Light from cosmic strings

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Light from Cosmic Strings

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The time-dependent metric of a cosmic string leads to an effective interaction between the string and photons – the "gravitational Aharonov-Bohm" effect – and causes cosmic strings to emit light. We evaluate the radiation of pairs of photons from cosmic strings and find that the emission from cusps, kinks and kink-kink collisions occurs with a flat spectrum at all frequencies up to the string scale. Further, cusps emit a beam of photons, kinks emit along a curve, and the emission at a kink-kink collision is in all directions. The emission of light from cosmic strings could provide an important new observational signature of cosmic strings that is within reach of current experiments for a range of string tensions.

I. INTRODUCTION

Cosmic strings are possible remnants from the early universe (for a review see [1]) and there is significant effort to try and detect them. A positive detection of cosmic strings will open up a window to very high energy fundamental physics and can potentially have strong implications for astrophysical processes. Hence it is of great interest to continue to discover new observational signatures of cosmic strings, as well as to refine features of known signatures. In this paper we address the radiation of photons by cosmic strings.

There is an extensive literature on gravitational radiation from cosmic strings, particularly motivated by upcoming and future gravitational wave detectors. More relevant to the work presented here, however, is the analysis in [2] and [3] of the emission of particles due to the time-dependent metric of cosmic strings from the viewpoint of Aharonov-Bohm radiation. The case of photon emission – which we treat in the present paper – was not explicitly discussed there. A crucial feature which emerged in these calculations is that cusps and kinks on cosmic strings emit radiation with a flat spectrum all the way up to the string scale. However, those results were based on studying two rather specific loop configurations with cusps and kinks. As we show here in more generality (namely for any loop configuration) the flat spectrum also applies to the emission of photons from cusps and kinks, as well as kink-kink collisions. Thus light emitted from cosmic strings in this way leads to a new and observable signature of cosmic strings that is completely independent of the details of the underlying particle physics model. As we shall see, the effect is small, however, being proportional to $(G\mu)^2$ where G is Newton's constant and μ the string tension. Despite that, since photons are being emitted, it may be more easily measurable than, say, the gravitational wave (GW) bursts also emitted by cusps and kinks.

The total power emitted in scalar particles from cosmic strings due to their gravitational coupling was first considered in [4], using formalism developed in [5]. In this paper we calculate the differential power emitted in photons from cosmic strings due to the gravitational coupling. We call this the "gravitational Aharonov-Bohm" effect because the metric is flat everywhere except at the location of the string, and is closely analogous to the case of the electromagnetic Aharonov-Bohm effect due to a thin solenoid. In Sec. II we set up the calculation and evaluate the invariant matrix element for the production of two photons. The emission is dominant in three cases – from cusps, kinks and kink-kink collisions. Integrals relevant to these cases are evaluated in Sec. III. In Sec. IV we find the power emitted from cusps, kinks and kink-kink collisions on strings. Our results are summarized in Sec. V, where we also consider observational signatures. Our numerical estimate in Eq. (81) indicates that light from cosmic strings may potentially be detectable by current detectors for a range of string tensions.

II. GRAVITATIONAL AHARONOV-BOHM

The gravitational field of a cosmic string is characterized by the parameter $G\mu$ which is constrained to be less than $\sim 10^{-7}$. Hence it is sufficient to consider the case of a weak gravitational field and linearize the metric around a Minkowski background

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \tag{1}$$

Then coupling between the gravitational field and the photon becomes

$$\mathcal{L}_{\rm int} = -\frac{1}{4}\sqrt{-g}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\gamma^{\mu\nu}F_{\mu\alpha}F_{\nu}{}^{\alpha} + \mathcal{O}(h^2) \tag{2}$$

where

$$\gamma_{\mu\nu} = h_{\mu\nu} - \frac{1}{4}\eta_{\mu\nu}h^{\alpha}_{\alpha} \tag{3}$$

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and the electromagnetic field strength is $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The coupling is quadratic in A_μ so that to lowest order in $h_{\mu\nu}$ photons are created in *pairs*. The metric perturbation, $h_{\mu\nu}$, due to the cosmic string energy-momentum tensor, $T_{\mu\nu}$, follows from the Einstein equations: in Fourier space (denoted by tildes),

$$\tilde{\gamma}_{\mu\nu} = \tilde{h}_{\mu\nu} - \frac{1}{4}\eta_{\mu\nu}\tilde{h}^\alpha_\alpha = -\frac{16\pi G}{k^2}\left[\tilde{T}_{\mu\nu} - \frac{1}{4}\eta_{\mu\nu}\tilde{T}^\alpha_\alpha\right] . \quad (4)$$

