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L. Frankfurt, M. Strikman, and C. Weiss

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# Transverse nucleon structure and diagnostics of hard parton–parton processes at LHC

L. Frankfurt,[1] M. Strikman,[2] and C. Weiss[3]

[1] *School of Physics and Astronomy, Tel Aviv University, Tel Aviv, Israel*
[2] *Department of Physics, Pennsylvania State University, University Park, PA 16802, USA*
[3] *Theory Center, Jefferson Lab, Newport News, VA 23606, USA*

We propose a new method to determine at what transverse momenta particle production in high–energy $pp$ collisions is governed by hard parton–parton processes. Using information on the transverse spatial distribution of partons obtained from hard exclusive processes in $ep/\gamma p$ scattering, we evaluate the impact parameter distribution of $pp$ collisions with a hard parton–parton process as a function of $p_T$ of the produced parton (jet). We find that the average $pp$ impact parameters in such events depend very weakly on $p_T$ in the range $2 < p_T <$ few 100 GeV, while they are much smaller than those in minimum–bias inelastic collisions. The impact parameters in turn govern the observable transverse multiplicity in such events (in the direction perpendicular to the trigger particle or jet). Measuring the transverse multiplicity as a function of $p_T$ thus provides an effective tool for determining the minimum $p_T$ for which a given trigger particle originates from a hard parton–parton process. Additional tests of the proposed geometric correlations are possible by measuring the dependence on the trigger rapidity. Various strategies for implementing this method are outlined.

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## I. INTRODUCTION

The first experimental results from LHC once again raise the question at what transverse momenta particle production in $pp$ collisions is dominated by hard parton–parton interactions. A quantitative understanding of the relevant mechanisms is important not only for future studies of QCD phenomena, but also for controlling the strong interaction background in new particle searches. The challenge lies in the fact that the growth of the average multiplicities makes it very difficult to observe jets with moderate $p_T$, while at the same time the properties of non–perturbative semi–hard dynamics and its ability to produce particles with $p_T \sim$ few GeV are not well understood.

In an earlier article [1], we demonstrated that the nucleon's transverse partonic structure plays an essential role in the theoretical analysis of $pp$ collisions with hard processes. Experiments in hard exclusive electroproduction of vector mesons $\gamma^* p \to V + p$ and photoproduction of heavy quarkonia $\gamma p \to J/\psi + p$ have shown that the gluons with $10^{-4} < x < 10^{-1}$ are localized at small transverse distances of $0.4 - 0.5\,\text{fm}$ (median, depending on $x$ and $Q^2$), much smaller than the characteristic range of soft interactions at high energies, see Fig. 1a. Qualitatively, this is explained by the fact that Gribov diffusion in the partonic wave function, which causes the range of soft interactions to grow with energy [2], is suppressed for highly virtual constituents. In $pp$ scattering this two–scale picture implies that hard processes mostly occur in central collisions, where the areas occupied by partons in the relevant $x$–range overlap. Peripheral collisions constitute the dominant part of the overall inelastic cross section without contributing much to inclusive jet production, see Fig. 1b. A trigger on a hard process thus, on average, selects central $pp$ collisions [1]. Numerical studies show that at a center–of–mass energy $\sqrt{s} = 14\,\text{TeV}$ a dijet trigger on $p_T \sim$ few 10 GeV reduces the median $pp$ impact parameter $b$ by a factor of $\sim 2$ compared to minimum–bias inelastic collisions; the reduction is nearly as strong at the current LHC energy of 7 TeV (see below).

Here we point out that these insights into the transverse geometry of $pp$ collisions can be used to address the

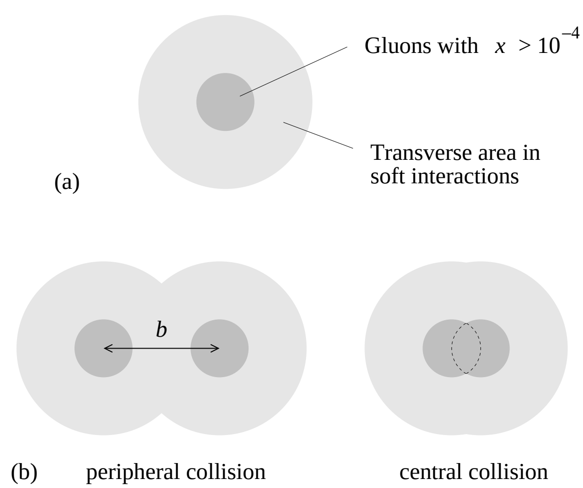


FIG. 1: (a) The two–scale picture of transverse nucleon structure at high energies. The drawing shows the transverse view; the fast nucleon's momentum points in the direction perpendicular to the plane. (b) Its implication for $pp$ collisions. Peripheral collisions constitute the dominant part of the overall inelastic cross section. Hard processes happen predominantly in central collisions, where the areas occupied by large–$x$ partons overlap. Here the momenta of the colliding nucleons point into and out of the plane, respectively.

question at what transverse momenta particle production is governed by hard parton–parton processes. The key observation is that the transverse multiplicity, measured in the direction perpendicular to the transverse momentum of the trigger particle (or jet), is correlated with the average impact parameter in the underlying $pp$ event. If the trigger particle originates from a hard parton–parton process, the underlying $pp$ collision is central, and the average impact parameter depends only weakly on $p_T$. The transverse multiplicity thus should be practically independent of $p_T$, and substantially larger than that in minimum bias events. If, however, the trigger particle is produced by soft interactions, the transverse multiplicity should be substantially smaller and reflect the average multiplicity in minimum–bias inelastic collisions. In this sense, the transverse multiplicity can serve as a diagnostic of the dynamics in the production of the trigger particle at given $p_T$. First theoretical suggestions for studies of the transverse multiplicity were put forward in Ref. [3]. Experimental investigation of the correlation between the jet production and the structure of the underlying event were pioneered by the CDF experiment at the Tevatron [4]. The first data on underlying event structure in collisions with hard processes at the LHC were recently reported by ATLAS [5] and CMS [6, 7].

Additional tests of the geometric correlations described here become possible with measurements of the dependence of the transverse multiplicity on the rapidity of the trigger jets. In particular, we predict that in the rapidity region not affected by the fragmentation of the trigger jets the enhancement of the multiplicity will persist and be isotropic in transverse space. In this way one could verify the universality of particle production in the central $pp$ collisions selected by hard processes.

This article is organized as follows. In Sec. II we summarize our knowledge of the nucleon's transverse partonic structure and update our parametrization of the transverse gluonic size as a function of $x$. In Sec. III we use this information to study the impact parameter distributions of $pp$ events with hard processes in dependence on the transverse momentum $p_T$ of the trigger particle (jet), for the kinematics currently covered by LHC, and find that the median $b$ weakly depends on the trigger $p_T$ and is substantially smaller than that in minimum–bias inelastic collisions. In Sec. IV we discuss the connection between centrality and the transverse multiplicity, and how measurement of the latter provides an effective means of quantifying at what $p_T$ particle production is dominated by hard QCD processes. In Sec. V we consider the dependence of the multiplicity on the rapidity of the trigger, and how it can be used for additional tests of the dominance of small impact parameters in $pp$ collisions with hard processes. In Sec. VI we present several suggestions for further analysis of the $pp$ event structure data. A summary and discussion of our results are given in Sec. VII.

