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Relation between transverse momentum dependent distribution functions and parton distribution functions in the covariant parton model approach

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We derive relations between transverse momentum dependent distribution functions (TMDs) and the usual parton distribution functions (PDFs) in the 3D covariant parton model, which follow from Lorentz invariance and the assumption of a rotationally symmetric distribution of parton momenta in the nucleon rest frame. Using the known PDFs $f_1^a(x)$ and $g_1^a(x)$ as input we predict the x- and $\mathbf{p}_T$-dependence of all twist-2 T-even TMDs.

I. INTRODUCTION

TMDs [1, 2] open a new way to a more complete understanding of the quark-gluon structure of the nucleon. Indeed, some experimental observations can hardly be explained without a more accurate and realistic 3D picture of the nucleon, which naturally includes transverse motion. The azimuthal asymmetry in the distribution of hadrons produced in deep-inelastic lepton-nucleon scattering (DIS), known as the Cahn effect [3], is a classical example. The intrinsic (transversal) parton motion is also crucial for the explanation of some spin effects [4–16].

In previous studies we discussed the covariant parton model, which is based on the 3D picture of parton momenta with rotational symmetry in the nucleon rest frame [17–26]. An important feature of this approach is the implication of relations among various PDFs, such as the Wandzura-Wilczek approximation between $g_1^a(x)$ and $g_2^a(x)$ which was proven in [18] together with some other sum rules. Assuming $SU(6)$ symmetry (in addition to Lorentz invariance and rotational symmetry) relations between the polarized and unpolarized structure functions were found [19], which agree very well with data. In [20] transversity was studied in the framework of this model and a relation between transversity $h_1^a(x)$ and helicity $g_1^a(x)$ was obtained. In a next step we generalized the model to the description of TMDs. We derived a relation between the pretzelosity distribution $h_{1T}^{\perp a}$, transversity and helicity [22]. Finally, with the same model we studied all time-reversal even (T-even) TMDs and derived a set of relations among them [23]. Moreover, it was also shown that the 3D picture of parton momenta inside the nucleon provides a basis for a consistent description of quark orbital angular momentum [21], which is related to pretzelosity [26]. It should be remarked that some of the relations among different TMDs were found (sometimes before) also in other models [27–35].

The comparison of the obtained relations and predictions with experimental data is very important and interesting from phenomenological point of view. It allows us to judge to which extent the experimental observation can be interpreted in terms of simplified, intuitive notions. The obtained picture of the nucleon can be a useful supplement to the exact but more complicated theory of the nucleon structure based on QCD. For example, the covariant parton model can be a useful tool for separating effects of QCD from effects of relativistic kinematics.

In this paper we further develop and broadly extend our studies [24, 25] of the relations between TMDs and PDFs. The formulation of the model in terms of the light-cone formalism [23] allows us to compute the leading-twist TMDs by means of the light-front correlators $\phi(x, \mathbf{p}_T)_{ij}$ [2] as:

$$\frac{\mathrm{tr}\left[\gamma^+ \phi(x,\mathbf{p}_T)\right]}{2} = f_1^a(x,\mathbf{p}_T) - \frac{\varepsilon^{jk} p_T^j S_T^k}{M} f_{1T}^{\perp a}(x,\mathbf{p}_T), \tag{1}$$

$$\frac{\mathrm{tr}\left[\gamma^+ \gamma_5 \phi(x,\mathbf{p}_T)\right]}{2} = S_L g_1^a(x,\mathbf{p}_T) + \frac{\mathbf{p}_T \mathbf{S}_T}{M} g_{1T}^{\perp a}(x,\mathbf{p}_T), \tag{2}$$

$$\begin{aligned}\frac{\mathrm{tr}\left[i\sigma^{j+}\gamma_5 \phi(x,\mathbf{p}_T)\right]}{2} &= S_T^j\, h_1^a(x,\mathbf{p}_T) + S_L \frac{p_T^j}{M} h_{1L}^{\perp a}(x,\mathbf{p}_T) \\ &+ \frac{(p_T^j p_T^k - \frac{1}{2}\mathbf{p}_T^2 \delta^{jk}) S_T^k}{M^2} h_{1T}^{\perp a}(x,\mathbf{p}_T) \\ &+ \frac{\varepsilon^{jk} p_T^k}{M} h_1^{\perp a}(x,\mathbf{p}_T)\ .\end{aligned} \tag{3}$$

The main goal of this work is to derive relations between these TMDs and the usual PDFs $f_1(x)$ and $g_1(x)$. Similar tasks were recently addressed also in other approaches [36, 37]. The Section II is devoted to the case of the unpolarized TMD, and in Section III we discuss polarized TMDs. We use the obtained relations to calculate and discuss the numerical predictions for TMDs. The Section IV is devoted to our concluding remarks.