From the Nambu-Goto action, and using the conformal gauge [1]

$$T_{\mu\nu}(x) = \mu \int d^2\sigma(\dot{X}_\mu \dot{X}_\nu - X'_\mu X'_\nu)\delta^4(x - X) \quad (5)$$

where $X^\mu(\sigma, t)$ is the string world-sheet. A cosmic string loop trajectory can be written in terms of left- and right-movers

$$X^\mu(\sigma, t) = \frac{1}{2}[a^\mu(\sigma_-) + b^\mu(\sigma_+)] \quad (6)$$

where $\sigma_\pm = \sigma \pm t$, and we will adopt world-sheet coordinates such that

$$\begin{aligned} a^0 &= -\sigma_- \ , \quad b^0 = \sigma_+ \\ |\boldsymbol{a}'| &= 1 = |\boldsymbol{b}'| \end{aligned} \quad (7)$$

where primes denote derivatives with respect to the argument. Substituting (6) into (5) yields

$$T_{\mu\nu}(x) = -\frac{\mu}{4}\int d\sigma_+ d\sigma_- ({a_\mu}'{b_\nu}' + {a_\nu}'{b_\mu}')\delta^4(x - X) \quad (8)$$

which, when Fourier transformed, is

$$\tilde{T}_{\mu\nu}(k) = -\frac{\mu}{4}\left(I_{+,\mu}I_{-,\nu} + I_{+,\nu}I_{-,\mu}\right) . \quad (9)$$

Here, for the periodic oscillations of a loop of length L

$$I^\mu_+ = \sum_{n=1}^{\infty} \delta\left(\frac{k^0 L}{4\pi} - n\right)\int_0^L d\sigma_+ b'^\mu e^{-ik\cdot b/2} \quad (10)$$

$$I^\mu_- = \sum_{n=1}^{\infty} \delta\left(\frac{k^0 L}{4\pi} - n\right)\int_0^L d\sigma_- a'^\mu e^{-ik\cdot a/2} . \quad (11)$$

It will be important in the following to notice that as a result of the periodicity of the loop,

$$k \cdot I_\pm = 0 . \quad (12)$$

We can now calculate the amplitude for the pair creation of two outgoing photons of momentum p and p', and polarisation ϵ and ϵ' respectively, where

$$p^2 = 0 = p'^2 ; \qquad p \cdot \epsilon = 0 = p' \cdot \epsilon' . \quad (13)$$

This is given by the tree level process shown in Fig. 1, and on using equations (2), (4) and (9) we find

$$\mathcal{M}(p, p') = -\frac{4\pi G\mu}{k^2} I^\mu_+(k) I^\nu_-(k) Q_{\mu\nu}(p, \epsilon; p', \epsilon') \quad (14)$$

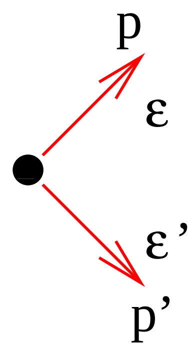

FIG. 1: Feynman diagram showing two photon production from a classical string.

where momentum conservation imposes that

$$k = p + p' , \quad (15)$$

while

$$\begin{aligned} Q_{\mu\nu} &= P_{\mu\nu} + P_{\nu\mu} - \frac{1}{2}\eta_{\mu\nu}P^\alpha_\alpha & (16) \\ P_{\mu\nu} &= (p_\mu \epsilon^*_\alpha - p_\alpha \epsilon^*_\mu)(p'_\nu \epsilon'^{\alpha *} - p'^\alpha \epsilon'^*_\nu) & (17) \end{aligned}$$

The number of photon pairs produced in a phase space volume, the "pair production rate", is given by (*e.g.* Sec. 4.5 of [6])

$$dN = \sum_{\epsilon,\epsilon'} \frac{d^3p}{(2\pi)^3}\frac{1}{2\omega}\frac{d^3p'}{(2\pi)^3}\frac{1}{2\omega'}|\mathcal{M}|^2 \quad (18)$$

where ω and ω' are the energies of the two outgoing photons, while the energy emitted in the pairs is

$$dE = \sum_{\epsilon,\epsilon'} \frac{d^3p}{(2\pi)^3}\frac{1}{2\omega}\frac{d^3p'}{(2\pi)^3}\frac{1}{2\omega'}(\omega + \omega')|\mathcal{M}|^2 . \quad (19)$$

Clearly the crucial relevant quantity is

$$\begin{aligned} |\mathcal{M}|^2_{\rm tot} &\equiv \sum_{\epsilon,\epsilon'} |\mathcal{M}|^2 \\ &= \left(\frac{4\pi G\mu}{k^2}\right)^2 \sum_{\epsilon,\epsilon'} I^\alpha_+ I^{\mu *}_+ I^\beta_- I^{\nu *}_- Q_{\alpha\beta} Q^*_{\mu\nu} . \end{aligned} \quad (20)$$

Note that in this paper we only study the total radiation rate from a cosmic string loops: a discussion of any polarization signatures is left for subsequent work.

