$ and set upper bounds on the cross-section as a function of M_H in Ref. [38]. We can adapt this data to our model by appropriately rescaling the couplings involved in the following way: Because of the $\sin \omega$ factor in the denominator of Eq. (21) for λ_t above, couplings of H_t to top quarks are enhanced compared to those of the SM Higgs, particularly for small $\sin \omega$. This leads to an enhanced cross section for H_t production in gluon fusion, scaling proportional to $(\lambda_t/\lambda_t^{\text{SM}})^2$. Simultaneously, due to the absence of the decay mode $H_t \rightarrow b\bar{b}$ at low H_t masses, the branching ratio for $H_t \rightarrow WW$ is larger than for the SM Higgs for masses below about 160 GeV. These two features lead to an enhancement of the predicted rate for $gg \rightarrow H_t \rightarrow WW$ compared to the corresponding SM process, which is already constrained by Tevatron data.

We can now translate the Tevatron bounds on the cross-section [38] into constraints on the $\sin \omega$ - M_{H_t} parameter space as follows. We compute the cross section for $gg \rightarrow H_t \rightarrow WW$ according to the approximation

$$\begin{aligned} \sigma(gg \rightarrow H_t \rightarrow WW) &= \sigma^{\text{SM}}(gg \rightarrow H) \frac{\Gamma(H_t \rightarrow gg)}{\Gamma^{\text{SM}}(H \rightarrow gg)} \text{BR}(H_t \rightarrow WW) \\ &= \sigma^{\text{SM}}(gg \rightarrow H) \frac{\text{BR}(H_t \rightarrow gg) \Gamma(H_t)}{\text{BR}(H_{\text{SM}} \rightarrow gg) \Gamma(H_{\text{SM}})} \text{BR}(H_t \rightarrow WW), \end{aligned} \quad (22)$$

where $\Gamma^{\text{SM}}(H \rightarrow gg)$ is the SM partial width of H to gluons computed using HDECAY [39], $\Gamma(H_t \rightarrow gg)$ and $\text{BR}(H_t \rightarrow WW)$ are the partial width of H_t to gluons and the branching ratio of H_t to WW , respectively, computed using our modified version of HDECAY, and $\sigma^{\text{SM}}(gg \rightarrow H)$ is the SM Higgs gluon fusion cross section, which we take from Table 2 of Ref. [40] for $M_H \leq 200$ GeV and compute using the public code RGHIGGS [41–43] for $M_H > 200$ GeV. For each value of $\sin \omega$, a specific range of masses for the top-Higgs is excluded by the Tevatron data. For example, for the illustrative value $\sin \omega = 0.5$, the data implies that the mass range $140 \text{ GeV} < M_{H_t} < 195 \text{ GeV}$ is excluded. We present this in Fig. 3 - as can clearly be seen, the scaling of the Yukawa enhances the production cross-section in our model. As we move to larger M_{H_t} values, $\sigma \cdot \text{BR}$ declines toward zero as the parton distribution function of the gluon falls rapidly.

Turning to the top-pion, we find that there are more important constraints on the Π_t masses than those derived from the Tevatron Higgs search limit. These constraints come from limits on rare top decays, from $Zb\bar{b}$ couplings, and from B-physics. We discuss these in turn below.

2. Lower bound on the top-pion mass

If the charged top-pion Π_t^+ is lighter than the top quark, it can appear in top decays, $t \rightarrow \Pi_t^+ b$. The Tevatron experiments have searched for this process in the context of two-Higgs-doublet models and set upper bounds of about 10–20% on the branching fraction of $t \rightarrow H^+ b$, with H^+ decaying to $\tau\nu$ or $c\bar{s}$ [44, 45] - we can use this to set a lower bound on the top-pion mass. In our model, below the $t\bar{b}$ threshold Π_t^+ decays via its mixing with π_0^+ to lighter SM fermions, with couplings controlled by the fermions' SM Yukawa couplings. The branching fraction of Π_t^+ to $\tau\nu$ is therefore about 70%, with the remainder of decays to $c\bar{s}$. The Tevatron studies can then be applied directly to the top-pion. The relevant limit is $\text{BR}(t \rightarrow \Pi_t^+ b) \lesssim 0.2$ based on D0 data [45].

In our model, the branching fraction of $t \rightarrow \Pi_t^+ b$ is controlled by the top-pion mass, the pion mixing angle ω , and the coupling λ_t :

$$\Gamma(t \rightarrow \Pi_t^+ b) = \frac{G_F m_t}{8\sqrt{2}\pi} [m_t^2 R^2 + \mathcal{O}(m_b^2)] \left[1 - \frac{M_{\Pi_t^+}^2}{m_t^2} \right]^2, \quad (23)$$

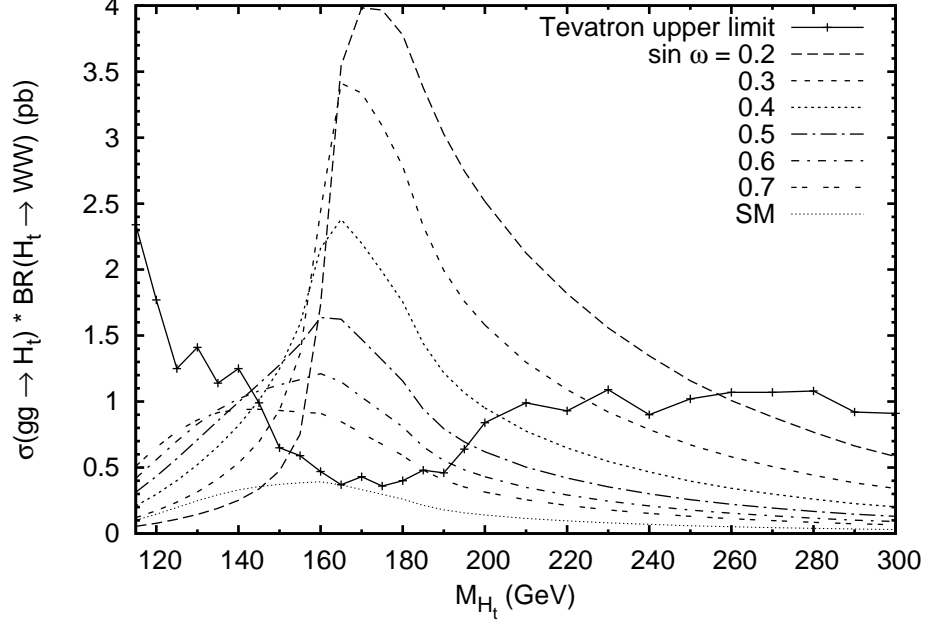


FIG. 3. Constraints on $\sigma \cdot BR$ for top-Higgs production at the Tevatron as a function of M_{H_t} and $\sin \omega$. One can clearly see how $\sin \omega < 1$ enhances the production cross-section. For a given value of $\sin \omega$, the range of masses for which the corresponding curve lies above the “Tevatron upper limit” curve is excluded. Each curve for fixed $\sin \omega$ falls off at small M_{H_t} due to a decline in $BR(H_t \rightarrow WW)$ and drops off at high M_{H_t} because the falling gluon pdf reduces the production cross-section $\sigma(gg \rightarrow H_t)$.

with

$$R \equiv \cos \omega \frac{\lambda_t}{\lambda_t^{\text{SM}}} = \cot \omega \left[\frac{M_D^2 (\epsilon_L^2 + 1) - m_t^2}{M_D^2 - m_t^2} \right]. \quad (24)$$

To evaluate this expression numerically, we choose the illustrative values: $\sin \omega = 0.5$, $M_D = 400$ GeV. Plugging in these values in Eq. (23) leads to the constraint $M_{\Pi_t^+} \gtrsim 150$ GeV from top quark decay³. Because Π_t^0 and Π_t^+ are degenerate, this sets a lower bound on both particles’ masses. Having established that M_{Π_t} cannot be much lighter than m_t , we will assume in the rest of the paper that $M_{\Pi_t} > m_t$, so that decays of $\Pi_t^+ \rightarrow t\bar{b}$ dominate⁴.

3. R_b and non-ideal delocalization of the left-handed top quark

Next, we consider how data on the $Zb\bar{b}$ coupling constrains the allowed values of $\sin \omega$ and M_{H_t} . At tree level, the Z coupling to left-handed bottom quarks is generically modified due to the profile of the b_L wavefunction at the three sites, yielding [30],

$$g_L^{Zbb} = -\frac{e}{s_W c_W} \left[\left(1 + \frac{x^2}{4} - \frac{\epsilon_{tL}^2}{2} \right) T_3 - Q s_W^2 \right]. \quad (25)$$

³This bound gets stronger as $\sin \omega$ becomes smaller.

⁴LHC experiments should be able to reduce the upper limit on $BR(t \rightarrow \Pi_t^+ b)$ to about 10^{-2} [46, 47], which would push the lower bound on the top-pion mass above 170 GeV for the parameter point considered here. However, the studies of the LHC reach have been done only for charged Higgs masses below 150 GeV; for higher masses, off-shell decays to $t^* b$ should also be considered.

The tree-level shift in R_b can be eliminated by imposing *ideal delocalization* on the left-handed fermions [30], which means setting:

$$\epsilon_{tL}^2 = (\epsilon_{tL}^{ideal})^2 \equiv \frac{x^2}{2}. \quad (26)$$

This has the additional benefit of decoupling the SM fermions from the W' , Z' gauge bosons, eliminating a potentially dangerous source of 4-fermion operators.

At one loop, R_b receives additional contributions which we parameterize as δg_L according to

$$g_L^{Zbb} = -\frac{e}{s_W c_W} \left[\left(1 + \frac{(\epsilon_{tL}^{ideal})^2}{2} - \frac{\epsilon_{tL}^2}{2} \right) T_3 + \delta g_L - Q s_W^2 \right]. \quad (27)$$

The one-loop corrections come from:

- Loops involving W , W' , SM fermions, and/or heavy vector-like fermions - these were computed for the three-site model [29] in Ref. [48].
- Loops involving the charged top-pion and at least one vector-like heavy fermion. Note that the couplings of Π_t^- to one (two) vector-like heavy fermions are suppressed by one (two) powers of x or ϵ_{tR} .
- Loops involving the charged top-pion and SM fermions, including contributions from the Goldstone boson eaten by the W in the Top Triangle Moose model, which contains an admixture of the original top-Higgs doublet. These contributions were studied in Ref. [49] for a generic topcolor model based on the calculation of the contribution to R_b in the two-Higgs-doublet model done in Ref. [50].

In the Top Triangle Moose model that we consider in this paper, most of the top quark mass comes from the topcolor mechanism and the contribution from ϵ_{tR} is small, no more than a few GeV. Therefore the contributions to δg_L given by the first two sources will be negligible, and the dominant correction comes from the charged top-pion loops. These give a λ_t^2 -enhanced correction to R_b given by

$$\delta g_L^{\Pi_t^-} = \frac{m_t^2}{16\pi^2 v^2} \cot^2 \omega \left[\frac{R}{R-1} - \frac{R \ln R}{(R-1)^2} \right], \quad (28)$$

where $R \equiv m_t^2/M_{\Pi_t^-}^2$; note that M_Z and m_b have been neglected in the loop calculation relative to m_t and $M_{\Pi_t^-}$.