II. THE UNPOLARIZED TMD

The distribution $f_1^a(x, \mathbf{p}_T)$ is given in the covariant parton model approach by the expression [23]

$$\begin{aligned} f_1^a(x,\mathbf{p}_T) &= xM \int \frac{\mathrm{d}p^1}{p^0}\, G^a(p^0)\, \delta\left(\frac{p^0 - p^1}{M} - x\right) \\ &= M\, G^a(\bar{p}^0)\ . \end{aligned} \tag{4}$$

In the final step of (4) we performed the p^1-integration by rewriting the δ-function as

$$x\,\delta\left(\frac{p^0-p^1}{M}-x\right)=\bar{p}^0\,\delta(p^1-\bar{p}^1)\;; \qquad (5)$$
$$\bar{p}^0=\frac{1}{2}\,xM\left(1+\frac{\mathbf{p}_T^2+m^2}{x^2M^2}\right)\;,$$
$$\bar{p}^1=-\frac{1}{2}\,xM\left(1-\frac{\mathbf{p}_T^2+m^2}{x^2M^2}\right)\;.$$

The remarkable feature of the present parton model approach is that one can predict unambiguously the x- and $\mathbf{p}_T$-dependence of TMDs from the x-dependence of the corresponding ("integrated") parton distribution functions. The deeper reason for that is the equal (3D-symmetric in the nucleon rest frame) description of longitudinal (i.e. p^1-dependence) and transverse ($\mathbf{p}_T$-dependence) parton momenta. Let us remark, that the invariant parameter x (Bjorken x) is tightly connected to both longitudinal and transverse parton through the δ-function. In the nucleon rest frame transverse momenta play for x an important role according to (5).

The 3D momentum distribution $G^a(p^0)$ was expressed in terms of $f_1^a(x)$ in previous works [18, 21]. In order to make this work self-contained we present here an independent derivation. We start from the model expression for $f_1^a(x)$ which follows from the first equality in Eq. (4). For the remainder of this section we set the parton mass $m\to 0$. Besides being a reasonable approximation, this step greatly simplifies the calculation though finite m-effect can be included [19]. Notice that if m is neglected then $p^0=\sqrt{p_1^2+\mathbf{p}_T^2}\equiv p$. Now, instead of integrating over p^1 as we did in Eq. (4), it is convenient to use spherical coordinates, and define the angles such that $p^1=p\cos\theta$, i.e.

$$\begin{aligned}f_1^a(x)&=xM\int\frac{\mathrm{d}^3p}{p}\,G^a(p)\,\delta\left(\frac{p-p\,\cos\theta}{M}-x\right)\\&=xM\int_0^{2\pi}\mathrm{d}\phi\int_0^{\infty}\mathrm{d}p\;p\,G^a(p)\\&\times\int_{-1}^{1}\mathrm{d}\cos\theta\;\delta\left(\frac{p-p\,\cos\theta}{M}-x\right)\\&=2\pi\,xM^2\int_0^{\infty}\mathrm{d}p\;G^a(p)\;\Theta\left(p-\frac{1}{2}\,xM\right)\;.\end{aligned}\qquad (6)$$

The Θ-function emerges because the integral over the δ-function obviously yields a non-zero result only if $|\cos\theta|<1$, and implies a lower limit for p-integral. Notice that there is also an upper limit, namely $p<\frac{1}{2}M$, related to the fact that $x<1$ [18]. This upper limit is natural in the covariant parton model in the nucleon rest frame because an on-shell parton can carry at most the momentum $p_{\max}=\frac{1}{2}M$ which must be compensated by all other partons going in the opposite direction, such that the center of mass of the nucleon remains at rest. Since this is an unlikely constellation the momentum distribution $G^a(p)$ vanishes as $p\to p_{\max}$ similarly as $f_1^a(x)$ drops to zero with $x\to 1$. Thus we obtain

$$\frac{f_1^a(x)}{x}=2\pi M^2\int_{\frac{1}{2}\,xM}^{\frac{1}{2}\,M}\mathrm{d}p\;G^a(p) \qquad (7)$$

and we reproduce the identity [18, 21]

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{f_1^a(x)}{x}\right]=-\pi M^3\;G^a\left(\frac{xM}{2}\right). \qquad (8)$$