After substitution of $Q_{\mu\nu}$ given in Eq. (16), the sum over photon polarizations in (20) can be simplified through the replacement

$$\sum_{\epsilon,\epsilon'} \epsilon^*_\mu \epsilon_\nu \to -\eta_{\mu\nu} , \quad (21)$$

provided certain conditions hold [6]. More specifically, let us define $\mathcal{M}_{\rho\sigma\mu\nu}$ via

$$Q_{\mu\nu} = \epsilon^{\rho *}\epsilon'^{\sigma *}\mathcal{M}_{\rho\sigma\mu\nu} \quad (22)$$

so that

$$\mathcal{M}_{\rho\sigma\mu\nu} = N_{\rho\sigma\mu\nu} + N_{\rho\sigma\nu\mu} - \frac{1}{2}\eta_{\mu\nu}N_{\rho\sigma}{}^{\alpha}{}_{\alpha} \quad (23)$$

$$N_{\rho\sigma\mu\nu} \equiv (p_\mu\eta_{\alpha\rho} - p_\alpha\eta_{\mu\rho})(p'_\nu\delta^\alpha_\sigma - p'^\alpha\eta_{\sigma\nu}) \quad (24)$$

Then the required condition [6] is that

$$p^\rho\mathcal{M}_{\rho\sigma\mu\nu} = 0 = p'^\sigma\mathcal{M}_{\rho\sigma\mu\nu}. \quad (25)$$

However, it is straightforward to check that this condition is satisfied, since it is an immediate consequence of the definition of $\mathcal{M}_{\rho\sigma\mu\nu}$ in (23)-(24).

Then, after quite a bit of algebra and on using (21), we find that $|\mathcal{M}|^2_{\rm tot}$ is given by

$$\begin{aligned}|\mathcal{M}|^2_{\rm tot} &= \left(\frac{2\pi G\mu}{p\cdot p'}\right)^2 \Big[\, 8|p\cdot I_+|^2|p\cdot I_-|^2 \\ &+4p\cdot p'\,\{\,|p\cdot I_+|^2|I_-|^2 + |p\cdot I_-|^2|I_+|^2 \\ &\quad +p\cdot I_+^*\; p\cdot I_-\; I_+\cdot I_-^* + p\cdot I_+\; p\cdot I_-^*\; I_+^*\cdot I_- \\ &\quad -p\cdot I_+\; p\cdot I_-\; I_+^*\cdot I_-^* - p\cdot I_+^*\; p\cdot I_-^*\; I_+\cdot I_-\,\} \\ &+2(p\cdot p')^2\left\{|I_+|^2|I_-|^2 + |I_+^*\cdot I_-|^2 - |I_+\cdot I_-|^2\right\}\Big] \quad (26)\end{aligned}$$

where $|I_\pm|^2 \equiv I_\pm^{*\mu}I_{\pm\mu}$, and in order to simplify the result we have made extensive use of $p\cdot I_\pm = -p'\cdot I_\pm$ which follows (12) since $k = p + p'$. Finally, we have expressed the answer in powers of $k^2 = 2p\cdot p'$: this will be important later when we will see that the $k^2 \to 0$ limit plays a crucial role.

III. EVALUATION OF $|\mathcal{M}|^2_{\rm tot}$

In order to calculate the energy radiation in photon pairs, we need to evaluate $|\mathcal{M}|^2_{\rm tot}$, where the dynamics of cosmic string loops enters Eq. (26) through the integrals $I_\pm(k)$. These integrals, defined in (10)-(11), also occur in the calculation of other forms of radiation from strings and have been discussed in the past (*e.g.* [7, 8]). There is, however, a key difference between Aharonov-Bohm (AB) radiation and other forms of radiation from strings that are commonly studied: namely AB radiation involves *two* particle final states. As a result, the kinematics of the problem is potentially more complicated.

Generally, however, it is well known that $I_\pm(k)$ decay exponentially with k^0L, where L is the length of the loop, unless either the phase in these integrals has a saddle point on the real line, or there is a discontinuity in the integrand due to kinks in b'^μ and/or a'^μ (see *e.g.* Ch. 6 of Ref. [9]). Below we study these cases in turn, and we will see that despite the two-particle nature of the final state, a saddle point in both I_+^μ and I_-^μ corresponds to a cusp on the string – namely $|\dot{\boldsymbol{X}}| = 1$; a saddle point in one of the integrals and a discontinuity in the other occurs at a kink. Finally, when two kinks collide, there is a discontinuity in both integrands. In all three possibilities – cusp, kink, kink-kink collision – $I_\pm^\mu$ decay as a powerlaw, $(k^0L)^{-q}$, where the index q will be determined below.