We now consider the size of the dominant new-physics correction $\delta g_L^{\Pi_t^-}$ and compare it to the experimental constraints on R_b . We can express the new physics contribution to R_b in terms of δg_L^{new} according to [51],

$$\delta R_b = 2R_b(1 - R_b) \frac{g_L}{g_L^2 + g_R^2} \delta g_L^{\text{new}}, \quad (29)$$

where g_L and g_R are the SM Zbb couplings at leading order,

$$g_L = -\frac{1}{2} + \frac{s_W^2}{3}, \quad g_R = \frac{s_W^2}{3}, \quad \text{with } s_W^2 \simeq 0.23. \quad (30)$$

To leading order we can insert the SM prediction for R_b [52],

$$R_b^{\text{SM}} = 0.21584 \pm 0.00006, \quad (31)$$

in the right-hand side of Eq. (29). This yields the convenient numerical expression,

$$\delta R_b \simeq -0.774 \delta g_L^{\text{new}}. \quad (32)$$

The current experimental value of R_b is [52],

$$R_b^{\text{expt}} = 0.21629 \pm 0.00066. \quad (33)$$

Subtracting this from the SM prediction Eq. (31) gives us a value for the left-hand side of Eq. (32), yielding a constraint on the new physics contribution,

$$\delta g_L^{\text{new}} = (-5.8 \pm 8.6) \times 10^{-4}. \quad (34)$$

This, in turn, implies a 2σ (3σ) upper bound on δg_L^{new} of 11.4×10^{-4} (20.0×10^{-4}).

Now let us see what we can deduce about constraints on the parameter space of our model. Let us first consider ideal delocalization (i.e., no tree-level contribution to R_b from the distribution of the light fermion wavefunction among the sites). The coefficient in Eq. (28) is numerically,

$$\frac{m_t^2}{16\pi^2 v^2} = 32 \times 10^{-4}. \quad (35)$$

We take $\sin \omega = 0.5$, which yields $\cot^2 \omega = 3$. When $M_{\Pi_t} = m_t$ (i.e., $R = 1$), the function of R in square brackets in Eq. (28) is equal to $1/2$. At this parameter point we thus have,

$$\delta g_L^{\Pi_t^-} = 48 \times 10^{-4} \quad (\cot^2 \omega = 3, \quad M_{\Pi_t} = m_t), \quad (36)$$

which is forbidden. The function of R in square brackets in Eq. (28) falls with increasing M_{Π_t} . This allows us to put a lower bound on M_{Π_t} assuming ideal fermion delocalization and taking $\sin \omega = 0.5$:

$$M_{\Pi_t} \gtrsim 760 \text{ (480) GeV} \quad \text{at } 2\sigma \text{ (3}\sigma\text{)}. \quad (37)$$

A lighter top-pion can be allowed if we shift the left-handed third generation quarks away from ideal delocalization. A positive one-loop δg_L^{new} can be compensated by choosing a smaller ϵ_{tL} . At our parameter point we have chosen $M_{W'} = 500$ GeV, which (with $\sin \omega = 0.5$) yields,

$$x = \frac{2 \cos \omega M_W}{M_{W'}} \simeq 0.28, \quad \text{or} \quad (\epsilon_{tL}^{\text{ideal}})^2 \simeq 0.039. \quad (38)$$

Returning to Eq. (27), the combined tree-level and one-loop new physics contribution can be eliminated by choosing ϵ_{tL} to satisfy

$$\frac{1}{4} (\epsilon_{tL}^2 - (\epsilon_{tL}^{\text{ideal}})^2) + \delta g_L^{\Pi_t^-} = 0. \quad (39)$$

More generally, if we define

$$\Delta \epsilon_{tL}^2 = \epsilon_{tL}^2 - (\epsilon_{tL}^{\text{ideal}})^2 \quad (40)$$

then we can deduce from Eq. (34) that the value of ϵ_{tL} must satisfy

$$\frac{1}{4} \Delta \epsilon_{tL}^2 + \delta g_L^{\Pi_t^-} < 11.4 \times 10^{-4} \quad (20.0 \times 10^{-4}) \quad (41)$$

in order for the predicted value of R_b to agree with experiment at the 2σ (3σ) level.

Figure 4 shows a contour plot of the fractional deviation $|\Delta \epsilon_{tL}^2 / (\epsilon_{tL}^{\text{ideal}})^2|$ from ideal fermion delocalization required in order for top quark delocalization to compensate for top-pion corrections to R_b (meaning agreement at the 90% CL level). Note that for a fractional deviation of order 1, essentially the entire $\sin \omega$ vs M_{Π_t} plane is allowed. The illustrative value $M_{W'} = 500$ GeV was used in making this plot; since $\epsilon_{tL}^{\text{ideal}} \propto M_{W'}^{-1}$, for heavier W' bosons the contours would retain their shape and label but correspond to a larger value of ϵ_{tL} .

Finally, one may worry that changing the value of ϵ_{tL} from its ideal value might cause problems with flavor changing neutral current constraints. We demonstrate in Appendix D that, in the case of “next-to-minimal” flavor violation [53], these do not rule out compensating for the deviation in R_b resulting from top-pion exchange by modifying the delocalization of the third-generation quarks.

B. Top Higgs production and decay

Having derived the relevant interactions between matter and the top-Higgs/top-pions and understood current constraints on the Top-Triangle moose parameter space, we are ready to move on to phenomenology. In this section and the following, we present the dominant production and decay rates for the top-Higgs and top-pions respectively in a viable region of parameter space. For both the H_t and Π_t we consider both direct production $pp \rightarrow H_t, \Pi_t$ and indirect production – top-Higgses/top-pions which arise from the decays of T quarks.

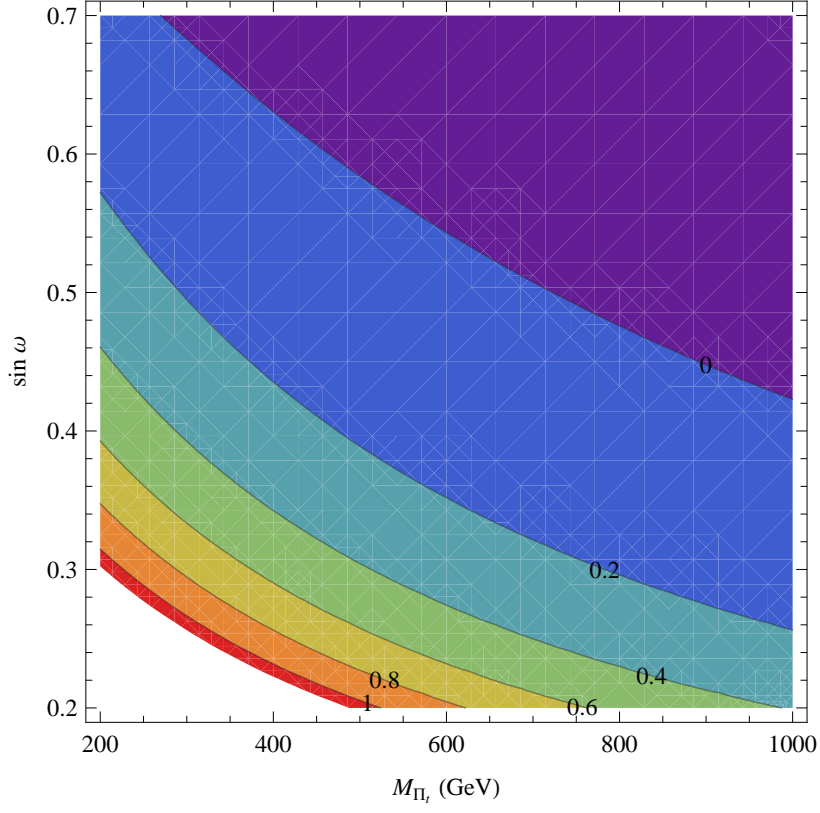


FIG. 4. A contour plot of the deviation $|\Delta\epsilon_{tL}^2/(\epsilon_{tL}^{ideal})^2|$ required to compensate for the top-pion contribution to R_b , for $M_{W'} = 500$ GeV. The contour boundaries correspond to $|\Delta\epsilon_{tL}^2/(\epsilon_{tL}^{ideal})^2|$ equal to 0 to 1.0 in increments of 0.2. As discussed in the text, for larger values of $M_{W'}$, each contour would correspond to a larger value of $|\Delta\epsilon_{tL}^2/(\epsilon_{tL}^{ideal})^2|$. Note that, for a deviation $|\Delta\epsilon_{tL}^2/(\epsilon_{tL}^{ideal})^2|$ of order 1, essentially the entire plane of values is allowed.

1. Decay Branching Ratios

The major two-body decay modes of the top-Higgs are the $t\bar{t}$ channel, gauge boson pair modes, and (when kinematically allowed), the $W\Pi_t$ and $\Pi_t\Pi_t$ modes. In Fig. 5, we present a plot of the branching ratios of the top-Higgs including only the dominant decay modes for the illustrative set of parameter values:

$$\begin{aligned} M_{\Pi_t} &= 200 \text{ GeV}, & M_D &= 400 \text{ GeV} \\ M_{W'} &= 500 \text{ GeV}, & \sin \omega &= 0.5. \end{aligned} \quad (42)$$

Note that for M_{H_t} below the WW threshold, the top-Higgs tends to decay to gg (through a top loop), or to virtual W 's and Z 's, as shown in the left-hand pane of Fig. 5 (computed using a modified version of HDECAY [39]).

2. Top Higgs production: Direct

The direct production of the top-Higgs, $pp \rightarrow H_t$ occurs at the LHC via gluon fusion (Fig. 2, left) just as for its SM counterpart. Within the Top-Triangle Moose model this process is completely dominated by loops of top quarks; the heavy top contribution is negligibly small. The production cross sections at the LHC are presented in Fig. 6 for two different center of mass energies, $\sqrt{s} = 7$ TeV and $\sqrt{s} = 14$ TeV.

As expected, the top-Higgs cross section is significantly larger than that for a standard model Higgs of equivalent mass. The enhancement is roughly a factor of four for our current parameter choice, though the actual value does depend somewhat on the width, and hence the mass, of the top-Higgs. Once the top-Higgs is sufficiently heavy that it can decay into a pair of top pions it becomes considerably wider than its SM equivalent, bringing down the cross section slightly.

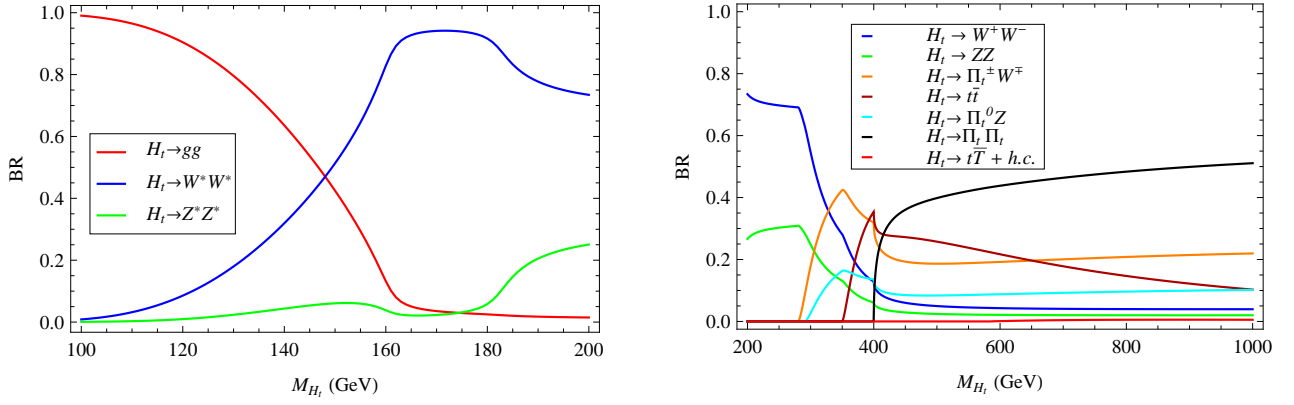


FIG. 5. Branching ratios for the dominant decay modes of the top-Higgs H_t . At left, the key shows (from top to bottom) the order of the curves at low M_{H_t} . At right, the key shows (from top to bottom) the order of the curves for M_{H_t} greater than the WW threshold. We fixed the following: $M_D = 400$ GeV, $M_{W'} = 500$ GeV, $M_{\Pi_t} = 200$ GeV, and $\sin \omega = 0.5$.