This result inserted in (4) enables us to predict uniquely $f_1^a(x,\mathbf{p}_T)$ from $f_1^a(x)$ as follows

$$f_1^a(x,\mathbf{p}_T)=-\,\frac{1}{\pi M^2}\,\frac{\mathrm{d}}{\mathrm{d}y}\left[\frac{f_1^a(y)}{y}\right]_{y=\xi(x,\mathbf{p}_T^2)} \qquad (9)$$

with the dependence on x, $\mathbf{p}_T$ given through the variable

$$\xi(x,\mathbf{p}_T^2)=\lim_{m\to 0}\frac{2\bar{p}^0}{M}=x\left(1+\frac{\mathbf{p}_T^2}{x^2M^2}\right)\;. \qquad (10)$$

The variable $\xi(x,\mathbf{p}_T^2)$, first suggested in [17], relates the (very different!) dependencies on longitudinal and transverse momenta so that the factorization of these dependencies can be only approximate. It is in some sense similar to the Nachtmann variable in DIS which controls some (kinematical) part of higher twist contributions. The reason for the appearance of such a variable is deeply related to the general properties of our model like Lorentz invariance [17, 36] and the on-shellness of partons.

Using as input for $f_1^a(x)$ the LO parameterization of [38] at the scale $4\,\mathrm{GeV}^2$, we obtain for $u, d, \bar{u}$ and $\bar{d}$-quarks the results shown in Fig. 1. The right panel of this figure is shown again, in a different scale in Fig. 2.

We make the following observations:

i) For fixed x, the p_T-distributions are similar to the Gauss Ansatz $f_1^a(x,p_T)\propto\exp\left(-p_T^2/\langle p_T^2\rangle\right)$. This is an interesting result, since the Gaussian shape is supported by phenomenology [41].

ii) The width $\langle p_T^2\rangle$ depends on x. This result reflects the fact, that in our approach the parameters x and p_T are not independent due to rotational symmetry. The x-dependence of $\langle p_T^2\rangle$ is, however, moderate over a wide range of x.

iii) The widths $\langle p_T^2\rangle$ also depend on the parton type. The widths of u- and d-quarks are similar. Also the widths of $\bar{u}$ and $\bar{d}$ are similar. But the widths of antiquarks are systematically smaller than those of quarks. This is opposite to the results from the chiral quark soliton model [32].

iv) The Figs. 1, 2 suggest that the typical values for transverse momenta of u and d-quarks are $\langle p_T\rangle\approx 0.1\,\mathrm{GeV}$. These values correspond to the estimates based on the different analyses of the structure function $F_2(x,Q^2)$ [24], or higher twist terms [39, 40]. On the other hand, much larger values $\langle p_T\rangle\sim 0.6\,\mathrm{GeV}$ are

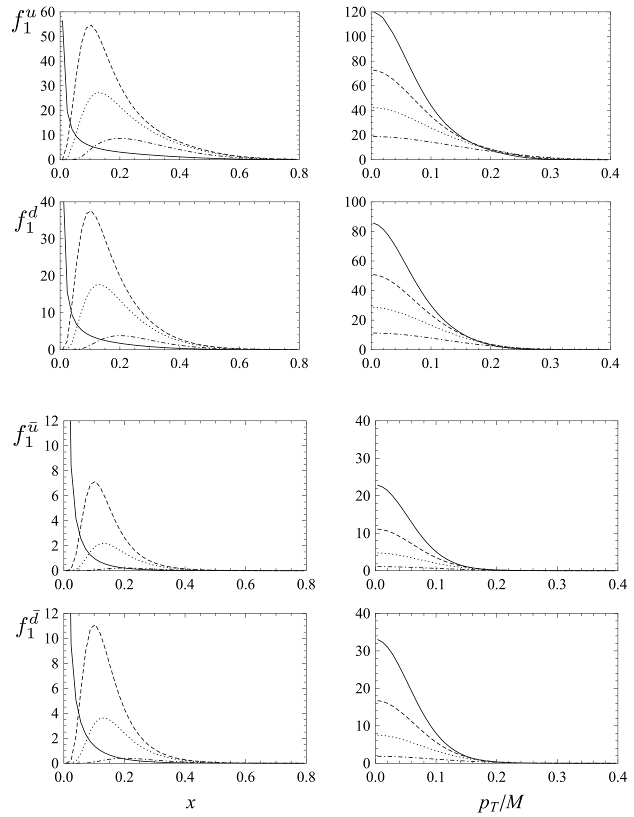

FIG. 1: The TMDs $f_1^a(x, \mathbf{p}_T)$ for u, d (*upper part*) and $\bar{u}, \bar{d}$-quarks (*lower part*). **Left panel**: $f_1^a(x, \mathbf{p}_T)$ as function of x for $p_T/M = 0.10$ (dashed), 0.13 (dotted), 0.20 (dashed-dotted line). The solid line corresponds to the input distribution $f_1^a(x)$. **Right panel**: $f_1^a(x, \mathbf{p}_T)$ as function of p_T/M for $x =$ 0.15 (solid), 0.18 (dashed), 0.22 (dotted), 0.30 (dashed-dot line).