A. Saddle points and cusps

As a first step in evaluating $I_\pm$, we establish certain relations between the momenta of particles emitted when there are saddle points in these integrals. On recalling that both $p^2 = 0 = p'^2$, let us write

$$\begin{aligned} p &= (\omega, \boldsymbol{p}) = \omega(1, \hat{\boldsymbol{p}}) \equiv \omega\hat{p} \\ p' &= (\omega', \boldsymbol{p}') = \omega'(1, \hat{\boldsymbol{p}}') \equiv \omega'\hat{p}' \end{aligned} \quad (27)$$

where $|\hat{\boldsymbol{p}}| = 1 = |\hat{\boldsymbol{p}'}|$. Since $k = p + p'$ then

$$k^2 = 2p\cdot p' = 2\omega\omega'(1 - \hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{p}}'). \quad (28)$$

We also define

$$k = (\Omega, \boldsymbol{k}) = (\omega + \omega', \boldsymbol{p} + \boldsymbol{p}'). \quad (29)$$

Next consider the integral $I_+(k)$ in Eq. (10) when there is a saddle point. This implies that there is a point such that

$$k\cdot b' = 0 \quad (30)$$

where from the gauge conditions Eq. (7) $b'^2 = 0$. Thus at a saddle point

$$\hat{k} = b', \quad \text{where} \quad \hat{k} \equiv \frac{k}{\Omega} \quad (31)$$

so that

$$k^2 = \Omega^2\hat{k}^2 = \Omega^2 b'^2 = 0. \quad (32)$$

Thus k^2 is *null*. Furthermore, from Eq. (28),

$$\hat{p} = \hat{p}' = \hat{k} = b', \quad (33)$$

where $\hat{p}$ and $\hat{p'}$ were defined in Eq. (27).

Apart from a sign, all the above goes through for a saddle point in I_-. Again k^μ must be light-like, but now $k\cdot a' = 0 = \Omega(-1 + \hat{\boldsymbol{k}}\cdot\boldsymbol{a}')$ (since $a^0 = -\sigma_-$). Thus

$$\hat{p} = \hat{p}' = \hat{k} = -a'. \quad (34)$$

Evaluation of the integrals around the saddle points, in the $\Omega L \gg 1$ limit can then be carried out in the standard way (see *e.g.* [9]) and, using the kinematic relations given above, leads to

$$I^{\rm saddle}_{+,n} = A_+L\frac{b'_s}{(\Omega L)^{1/3}} + iB_+L^2\frac{b''_s}{(\Omega L)^{2/3}} + \ldots \quad (35)$$

$$I^{\rm saddle}_{-,n} = A_-L\frac{a'_s}{(\Omega L)^{1/3}} + iB_-L^2\frac{a''_s}{(\Omega L)^{2/3}} + \ldots \quad (36)$$

where the subscript n on $I_\pm$ refers to the $n^{\rm th}$ term in the sum in Eqs. (10) and (11), and the subscript s on a^μ and b^μ refers to evaluation at a saddle point. The delta functions in Eqs. (10) and (11) enforce $\Omega L = 4\pi n$. We have dropped an overall phase factor which is irrelevant because it is the square of the amplitude that gives a rate. The coefficients can be evaluated explicitly;

$$A_+ = \left(\frac{12}{L^2|b''|^2}\right)^{1/3}\frac{2\pi}{3\Gamma(2/3)},$$
$$B_+ = \left(\frac{12}{L^2|b''|^2}\right)^{2/3}\frac{1}{\sqrt{3}}\Gamma(2/3) \qquad (37)$$

and A_- and B_- are given by identical expressions except that b'' is replaced by a''.

A cusp on a string loop occurs when $|\dot{\boldsymbol{X}}| = 1$ and hence, from (6), when $\boldsymbol{a}' = -\boldsymbol{b}'$. However, from Eqs. (33) and (34) this condition requires a saddle point contribution to both I_+ and I_-. In the vicinity of the beam of the cusp, from Eqs. (31), (33), we get $\hat{\boldsymbol{p}} \sim \boldsymbol{b}'_c = -\boldsymbol{a}'_c$, and similarly for p'. Therefore from (35) and (36), we estimate

$$p \cdot I_\pm^{\rm saddle} \sim \mathcal{O}(k^2)\ ,\quad p' \cdot I_\pm^{\rm saddle} \sim \mathcal{O}(k^2). \qquad (38)$$

This is an important result: consider $|\mathcal{M}|^2_{\rm tot}$ given in Eq. (26). In the beam of the cusp $k^2 = 2p\cdot p' \to 0$ (Eq. (32)), so one might worry that Eq. (26) diverges due to the overall factor of $1/(k^2)^2$. However, this divergence is rendered harmless by the scaling in Eq. (38). Indeed, in the $k^2 = 0$ limit, only the last line in Eq. (26) gives the dominant contribution to the emission from the cusp:

$$(|\mathcal{M}|^2_{\rm tot})_{\rm cusp} \to (2\pi G\mu)^2$$
$$\times 2[\,|I_+|^2|I_-|^2 + |I_+^*\cdot I_-|^2 - |I_+\cdot I_-|^2\,] \qquad (39)$$

with $I_\pm$ in Eqs. (35) and (36) (we have dropped the label "saddle"). The other terms in Eq. (26) all contain factors such as $|p\cdot I_\pm|$ and are higher order in k^2.