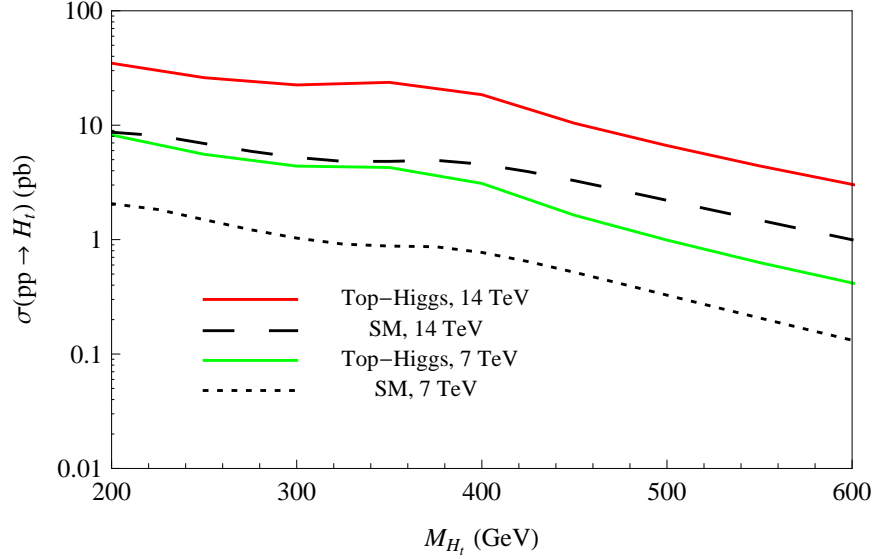


FIG. 6. The production cross-section for $pp \rightarrow H$ at the LHC. The dashed lines indicate the LO production cross sections for a standard model Higgs at 7 TeV and 14 TeV at the LHC and are taken from [39], while the solid lines are the leading order (LO) top-Higgs production cross sections at the same energy values. The top-Higgs cross sections were calculated using CTEQ5L parton distribution functions, with factorization scale $\mu_F = m_{H_t}/2$ and renormalization scale $\mu_R = m_{H_t}$.

3. Top Higgs production: Indirect

Since the top-Higgs has a non-zero off-diagonal coupling to a light and a heavy top, we could look for it in the decays of the heavy top. To see when this strategy might be useful, we examine the decays of the heavy top, shown in Fig. 7. We see that the Wb mode dominates for Dirac masses up to about a TeV. This suggests that one could look at the pair production of the heavy tops, with one of them decaying to Wb , and the other decaying to a top-Higgs, i.e., $pp \rightarrow T\bar{T} \rightarrow WbH_t t$, as shown in Fig. 2. This strategy is identical to the indirect Higgs-production mechanism proposed previously in the context of vector-like fermion extensions of the standard model [54–57]. To get an idea for the size of indirect top-Higgs production in the Top-Triangle Moose model, we present the rate for $pp \rightarrow WbH_t t$ at the LHC (14 TeV) in Fig. 8 below.

In this plot, we have fixed $M_D = 650$ GeV, $M_{\Pi_t} = 200$ GeV, and scanned over top-Higgs mass values from 100 GeV up to⁵ 600 GeV. We will discuss the implications of top-Higgs production and decay modes in more detail

⁵We choose 600 GeV as the upper limit because the top-Higgs becomes a broad resonance beyond this point.

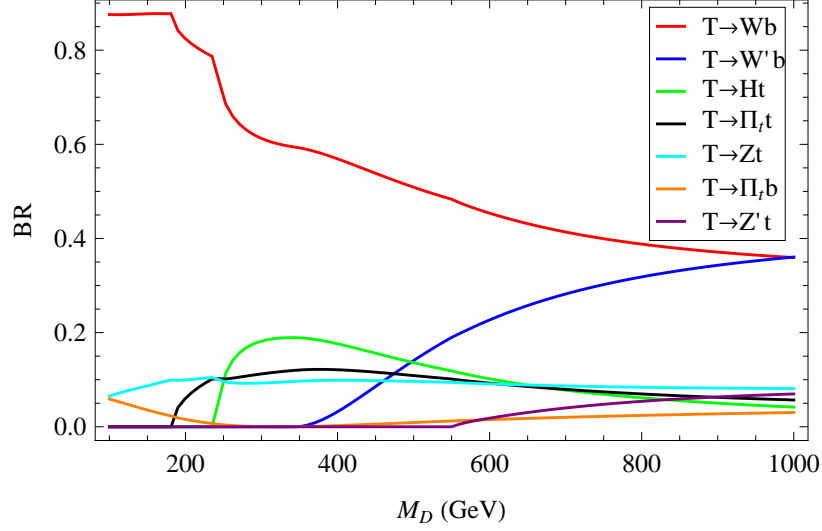


FIG. 7. The decay branching ratios of the heavy top for $M_{H_t} = 250$ GeV, $M_{\Pi_t} = 200$ GeV and $M_{W'} = 500$ GeV. The key shows (from top to bottom) the order of the curves in the middle of the plot. We see that the decay modes involving the top higgs and top pion are comparable, while the Wb mode dominates for a wide range of Dirac masses.

in sub-section (IV D).

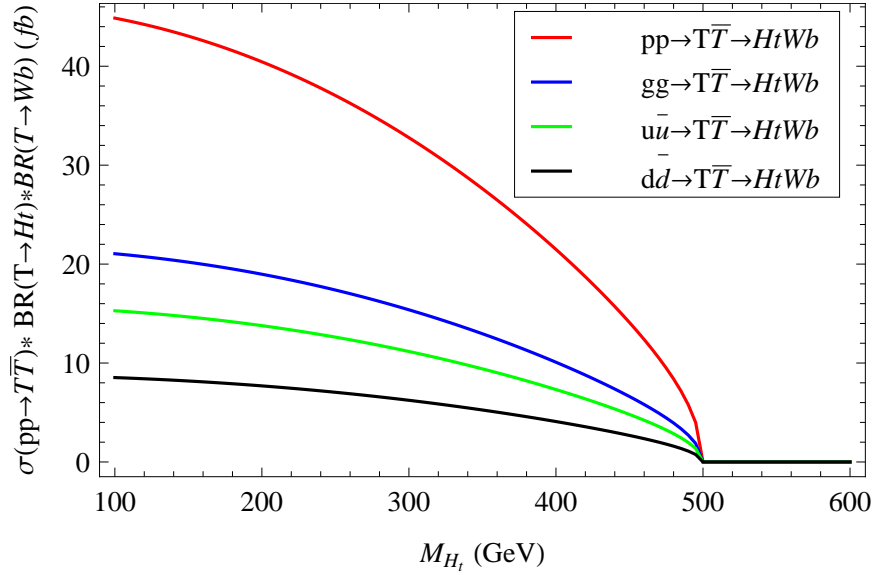


FIG. 8. $\sigma \cdot \text{BR}$ for the process $pp \rightarrow T\bar{T} \rightarrow WbH_t t$ for $M_{\Pi_t} = 200$ GeV, $M_D = 650$ GeV, $M_{W'} = 500$ GeV and $\sin \omega = 0.5$. This choice of final state takes advantage of the high BR for $T \rightarrow Wb$. The key shows (from top to bottom) the order of the curves in the middle of the plot. The cross section were calculated using an implementation of the Top-Triangle Moose model in CalcHEP [58] and assumed a LHC center of mass energy of 14 TeV.

C. Top Pion production and decay

We now turn to the top-pions. Before discussing their production channels, which are similar to the ones discussed for the top-Higgs, we will first work out the decay branching ratios of the charged and neutral top-pion.

1. Decay Branching Ratios

The charged (neutral) top pion, when produced, decays to tb (tt), Wh (Zh), or Tb (tT). The decays involving heavy gauge bosons or two heavy fermions are suppressed. We show the plot of branching ratios of the Π_t^- and Π_t^0 in Fig. 9 for the following illustrative set of parameter values:

$$\begin{aligned} M_{H_t} &= 250 \text{ GeV}, & M_D &= 400 \text{ GeV} \\ M_{W'} &= 500 \text{ GeV}, & \sin \omega &= 0.5. \end{aligned} \quad (43)$$

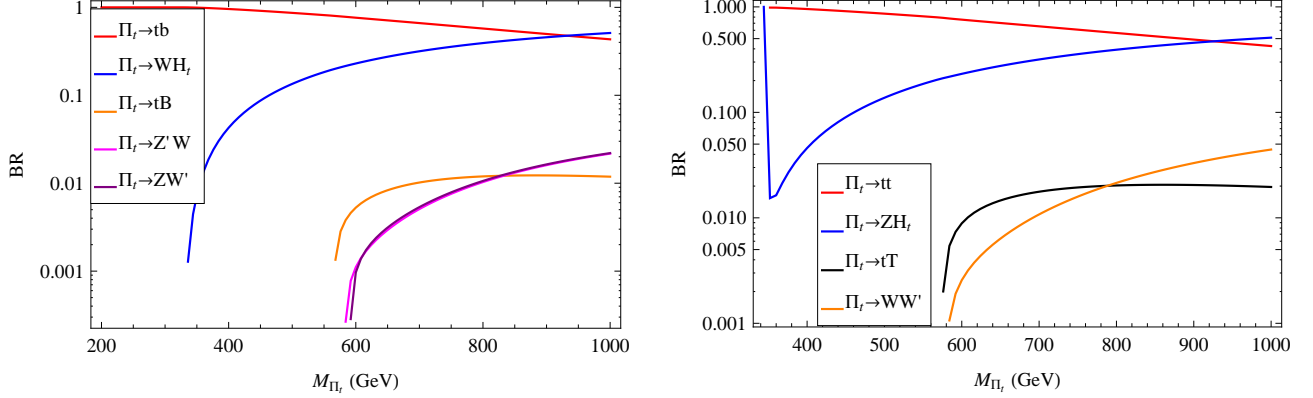


FIG. 9. The decay branching ratios of the charged top-pions (left) and the neutral top-pion (right). The key shows (from top to bottom) the order of the curves in the middle of the plot. The dominant decay modes are to tb , tB , WH_t , ZW' , and WZ' for the Π_t^\pm , and tt , tT , WW' , and ZH_t for the Π_t^0 . Below the $t\bar{t}$ threshold, the Π_t^0 will decay almost exclusively into a pair of gluons.

As Π_t is a pseudoscalar, it cannot decay into longitudinally polarized gauge bosons. With the longitudinal W/Z modes forbidden, the dominant decay mode of Π_t below the top-pair threshold is $\Pi_t \rightarrow gg$. Decays to pairs of (transversely polarized) electroweak bosons are present but suppressed by small coupling. Similarly, phase space suppresses three and four-body decay modes like $\Pi_t \rightarrow t\bar{t}^*$. As a result, the neutral top-pion is quite narrow below the top-pair threshold.