Our analysis relies on information on the nucleon's transverse partonic structure obtained from hard exclusive processes in $ep/\gamma p$ scattering. Extending our previous study [1], we present here an updated parametrization of the transverse distribution of gluons, which takes into account the more recent HERA data [8, 9] and permits realistic uncertainty estimates. The numerical results are nevertheless close to those obtained in our previous study.

Current Monte Carlo (MC) generators for $pp$ events usually do not take into account the available experimental information on transverse nucleon structure and treat the distribution of gluons over transverse position as a free function. The typical setting for PYTHIA [10] and HERWIG [11] correspond to a transverse area occupied by gluons which is a factor $\sim 2$ smaller than what is indicated by the HERA data (see below). In the analysis of experimental data, the shape of the transverse gluon distribution is usually treated as one of the tuning parameters, see e.g. Refs. [4, 6]. While we do not directly address these technical issues here, our results certainly have implications for the design of future MC generators for $pp$ events at LHC.

## II. TRANSVERSE PARTONIC STRUCTURE OF THE NUCLEON

Information on the transverse spatial distribution of gluons in the nucleon comes from the study of hard exclusive processes such as electroproduction of vector mesons, $\gamma^* p \to V + p$, or the photoproduction of heavy quarkonia, $\gamma p \to J/\psi + p$. Thanks to a QCD factorization theorem [12], the amplitude of these processes in the leading–twist approximation can be expressed in terms of the gluon generalized parton distribution (or GPD), which parametrizes the matrix element for the emission and absorption of a gluon by the target at a given normalization scale $Q^2$. Of particular interest is the GPD in the "diagonal" case, $g(x, t|Q^2)$, where the longitudinal momentum fractions of the emitted and absorbed gluon are the same and denoted by $x$, and the momentum transfer to the nucleon, $\boldsymbol{\Delta}_\perp$, is in the transverse direction, with $t = -\boldsymbol{\Delta}_\perp^2$ (we follow the notation of Ref. [1]). This function reduces to the usual gluon density in the nucleon in the limit of zero momentum transfer, $g(x, t = 0|Q^2) = g(x|Q^2)$. The two–dimensional Fourier transform of the diagonal GPD,

$$g(x, \rho|Q^2) \;\equiv\; \int \frac{d^2\Delta_\perp}{(2\pi)^2}\, e^{i(\boldsymbol{\Delta}_\perp \boldsymbol{\rho})}\, g(x, t = -\boldsymbol{\Delta}_\perp^2|Q^2), \quad (1)$$

describes the one–body density of gluons with given longitudinal momentum fraction $x$ in transverse space, with $\rho \equiv |\boldsymbol{\rho}|$ measuring the distance from the transverse center–of–momentum of the nucleon, and is normalized such that

$$\int d^2\rho\, g(x, \rho|Q^2) \;=\; g(x|Q^2). \quad (2)$$

For the purpose of our analysis it is convenient to separate the information on the total density of gluons from their spatial distribution and parametrize the diagonal GPD in the form

$$g(x, t|Q^2) \;=\; g(x|Q^2)\, F_g(x, t|Q^2), \tag{3}$$

where the latter function satisfies $F_g(x, t = 0|Q^2) = 1$ and is known as the two–gluon form factor of the nucleon [1]. Its Fourier transform describes the normalized spatial distribution of gluons with given $x$,

$$F_g(x, \rho|Q^2) \;\equiv\; \int \frac{d^2\Delta_\perp}{(2\pi)^2}\, e^{i(\boldsymbol{\Delta}_\perp \boldsymbol{\rho})}\, F_g(x, t = -\boldsymbol{\Delta}_\perp^2|Q^2), \tag{4}$$

with $\int d^2\rho\, F_g(x, \rho|Q^2) = 1$ for any $x$.

Experiments in hard exclusive processes actually probe the gluon GPD in the non–diagonal case (different momentum fraction of the emitted and absorbed gluon), because of the longitudinal momentum transfer required by kinematics. At $x \lesssim 10^{-2}$, QCD evolution allows one to relate the non-diagonal gluon GPD to the diagonal one at the input scale, and the diagonal two–gluon form factor can be directly inferred from the $t$–dependence of the measured cross sections. At larger $x$, the relation between the non–diagonal and diagonal GPDs generally becomes model–dependent, but useful information can still be extracted using GPD parametrizations.

The $t$–dependence of the measured differential cross sections of exclusive processes at $|t| < 1\,\mathrm{GeV}^2$ is commonly described either by an exponential, or by a dipole form inspired by analogy with the nucleon elastic form factors. Correspondingly, we consider here two parametrizations of the two–gluon form factor:

$$F_g(x, t|Q^2) \;=\; \begin{cases} \exp(B_g t/2), \\ (1 - t/m_g^2)^{-2}, \end{cases} \tag{5}$$

where the parameters $B_g$ and $m_g$ are functions of $x$ and $Q^2$. The two parametrizations give very similar results if the functions are matched at $|t| = 0.5\,\mathrm{GeV}^2$, where they are best constrained by present data (see Fig. 3 of Ref. [13]); this corresponds to

$$B_g \;=\; 3.24/m_g^2. \tag{6}$$

We use both parametrizations in our studies below; the difference between the results serves as an estimate of the uncertainty due to our lack of precise knowledge of the shape. The corresponding spatial distributions of gluons in the transverse plane, Eq. (4), are given by

$$F_g(x, \rho|Q^2) \;=\; \begin{cases} (2\pi B_g)^{-1} \exp[-\rho^2/(2B_g)], \\ [m_g^2/(2\pi)]\, (m_g\rho/2)\, K_1(m_g\rho), \end{cases} \tag{7}$$

where $K_1$ denotes the modified Bessel function.

Most of the experimental information on the nucleon's two–gluon form factor comes from $J/\psi$ photoproduction, which probes the gluon GPD at an effective scale $Q^2 \approx 3\,\mathrm{GeV}^2$, determined by the average transverse size