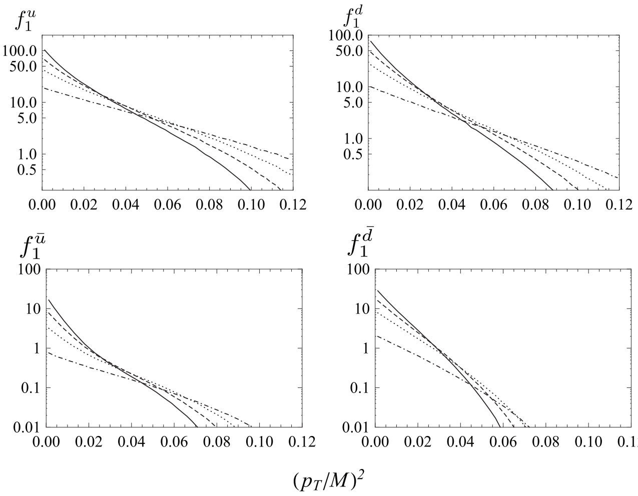

FIG. 2: $f_1^a(x, \mathbf{p}_T)$ as function of $(p_T/M)^2$ for $x = 0.15$ (solid), 0.18 (dashed), 0.22 (dotted), 0.30 (dash-dotted line).

inferred from SIDIS data referring to comparable scales [41–43].

It is interesting to remark that in applications of the Collins-Soper-Sterman formalism [45] for evolving TMDs, such as [46], the Gaussian width of the non-perturbative transverse momentum dependence is only weakly x-dependent, and the widths of quarks and antiquarks are the same when taking into consideration proton-proton data. The first remark is in qualitative agreement with our observation in point *ii)*. The second remark, however, seems in conflict with our observation in *iii)*. Noteworthy, the chiral quark soliton model offers yet another scenario and an opposite picture to the present approach, namely $\langle p_T^2 \rangle$ of sea quarks is there larger than for quarks [32]. More phenomenological work is needed to clarify the situation. An important step in this context was recently done in [47].

Another interesting point concerns the question why the $\langle p_T \rangle$ in the present approach are so much smaller than what seems required in SIDIS phenomenology [41–43]. We notice that this seems to be a more general feature of the parton model approach [24, 39, 40], including its extended version along the lines of [48], and could be intuitively understood by recalling that parton models imply the absence of interactions and confinement, which would localize the quarks (in the transverse plane) and imply larger transverse momenta by means of the uncertainty principle.

These very interesting points deserve further studies which, however, would go beyond the scope of the present work. We also note that in the statistical model of TMDs [37, 44] the parameter $\langle p_T \rangle$ may be interpreted as an effective temperature of the partonic "ensemble" [49]. It is instructive to compare this number to the lattice calculations [50] of the QCD phase transition temperature $T \approx 175$ MeV.

III. THE POLARIZED TMDs

All polarized leading-twist T-even TMDs are described in terms of the *same* polarized covariant 3D distribution $H^a(p^0)$. This follows from the compliance of the approach with relations following from QCD equations of motion [23]. As a consequence all polarized TMDs can be expressed in terms of a single "generating function"

$K^a(x, \mathbf{p}_T)$ as follows

$$
\begin{aligned}
g_1^a(x,\mathbf{p}_T) &= \frac{1}{2x}\left(\left(x+\frac{m}{M}\right)^2-\frac{\mathbf{p}_T^2}{M^2}\right) \times K^a(x,\mathbf{p}_T),\\
h_1^a(x,\mathbf{p}_T) &= \frac{1}{2x}\left(x+\frac{m}{M}\right)^2 \times K^a(x,\mathbf{p}_T),\\
g_{1T}^{\perp a}(x,\mathbf{p}_T) &= \frac{1}{x}\left(x+\frac{m}{M}\right) \times K^a(x,\mathbf{p}_T),\\
h_{1L}^{\perp a}(x,\mathbf{p}_T) &= -\frac{1}{x}\left(x+\frac{m}{M}\right) \times K^a(x,\mathbf{p}_T),\\
h_{1T}^{\perp a}(x,\mathbf{p}_T) &= -\frac{1}{x} \times K^a(x,\mathbf{p}_T).
\end{aligned} \tag{11}
$$

with the "generating function" $K^a(x, \mathbf{p}_T)$ defined (in the compact notation introduced in Eq. (32) of [23]) by

$$
K^a(x,\mathbf{p}_T) = M^2 x \int \mathrm{d}\{p^1\}\,; \tag{12}
$$

$$
\mathrm{d}\{p^1\} \equiv \frac{\mathrm{d}p^1}{p^0}\,\frac{H^a(p^0)}{p^0+m}\,\delta\left(\frac{p^0-p^1}{M}-x\right)\,.
$$