2. Top pion: Direct, indirect and associated single production

The neutral top pion, by analogy with the top-Higgs, can either be produced directly via $gg \rightarrow \Pi_t$, or could show up as a decay product of the heavy top quarks. The production cross section for the first process $gg \rightarrow \Pi_t$ is shown in Fig. 10 for two different LHC energies. As with top-Higgs production, the top-quark loop contribution is dominant. We see that there is a small sharp peak at $M_\Pi \sim 350$ GeV - this is due to the effect of the $t\bar{t}$ in the loop going on-shell. In Fig. 11, we present $\sigma \cdot \text{BR}$ for indirect production, again looking at the case where one of the heavy tops decays to Wb and the other decays to $\Pi_t t$. Here, we fixed $M_D = 650$ GeV, and $M_{H_t} = 250$ GeV.

In addition, the top-pion can also be produced in association with a top-quark - see Fig. 12. We present the cross-section for this process in Fig. 13 as a function of the top-pion mass, summing over Π_t^+ and Π_t^- production.

3. Pair production of H_t and Π_t

In addition to the processes considered above, one could also look at pair production of two top-pions or production of one top-pion and one top-Higgs at the LHC. The latter occurs via a W^* exchange, e.g. the process $pp \rightarrow W^* \rightarrow \Pi_t^\pm H_t$; see Fig. 14. We present the cross-section for this process on the left-hand side in Fig. 15 as a function of the top-pion mass (keeping $M_{H_t} = 250$ GeV) and summing over the Π_t^+ and Π_t^- production. We also show the pair production of a neutral and a charged top-pion in the same plot. We have isolated the pair production of charged top-pions (the right-hand pane in Fig. 15) - one can see that the cross-section for this process is higher than the rest. This is because of the contribution of additional t -channel diagrams involving the top-quark (and its heavy partner) when we include the bottom quark parton distribution function.

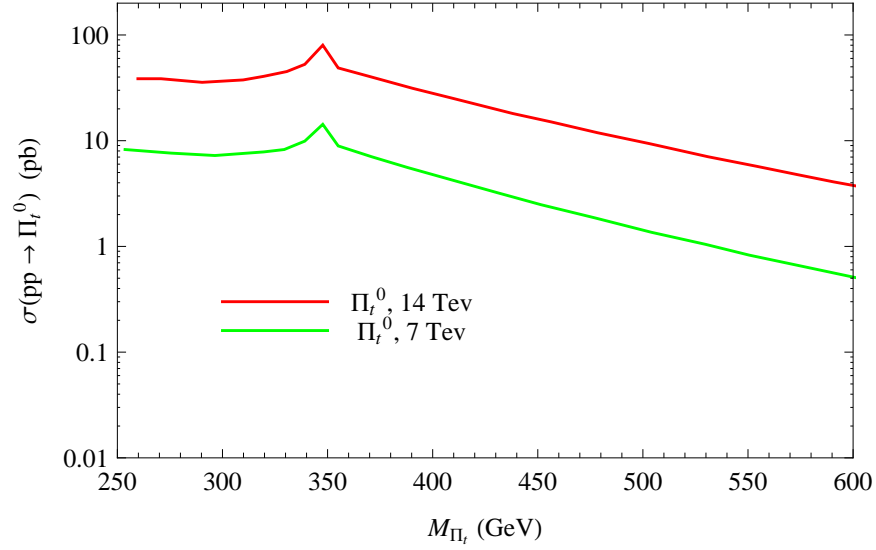


FIG. 10. The production cross-section for $pp \rightarrow \Pi_t$ at the LHC at 14 TeV (top curve) and 7 TeV (bottom curve). Parton distribution functions and parameters are the same as in Fig. 6.

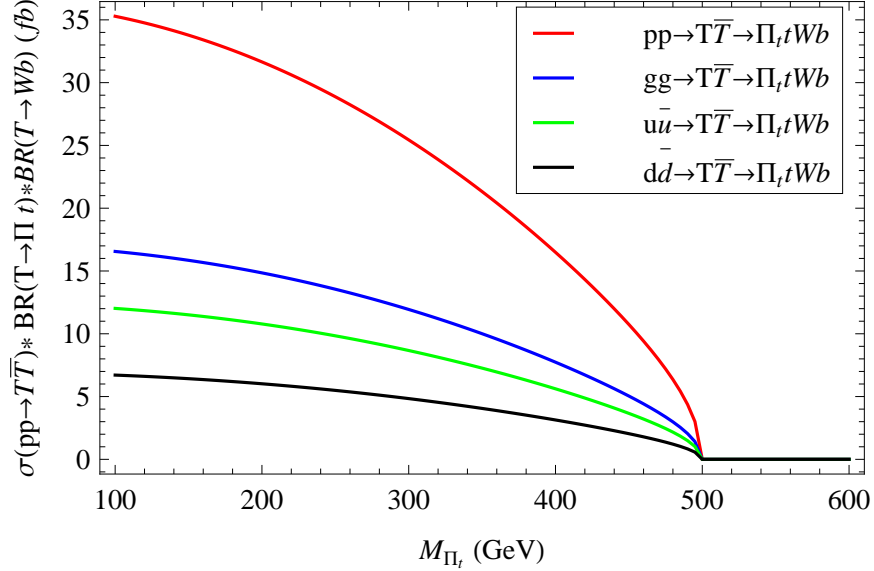


FIG. 11. $\sigma \cdot \text{BR}$ for the process $pp \rightarrow T\bar{T} \rightarrow \Pi_t t Wb$. The final state was chosen to take advantage of the high BR for $T \rightarrow Wb$. The key shows (from top to bottom) the order of the curves. This plot was made using the same tools and assumptions as Fig. 8.

D. Discovery prospects at the LHC

Now that we have discussed the production and decay of the top-Higgs, top-pion and the heavy T -quark in the model, we survey their discovery prospects at the LHC. We identify channels with clear discovery prospects and estimate their LHC reach. We also point out which channels are promising enough to warrant detailed investigation in future work. Since heavy scalars can be produced indirectly, through the decay of the heavy T -quark, we start by commenting on the visibility of this heavy fermion at the LHC.

Heavy T -quark: The LHC phenomenology of the heavy partners of the first and second generation quarks in this model was already discussed in [30] - the essential conclusion of that analysis is that, by considering both single and pair productions and subsequently letting the quarks decay to SM gauge bosons, we can discover them at the 5σ

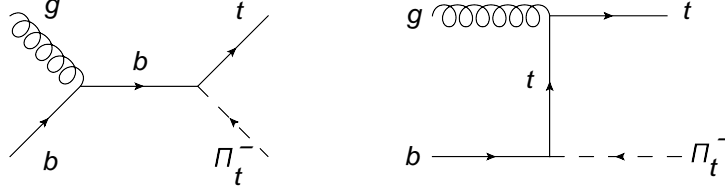


FIG. 12. Feynman diagrams for the process $pp \rightarrow \Pi_t^- t$ at the LHC ($\sqrt{s} = 14$ TeV).

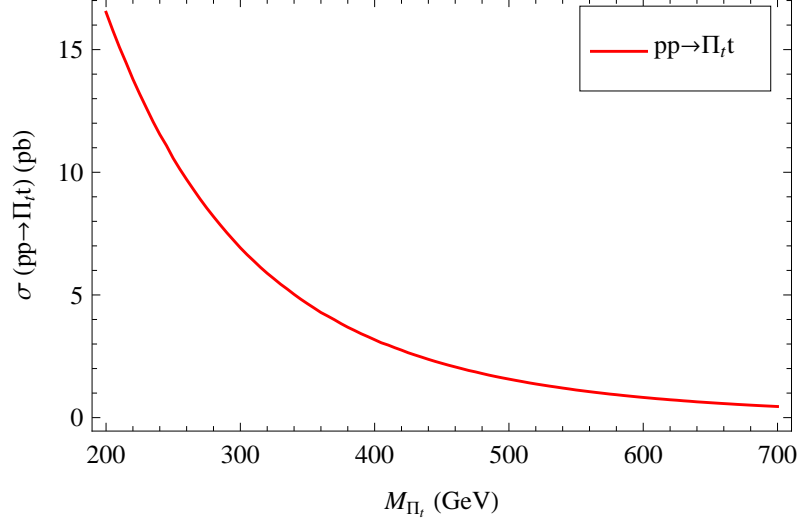


FIG. 13. The cross-section for the process $pp \rightarrow t\Pi_t^\pm$ at the LHC ($\sqrt{s} = 14$ TeV) as a function of the top-pion mass.

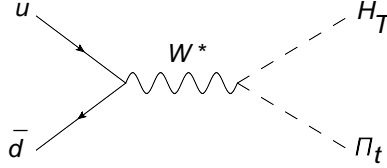


FIG. 14. Feynman diagram for the production of $H_t + \Pi_t$
- the process occurs through an s channel W^* .

level for a 14 TeV LHC with $\approx 300 \text{ fb}^{-1}$ luminosity for masses up to ~ 1 TeV. Light masses, naturally, require less integrated luminosity.

Here, we discuss the prospects for discovering the heavy partner of the top-quark. The T state decays predominantly to Wb for a wide range of M_D . Thus, for a wide range of the heavy- T mass, the best possible discovery channel, based on branching fraction considerations, seems to be $pp \rightarrow TT \rightarrow WbWb$, with the W 's decaying to either leptons or quarks. If both W 's decay leptonically, we would have two sources of missing energy, and reconstructing the heavy- T mass would be problematic. Hence, the best bet⁶ seems to be $pp \rightarrow TT \rightarrow WbWb \rightarrow l\nu + 4j$. But in order to facilitate comparison with [30], we first consider the process $pp \rightarrow \bar{T}T \rightarrow WbWb \rightarrow l\nu l\nu jj$, ignoring for the moment the complication arising due to the presence of two neutrinos in the final state. In order to make definitive statements regarding discovery prospects, we would have to calculate the complete SM background. But it is conceivable that once we impose hard p_T cuts on the jets, the SM background reduces to almost zero, as was the case in [30]. In this case, one could translate the results of that analysis by scaling the couplings. Thus, comparing the process of interest

⁶We could also consider one or both of the heavy quarks decaying to a Z , but this would introduce extra top decays in the final state, and is not likely to compete with the charged current channel.

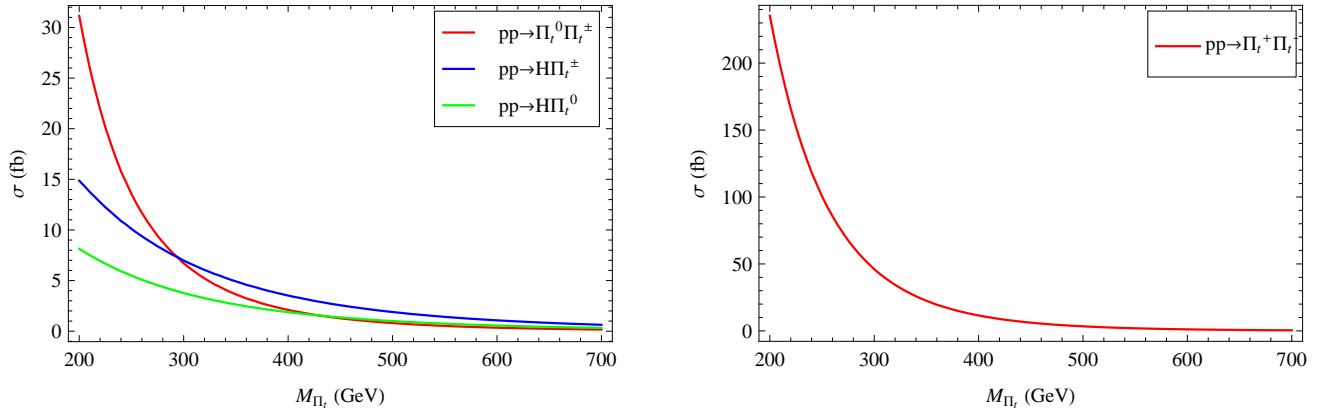


FIG. 15. Left: The production cross-section for the processes $pp \rightarrow \Pi_t H_t$ and $pp \rightarrow \Pi_t^0 \Pi_t^\pm$ for $M_{H_t} = 250$ GeV. The key shows (from top to bottom) the order of the curves. Right: Cross-section for the pair productions of charged top-pions - this is seen to be significantly higher than the values in the plot on the left because of the contribution of additional diagrams involving the top-quark (and its heavy partner) in the t channel once we include bottom quark pdf's. Both plots were made assuming a $\sqrt{s} = 14$ TeV LHC.