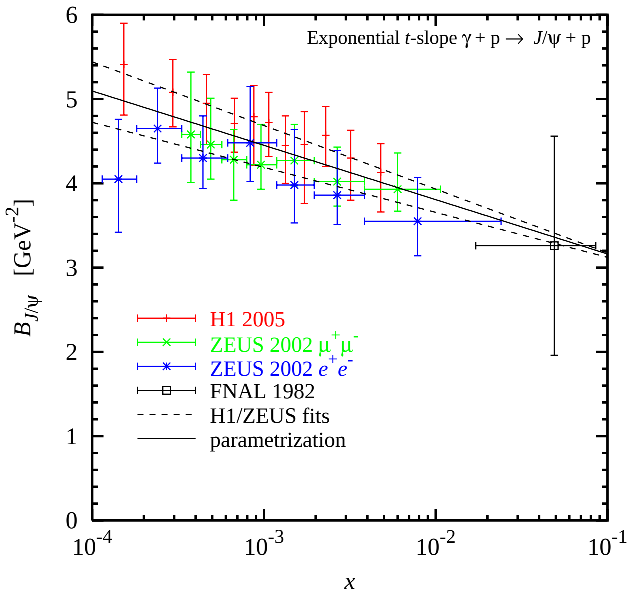


FIG. 2: The exponential $t$–slope, $B_{J/\psi}$, of the differential cross section of exclusive $J/\psi$ photoproduction measured in the FNAL E401/E458 [16], HERA H1 [9], and ZEUS [17] experiments, as a function of $x = M_{J/\psi}^2/W^2$. (In the H1 and ZEUS results the quoted statistical and systematic uncertainties were added linearly.) The dashed lines represent the published two–dimensional fits to the H1 and ZEUS data [9, 17]. The parameter $B_g$ in the exponential two–gluon form factor Eq. (5) is related to the measured $J/\psi$ slope by Eq. (8). Our parametrization Eqs. (9)–(12) is shown by the solid line.

of the $c\bar{c}$ pair during its interaction with the target, and momentum fractions of the order $x \sim M_{J/\psi}^2/W^2$ [14]. When extracting the two–gluon form factor from the slope of the $J/\psi$ differential cross section, a correction is made for the effect of the finite $J/\psi$ size on the observed $t$–dependence,

$$B_{J/\psi} \;=\; B_g + \Delta B, \tag{8}$$

where $\Delta B \approx 0.3\,\mathrm{GeV}^2$ from the dipole model estimate of Ref. [14] (see also Ref. [15]).

The data from the FNAL E401/E458 broadband beam experiment at $\langle x \rangle = 0.05$ [16], in which the recoiling proton was detected, are described by an exponential two–gluon form factor with $B_g = 3.0\,\mathrm{GeV}^{-2}$ (see Fig. 2), albeit with large errors, or a corresponding dipole form factor with $m_g^2 = 1.1\,\mathrm{GeV}^2$. Comparison with the mass parameter in the dipole parametrization of the nucleon's electromagnetic (Dirac) form factor, $m_{\mathrm{em}}^2 = 0.7\,\mathrm{GeV}^2$, indicates that at these values of $x$ the average transverse gluonic radius squared is only $\sim 0.6$ of the transverse electromagnetic radius squared.

The HERA data at $x < 10^{-2}$ [9, 17] indicate that the transverse gluonic size of the nucleon increases with decreasing $x$ (see Fig. 2). In the region $x \sim \mathrm{few}10^{-2}$ this effect is partly due to the contribution of the nucleon's pion cloud, which is strongly suppressed at $x > 0.1$ and fully developed only for $x < 0.01$ [18]. Another contribution may arise from Gribov diffusion, which is suppressed by the hard scale $Q^2$ but still not negligible. Over the range covered by HERA, the rate of increase of the gluonic size can be parametrized by an effective Regge slope, $\alpha'_g$, which is smaller than that for corresponding soft processes, $\alpha'_{\text{soft}} = 0.25\,\text{GeV}^{-2}$. Averaging the fits to the HERA H1 and ZEUS data [9, 17], and accounting for the correction Eq. (8), we parametrize the $x$–dependence of the gluonic exponential slope as (here $Q^2 = 3\,\text{GeV}^2$)

$$
\begin{aligned}
B_g(x) &= B_{g0} + 2\alpha'_g \ln(x_0/x), &(9)\\
x_0 &= 0.0012, &(10)\\
B_{g0} &= 4.1\,(^{+0.3}_{-0.5})\ \text{GeV}^{-2}, &(11)\\
\alpha'_g &= 0.140\,(^{+0.08}_{-0.08})\ \text{GeV}^{-2}. &(12)
\end{aligned}
$$

The uncertainties in parentheses represent a rough estimate based on the range of values spanned by the H1 and ZEUS fits, with statistical and systematic uncertainties added linearly. One sees from Fig. 2 that the fit to the HERA data consistently extrapolates to the FNAL data point. The corresponding dipole parametrization obtained via Eq. (6) is close to the one used in our previous study [1].

The transverse spatial distribution of partons also changes with the resolution scale, $Q^2$, as a result of DGLAP evolution. Generally, the partons observed at a given momentum fraction $x$ and scale $Q^2$ are decay products of partons with $x' > x$ which existed at a lower scale, $Q_0^2$. In the leading–twist approximation the decay happens locally in transverse space. As a result, the transverse size observed at fixed $x$ shrinks with increasing $Q^2$, because the decaying partons at the lower scale had larger momentum fractions and were localized in a smaller transverse area. In order to calculate the change of the transverse spatial distribution of gluons with $Q^2$ one would need to know the spatial distributions of both gluons and singlet quarks at the input scale for all $x' > x$, where they are only poorly constrained by present data. Numerical studies based on a simple parametrization [1] suggest that the evolution effect is small for $Q^2 > 3\,\text{GeV}^2$. The average transverse size $\langle\rho^2\rangle_g$ at $x \sim 10^{-3}$ decreases by $\sim 15\%$ between $Q^2 = 3$ and $10^4\,\text{GeV}^2$, while the effective value of $\alpha'_g$ in this $x$–region drops by about half. We note that the $J/\psi$ electroproduction data from HERA [8, 9] provide some indication that the effective $\alpha'_g$ may be smaller than in photoproduction, although the results are not fully conclusive. In any case, the $\alpha'_g$ in the parametrization Eqs. (9)–(12) can be considered as an upper limit at values of $Q^2 \gtrsim 10\,\text{GeV}^2$, as are of interest in the applications here.

Comparatively little is known about the transverse distribution of singlet quarks $(q + \bar{q})$ at small $x$. Comparison of the HERA deeply–virtual Compton scattering [19] and $J/\psi$ production data indicates that singlet quarks at $x < 10^{-2}$ are distributed over a larger transverse area than the gluons, in qualitative agreement with theoretical arguments based on the pion cloud contribution to the parton densities at large $b$ [20]. In the applications here we are concerned with gluon–induced processes; parametrizations similar to Eq. (5) could be formulated also for the quark distributions.

## III. IMPACT PARAMETER DISTRIBUTION OF PROTON–PROTON COLLISIONS

Using the information on the transverse spatial distribution of partons in the nucleon, one can infer the distribution of impact parameters in $pp$ collisions with hard parton–parton processes [1]. While not directly observable, the latter determines the spectator interactions in such collisions and thus can be studied indirectly through measurements of the correlation of hard processes with final–state properties.