We remark that in order to rewrite $g_1^a(x, \mathbf{p}_T) = \int \mathrm{d}\{p^1\}\,[(p^0 + m)xM - \mathbf{p}_T^2]$ [23] as shown in the first equation of (11), we used the identity $p^0 = (\mathbf{p}_T^2 + x^2M^2 + m^2)/(2xM)$ valid under the p^1-integral, which holds because in the model the partons are on-shell, i.e. $p_0^2 - p_1^2 - \mathbf{p}_T^2 = m^2$, and $p^0 - p^1 = xM$ due to the delta-function. The expressions for the other TMDs in (11) can be read off directly from Eqs. (35-38) in [23]. Using the identity (5) we perform the p^1-integration in (12) and obtain for the generating function, which depends on x, $\mathbf{p}_T$ only via $\bar{p}^0$ defined in Eq. (5):

$$
K^a(x,\mathbf{p}_T) = M^2\frac{H^a(\bar{p}^0)}{\bar{p}^0+m}\,. \tag{13}
$$

From (11–13) it is clear that we can predict all polarized TMDs if we know $H^a(p^0)$. The polarized 3D momentum distribution $H^a(p^0)$ could be determined in principle from any polarized TMD, but the helicity parton distribution function $g_1^a(x)$ plays a special role, because its x-dependence is known. The connection of $g_1^a(x)$ and $H^a(p^0)$ was derived previously in [18, 21]. In order to make this work self-contained we present here an independent derivation.

We start from the expression for $g_1^a(x)$ which follows from (11) and proceed as in Eq. (6), i.e. we neglect m and use spherical coordinates such that $p^1 = p\cos\theta$ and $\mathbf{p}_T^2 = p^2\sin^2\theta$, i.e.

$$
\begin{aligned}
g_1^a(x) &= \int \frac{\mathrm{d}^3p}{2\,p^2}\,H^a(p)\,(x^2M^2 - p^2\sin^2\theta)\\
&\quad\times\delta\left(\frac{p-p\cos\theta}{M}-x\right)\\
&= \int_0^{2\pi}\mathrm{d}\phi\int\frac{\mathrm{d}p}{2}\,H^a(p)\\
&\quad\times\int_{-1}^{1}\mathrm{d}\cos\theta\,(x^2M^2-p^2\sin^2\theta)\delta\left(\frac{p-p\cos\theta}{M}-x\right)\\
&= 2\pi M^2\int\mathrm{d}p\,H^a(p)\,\frac{x}{p}(xM-p)\,\Theta\left(p-\frac{1}{2}\,xM\right)\,.
\end{aligned} \tag{14}
$$

where the Θ-function emerges in the same way it did in Eq. (6). Consequently, we obtain

$$
g_1^a(x) = 2\pi\,M^2 x\left(xM\int_{\frac{1}{2}xM}^{\frac{1}{2}M}\frac{\mathrm{d}p}{p}\,H^a(p) - \int_{\frac{1}{2}xM}^{\frac{1}{2}M}\mathrm{d}p\,H^a(p)\right)\,. \tag{15}
$$

Differentiating this equation and integrating it by parts gives the following results

$$
\begin{aligned}
x\,\frac{\mathrm{d}g_1^a(x)}{\mathrm{d}x} &= 2\pi\,M^2x\left(2\,xM\int_{\frac{1}{2}xM}^{\frac{1}{2}M}\frac{\mathrm{d}p}{p}\,H^a(p)\right.\\
&\quad\left.-\int_{\frac{1}{2}xM}^{\frac{1}{2}M}\mathrm{d}p\,H^a(p) - \frac{xM}{2}H^a\left(\frac{xM}{2}\right)\right)\,,\\
\int_x^1\frac{\mathrm{d}y}{y}\,g_1^a(y) &= 2\pi\,M^2x\left(\frac{-xM}{2}\int_{\frac{1}{2}xM}^{\frac{1}{2}M}\frac{\mathrm{d}p}{p}\,H^a(p)\right.\\
&\quad\left.+\int_{\frac{1}{2}xM}^{\frac{1}{2}M}\mathrm{d}p\,H^a(p)\right)\,.
\end{aligned} \tag{16}
$$