to one that was analyzed, we see that the particular ratio we are after is:

$$\frac{pp \rightarrow T\bar{T} \rightarrow WbW\bar{b} \rightarrow \ell\nu\ell\nu jj}{pp \rightarrow Q\bar{Q} \rightarrow WZjj \rightarrow \ell\nu\ell\nu jj} = \frac{BR(T \rightarrow Wb)^2}{BR(Q \rightarrow Wj)BR(Q \rightarrow Zj)} \frac{BR(W \rightarrow \ell\nu)}{BR(Z \rightarrow \ell\ell)}. \quad (44)$$

The branching ratios of the heavy quarks depend on the Dirac mass, but we can still make rough estimates. Comparing the branching ratio plot Fig. 7 to the one for the heavy- U in [30], we see that the branching ratio to Wj is enhanced for the heavy- T , while that to Zj is suppressed by roughly the same amount. Also, $BR(Q \rightarrow Wj) \approx 2BR(Q \rightarrow Zj)$, as can be readily verified from Fig. 3 in [30]. These two facts mean that the first ratio in the above equation is ≥ 2 . The second ratio is approximately 3.2 (using the SM values: $BR(W \rightarrow \ell\nu) = 0.108$, $BR(Z \rightarrow \ell\ell) = 0.033$). Thus, we see that the reach for the heavy- T is roughly enhanced by a factor of 6. But in the analysis for the heavy- U quarks, there is a factor of 4 included (for the heavy partners of the first two generations), and thus in our comparison, we have to divide out by the same factor. This gives an enhancement of 1.5. Considering all this, it is conservative to estimate that the reach for the heavy- T quarks, via pair production at the LHC, would be comparable to that of the heavy- U , and that the analysis of the pair production scenario in [30] applies here. Thus, referring to Fig. 12 in [30], we conclude that, for a fixed $M_{W'} = 500$ GeV, the heavy- T is discoverable at the LHC with a luminosity of 1 fb^{-1} for masses up to 450 GeV. This reach is extended to about 650 (850) GeV for 10 (100) fb^{-1} . This indicates that it would be worth doing a thorough analysis of the signal and background for the search for the T states; we plan to present this in forthcoming work.

Top-Higgs: Much as with the standard model Higgs, the detection prospects of the top-Higgs depend on its mass. Top-Higgses lighter than ~ 150 GeV decay dominantly into two gluons and will be impossible to see unless produced in association with a vector boson. Even when produced with a W/Z , the immense SM W/Z + jet backgrounds would make detection difficult, especially for lighter top-Higgses⁷. Above 160 GeV, top-Higgses produced via gluon fusion are detectable through leptonic WW/ZZ modes. Gluon fusion to top-Higgses is enhanced by $1/\sin^2 \omega \sim 4$ over a SM Higgs of equivalent mass, making the discovery prospects excellent. To get some idea of the accessible parameter range we can rescale SM Higgs discovery projections to account for the altered production rate and decay of the top-Higgs. This is most easily done for a 14 TeV collider, where many studies have been done for all Higgs masses (see, for example [60]). As an example, we can concentrate on top-Higgses heavier than 200 GeV where the 4-lepton ‘golden’ mode will be dominant. The $h \rightarrow ZZ$ significances found in [60, 61] are rescaled, then translated into a luminosity required for $S/\sqrt{B} = 5.0$ at a given top-Higgs mass. This gives us the top-Higgs discovery luminosity curve, which we show in Fig. (16). Discovery of top-Higgses lighter than 350 GeV using the leptonic mode alone is possible over a wide range of masses; top-Higgses with mass < 350 GeV (< 400 GeV) would be seen within the first fb^{-1} (10 fb^{-1}). Using the leptonic $H_t \rightarrow WW \rightarrow 2\ell 2\nu$ mode, we expect similar number for discovery prospects extending down to $m_{H_t} \sim 160$ GeV.

⁷Amusingly, the CDF collaboration does see a slight excess in the di-jet invariant mass distribution of W + jets events at ~ 150 GeV [59]. Though it is unlikely that the top-Higgs can be produced with sufficient rate to explain this excess, further study may be warranted.

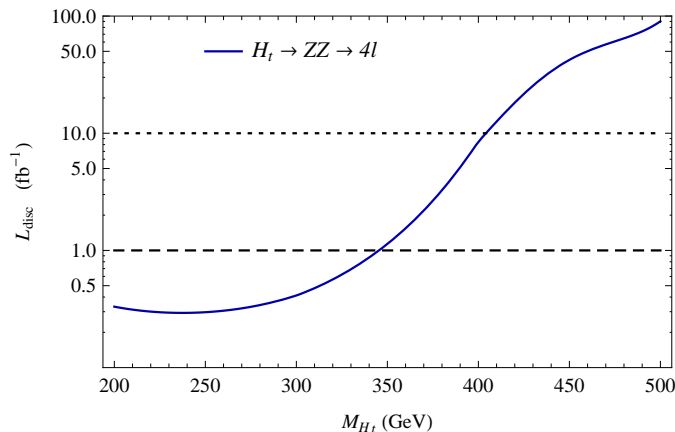


FIG. 16. Luminosity (in fb^{-1}) necessary for the discovery the top-Higgs via the fully leptonic mode $pp \rightarrow H_t \rightarrow ZZ \rightarrow 4\ell$ at a 14 TeV LHC.

While the discovery prospects of a $\sim 160 - 400$ GeV top-Higgs at a full-powered LHC are excellent, one may ask what the discovery prospects are during the initial, low-energy LHC run. Few phenomenology studies have been carried out for SM Higgs at this lower energy, however Ref. [62] has studied the leptonic WW mode for a 7 TeV collider for Higgses lighter than 200 GeV - however, as can be seen from Fig. 3, $145 \text{ GeV} < M_{H_t} < 195 \text{ GeV}$ has already been excluded by the Tevatron for $\sin \omega \leq 0.5$. A more thorough investigation of light top-Higgses including other modes would be interesting, however from this simple rescaling alone we can say that light top-Higgses are certainly detectable even during the initial LHC run.

For heavier top-Higgses, discovery becomes more challenging because the $t\bar{t}$ mode opens up. In the SM the $t\bar{t}$ mode is never discussed as a discovery mode since the Higgs branching fraction to $t\bar{t}$ never gets bigger than 10% and cannot compete with the cleaner leptonic WW/ZZ channels. In contrast, the branching fraction to $t\bar{t}$ for the top-Higgs can be much bigger than 10% because of the enhanced top - H_t coupling and, consequently, the WW/ZZ branching fractions drop at high M_{H_t} much more sharply than in the SM [60]. This, unfortunately, has a net negative effect on the discovery potential: $H_t \rightarrow t\bar{t}$ is unlikely to be a discovery mode due to large backgrounds and the signal size in the cleaner di-boson channels is reduced.

A better option for the discovery of heavy H_t is $H_t \rightarrow \Pi_t^0 Z$. Provided that $M_{\Pi_t} < 2m_t$, this decay mode yields the final state $\ell\ell jj$, where the jets are quite energetic and can be reconstructed to M_{Π_t} . Rejecting events with $\gtrsim 2$ jets or heavy flavor and by exploiting the kinematics of the $\Pi_t \rightarrow jj$ system, it may be possible to suppress SM $Z + \text{jets}$ and $t\bar{t}$ backgrounds to the point that the top-Higgs is discoverable. While this channel can arise any time there are multiple sectors which break EWS (such as 2HDM), we are not aware of any phenomenological studies. We plan to address this in forthcoming work. Note that the $Z\Pi_t^0$ mode is only potentially useful for top-Higgses lighter than $2M_{\Pi_t}$; above $2M_{\Pi_t}$, top-Higgses will decay primarily to $H_t \rightarrow \Pi_t \Pi_t$, where the top-pions can be charged or neutral. In either case, this final state will be extremely challenging to discover [63–65].

Charged top-pion: The charged top-pion is phenomenologically similar to a charged Higgs boson in a two-Higgs-doublet model with low $\tan \beta$ (i.e., enhanced top quark Yukawa coupling). Discovery prospects for a charged Higgs boson have previously been studied for the 14 TeV LHC in the context of supersymmetric models. The charged top-pion can be produced in association with a top quark through bottom-gluon fusion, $gb \rightarrow t\Pi_t^-$, and through gluon-gluon fusion, $gg \rightarrow \bar{b}t\Pi_t^-$. The cross section has been computed to next-to-leading order in QCD [66]; it grows proportional to $\cot^2 \omega$ (analogous to $\cot^2 \beta$ in the usual two-Higgs-doublet notation).

Due to the popularity of supersymmetric models, studies of tH^\pm , $H^\pm \rightarrow \bar{t}b$ at ATLAS [46] and CMS [67] have focused entirely on the large $\tan \beta$ regime. The major background comes from $t\bar{t}$ plus jets; the systematic uncertainty from the background normalization presents the biggest challenge to this search. The CMS study [67] gives values of $\sigma(pp \rightarrow tH^\pm) \times \text{BR}(H^\pm \rightarrow \bar{t}b)$ required for 5σ discovery in this channel as a function of the mass of the pseudoscalar A^0 in the MSSM, which is nearly degenerate with H^\pm in that model - the sensitivity depends very strongly on the systematic uncertainty on the $t\bar{t}$ background. We present a plot of the cross-section for the process $pp \rightarrow t\Pi_t^-$ in Fig. 13. For $M_{\Pi_t} \leq 600 \text{ GeV}$, the charged top-pion decays to tb 100% of the time (see Fig. 9), and so we can make a direct comparison between Fig. 13 and the CMS study. Doing so, we find that one can discover a charged top-pion

at the 5σ level at a luminosity of 30 fb^{-1} for $M_{\Pi_t} \leq 450 \text{ GeV}$, assuming a 0% uncertainty on the $t\bar{t}$ background⁸. For 1% (3%) systematic uncertainty, the reach goes down to 350 GeV (250 GeV)⁹.

Even when decays to other final states (WH_t , tB , ZW' , $Z'W$) are kinematically accessible, the branching fraction to $\bar{t}b$ remains high - see Fig. 9. Studies of this channel done in the context of the MSSM can thus still be applied, with the caveat that angular correlations among the final-state particles in the event may be different. In the MSSM at large $\tan\beta$, H^- decays to $\bar{t}_L b_R$ through the bottom Yukawa coupling. For the top-pion, however, $\Pi_t^- \rightarrow \bar{t}_R b_L$ through the top Yukawa coupling. This difference may affect details of the experimental acceptance for the signal. However, it is reasonable to conclude that this channel is quite promising for Π_t discovery and warrants further study.

Finally we note that ATLAS [46] combines this tH^- , $H^- \rightarrow \bar{t}b$ channel with tH^- , $H^- \rightarrow \tau\nu$ in the MSSM to present combined discovery reach contours at large $\tan\beta$, but the $H^- \rightarrow \bar{t}b$ contribution improves the reach only marginally [46]. We emphasize that while $\text{BR}(H^- \rightarrow \tau\nu) \simeq 10\%$ above the tb threshold in the MSSM at large $\tan\beta$, for the charged top-pion this decay mode is absent.