The above analysis assumes that $\boldsymbol{k}$ is in the direction of the cusp. If $\boldsymbol{k}$ is at some small angle, θ_+, to $\boldsymbol{b}'$, we can write $\hat{\boldsymbol{k}}\cdot\boldsymbol{b}' = \theta_+^2/2$ and repeat the above analysis as in [7]. The estimate is valid for

$$\theta_+ \le \theta_{m,+} \equiv \left(\frac{4L|\boldsymbol{b}''|^2}{\sqrt{3}\Omega}\right)^{1/3} \qquad (40)$$

Similarly in the case of I_-

$$\theta_- \le \theta_{m,-} \equiv \left(\frac{4L|\boldsymbol{a}''|^2}{\sqrt{3}\Omega}\right)^{1/3} \qquad (41)$$

This estimate assumes $k^2 = 0$ but it holds even if k is perturbed so that it is not precisely null. To summarize, the estimate (39) holds in a cone of opening angle $\theta_+ \sim \theta_- \sim (\Omega L)^{-1/3}$.

B. Discontinuities and kinks

Next we find the contribution of a discontinuity to the integrals $I_\pm$. On expanding the integrands on both sides of the discontinuity, which is say in a' at $\sigma_- = u_k$, one can extract the dominant contribution;

$$I_-^{\rm disc} \sim \int^{u_k} d\sigma_-\ a'_- e^{-ik\cdot(a_k + a'_-(\sigma_- - u_k)+\ldots)}$$
$$+\int_{u_k} d\sigma_-\ a'_+ e^{-ik\cdot(a_k + a'_+(\sigma_- - u_k)+\ldots)}$$
$$= -\frac{2}{i\Omega}\left(\frac{a'_+}{\hat{k}\cdot a'_+} - \frac{a'_-}{\hat{k}\cdot a'_-}\right)e^{-ik\cdot a_k} \qquad (42)$$

where $a'_\pm$ refers to the value of a' on either side of the discontinuity, and $a_k = a(\sigma_- = u_k)$. Similarly we can find I_+^μ, and the result is

$$I_+^{\rm disc} \sim -\frac{2}{i\Omega}\left(\frac{b'_+}{\hat{k}\cdot b'_+} - \frac{b'_-}{\hat{k}\cdot b'_-}\right)e^{-ik\cdot b_k}\,. \qquad (43)$$

It is important to observe that $I_\pm^{\rm disc} \sim \Omega^{-1}$ decays faster with frequency than $I_\pm^{\rm saddle}$ (Eqs. (35), (36)).

The estimates in eqns. (42) and (43) preserve the relation $k\cdot I_\pm = 0$. Hence *if* $k^2 \to 0$ then $\hat{p}^\mu = \hat{p}'^\mu = \hat{k}^\mu$ and again $p\cdot I_\pm^{\rm disc} \to 0$ and $p'\cdot I_\pm^{\rm disc} \to 0$. These relations are important to see that the expression for the invariant matrix element in Eq. (26) is not singular in the $k^2 \to 0$ limit. The case when $\hat{k}\cdot b'_\pm = 0$ or $\hat{k}\cdot a'_\pm = 0$ is very special because now there is a saddle point in addition to a discontinuity and we shall not consider its consequences.

While a saddle point in both I_+^μ and I_-^μ corresponds to a cusp on the string, a saddle point in one of the integrals and a discontinuity in the other occurs at a kink. Thus the dominant contribution to photon production from a kink takes place exactly when $k^2 \to 0$, namely in the forward direction, when $\hat{p}$ and $\hat{p}'$ are collinear. In this limit,

$$|p\cdot I_\pm^{\rm disc}|^2 \sim O(k^2), \qquad |p'\cdot I_\pm^{\rm disc}|^2 \sim O(k^2) \qquad (44)$$

which should be compared to Eq. (38) for a cusp. This can be most clearly seen by writing, for example,

$$p\cdot I_-^{\rm disc} \sim p\cdot\left(\frac{a'_+}{k\cdot a'_+} - \frac{a'_-}{k\cdot a'_-}\right)$$
$$= \left\{\frac{\omega\omega'}{\Omega^2}\frac{[(\hat{\boldsymbol{p}}\cdot\boldsymbol{a}'_+)\boldsymbol{a}'_- - (\hat{\boldsymbol{p}}\cdot\boldsymbol{a}'_-)\boldsymbol{a}'_+]}{\hat{k}\cdot a'_+\ \hat{k}\cdot a'_-}\right\}\cdot(\hat{\boldsymbol{p}}' - \hat{\boldsymbol{p}}) \qquad (45)$$

Therefore $p\cdot I_-^{\rm disc} = O(|\hat{\boldsymbol{p}}' - \hat{\boldsymbol{p}}|)$ from which (44) follows since $\omega\omega'(\hat{\boldsymbol{p}}' - \hat{\boldsymbol{p}})^2 = k^2$ (see Eq. (28)).