In high–energy $pp$ scattering angular momentum conservation implies that the impact parameter becomes a good quantum number, and it is natural to consider amplitudes and cross sections in the impact parameter representation. The nucleon's light–cone wave functions, describing the partonic structure at a low resolution scale, can be expressed in terms of the longitudinal momentum fractions of the partons and their transverse positions relative to the center–of–momentum. In a hard parton–parton processes such as dijet production with $p_T \gg 1\text{GeV}$ the transverse momenta of the final–state partons are much larger than the typical inverse hadronic size. The transverse momenta of partons in the initial state, ranging from the soft scale to $p_T$, are integrated over (collinear approximation). The hard process can therefore be regarded as happening locally in transverse space on the scale of variation of the transverse distributions of partons in the colliding nucleons. Using this basic fact, and assuming closure over partonic states, one obtains a simple expression for the cross section for inclusive production of a pair of high–$p_T$ partons ("jets") in a $pp$ collision at a given impact parameter $b \equiv |\boldsymbol{b}|$ [1]. Specifically, for the case of a $gg \to gg$ partonic process

$$
\begin{aligned}
&\frac{d\sigma_{pp\to gg+X}}{dx_1\,dx_2\,d\cos\hat{\theta}\,d^2b}\,(x_1,x_2,b)\\
&\equiv \int d^2\rho_1 \int d^2\rho_2\ \delta^{(2)}(\boldsymbol{b} - \boldsymbol{\rho}_1 + \boldsymbol{\rho}_2)\\
&\times\ g(x_1,\rho_1|Q^2)\ g(x_2,\rho_2|Q^2)\\
&\times\ I_{gg}(x_1,x_2)\ \frac{d\hat{\sigma}_{gg\to gg}}{d\cos\hat{\theta}}(\hat{s},\hat{\theta}); \qquad (13)
\end{aligned}
$$

analogous expressions are obtained for other types of partonic subprocesses. Here $x_{1,2}$ are the longitudinal momentum fractions of the colliding gluons, $\rho_{1,2} \equiv |\boldsymbol{\rho}_{1,2}|$ their transverse distances from the centers of the parent protons, $I_{gg}(x_1, x_2)$ denotes a kinematic factor defining the effective partonic flux, and $d\hat{\sigma}_{gg\to gg}/d\cos\theta$ is the cross section for the parton–parton subprocess, which depends on the subprocess invariant $\hat{s} = x_1x_2s$ and the

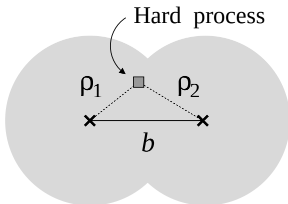


FIG. 3: Overlap integral of the transverse spatial parton distributions, defining the impact parameter distribution of *pp* collisions with a hard parton–parton process, Eq. (18). Shown is the transverse view; the momenta of the colliding nucleons point into and out of the plane, respectively.

center–of–mass scattering angle $\hat{\theta}$. (We do not indicate the dependence on the overall center–of–mass energy $s$ for brevity.) The scale at which the parton densities are probed is of the order $Q^2 \sim p_T^2$, with a coefficient which remains undetermined at leading–order accuracy. Equation 13 has a simple geometric interpretation (see Fig. 3). The inclusive cross section is proportional to the product of local gluon densities in transverse space, and the integral accounts for the possibility that the hard process may happen at any transverse position. The expression of Eq. (13) represents a straightforward generalization of the so–called Drell–Yan formula for inclusive production of a high–$p_T$ pair in *pp* collisions with unspecified impact parameter [21]. Indeed, when integrating Eq. (13) over the vector $\boldsymbol{b}$ the delta function disappears, and using Eq. (2) one obtains the well–known expression in terms of the total gluon densities:

$$\frac{d\sigma_{pp\to gg+X}}{dx_1\, dx_2\, d\cos\hat{\theta}}(x_1, x_2)$$

$$\equiv \int d^2 b\, \frac{d\sigma_{pp\to gg+X}}{dx_1\, dx_2\, d\cos\hat{\theta}\, d^2 b}(x_1, x_2, b) \tag{14}$$

$$= g(x_1|Q^2)\, g(x_2|Q^2)\, I_{gg}(x_1, x_2)\, \frac{d\hat{\sigma}_{gg\to gg}}{d\cos\theta}(\hat{s}, \hat{\theta}). \tag{15}$$

In the analysis of the underlying event characteristics in hard processes we are interested in the *relative* distribution of the cross section for a given hard process over *pp* impact parameters. We thus consider the ratio

$$P_2(x_1, x_2, b|Q^2) \equiv \frac{d\sigma_{pp\to gg+X}}{dx_1\, dx_2\, d\cos\hat{\theta}\, d^2 b}(x_1, x_2, b) \Bigg/ \frac{d\sigma_{pp\to gg+X}}{dx_1\, dx_2\, d\cos\hat{\theta}}(x_1, x_2), \tag{16}$$

which is normalized such that

$$\int d^2 b\, P_2(x_1, x_2, b|Q^2) = 1 \tag{17}$$

and can be interpreted as the probability distribution of *pp* impact parameters in events with a given hard process. This normalized distribution is independent of the total gluon densities and the partonic subprocess cross section and reflects the pure "geometric" probability for two gluons to collide at the same point in transverse space in a *pp* collision at given impact parameter (see Fig. 3) [1]. It can be computed directly from the normalized spatial distributions of gluons in the colliding protons, Eq.(4), as

$$P_2(x_1, x_2, b|Q^2) \equiv \int d^2\rho_1 \int d^2\rho_2\, \delta^{(2)}(\boldsymbol{b} - \boldsymbol{\rho}_1 + \boldsymbol{\rho}_2) \times F_g(x_1, \rho_1|Q^2)\, F_g(x_2, \rho_2|Q^2). \tag{18}$$

This distribution represents an essential tool in phenomenological studies of the underlying event in *pp* collisions [1, 22]. We note that the concept of impact parameter distribution is also used in MC generators of *pp* events with hard processes [10, 11], albeit without making the connection with GPDs, which allows one to import information on transverse nucleon structure obtained in independent measurements.

With the parametrizations of Eq. (5) the convolution integral in Eq. (18) can easily be evaluated analytically. In the case of symmetric collisions ($x \equiv x_1 = x_2$) we find

$$P_2(x, b|Q^2) = \begin{cases} (4\pi B_g)^{-1} \exp[-b^2/(4B_g)], \\ [m_g^2/(12\pi)]\, (m_g b/2)^3\, K_3(m_g b), \end{cases} \tag{19}$$

where the parameters $B_g$ and $m_g$ are taken at the appropriate values of $x$ and $Q^2$.