Now we see that if we take the linear combination $2\int_x^1 \mathrm{d}y\,g_1^a(y)/y + 3\,g_1^a(x) - x\,g_1^{a\,\prime}(x)$ the integral terms from the last three equations cancel out, and we obtain for $p^2H^a(p)$ at $p = \frac{M}{2}x$ the expression

$$
\pi x^2M^3H^a\left(\frac{M}{2}x\right) = 2\int_x^1\frac{\mathrm{d}y}{y}\,g_1^a(y)+3\,g_1^a(x)-x\,\frac{\mathrm{d}g_1^a(x)}{\mathrm{d}x}\,, \tag{17}
$$

which confirms previous works [18, 21]. For the generating function (13) we obtain, in the limit $m \to 0$, the result

$$
\begin{aligned}
K^a(x,\mathbf{p}_T) &= M^2\frac{H^a(\frac{M}{2}\xi)}{\frac{M}{2}\xi} \qquad (18)\\
&= \frac{2}{\pi\xi^3M^2}\left(2\int_\xi^1\frac{\mathrm{d}y}{y}\,g_1^a(y)+3\,g_1^a(\xi)-x\,\frac{\mathrm{d}g_1^a(\xi)}{\mathrm{d}\xi}\right)
\end{aligned}
$$

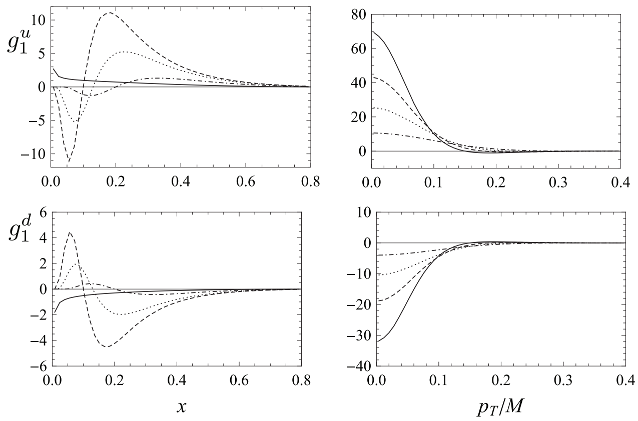

FIG. 3: $g_1^q(x, \mathbf{p_T})$ for u- (*upper panel*) and d-quarks (*lower panel*). **Left panel**: $g_1^q(x, \mathbf{p_T})$ as function of x for $p_T/M = 0.10$ (dashed), 0.13 (dotted), 0.20 (dash-dotted line). The solid line corresponds to the input distribution $g_1^q(x)$. **Right panel**: $g_1^q(x, \mathbf{p_T})$ as function of p_T/M for $x = 0.15$ (solid), 0.18 (dashed), 0.22 (dotted), 0.30 (dash-dotted line).

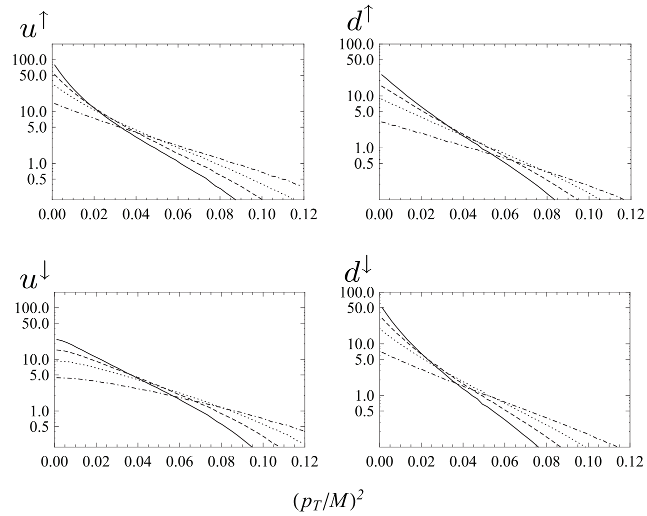

FIG. 4: The TMDs $q^\uparrow(x, \mathbf{p_T}) = \frac{1}{2}(f_1^q + g_1^q)(x, \mathbf{p_T})$ (*upper panel*) and $q^\downarrow(x, \mathbf{p_T}) = \frac{1}{2}(f_1^q - g_1^q)(x, \mathbf{p_T})$ (*lower panel*) as functions of p_T^2/M^2 for fixed values of $x = 0.15$ (solid), 0.18 (dashed), 0.22 (dotted), 0.30 (dash-dotted lines). **Left panel**: u-flavor. **Right panel**: d-flavor.

and from (11) we obtain (in agreement with the result reported in the proceeding [25] which was derived independently)

$$g_1^a(x, \mathbf{p}_T) = \frac{2x - \xi}{\pi \xi^3 M^2} \times \left(2 \int_\xi^1 \frac{\mathrm{d}y}{y}\, g_1^a(y) + 3\, g_1^a(\xi) - \xi\, \frac{\mathrm{d}g_1^a(\xi)}{\mathrm{d}\xi} \right). \tag{19}$$

In (18, 19) and also below in (20) we use the variable $\xi = \xi(x, \mathbf{p}_T)$ as defined in (10).