C. No saddle point or discontinuities

If a loop has neither cusps or kinks, then $I_\pm$ decay exponentially with ΩL. In that case the energy radiated by the loop per unit unit time, which is proportional to $|\mathcal{M}|^2_{\rm tot}$, also decays exponentially as $\dot{E}_n \propto e^{-\alpha n}$ where

$n = \Omega L/(4\pi)$ is the harmonic number and α is a coefficient. In other words, a loop with no kinks or cusps will radiate a *finite* amount of energy. We have checked explicitly that this is the case by considering the radiation from a chiral cosmic string loop for which there are no cusps and kinks. On the other hand, as we now show, the radiation from a cusp or a kink diverges and needs to be cut off due to the thickness of the string.

IV. POWER EMITTED

We now evaluate the power emitted from cusps, kinks and kink-kink collisions. We divide (19) by L to get the average power radiated in the $n^{\rm th}$ harmonic

$$\begin{aligned}\dot{E}_n &= \frac{2\pi}{L^2} n \int \frac{d^3p}{(2\pi)^3}\frac{1}{2\omega}\int \frac{d^3p'}{(2\pi)^3}\frac{1}{2\omega'} \\ &\quad \times |\mathcal{M}|^2_{n,\rm tot}\delta(n-(\omega+\omega')L/4\pi)\ , \qquad (46)\end{aligned}$$

while the total power is the sum over all harmonics. Note that $\Omega = \omega+\omega' = 4\pi n/L$.

A. Emission from cusp

In order to get the power radiated from a cusp, we first insert the expressions for $I_\pm^{\rm saddle}$ given in Eqs. (35) and (36) into Eq. (39), remembering to include the sum and delta functions in Eqs. (10) and (11). Not all the terms in $I_\pm^{\rm saddle}$ contribute since the string constraint equations (7) imply

$$|a'|^2 = 0 = |b'|^2\ ,\quad a'\cdot a'' = 0 = b'\cdot b'' \qquad (47)$$

and, in addition, since $\boldsymbol{a}' = -\boldsymbol{b}'$ at the cusp,

$$b'_c\cdot a''_c = 0 = a'_c\cdot b''_c \qquad (48)$$

(where the subscript c denotes cusp). Then to leading order we find $|I_+^*\cdot I_-| = |I_+\cdot I_-|$ and the non-vanishing contributions in (39) come from the $|I_+|^2|I_-|^2$ term which is proportional to $|a''_c|^2|b''_c|^2$, so that

$$\begin{aligned}(|\mathcal{M}|^2_{n,\rm tot})_{\rm cusp} &= 2(2\pi G\mu)^2|I_+|^2|I_-|^2 \\ &\sim (G\mu)^2L^4\frac{1}{n^{8/3}}|\boldsymbol{a}''_cL|^{-2/3}|\boldsymbol{b}''_cL|^{-2/3} \qquad (49)\end{aligned}$$

where we ignored an overall numerical factor of order 1.

In order to calculate (46) and estimate $\dot{E}_n$ we next rescale the momenta $\boldsymbol{p}$ and $\boldsymbol{p}'$ by $4\pi n/L$. Then

$$\int \frac{d^3p}{(2\pi)^3}\frac{1}{2\omega} = \frac{1}{2(2\pi)^3}\left(\frac{4\pi n}{L}\right)^2\int d\bar\omega\bar\omega\int d^2\hat{\boldsymbol{p}} \qquad (50)$$

where $\bar\omega = L|\boldsymbol{p}|/4\pi n$. The integration over $\bar\omega$ gives an order 1 numerical factor. The integration over the direction $\hat{\boldsymbol{p}}$ is estimated as

$$\begin{aligned}\int d^2\hat{\boldsymbol{p}}d^2\hat{\boldsymbol{p}}' &\sim \pi\theta^2_{m,+}\ \pi\theta^2_{m,-} \\ &\sim |\boldsymbol{a}''_cL|^{4/3}|\boldsymbol{b}''_cL|^{4/3}n^{-4/3} \qquad (51)\end{aligned}$$

where we have used $\theta_{m,\pm}$ given in Eqs. (40) and (41).