The impact parameter distribution in minimum–bias inelastic *pp* collisions can be inferred from the *pp* elastic scattering amplitude, which incorporates the information on the *pp* total cross section through the unitarity relation. It is given by

$$P_{\rm in}(s, b) = \left[1 - |1 - \Gamma(s, b)|^2\right] / \sigma_{\rm in}(s), \tag{20}$$

where $\Gamma(s, b)$ is the profile function of the *pp* elastic amplitude in the conventions of Ref. [1] and the inelastic cross section $\sigma_{\rm in}(s)$ is given by the integral $\int d^2 b$ of the expression in brackets, such that $\int d^2 b\, P_{\rm in}(s, b) = 1$. For the purpose of the present study we employ a simple analytic parametrization of the profile function which satisfies unitarity and reflects the approach to the black–disk regime ($\Gamma \to 1$) at small impact parameters:

$$\Gamma(s, b) = \Gamma_0 \exp\{-b^2/[2B(s)]\}, \tag{21}$$

where $\Gamma_0 = 1$ and the slope parameter is given in terms of the total cross section as $B(s) = \sigma_{\rm tot}(s)/(4\pi)$; the inelastic cross section for this profile is $\sigma_{\rm in}(s) = 3\pi B(s)$. For the total cross section we use the extrapolation suggested by the COMPETE Collaboration [23], which gives $B(s) = 20.2$ (22.8) GeV$^{-2}$ at $\sqrt{s} = 7$ (14) TeV. The uncertainty in the profile function at LHC energies is dominated by that of the total cross section. The impact parameter distributions calculated with Eq. (21) provide a fully satisfactory representation of those obtained with more elaborate parametrizations of the *pp* elastic amplitude, see Fig. 1 of Ref. [13] and references therein.

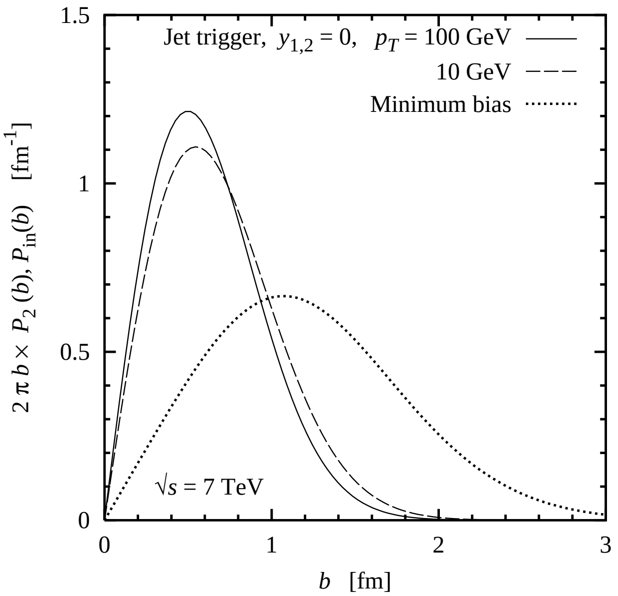


FIG. 4: Impact parameter distributions of inelastic $pp$ collisions at $\sqrt{s} = 7\,\text{TeV}$. *Solid (dashed) line:* Distribution of events with a dijet trigger at zero rapidity, $y_{1,2} = 0$, *cf.* Eq. (19), for $p_T = 100\,(10)\,\text{GeV}$ . *Dotted line:* Distribution of minimum–bias inelastic events, *cf.* Eq. (20).

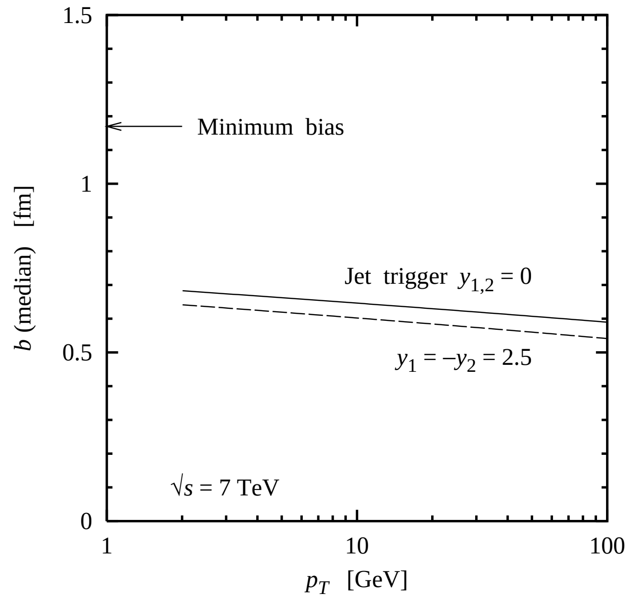


FIG. 5: Median impact parameter $b$(median) of events with a dijet trigger, as a function of the transverse momentum $p_T$, *cf.* Fig. 4. *Solid line:* Dijet at zero rapidity $y_{1,2} = 0$. *Dashed line:* Dijet with rapidities $y_{1,2} = \pm 2.5$. The arrow indicates the median $b$ for minimum–bias inelastic events.

Using the above expressions we can now study the influence of the trigger conditions on the impact parameter distribution of $pp$ events at the current LHC energy $\sqrt{s} = 7\,\text{TeV}$. The present experiments typically consider a jet trigger near zero rapidity, $y_1 \approx 0$, and study the characteristics of the underlying events as a function of the transverse momentum $p_T$ of the highest–momentum particle in the pseudorapidity interval $-2.5 < \eta < 2.5$. In this setting one integrates over the energy of the balancing jet (as well as that of other jets which might arise from higher–order processes), which effectively amounts to integrating over the momentum fraction of the second parton, $x_2$, at fixed $x_1$. Since the distribution is symmetric in the rapidity of the balancing jet, $y_2$, and the variation of the transverse distribution of partons with $x$ is small, *cf.* Eqs. (9)–(12) and Fig. 2, we can to a good approximation set $y_2 = 0$ and thus take $x_{1,2}$ at the average point

$$x_1 \;=\; x_2 \;=\; 2p_T/\sqrt{s}. \qquad (22)$$

The scale at which the parton densities are probed is of the order $Q^2 \sim p_T^2$. Generally, we expect the impact parameter distribution in events with such a jet trigger to become narrower with increasing $p_T$, because the transverse distribution of partons shrinks both with increasing $x_{1,2}$ and with increasing $Q^2$. The impact parameter distributions with a jet trigger of $p_T = 10$ and $100\,\text{GeV}$ are presented in Fig. 4. Shown are the results obtained with the exponential parametrization of Eq. (19) and Eqs. (9)–(12); the dipole form leads to comparable results. One sees that the change of the width of this distribution with $p_T$ is rather small, because the transverse distribution of gluons changes only little with $x$ in the range explored here; account of the $Q^2$ dependence of the transverse distribution of gluons would lead to an additional small change. One also sees that the impact parameter distributions with the jet trigger are much narrower than that in minimum–bias inelastic events at the same energy. This quantifies the two–scale picture of transverse nucleon structure summarized in Fig. 1.

The median impact parameter in dijet events, defined as the value of $b$ for which the integral over $P_2$ reaches the value 1/2, is shown in Fig. 5 as a function of $p_T$. For the parametrizations of Eq. (19) it is given by $b(\text{median}) = 1.67\,\sqrt{B_g}$ and $3.08\,m_g^{-1}$, respectively. The results obtained with the exponential and dipole form factors differ only by a few percent if the parameters are related by Eq. (6), indicating that the uncertainty resulting from our imperfect knowledge of the shape of the transverse spatial distribution of gluons is small. The uncertainty in $b$(median) resulting from the uncertainty of $B_{g0}$ in the parametrization Eqs. (9)–(12) is less than $\pm 10\%$ at $p_T \sim$ few GeV. It is seen that the median $b$ in jet events drops only very weakly as a function of $p_T$ for all values above $\sim 2\,\text{GeV}$. We estimate that account of the $Q^2$ dependence of the transverse distributions due to DGLAP evolution would change the results in Fig. 5 by less than $\sim 5\%$. Also shown is the median $b$ with a trigger on a jets at non-zero rapidity $y_1 = -y_2 = 2.5$, which amounts to an effective increase of $x_{1,2}$ by a factor $\cosh y \approx 6$, *cf.* Eq. (24) and the discussion in Sec. V. In all cases, the median impact parameter in jet events is far smaller than that in minimum–bias collisions, which is given by $b(\text{median}) = 1.32\sqrt{B}$ for the parametrization of Eq. (21).