V. DISCUSSION AND CONCLUSIONS

This paper has explored the collider physics of the new heavy fermionic top-quark partner (T), the top-Higgs boson (H_t) and the top-pion (Π_t) states in the deconstructed topcolor-assisted technicolor theory known as the Top Triangle Moose. After establishing the spectrum and the couplings of the new states to each other and to standard model particles in Section III, we turned to phenomenology. We showed in Section IV.A how existing Fermilab Tevatron data constrains M_{H_t} and M_{Π_t} as a function of the mixing angle $\sin\omega$ between the linear and nonlinear sigma model symmetry-breaking sectors: Π_t lighter than 150 GeV would likely have been seen already in $t \rightarrow \Pi_t b$ while H_t in the 150-200 GeV range would likely have been visible in WW decays.

We also established that the presence of relative light Π_t does not present insurmountable challenges related to third-generation flavor physics, as one might have feared. In particular, allowing the delocalization of the left-handed top quark to deviate from the value suggested by ideal delocalization can cancel contributions from one-loop diagrams involving Π_t exchange that would otherwise have shifted R_b from agreement with experiment. As shown in Figure 3, nearly the full $\sin\omega$ vs. M_{Π_t} parameter space can be accommodated in this way. Moreover, limits from third-generation FCNCs are consistent with this finding, as shown in Appendix D.

In Sections IV.B and IV.C, we laid the foundation for studies of LHC phenomenology by calculating the decay branching ratios of H_t and Π_t , as well as their production cross-sections, for a variety of key processes. This information allowed us to determine which channels are most promising for discovery of T , H_t and Π_t . Adapting previous work on the heavy partners Q of the first and second generation quarks enabled us to demonstrate that the T should be visible at the LHC for $M_T \leq 900 \text{ GeV}$ in the pair-production channel $pp \rightarrow T\bar{T} \rightarrow WbWb \rightarrow \ell\nu \ell\nu jj$. Hence, a full study of the detailed background processes and optimal cuts for this process is indicated (and is now underway). The alternative channel in which one W decays hadronically, so that the final state is $\ell\nu 4j$, should offer a larger signal along with the welcome possibility for full reconstruction of the top quark's heavy partner; we are also planning to study this channel.

In the case of a moderately light H_t , we found that the situation resembles that of the standard model Higgs. For $M_{H_t} \leq 160 \text{ GeV}$, the top-Higgs will be invisible because it decays almost exclusively to dijets, for which the background is overwhelming. For $160 \text{ GeV} \leq M_{H_t} \leq 400 \text{ GeV}$, the top-Higgs should actually be easier to find than the standard model Higgs, because the “golden” all-leptonic decay modes open up and the signal rate is enhanced by a factor of $1/\sin\omega$. In fact, for H_t in the lighter end of this mass range, discovery in the first 1fb^{-1} of LHC data would be possible. Top-Higgs bosons heavier than 400 GeV will be more challenging to find since $\text{BR}(H_t \rightarrow WW)$ will be below even the already-reduced diboson branching ratio for the standard model Higgs. The most promising decay channel for top-Higgs discovery in the window $2m_t \leq M_{H_t} \leq 2M_{\Pi_t}$ would be $H_t \rightarrow W\Pi_t$ and we plan to study this in detail in forthcoming work. Once $M_{H_t} > 2M_{\Pi_t}$, the primary decay mode is $H_t \rightarrow \Pi_t \Pi_t \rightarrow 4g$ and the large multijet background will make discovery difficult (though the methods advocated in [63–65] can be of help).

Adapting existing work on charged-Higgs search protocols suggests that Π_t^\pm with masses below 400 GeV should be visible in 30 fb^{-1} of LHC data through the process $pp \rightarrow t\Pi_t^\pm \rightarrow t\bar{t}b$. Further studies of final state particle angular correlations and the dependence on $\sin\omega$ are needed. In particular, most studies of charged-Higgs searches have focused on the case of large mixing angle (essentially, large $\tan\beta$) whereas the case of small $\sin\omega$ is of greatest interest in the Top Triangle Moose.

Finally, it is interesting to reflect on how one would know that the new states one had discovered, whether T , H_t or Π_t , were those of the Top Triangle Moose rather than some other model. The answer will surely lie in the overall

⁸Note that $\sigma(pp \rightarrow \Pi_t^- t) \propto \cot^2\omega$, and hence the reach becomes higher for lower $\sin\omega$.

⁹The CMS study only looks at charged Higgs of mass at least 250 GeV.

pattern of observable relationships among these three states. Consider, for instance, a top-Higgs boson of moderate mass. One would first find this state in single production followed by diboson decays, $pp \rightarrow H_t \rightarrow WW$; the fact that the signal rate noticeably exceeded the standard model prediction would show that one had found an exotic rather than a standard model Higgs state. As the LHC integrated luminosity grew, the T state would eventually be found in $pp \rightarrow T\bar{T} \rightarrow WbWb$ channels. Once the existence of that state is confirmed, it would be possible to measure the rarer $T \rightarrow H_t t$ decay path and confirm that the H_t found in T decays is the same particle that one had already discovered in $H_t \rightarrow WW$. This would show that the H_t was both part of the electroweak sector (as witnessed by its diboson coupling) and strongly coupled to the top quark sector. In the case of top-pions, one might begin by establishing their presence in associated production with top quarks; this would help show that they were strongly coupled to the top sector, which a measurement of $T \rightarrow \Pi_t t$ could also confirm. Then finding either joint production of H_t and Π_t through an off-shell W boson ($pp \rightarrow W^* \rightarrow H_t \Pi_t$) or one of the decay paths $H_t \rightarrow \Pi_t W$ or $\Pi_t \rightarrow H_t W$ would demonstrate the relationship of Π_t to the electroweak sector, including the top-Higgs.

As the LHC data set grows, it will be interesting to watch for signs of these new states, heralding the presence of new strong dynamics in the top quark sector.

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Appendix A: Masses and Eigenstates

1. Gauge Bosons

The neutral gauge boson mass matrix is given by:

$$M_Z^2 = \frac{e^2 v^2}{4 x^2 \sin^2 \theta} \begin{pmatrix} \frac{x^2}{1-x^2}(1 + \cos^2 \omega) & -\frac{2x}{\sqrt{1-x^2}}\cos^2 \omega & -\frac{x^2}{\sqrt{1-x^2}}\sin^2 \omega \tan \theta \\ -\frac{2x}{\sqrt{1-x^2}}\cos^2 \omega & 4 \cos^2 \omega & -2x \cos^2 \omega \tan \theta \\ -\frac{x^2}{\sqrt{1-x^2}}\sin^2 \omega \tan \theta & -2x \cos^2 \omega \tan \theta & x^2(1 + \cos^2 \omega)\tan^2 \theta \end{pmatrix}. \quad (\text{A1})$$

Diagonalizing perturbatively in the small parameter x yields the following masses for the Z and the Z' [30]:

$$M_Z^2 = \frac{e^2 v^2}{4 \sin^2 \theta \cos^2 \theta} \left(1 + x^2 \left(1 - \frac{\sec^2 \theta}{4} \right) \right) \quad (\text{A2})$$

$$M_{Z'}^2 = \frac{e^2 v^2 \cos^2 \omega}{4 \sin^2 \theta x^2} (4 + x^2 \sec^2 \theta), \quad (\text{A3})$$

while the photon remains massless. The eigenvector of the Z is given by:

$$Z^\mu = v_z^0 W_0^\mu + v_z^1 W_1^\mu + v_z^2 B^\mu, \quad (\text{A4})$$

where

$$v_z^0 = \frac{1}{8} (4(-2 + x^2)\cos \theta - 3x^2 \sec \theta), \quad v_z^1 = \frac{1}{2} x(-2\cos^2 \theta + 1)\sec \theta, \quad v_z^2 = \sin \theta - \frac{1}{2} x^2 \sec \theta \tan \theta.$$

The eigenvector of Z' is the orthogonal combination. The charged gauge boson mass matrix is the upper 2×2 block of Eq. (A1). The masses of the physical gauge bosons are given by:

$$M_W^2 = \frac{e^2 v^2}{4 \sin^2 \theta} \left(1 + \frac{3x^2}{4} \right) \quad (\text{A5})$$

$$M_{W'}^2 = \frac{e^2 v^2 \cos^2 \omega}{4 \sin^2 \theta x^2} (4 + x^2), \quad (\text{A6})$$

with the respective eigenvectors:

$$W^\mu = \left(1 - \frac{x^2}{8}\right) W_0^\mu + \frac{1}{2}x W_1^\mu \text{ and} \quad (\text{A7})$$

$$W'^\mu = -\frac{1}{2}x W_0^\mu + \left(1 - \frac{x^2}{8}\right) W_1^\mu. \quad (\text{A8})$$

We are now in a position to define the weak mixing angle, $1 - \sin^2 \theta_W \equiv M_W^2/M_Z^2$. Including corrections up to $\mathcal{O}(x^2)$, we obtain,

$$\sin \theta_W = \left(1 - \frac{x^2}{8}\right) \sin \theta. \quad (\text{A9})$$

2. Fermions

The light fermion mass matrix is derived from the Lagrangian:

$$\mathcal{L} = M_D \left[\epsilon_L \bar{\psi}_{L0} \Sigma_{01} \psi_{R1} + \bar{\psi}_{R1} \psi_{L1} + \bar{\psi}_{L1} \Sigma_{12} \begin{pmatrix} \epsilon_{uR} & 0 \\ 0 & \epsilon_{dR} \end{pmatrix} \begin{pmatrix} u_{R2} \\ d_{R2} \end{pmatrix} \right], \quad (\text{A10})$$

and is given by:

$$M_{u,d} = M_D \begin{pmatrix} \epsilon_L & 0 \\ 1 & \epsilon_{uR,dR} \end{pmatrix}. \quad (\text{A11})$$

This can be diagonalized in the small parameters ϵ_L and ϵ_{fR} to yield the masses of the light fermion and its heavy Dirac partner:

$$m_f = \frac{M_D \epsilon_L \epsilon_{fR}}{\sqrt{1 + \epsilon_{fR}^2}} \left[1 - \frac{\epsilon_L^2}{2(1 + \epsilon_{fR}^2)} + \dots \right] \quad (\text{A12})$$

$$m_F = M_D \sqrt{1 + \epsilon_{fR}^2} \left[1 + \frac{\epsilon_L^2}{2(1 + \epsilon_{fR}^2)^2} + \dots \right]. \quad (\text{A13})$$

The left- and right-handed eigenstates of the light fermion can be derived to be:

$$u_L = \left(-1 + \frac{\epsilon_L^2}{2(1 + \epsilon_{uR}^2)^2} \right) \psi_{L0} + \left(\frac{\epsilon_L}{1 + \epsilon_{uR}^2} \right) \psi_{L1}, \quad (\text{A14})$$

$$u_R = \left(-\frac{\epsilon_{uR}}{\sqrt{1 + \epsilon_{uR}^2}} + \frac{\epsilon_L^2 \epsilon_{uR}}{(1 + \epsilon_{uR}^2)^{5/2}} \right) \psi_{R1} + \left(\frac{1}{\sqrt{1 + \epsilon_{uR}^2}} + \frac{\epsilon_L^2 \epsilon_{uR}^2}{(1 + \epsilon_{uR}^2)^{5/2}} \right) u_{R2}. \quad (\text{A15})$$

The eigenvector of the left- and right-handed heavy quark are the orthogonal combinations.