Now putting together results in Eqs. (46), (49), and (51) we finally obtain

$$\dot{E}_n\bigg|_{\rm cusp} \approx \left(\frac{G\mu}{L}\right)^2|\boldsymbol{a}''_cL|^{2/3}|\boldsymbol{b}''_cL|^{2/3}. \qquad (52)$$

The most important feature of this estimate [13] is that it does not depend on the harmonic number n: all factors of n have cancelled! This result was noted earlier in [2] in the context of scalar radiation and in [3] in the context of fermion radiation, though for specific loop trajectories. Our analysis here is more general. Additional features of interest are the $1/L^2$ dependence that will result in larger radiation from smaller loops, and the dependence on the (dimensionless) cusp acceleration vectors, $L\boldsymbol{a}''_c$ and $L\boldsymbol{b}''_c$.

The n-independent spectrum of gravitational Aharonov-Bohm radiation implies that photons of arbitrarily high frequencies will be emitted from cusps. Naively this would imply a divergence in the emitted power. However, the Nambu-Goto action ignores the thickness of a field theory string, and this suggests a cutoff for the highest harmonic that can be emitted – the highest harmonic emitted from a cusp is the string width in the rest frame of the string. To work out the Lorentz factor we start out by noting that the saddle point analysis gives contributions from the region around the cusp of size $|k\cdot a'| < 1$ (in the case of I_-) where the phase in the integrand is not oscillating rapidly. (We follow the discussion in [10].) This gives a time and length interval on the string world-sheet

$$|\Delta t|, |\Delta\sigma| < \frac{L}{(\Omega L)^{1/3}}\ . \qquad (53)$$

In this region

$$1-\dot{\boldsymbol{x}}^2 \sim \frac{(\Delta\sigma)^2}{L^2} \sim (\Omega L)^{-2/3} \qquad (54)$$

Requiring that Ω be less than the Lorentz boost factor times the inverse string width, and ignoring numerical factors we get

$$\Omega < M\sqrt{ML} \qquad (55)$$

where M is the mass scale associated with the string. In terms of the harmonic number, this gives a cutoff

$$n_c \sim (ML)^{3/2}\ . \qquad (56)$$

Hence Eq. (52) applies for $n \le n_c$.

The result in Eq. (52) gives the power emitted in the $n^{\rm th}$ harmonic. Observationally, it is the energy flux at the observer's location that is relevant. To get this flux we divide $\dot{E}_n$ by the cross-sectional area of the beam emitted from the cusp at a distance r from the cusp

$$\frac{\dot{E}_n}{\pi\theta_m^2r^2} \sim \frac{1}{r^2}\left(\frac{G\mu}{L}\right)^2 n^{2/3} \qquad (57)$$

where we have estimated the dimensionless cusp acceleration to be order unity, and the width of the beam as in (41).

The flux grows with n suggesting that it is advantageous to observe at high frequencies. However, the beam at higher n is narrower and hence event rates are lower at high frequencies. These are observational issues that we will return to in future work.

B. Emission from kink

Now we estimate the power emitted from a kink (Eq. (46)), say when there is a discontinuity in $\boldsymbol{a}'$ that contributes to I_-, and a saddle point that contributes to I_+. Then $I_- = I_-^{\rm disc}$ is given by (42) and $I_+ = I_+^{\rm saddle}$ by (35). Note that, for a given k^μ, there will be a saddle point contribution to I_+ if there exists a σ_+ such that $k \cdot b'(\sigma_+) = 0$. Since $\boldsymbol{b}'(\sigma_+)$ describes a curve parametrized by σ_+, there is a saddle point contribution for every $\boldsymbol{k}$ in the direction of a point on the $\boldsymbol{b}'$ curve. Hence the saddle point contribution applies to emission along a curve of directions.

Since there is a saddle point in I_+, the kinematic relations discussed in Sec. IV A (Eqs. (27)-(33)) apply. Noting the estimate in Eq. (44), we find that the dominant terms in (26) in the $\Omega L \gg 1$ limit are given by

$$(|\mathcal{M}|^2_{n,{\rm tot}})_{\rm kink} \to 2\frac{(2\pi G\mu)^2}{p\cdot p'}|I_+|^2 \left[2|p\cdot I_-|^2 + p\cdot p'|I_-|^2\right] \tag{58}$$

where we have also used $|I_+^* \cdot I_-| = |I_+ \cdot I_-|$. Now we can substitute I_- from Eq. (42) and I_+ from (35). Ignoring numerical factors, the result can be written as

$$(|\mathcal{M}|^2_{n,{\rm tot}})_{\rm kink} \sim (G\mu)^2 L^4 \frac{1}{n^{10/3}} (\text{shape factors}) \tag{59}$$

where the "shape factors" include various scalar products that depend on $a'_\pm$ – the shape of the loop – and the directions of the momenta. The shape factors can be written down using the expression in (58) but the result is not illuminating. Expression (59) should be compared with the analogous one, Eq. (49), for a cusp.

We now turn to the phase space integrals in (46). The phase space volume is given in (50) except that, unlike in the case of the cusp, there is a whole curve of saddle points that are relevant, and the emission is along a curve of directions. Hence the integration over momentum direction is estimated as

$$\int d^2\hat{\boldsymbol{p}} \sim 2\pi\theta_{m,\pm} \tag{60}$$

where $\theta_{m,\pm}$ are given in (40), (41).