To conclude this discussion, a comment is in order concerning the interpretation of the impact parameter distributions in *pp* events with hard processes. Our analysis based on Eq. (18) shows that *pp* events with at least one hard process (and no other requirements) are on average more central than minimum–bias inelastic events. This statement concerns the *relative* distribution of impact parameters in a collective of inelastic *pp* events and how it is changed by imposing the requirement of a hard process. One should not confuse this with statements about the *absolute* probability for a hard process (in a certain rapidity interval) in a *pp* collision at certain impact parameters. In fact, the analysis of Refs. [24, 25] shows that there can be a substantial absolute probability for a hard process in *pp* collisions at large $b$, and that unitarity places non–trivial restrictions on the dynamics of hard interactions in peripheral collisions.

## IV. TRANSVERSE MULTIPLICITY AS AN INDICATOR OF HARD DYNAMICS

The estimates of the previous section show that *pp* events with a hard parton–parton collision are much more central than minimum–bias events, and that the average impact parameters change only very little for $p_T$ above $\sim 2\,\text{GeV}$. At the same time, it is known that the underlying event characteristics, such as the average multiplicity, depend strongly on the centrality of the *pp* collision. Combining these two observations, we can devise a practical method to determine down to which values of $p_T$ mid–rapidity particle production is predominantly due to hard parton–parton collisions. The observable of interest here is the transverse multiplicity, defined as the multiplicity of particles with transverse momenta in a certain angular region perpendicular to the transverse momentum of the trigger particle or jet (the standard choice is the interval $60^\circ < |\Delta\phi| < 120^\circ$ relative to the jet axis; see Ref. [4] for an illustration and discussion of the experimental definition). The transverse multiplicity is not *directly* affected by the multiplicity associated with the trigger or balancing jets, but is *indirectly* correlated with the presence of a hard process because of its dependence on the centrality.

Based on the results of Figs. 4 and 5 we predict that the transverse multiplicity should be practically independent of $p_T$ of the trigger as long as the trigger particle originates from a hard parton–parton collision which "centers" the *pp* collision. Furthermore, the transverse multiplicity in such events should be significantly higher than in minimal–bias inelastic events, since the known mechanisms of particle production — minijet interactions, multiple soft interactions, *etc.* — are much more effective in central collisions. When measuring the transverse multiplicity as a function of $p_T$ of the trigger, we thus expect

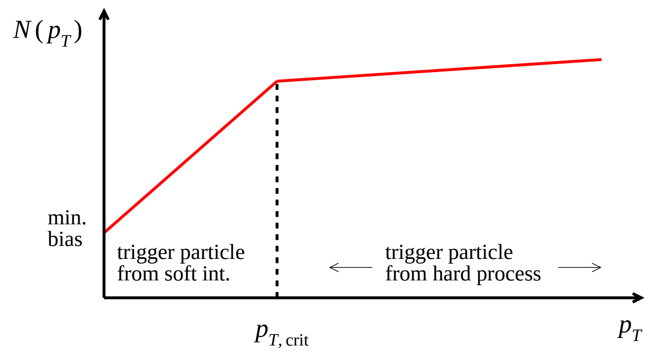


FIG. 6: Schematic illustration of the expected dependence of the transverse multiplicity, $N(p_T)$, on the $p_T$ of the trigger.

it to increase from its minimum–bias value at low $p_T$ and become approximately constant at $p_T \sim$ few GeV, as indicated in Fig. 6. (A small residual increase at large $p_T$ is expected due to color flow in the jet; see below). The point where the transition happens, $p_{T,\text{crit}}$, indicates the critical value of $p_T$ above which particle production is dominated by hard parton–parton processes.

Interestingly, the predicted increase and eventual flattening of the transverse multiplicity agrees well with the pattern observed in the existing data. At $\sqrt{s} = 0.9\,\text{TeV}$ the transition occurs approximately at $p_{T,\text{crit}} \approx 4\,\text{GeV}$ [6], at $\sqrt{s} = 1.8\,\text{TeV}$ at $p_{T,\text{crit}} \approx 5\,\text{GeV}$ [4], and the preliminary data at 7 TeV indicate somewhat larger values of $p_{T,\text{crit}} = 6 - 8\,\text{GeV}$ [5, 7]. We thus conclude that the minimum $p_T$ for hard particle production increases with the collision energy. Note that we consider here an inclusive trigger; the procedure adopted in the experimental analysis (selection of the fastest particle in the measured rapidity interval) somewhat enhances the contribution of soft mechanisms in particle production.

It is worth noting that the overall pattern described here is reproduced by the tunes of current MC models; *cf.* the comparisons in Refs. [4–7]. This is because these models effectively include the key feature used in our analysis — the narrow impact parameter distribution of dijet events (although $\langle b^2\rangle$ in these models is too small by a factor $\sim 2$), and impose a cutoff on the minimal $p_T$ of the minijets. Our point here is that the observed pattern can be explained naturally on the basis of the transverse geometry of *pp* collisions with hard processes, without involving detailed models. This allows one to determine in a model–independent way where the dominant dynamics in particle production changes from soft interactions to hard parton–parton processes.

For $p_T$ lower than $p_{T,\text{crit}}$ the relative contribution of hard processes to particle production starts to decrease. In terms of the transverse geometry, this means that the observed trigger particle can, with some probability, originate from either peripheral or central collisions in the sense of Fig. 1. We can estimate the fraction of particles produced by hard interactions in this "mixed" region in a simple two–component model, based on the observation that the effective impact parameters in soft collisions are much larger than those in hard events and do not change much with transverse momenta of the produced particles [32]. Thus, we assume that: ($i$) a trigger particle observed at given $p_T$ originated with a probability $\lambda_{\rm hard}(p_T)$ from a hard process, and with probability $1 - \lambda_{\rm hard}(p_T)$ from soft interactions; ($ii$) the average impact parameters in both classes of collisions do not depend of the $p_T$ of the trigger. This allows us to write the $p_T$ dependence of the transverse multiplicity in the form

$$N(p_T) \;\; = \;\; \lambda_{\rm hard}(p_T) N_{\rm hard} + [1 - \lambda_{\rm hard}(p_T)] N_{\rm soft}, \quad (23)$$

where $N_{\rm hard}$ and $N_{\rm soft}$ are independent of $p_T$. Assuming that for some sufficiently small $p_T$ cutoff $\lambda_{\rm hard}(p_T)$ is close to zero, we can determine $N_{\rm soft}$, which corresponds to the minimum–bias impact parameter distribution presented in Fig.4, and use it to determine $\lambda_{\rm hard}(p_T)$ for $p_T$ smaller than $p_{T,{\rm crit}}(s)$ via Eq. (23). The data indeed indicate that $N_{\rm hard} \gg N_{\rm soft}$; so in the region of $p_T$ where $N(p_T)/N_{\rm hard} \geq 1/3$ our estimate is not sensitive to the exact value of $N_{\rm soft}$. By inspection of the data we conclude that the contribution of the hard mechanism drops to about half of the total yield for $p_T \approx 1.5 - 2, 2 - 2.5, 3 - 4\,{\rm GeV}$ for $\sqrt{s} = 0.9, 1.8, 7\,{\rm TeV}$.