For the top, the mass term is dominated by the top-Higgs contribution. The mass matrix is given by:

$$M_t = M_D \begin{pmatrix} \epsilon_{tL} & a \\ 1 & \epsilon_{tR} \end{pmatrix}, \quad (\text{A16})$$

where the parameter a is defined as $a \equiv v \sin \omega / \sqrt{2} M_D$. Diagonalizing Eq. (A16) perturbatively in ϵ_{tL} and ϵ_{tR} , we get the mass of the SM top-quark:

$$m_t = \lambda_t v \sin \omega \left[1 + \frac{\epsilon_{tL}^2 + \epsilon_{tR}^2 + \frac{2}{a} \epsilon_{tL} \epsilon_{tR}}{2(-1 + a^2)} \right]. \quad (\text{A17})$$

Thus, we see that m_t depends only slightly on ϵ_{tR} , in contrast to the light fermion mass, Eq. (A12), where the dominant term is ϵ_{fR} dependent. The mass of the heavy partner of the top is given by:

$$m_T = M_D \left[1 - \frac{\epsilon_{tL}^2 + \epsilon_{tR}^2 + 2a\epsilon_{tL}\epsilon_{tR}}{2(-1 + a^2)} \right]. \quad (\text{A18})$$

The left- and right-handed eigenvectors of the SM top are given by:

$$t_L = \left(1 - \frac{\epsilon_{tL}^2 + a^2\epsilon_{tR}^2 + 2a\epsilon_{tL}\epsilon_{tR}}{2(-1 + a^2)^2} \right) \psi_{L0}^t + \left(\frac{\epsilon_{tL} + a\epsilon_{tR}}{-1 + a^2} \right) \psi_{L1}^t \quad \text{and} \quad (\text{A19})$$

$$t_R = \left(1 - \frac{a^2\epsilon_{tL}^2 + \epsilon_{tR}^2 + 2a\epsilon_{tL}\epsilon_{tR}}{2(-1 + a^2)^2} \right) \psi_{R1}^t + \left(\frac{a\epsilon_{tL} + \epsilon_{tR}}{-1 + a^2} \right) t_{R2}. \quad (\text{A20})$$

Appendix B: The Lagrangian

In order to derive the terms in the Lagrangian describing the interaction of the top-Higgs and the top pions with the gauge bosons, we start by plugging Eq. (14) in Eq. (2), and writing the covariant derivative of Σ_{01} as

$$D_\mu \Sigma_{01} = \frac{i}{F} \partial_\mu \pi_0 + igW_{0\mu} - \frac{g}{F} W_{0\mu} \pi_0 - i\tilde{g}W_{1\mu} - \frac{\tilde{g}}{F} \pi_0 W_{1\mu}, \quad (\text{B1})$$

where we have denoted $\pi_0 = \pi_0^a \sigma^a$. The product can be evaluated to be:

$$\begin{aligned} (D_\mu \Sigma_{01})^\dagger (D_\mu \Sigma_{01}) &= \frac{1}{F^2} (\partial_\mu \pi_0)^2 + [g^2 W_{0\mu}^2 + \tilde{g}^2 W_{1\mu}^2 - g\tilde{g} W_0^\mu W_{1\mu} - g\tilde{g} W_1^\mu W_{0\mu}] \\ &+ \frac{1}{F} (\partial^\mu \pi_0) [gW_{0\mu} - \tilde{g}W_{1\mu}] + \left[\frac{g}{F} W^{0\mu} (\partial_\mu \pi_0) - \frac{\tilde{g}}{F} W^{1\mu} (\partial_\mu \pi_0) \right] \\ &- \frac{i}{F^2} (\partial^\mu \pi_0) [-gW_{0\mu} \pi_0 + \tilde{g}\pi_0 W_{1\mu}] - \left[\frac{ig}{F^2} \pi_0 W_0^\mu (\partial_\mu \pi_0) - \frac{i\tilde{g}}{F^2} W_1^\mu \pi_0 (\partial_\mu \pi_0) \right] \\ &+ \frac{i}{F} [-gW_0^\mu + \tilde{g}W_1^\mu] [-gW_{0\mu} \pi_0 + \tilde{g}\pi_0 W_{1\mu}] - \frac{1}{F} [g\pi_0 W_0^\mu - \tilde{g}W_1^\mu \pi_0] [igW_{0\mu} - i\tilde{g}W_{1\mu}] \\ &+ \frac{1}{F^2} [-g\pi_0 W_0^\mu + \tilde{g}W_1^\mu \pi_0] [-gW_{0\mu} \pi_0 + \tilde{g}\pi_0 W_{1\mu}]. \end{aligned} \quad (\text{B2})$$

The first line gives the kinetic energy term for the pions and the gauge bosons masses. The second line gives the mixing between the gauge and the Goldstone bosons. The third and fourth lines give the $\pi\pi V$ and πVV couplings respectively, while the last line gives the four point coupling, $\pi\pi VV$. Plugging in the matrix definitions of the fields and taking the trace, we get,

$$\begin{aligned} \mathcal{L}_{\pi KE}^{(2)} &= \frac{1}{2} (\partial_\mu \pi_1^a)^2 \\ \mathcal{L}_{\text{mixing}}^{(2)} &= \frac{F}{2} [\tilde{g}W_{1\mu}^a (\partial_\mu \pi_1^a) - g'B_{2\mu} (\partial_\mu \pi_1^3)] \\ \mathcal{L}_{\pi\pi V}^{(2)} &= -\frac{\tilde{g}}{2} \epsilon_{abc} (\partial^\mu \pi_1^a) W_{1\mu}^b \pi_1^c + \frac{g'}{2} \epsilon_{ab3} (\partial^\mu \pi_1^a) \pi_1^b B_{2\mu} \\ \mathcal{L}_{\pi VV}^{(2)} &= \frac{g'\tilde{g}F}{2} \epsilon_{ab3} W_1^{a\mu} B_{2\mu} \pi_1^b \\ \mathcal{L}_{\pi\pi VV}^{(2)} &= \mathcal{M}_{abcd} \tilde{g}^2 \pi_1^a W_{1\mu}^{b\mu} W_{1\mu}^c \pi_1^d - 2\tilde{g}g' \mathcal{M}_{abc3} \pi_1^a W_1^{b\mu} \pi_1^c B_{2\mu} + \frac{g'^2}{8} B_{2\mu}^2 (\pi_1^a)^2 \end{aligned}$$

where $\mathcal{M}_{abcd} = \frac{1}{8} (\delta_{ab}\delta_{cd} - \delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc})$.

The corresponding terms from the kinetic term of the other nonlinear sigma model field can be read off by relabeling the fields and couplings as follows:

$$gW_{0\mu} \rightarrow \tilde{g}W_{1\mu}; \tilde{g}W_{1\mu} \rightarrow g'B_{2\mu}; \pi_0 \rightarrow \pi_1. \quad (\text{B3})$$

We summarize the results for the sake of completeness:

$$\begin{aligned} \mathcal{L}_{\pi KE}^{(2)} &= \frac{1}{2} (\partial_\mu \pi_1^a)^2 \\ \mathcal{L}_{\text{mixing}}^{(2)} &= \frac{F}{2} [\tilde{g}W_{1\mu}^a (\partial_\mu \pi_1^a) - g'B_{2\mu} (\partial_\mu \pi_1^3)] \\ \mathcal{L}_{\pi\pi V}^{(2)} &= -\frac{\tilde{g}}{2} \epsilon_{abc} (\partial^\mu \pi_1^a) W_{1\mu}^b \pi_1^c + \frac{g'}{2} \epsilon_{ab3} (\partial^\mu \pi_1^a) \pi_1^b B_{2\mu} \\ \mathcal{L}_{\pi V V}^{(2)} &= \frac{g' \tilde{g} F}{2} \epsilon_{ab3} W_1^{a\mu} B_{2\mu} \pi_1^b \\ \mathcal{L}_{\pi\pi V V}^{(2)} &= \mathcal{M}_{abcd} \tilde{g}^2 \pi_1^a W_1^{b\mu} W_{1\mu}^c \pi_1^d - 2\tilde{g}g' \mathcal{M}_{abc3} \pi_1^a W_1^{b\mu} \pi_1^c B_{2\mu} + \frac{g'^2}{8} B_{2\mu}^2 (\pi_1^a)^2 \end{aligned}$$

where \mathcal{M}_{abcd} is given as before.

Turning to the kinetic energy term of Φ , we see that its covariant derivative

$$D_\mu \Phi = \partial_\mu \Phi + igW_{0\mu}^1 \Phi - \frac{ig'}{2} B_{2\mu} \Phi \quad (\text{B4})$$

can be expanded by plugging in Eq. (6):

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}} (\partial_\mu H + i\partial_\mu \pi_t^0) \\ i\partial_\mu \pi_t^- \end{pmatrix} + \frac{ig}{2} \begin{pmatrix} \frac{W_{0\mu}^3}{\sqrt{2}} (f + H + i\pi_t^0) + \sqrt{2} i W_{0\mu}^+ \pi_t^- \\ W_0^- (f + H + i\pi_t^0) - i W_{0\mu} \pi_t^- \end{pmatrix} - \frac{ig'}{\sqrt{2}} \begin{pmatrix} \frac{B_{2\mu}}{\sqrt{2}} (f + H + i\pi_t^0) \\ i B_{2\mu} \pi_t^- \end{pmatrix}. \quad (\text{B5})$$

In order to make the expressions more compact, we will introduce the following notation:

$$Z_\mu = gW_{3\mu} - g'B_{2\mu}, \quad (\text{B6})$$

$$A_\mu = gW_{3\mu} + g'B_{2\mu}. \quad (\text{B7})$$

The Z and A appearing in the above formulas are convenient aids to make the expressions look simple, and are *not* the physical Z_μ and A_μ . Using this, the product can be evaluated to be:

$$\begin{aligned} D_\mu \Phi^\dagger D_\mu \Phi &= \frac{1}{2} (\partial_\mu H)^2 + \frac{1}{2} (\partial_\mu \pi_t^0)^2 + (\partial^\mu \pi_t^+) (\partial_\mu \pi_t^-) + \frac{Z_\mu}{2} [(f + H) (\partial_\mu \pi_t^0) - \pi_t^0 \partial_\mu H] \\ &\quad - \frac{g}{2} (\partial^\mu H) (W_{0\mu}^+ \pi_t^- + W_{0\mu}^- \pi_t^+) + \frac{ig}{2} (\partial^\mu \pi_t^0) (W_{0\mu}^+ \pi_t^- - W_{0\mu}^- \pi_t^+) + \frac{(Z_\mu)^2}{8} [(f + H)^2 + (\pi_t^0)^2] \\ &\quad + \frac{g}{2} [(f + H) [W_0^{-\mu} (\partial_\mu \pi_t^+) + W_0^{+\mu} (\partial_\mu \pi_t^-)] + \frac{ig}{2} \pi_t^0 [W_0^{-\mu} (\partial_\mu \pi_t^+) - W_0^{+\mu} (\partial_\mu \pi_t^-)]] \\ &\quad + \frac{ig}{4} Z_\mu [(f + H) (W_0^{+\mu} \pi_t^- - W_0^{-\mu} \pi_t^+) - i\pi_t^0 (W_0^{+\mu} \pi_t^- + W_0^{-\mu} \pi_t^+)] \\ &\quad - \frac{iA_\mu}{2} [(\partial^\mu \pi_t^+) \pi_t^- - (\partial^\mu \pi_t^-) \pi_t^+] + \frac{g^2}{4} W_0^{-\mu} W_{0\mu}^+ [(f + H)^2 + (\pi_t^0)^2] \\ &\quad - \frac{ig}{4} A_\mu [(f + H) (W_{0\mu}^+ \pi_t^- - W_{0\mu}^- \pi_t^+) - i\pi_t^0 (W_{0\mu}^+ \pi_t^- + W_{0\mu}^- \pi_t^+)] \\ &\quad + \frac{1}{2} \pi_t^+ \pi_t^- (A_\mu)^2 + \frac{1}{2} \pi_t^+ \pi_t^- W_0^{+\mu} W_{0\mu}^-, \end{aligned} \quad (\text{B8})$$

where $W^\pm = (W^1 \mp iW^2)/\sqrt{2}$, and similarly for the π_t^\pm . Eq. (B8) gives us the coupling of the top-Higgs and the pions to the gauge bosons, and the gauge-Goldstone mixing terms. Let us pick the latter contribution to the Lagrangian.

$$\mathcal{L}_{\text{mixing}}^{(3)} = \frac{gf}{2} [W_0^{-\mu}(\partial_\mu \pi_t^+) + W_0^{+\mu}(\partial_\mu \pi_t^-)] + \frac{f}{2} Z_\mu(\partial_\mu \pi_t^0). \quad (\text{B9})$$

Plugging in the definitions of the fields, this becomes:

$$\mathcal{L}_{\text{mixing}}^{(3)} = \frac{f}{2} [g(\partial_\mu \pi_t^a) W_0^{a\mu} - g'(\partial_\mu \pi_t^3) B_2^\mu]. \quad (\text{B10})$$

Appendix C: Four point couplings

We present the four point couplings involving two gauge bosons and top-pions/top-Higgs in Table V.