Eq. (19) yields for $g_1^a(x, \mathbf{p}_T)$, with the LO parameterization of [51] for $g_1^a(x)$ at $4\,\mathrm{GeV}^2$, the results shown in Fig. 3. The remarkable observation is that $g_1^a(x, \mathbf{p}_T)$ changes sign at the point $p_T = Mx$, which is due to the prefactor

$$2x - \xi = x \left(1 - \left(\frac{p_T}{Mx} \right)^2 \right) = -2\bar{p}^1/M \tag{20}$$

in (19). The expression in (20) is proportional to the quark longitudinal momentum $\bar{p}^1$ in the proton rest frame, which is determined by x and p_T, see Eq. (5). This means, that the sign of $g_1^a(x, p_T)$ is controlled by sign of $\bar{p}^1$. To observe these dramatic sign changes one may look for multi-hadron jet-like final states in SIDIS. Performing the cutoff for transverse momenta from below and from above, respectively, should effect the sign of asymmetry.

There is some similarity to $g_2^a(x)$ which also changes sign, and is given in the model by the expression [21]

$$g_2^a(x) = \frac{1}{2} \int H^a(p^0) \left(p^1 - \frac{(p^1)^2 - p_T^2/2}{p^0 + m} \right) \times \delta \left(\frac{p^0 - p^1}{M} - x \right) \frac{d^3 p}{p^0}. \tag{21}$$

The δ−function implies that, for our choice of the light-cone direction, large x are correlated with large and negative p^1, while low x are correlated with large and positive p^1. Thus, $g_2^a(x)$ changes sign, because the integrand in (21) changes sign between the extreme values of p^1. Let us remark, that the calculation of $g_2(x)$ based on the relation (21) well agrees [19] with the experimental data.

While the $g_1^q(x, \mathbf{p_T})$ exhibit zeros and show no Gaussian behavior, the functions $q^\uparrow(x, \mathbf{p_T}) = \frac{1}{2}(f_1^q + g_1^q)(x, \mathbf{p_T})$ and $q^\downarrow(x, \mathbf{p_T}) = \frac{1}{2}(f_1^q - g_1^q)(x, \mathbf{p_T})$ approximately support it, see Fig. 4, though again with x-dependent Gauss widths $\langle \mathbf{p}_T^2 \rangle$ which reflects the correlation of longitudinal and transverse momenta in the approach. We refrain from discussing analogous antiquark distributions $\bar{q}^\uparrow(x, \mathbf{p_T})$ and $\bar{q}^\downarrow(x, \mathbf{p_T})$ which are not well constrained due to the less well known helicity sea quark distributions.

The other TMDs (11) can be calculated similarly and differ, in the limit $m \to 0$, by simple x-dependent pref-