It is also of interest that for $p_T > p_{T,{\rm crit}}$ the transverse multiplicity appears to increase with the collision energy faster than the average multiplicity [5, 7]. In the leading–twist approximation the perturbative contribution is proportional to the product of the gluon densities at small $x_{1,2}$ and thus scales as $(\sqrt{s})^{2\lambda}$, where $\lambda$ is the exponent of the gluon density, $xg(x, Q^2) \propto x^{-\lambda}$, and takes on values $\lambda \sim 0.2 - 0.3$ in the $Q^2$ region of interest here. This is roughly consistent with the factor $\sim 2$ increase of the observed transverse multiplicity between $\sqrt{s} = 0.9$ and 7 TeV, which suggests scaling as $(\sqrt{s})^{0.34}$ [33]. We note that at very small values of $x_1$ or $x_2$ the leading–twist approximation breaks down because of the onset of the black–disk regime in hard interactions, which generates a new dynamical scale in the form of the gluon density per transverse area; see Ref. [1] for an estimate of the relevant values of $x_{1,2}$ and $p_T$.

At transverse momenta above $p_{T,{\rm crit}}$ a small residual increase of the transverse multiplicity with $p_T$ should occur due to color flow between the jets, i.e., production of particles at large angles relative to the jet direction through higher–order QCD processes [26]. This flow can be calculated within perturbative QCD and is expected to become significant only at transverse momenta substantially larger than the values $p_{T,{\rm crit}} \sim$ few GeV suggested by the data. A similar pattern is obtained as the result of the "color drag" phenomenon in the string fragmentation picture [27]. That this contribution to the transverse multiplicity is small in the kinematics considered here is seen from the fact that it should be primarily a function of $p_T$ and not of $\sqrt{s}$, while the data show a fast increase of the transverse multiplicity with $\sqrt{s}$.

## V. RAPIDITY DEPENDENCE AS A TEST OF UNIVERSALITY

The basic idea of our approach is that hard processes influence the event characteristics by selecting $pp$ collisions with small impact parameters. Further interesting tests of these geometric correlations can be performed by measuring the dependence of event characteristics on the rapidity of the jets in the trigger.

In production of dijets at non-zero rapidity only part of the center–of–mass energy of the colliding partons is converted to transverse energy, allowing one to probe larger momentum fractions $x_{1,2}$ at the same $p_T$. For jets with symmetric rapidities $y_1 = -y_2 \equiv y$,

$$x_1 \;=\; x_2 \;=\; (2 p_T \cosh y)/\sqrt{s}. \quad (24)$$

Because partons with larger $x_{1,2}$ sit at smaller transverse distances, the average impact parameters in $pp$ collisions with a dijet trigger decrease with increasing $y$; however, the effect is small (see Fig.5). Observing the approximate $y$–independence of the transverse multiplicity would test that the selection of central collisions does not depend on the details of the hard process. Beyond that, we predict a small increase in the transverse multiplicity if $y$ is increased away from zero at fixed $p_T$. In particular, such measurements could separate the effects of the $x$– and $Q^2$–dependence of the transverse distribution of partons on the average impact parameters. At lower $p_T$, the dependence of the transverse multiplicity on the rapidity would help to distinguish between the minijet and soft mechanisms of hadron production, as minijets are much more centered at small rapidities, while typical soft multi–ladder–type interactions lead to correlations over large rapidity intervals. An additional advantage of measurements with the $y \neq 0$ trigger is that the difference between the transverse, forward, and away–side regions of particles produced at mid–rapidity is much smaller than for the $y = 0$ trigger.

The selection of central $pp$ impact parameters by hard parton–parton processes could in principle be verified not only through the transverse multiplicity, but also by measuring event characteristics in *rapidity regions* which are not directly affected by the jet fragmentation of the partons in the trigger process. The identification of the jet fragmentation regions requires detailed modeling beyond the scope of the present investigation. Assuming that one could reliably remove the fragmentation regions, several types of interesting correlation measurements become possible. First, we predict that in the remaining rapidity region the multiplicity should become, on average, isotropic in the transverse direction, *i.e.*, it should attain the value of the previously considered transverse multiplicity in all directions and be substantially higher than in minimum bias events. Second, this multiplicity should not depend on the rapidity of the trigger, $y$, if $p_T > p_{T,{\rm crit}}$. Both measurements would directly attest to the universality of particle production in central $pp$ collisions.

The present LHC experiments use the central detectors to study the underlying event structure in the production of high–$p_T$ particles in the pseudorapidity interval $\eta = \pm 2.5$ (corresponding roughly to the same range in rapidity proper, $y$); the measurements may be extended to $-5 \leq \eta \leq 5$ (CMS) and $-4.9 \leq \eta \leq 4.9$ (ATLAS) using forward detectors. In production of two jets at $y_1 = -y_2 = y \approx 2$, assuming a rapidity interval of approximately $\pm 0.7$ for the fragmentation of either jet, the rapidity region $\pm 1$ should be free of direct jet fragments and could be used for the envisaged multiplicity measurements. Alternatively, one may consider a pair of jets at the same positive rapidity, $y_1 = y_2 \equiv \bar{y} > 0$, and study the multiplicity in the negative rapidity region as a function of $\bar{y}$. The latter choice would have the advantage that the parton momentum fractions $x_{1,2}$ change in different directions when increasing $\bar{y}$ from zero, compensating the effect on the width of the impact parameter distribution to first order.

An interesting phenomenon should occur when extending such measurements with symmetric jets at $y_1 = -y_2 = y$ to larger rapidities. As discussed in Sec. II, the transverse size of the parton distribution decreases with increasing $x$. This leads to a seemingly paradoxical prediction, that the larger the rapidity interval $y_1 - y_2 = 2y$ between the jets, the larger the multiplicity in the mid–rapidity region. In other words, one expects long–range correlations in rapidity which are becoming stronger with increase of the rapidity interval. However, the effect is rather small; at $p_T = 5\,\mathrm{GeV}$ the median $pp$ impact parameter changes from 0.66 fm at $y = 0$ to 6.1 (0.55) fm at $y = 2.5\,(5)$. Therefore the ability to measure over a wide range of pseudorapidities $|\eta| < 5$ would be very helpful for studying this effect.