Vertex	Strength
$H_t H_t W W$	$\frac{g_0^2}{4} \left(1 + \frac{3x^2}{4}\right)$
$H_t H_t W' W$	$-\frac{g_0^2 x}{8}$
$H_t H_t W' W'$	$\frac{g_0^2 x^2}{16}$
$H_t H_t Z Z$	$\frac{e^2}{2} \left(\text{cosec}^2 2\theta + \frac{x^2}{16} [1 + 2 \cos 2\theta] \text{cosec}^2 \theta \sec^4 \theta \right)$
$H_t H_t Z' Z$	$-\frac{g_0^2 x}{8} \sec^3 \theta \cos 2\theta$
$H_t H_t Z' Z'$	$\frac{g_0^2 x^2}{32} \sec^4 \theta \cos^2 2\theta$
$H_t Z W^- \Pi_t^+$	$-\frac{ig_0^2}{2} \cos \omega \tan \theta \left(\sin \theta + \frac{x^2}{16} [1 + 3 \cos 2\theta] \sec \theta \tan \theta \right)$
$H_t Z W'^- \Pi_t^+$	$\frac{ig_0^2}{4} x \cos \omega \sin \theta \tan \theta$
$H_t Z' W^- \Pi_t^+$	$-\frac{ig_0^2}{4} x \cos \omega \tan^2 \theta$
$H_t Z' W'^- \Pi_t^+$	$\frac{ig_0^2}{8} x^2 \cos \omega \tan^2 \theta$
$H_t A W^- \Pi_t^+$	$-\frac{ig_0^2}{2} \cos \omega \sin \theta \left(1 + \frac{3x^2}{8}\right)$
$H_t A W'^- \Pi_t^+$	$\frac{ig_0^2}{4} x \cos \omega \sin \theta$

TABLE V. Four point couplings involving the top-Higgs, again calculated to $\mathcal{O}(x^2)$.

Appendix D: FCNC constraints and ideal delocalization

1. Limits on $\Delta \epsilon_{iL}^2$: $\Delta F = 2$

Limits on the deviation of ϵ_{iL} from ideal comes from the minimal size of tree-level flavor-changing neutral currents from Z -exchange. Consider re-writing Eq. (27) for the left-handed quarks of the i th family (where $i = u, c, t$) as

$$g_L^{Zii} = -\frac{e}{s_W c_W} \left[\left(1 - \frac{\Delta \epsilon_{iL}^2}{2}\right) T_3 - Q s_W^2 \right], \quad (\text{D1})$$

where $\Delta \epsilon_{iL}^2$ denotes the deviation from ideal delocalization of the i th family in the top-quark mass-eigenstate basis

$$\Delta \epsilon_{iL}^2 = \epsilon_{iL}^2 - \frac{x^2}{2}. \quad (\text{D2})$$

In this notation t_L is the left-handed quark (in the top-quark mass eigenstate basis) whose “down” component receives a large correction from top-pion exchange. In general, this “down” component may be written

$$d_{tL}^t = U_{3j} d_{jL}, \quad (\text{D3})$$

where d_{iL}^t represent the “down” components of the left-handed doublet fields in the top-quark mass eigenstate basis, and d_{jL} are the same fields in the down-quark mass-eigenstate basis, and the U_{3j} are the third row of a unitary matrix. The minimal size of the U_{3j} , corresponding to “next-to-minimal” flavor violation [53], is

$$U_{3j} = \mathcal{O}(V_{tj}^{CKM}), \quad (D4)$$

where V^{CKM} is the usual CKM flavor-mixing matrix in the standard model. Since GIM cancellation is exact when $\Delta\epsilon_{tL}^2 = 0$, we find the tree-level flavor changing Z -boson couplings to down-quarks

$$g_Z^{Zb_L s_L} = \frac{e\Delta\epsilon_{tL}^2 V_{ts}^{CKM}}{4s_W c_W} \quad (D5)$$

$$g_Z^{Zb_L d_L} = \frac{e\Delta\epsilon_{tL}^2 V_{td}^{CKM}}{4s_W c_W}. \quad (D6)$$

Z -exchange then produces the $\Delta F = 2$ effective operators

$$C_K^1 (\bar{s}_L \gamma^\mu d_L) (\bar{s}_L \gamma_\mu d_L) \quad (D7)$$

$$C_{B_d}^1 (\bar{b}_L \gamma^\mu d_L) (\bar{b}_L \gamma_\mu d_L) \quad (D8)$$

$$C_{B_s}^1 (\bar{b}_L \gamma^\mu s_L) (\bar{b}_L \gamma_\mu s_L), \quad (D9)$$

where, since GIM cancellation is exact when $\Delta\epsilon_{tL}^2 = 0$, we find

$$|\Re(C_K^1)| = \left| \Re \left(\frac{e^2 (\Delta\epsilon_{tL}^2 V_{ts}^{CKM} V_{td}^{CKM})^2}{8(s_W c_W)^2 M_Z^2} \right) \right| < \frac{1}{(1.0 \times 10^6 \text{ GeV})^2} \quad (D10)$$

$$|\Im(C_K^1)| = \left| \Im \left(\frac{e^2 (\Delta\epsilon_{tL}^2 V_{ts}^{CKM} V_{td}^{CKM})^2}{8(s_W c_W)^2 M_Z^2} \right) \right| < \frac{1}{(1.5 \times 10^7 \text{ GeV})^2} \quad (D11)$$

$$|C_{B_d}^1| = \left| \frac{e^2 (\Delta\epsilon_{tL}^2 V_{tb}^{CKM} V_{td}^{CKM})^2}{8(s_W c_W)^2 M_Z^2} \right| < \frac{1}{(2.1 \times 10^5 \text{ GeV})^2} \quad (D12)$$

$$|C_{B_s}^1| = \left| \frac{e^2 (\Delta\epsilon_{tL}^2 V_{tb}^{CKM} V_{td}^{CKM})^2}{8(s_W c_W)^2 M_Z^2} \right| < \frac{1}{(3 \times 10^4 \text{ GeV})^2}, \quad (D13)$$

where the bounds given by the last inequality in each expression come from Ref. [68]. The strongest constraint arises from limits on extra contributions to CP-violation in K -meson mixing, for which we find

$$\Delta\epsilon_{tL}^2 < 7.2 \times 10^{-2}. \quad (D14)$$

We plot this bound as a limit on $\Delta\epsilon_{tL}^2 / (\epsilon_{tL}^{ideal})^2$ in the upper curve in Fig. 17, as a function of $\sin \omega$ for $M_{W'} = 500$ GeV.

2. Limits on $\Delta\epsilon_{tL}^2$: $\Delta F = 1$

The strongest limits from $\Delta F = 1$ processes come from constraints on the B -meson decays $B_{d,s} \rightarrow \mu^+ \mu^-$. The limits arising from experimental constraints have been summarized in [69]. In Table 2 of that reference, we find the strongest constraint arising from Tevatron limits on $B_s \rightarrow \mu^+ \mu^-$ and, for the operator

$$\frac{2\varepsilon}{v^2} (\bar{b}_L \gamma^\nu s_L) (\bar{\mu}_L \gamma_\nu \mu_L), \quad (D15)$$

where $\sqrt{2}G_F = v^{-2}$ and $v \approx 246$ GeV is the weak scale. In our case, using Eq. (D5) and $M_Z = ev/(2s_W c_W)$, we find

$$\varepsilon = \frac{1}{2} \Delta\epsilon_{tL}^2 V_{ts}^{CKM} V_{tb}^{CKM} \approx \frac{\lambda^2 \Delta\epsilon_{tL}^2}{2}. \quad (D16)$$

Using the limit $BR(B_s \rightarrow \mu^+ \mu^-) < 4.3 \times 10^{-8}$ [70] and the techniques¹⁰ of ref. [69], we find the bound $\varepsilon < 7.6 \times 10^{-4}$. From eqn. (D15), we then obtain

$$\Delta\epsilon_{tL}^2 < 3.8 \times 10^{-2}, \quad (D17)$$

¹⁰We disagree with the numerical extraction of the bound on ε presented in [69], though we agree with their method.

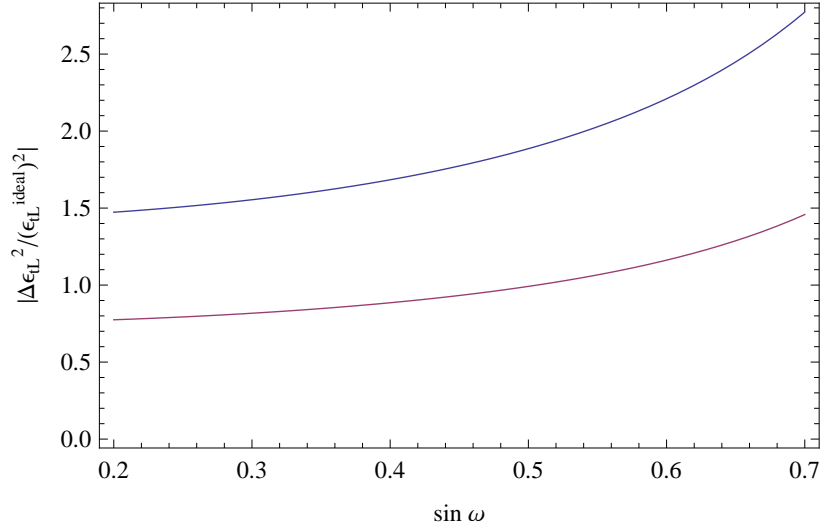


FIG. 17. The upper bounds in the deviation $|\Delta\epsilon_{tL}^2/(\epsilon_{tL}^{ideal})^2|$ arising from limits on extra contributions to CP-violation in K -meson mixing (upper curve) and from bounds on the rare decay $B_s \rightarrow \mu^+\mu^-$ (lower curve), as a function of $\sin \omega$ for $M_{W'} = 500$ GeV.

a constraint roughly twice as small as that given by limiting contributions to CP-violation in K -meson mixing in Eq. (D14).

Comparing Figs. 4 and 17, we see that compensating for the deviation in R_b resulting from top-pion exchange by modifying the delocalization of the third-generation quarks is not, in the context of “next-to-minimal” flavor violation [53], ruled out by flavor changing neutral current constraints.

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