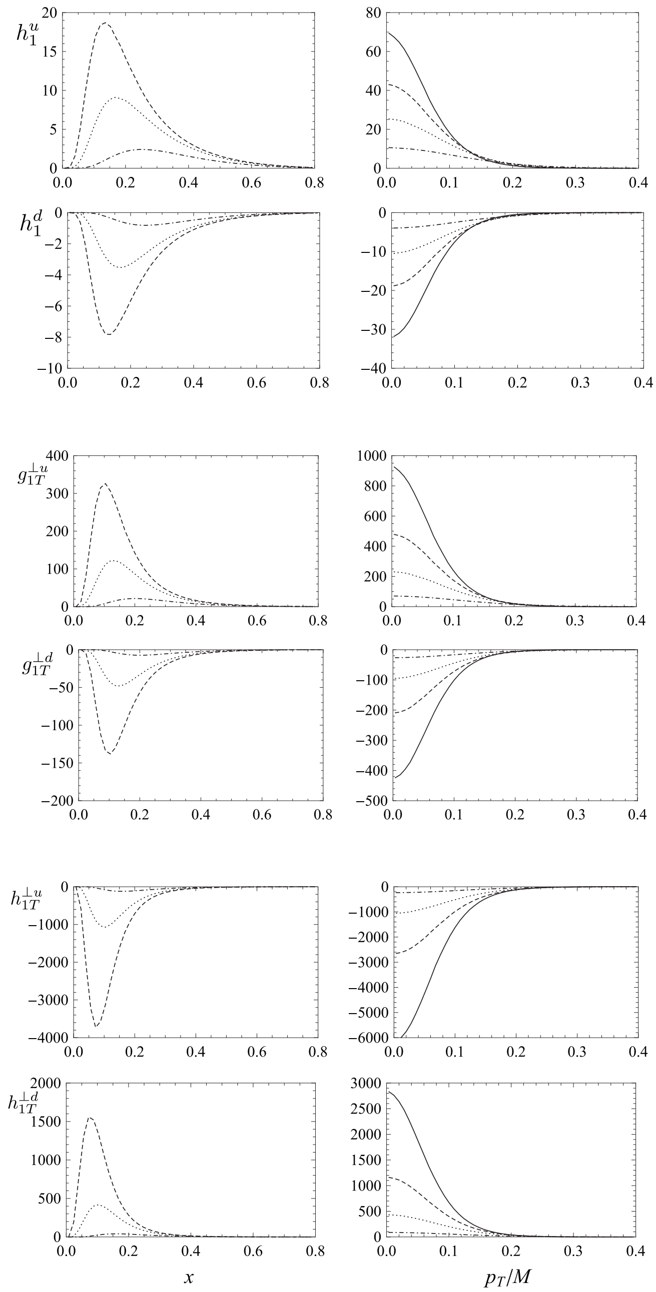

FIG. 5: The TMDs $h_1^q(x, \mathbf{p_T})$, $g_{1T}^{\perp q}(x, \mathbf{p_T})$, $h_{1T}^{\perp q}(x, \mathbf{p_T})$ for u- and d-quarks. **Left panel**: The TMDs as functions of x for $p_T/M = 0.10$ (dashed), 0.13 (dotted), 0.20(dash-dotted lines). **Right panel**: The TMDs as functions of p_T/M for $x = 0.15$ (solid), 0.18 (dashed), 0.22 (dotted), 0.30 (dash-dotted lines).

actors

$$\begin{aligned} h_1^a(x, \mathbf{p}_T) &= \frac{x}{2} K^a(x, \mathbf{p}_T) \,, \qquad (22) \\ g_{1T}^{\perp a}(x, \mathbf{p}_T) &= K^a(x, \mathbf{p}_T) \,, \\ h_{1L}^{\perp a}(x, \mathbf{p}_T) &= -K^a(x, \mathbf{p}_T) \,, \\ h_{1T}^{\perp a}(x, \mathbf{p}_T) &= -\frac{1}{x} K^a(x, \mathbf{p}_T) \,. \end{aligned}$$

The results are shown in Fig. 5. We do not plot $h_{1L}^{\perp a}$ since this TMD is equal to $-g_{1T}^{\perp a}$ in our approach [23]. Let us remark that $g_1^a(x, \mathbf{p}_T)$ is the only TMD which can change sign. The other TMDs have all definite signs, which follows from (11),(22). Note also that pretzelosity $h_{1T}^{\perp a}(x, \mathbf{p}_T)$, due to the prefactor $1/x$, has the largest absolute value among all TMDs.

IV. CONCLUDING REMARKS

We have studied relations between the TMDs $f_1^a(x, \mathbf{p}_T)$, $g_1^a(x, \mathbf{p}_T)$, $h_1^a(x, \mathbf{p}_T)$, $g_{1T}^{\perp a}(x, \mathbf{p}_T)$, $h_{1L}^{\perp a}(x, \mathbf{p}_T)$, $h_{1T}^{\perp a}(x, \mathbf{p}_T)$ and the PDFs $f_1^a(x)$, $g_1^a(x)$. These relations follow in the covariant parton model as a consequence of Lorentz invariance and rotational symmetry of parton momenta in the nucleon rest frame. These relations used with parameterizations of PDFs give numerical predictions for all T-even (in this work: leading twist) TMDs. In fact we have demonstrated, that in the framework of the model there are fundamental distributions covariant momentum distributions $G^a(p)$ and $H^a(p)$ from which all the considered unpolarized and polarized PDFs and TMDs can be uniquely obtained. And vice versa, from any PDF or TMD one can unambiguously calculate the corresponding G^a or H^a.

Some of our results are compatible with the results of the recent paper [36]. In spite of some differences, both approaches have an important common basis consisting in the Lorentz invariance. For a more detailed comparison of the two approaches we refer to [24]. Our predictions are consistent also with the results obtained in the recent study [37], some quantitative differences between these two approaches are discussed in the cited paper.

To conclude, let us remark that an experimental check of the predicted TMDs requires care. In fact, TMDs are not directly measurable quantities unlike structure functions. What one can measure for instance in semi-inclusive DIS is a convolution with a quark fragmentation function. This naturally "dilutes" the effects of TMDs, and makes it difficult to observe for instance the prominent sign change in the helicity distribution, see Fig. 3. A dedicated study of the phenomenological implications of our results is in progress.

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