Deviations from the predicted universality can arise as a result of spatial correlations between partons involved in the hard collisions and those participating in spectator interactions [28]. At the relatively small $x$–values probed with the central detectors at LHC ($x < 10^{-2}$), such correlations are likely to depend weakly on $x$ and would not significantly affect the rapidity dependence. In principle, the study of these deviations from universality may provide a new window on correlations in the nucleon's partonic wave function.

Further corrections to the universality pattern could come from the contribution of branching processes, such as parton–parton $\rightarrow$ 3 jets, to events with a two–jet trigger. These contributions could be subtracted using current event generators for NLO jet production. The contribution of such processes can also be suppressed experimentally, by selecting pairs of jets with a small transverse momentum imbalance $|\boldsymbol{p}_T(\text{jet 1}) + \boldsymbol{p}_T(\text{jet 2})|$.

## VI. SUGGESTIONS FOR FUTURE MEASUREMENTS

In addition to the studies described in Secs. IV and V, several other kinds of measurements could further explore the proposed connection between hard processes and the transverse multiplicity, or use it to investigate interesting aspects of QCD and nucleon structure.

*Energy dependence of transverse multiplicity.* Measurement of the energy dependence of the transverse multiplicity in jet events would, in effect, reveal the energy dependence of the average multiplicity in central $pp$ collisions, which is of interest beyond the specific applications considered here. In order to avoid change of the average impact parameters due to the $x$–dependence of the transverse distribution of partons, it would be desirable to compare data at the same values of $x_{1,2}$. When increasing the collision energy from $\sqrt{s_0}$ to $\sqrt{s}$ one thus needs to trigger on jets with rapidities scaled by $(1/2)\ln(s/s_0)$.

*Forward energy flow.* In the central $pp$ collisions selected by a hard process there is a larger probability for the large–$x$ partons in one proton to scatter inelastically from small–$x$ gluons in the other proton as compared to minimum bias events. Such inelastic interactions are close to the maximum strength allowed by unitarity and result in strong break–up of the nucleon [1, 29]. One thus expects a larger forward energy flow in dijet events than in minimum–bias collisions. Recent data from the CMS experiment seem to confirm this expectation [30]. As with the other underlying event observables considered here, we expect the increase of the energy flow to be a weak function of the $p_T$ of the trigger.

*Double dijet trigger.* Further reduction of the effective impact parameters in $pp$ collisions can be achieved with a trigger on multiple hard processes [1]. In the absence of transverse correlation between partons, the effective impact parameter distribution of events with two dijets would be given by the (properly normalized) product of distributions $P_2$ in Eq. (18). In the simplest case of two dijets with $x_1 = x_2 = x_3 = x_4$, the median $b$ in such events would be $1.18\sqrt{B_g}$ $(1.97\,m_g^{-1})$, to be compared to $1.67\sqrt{B_g}$ $(3.08\,m_g^{-1})$ for single jet events. Account of transverse correlations between partons reduces the difference in effective impact parameters by about half [1]. In all, we expect a $15-20\,\%$ reduction of the median $b$ with a double dijet trigger, which should manifest itself in a further increase of the transverse multiplicity compared to single dijet events.

*Other centrality triggers.* Knowledge of the dependence of the transverse multiplicity on $b$ would allow one to calibrate other triggers on central $pp$ collisions. In particular, it would be interesting explore triggers related to particle production in the nucleon fragmentation regions, for example leading neutrons. Ultimately one aims here at designing a trigger on ultra–central $pp$ collisions, in which the effective gluon densities would be comparable to those reached in heavy–ion collisions [29].

*Quark vs. gluon–induced jets.* It would be interesting to compare the transverse multiplicities and other underlying event characteristics for quark–antiquark induced hard processes like $W^{\pm}, Z^0$ production and gluon–gluon induced processes. Another possibility would be to consider large–$|\eta|$ dijet production and separate quark– and gluon induced jets using the different jet shapes. This would allow one to observe a possible difference between the transverse distributions of quarks and gluons, complementing studies of hard exclusive processes in $ep/\gamma p$ scattering.

*Deconvolution of impact parameter dependence.* If the fluctuations of the multiplicity between events at a given impact parameter are not too large, one can attempt a program of deconvolution of the impact parameter dependence of the multiplicity, using information on the impact parameter dependence of dijet and 4–jet rates [25].

## VII. SUMMARY AND DISCUSSION

The transverse multiplicity in $pp$ collisions with a jet trigger provides an effective way to determine the rate of processes initiated by hard parton–parton interactions down to rather small transverse momenta. Further analysis of the data can then establish whether the observed particle production rates are consistent with perturbative QCD predictions. Studies of the dependence of the transverse multiplicity on the transverse momenta and rapidities of the jets, and on the collision energy, can provide a better understanding of the impact parameter dependence of the underlying event characteristics and allow one to refine the use of the transverse multiplicity as an indicator of the dynamics in particle production.

The results of the present study have implications also for the use of hard processes in new particle searches. In the production of a Higgs boson with a mass $M_H \sim 100\,\text{GeV}$ the parton momentum fractions $x_{1,2}$ are the same as in dijet events at zero rapidity with $p_T = M_H/2 = 50\,\text{GeV}$. The two types of events thus involve the same average $pp$ impact parameters. This allows one to describe the background to new particle production processes much more accurately than on the basis of minimum–bias event characteristics.

A more quantitative understanding of the dynamics of particle production in $pp$ scattering would impact also on the analysis of heavy–ion collisions at LHC energies. If a value of $p_{T,\text{crit}} = 6 - 8\,\text{GeV}$ is confirmed by the forthcoming $pp$ data at $\sqrt{s} = 7\,\text{TeV}$ (see Sec. IV), it would mean that soft interactions play a significant role in particle production in $pp$ collisions up to transverse momenta of this magnitude. This would influence the interpretation of the nuclear–to–proton ratios of particle production measured in heavy–ion collisions.

Finally, our analysis relies crucially on information about the transverse spatial distribution of gluons from exclusive $J/\psi$ photo/electroproduction and similar processes. While the region $x < 0.01$ has been covered by HERA, no data of comparable precision are available at larger $x$ (see Sec. II). This is unfortunate, as the production of new particle with masses $\sim 10^2\,\text{GeV}$ requires partonic collisions at precisely such momentum fractions. The region $x > 0.01$ is also particularly interesting for nucleon structure studies [18, 20]. The $J/\psi$ results expected from the COMPASS experiment, as well as measurements with a future Electron–Ion Collider (EIC), thus would have a major impact on both areas of study.

### Note added

After submission of this article for publication the ATLAS Collaboration published results of measurements of the underlying event characteristics using charged particles in $pp$ collisions at LHC at $\sqrt{s} = 0.9$ and $7\,\text{TeV}$ [31]. The data show that the transverse multiplicity and the scalar sum of transverse momenta $\sum p_T$ in the transverse region are approximately independent of the rapidity of the leading particle in the jet trigger; see Sec. VIII I and Fig. 12 of that article. This observation confirms the universality of central impact parameters in jet events discussed in Sec. V and supports the geometric picture of $pp$ collisions proposed here.

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