Putting together all the pieces in Eq. (46) and using (50), we obtain

$$\dot{E}_n\Big|_{\rm kink} \sim \left(\frac{G\mu}{L}\right)^2 (\text{shape factors}) \tag{61}$$

As in the cusp case, an important feature of this result is that it is independent of the frequency of emission.

We now denote the cutoff in harmonic number by n_k, and estimate it by the thickness of the string. What is different from the cusp is that the velocity of the string at the kink is not ultra-relativistic and there is no corresponding large Lorentz boost factor. Therefore the estimate in (61) applies for

$$n < n_k \sim ML \ . \tag{62}$$

The energy flux from a kink at distance r is now given by

$$\frac{\dot{E}_n}{\theta_m r^2} \sim \frac{1}{r^2}\left(\frac{G\mu}{L}\right)^2 n^{1/3} \ . \tag{63}$$

C. Emission from kink-kink collision

This is the situation in which both I_- and I_+ get contributions from discontinuities in a' and b' respectively. Therefore, in the general formula (26) we have to insert (42), (43). This gives

$$\begin{aligned}|\mathcal{M}|^2_{\rm tot} \ \to\ 2\left(\frac{2\pi G\mu}{p\cdot p'}\right)^2 \Big[& 4|p\cdot I_+|^2|p\cdot I_-|^2 \\ & +2(p\cdot p')^1\{|p\cdot I_+|^2|I_-|^2 + |p\cdot I_-|^2|I_+|^2\} \\ & +(p\cdot p')^2|I_+|^2|I_-|^2\Big] \ .\end{aligned} \tag{64}$$

The estimate in Eq. (44) shows that all the terms are non-singular if we take $k^2 \to 0$.

To evaluate the power radiated from a kink-kink collision, we see from Eqs. (43) and (42) that $I_\pm$ are $O(1/n)$. Hence $|\mathcal{M}|^2 \sim n^{-4}$. Also, since there is no beaming in the kink-kink collision

$$\int d^2\hat{\boldsymbol{p}} \sim \pi \ . \tag{65}$$

Then, putting together factors in Eq. (46) and using (50), we obtain

$$\dot{E}_n\Big|_{\rm k-k} \sim \left(\frac{G\mu}{L}\right)^2 (\text{shape factors}) \tag{66}$$

exactly as in the estimate for the cusp and the kink, though the shape factors are different in all three cases and in this kink-kink case they may vary with direction of the momenta. Once again, the result is independent of the harmonic number n and holds up to $n_k = ML$, as in the kink case.

The energy flux from a kink-kink collision at distance r is

$$\frac{\dot{E}_n}{r^2} \sim \frac{1}{r^2}\left(\frac{G\mu}{L}\right)^2 \ . \tag{67}$$

The estimate (66) is due to contribution of the discontinuities at fixed values of both σ_+ and σ_-. Thus the emission is coming from a single point on the string at one instant of time. This corresponds to the point where a left-moving kink and a right-moving kink collide. Hence the temporal duration of the burst is set by the string thickness. On the other hand, emission at a frequency ω cannot be temporally resolved in a time interval less than $\sim \omega^{-1}$. For this reason, the observed burst duration at frequency ω is set by ω^{-1}.

The total energy emitted in harmonic n during a kink-kink collision can be estimated from (66), which is the emitted power averaged over a time period L. Hence the total energy emitted in the $n^{\rm th}$ harmonic in one kink-kink collision is

$$E_n \bigg|_{\rm k-k} \sim \frac{(G\mu)^2}{L} (\text{shape factors}). \tag{68}$$

The continuum version may now be written as

$$\frac{dE}{d\omega}\bigg|_{\rm k-k} \sim (G\mu)^2 \psi_a \psi_b (\text{other shape factors}). \tag{69}$$

where the "sharpness" ψ_a is defined by [11]

$$\psi_a = \frac{1}{2}(1 - \boldsymbol{a}'_+ \cdot \boldsymbol{a}'_-) = \frac{1}{4}(\boldsymbol{a}'_+ - \boldsymbol{a}'_-)^2 \tag{70}$$

and similarly for ψ_b. In Eq. (69) we have pulled out factors of the sharpness since the result must vanish if the sharpness vanishes. The "other shape factors" will in general also depend on the direction of emission.

V. CONCLUSIONS

In this paper we have calculated the flux of photons from cosmic strings. In general this falls off exponentially with harmonic number n. However, as we have shown, the power emitted from cusps, kinks and kink-kink collisions does not fall off with n – rather, it is n-*independent*. Thus the emission from these features on the string dominates over the emission from the rest of the string, at least at high frequencies. If we denote the differential energy flux at frequency ω_0 by F *i.e.*

$$F \equiv \frac{d^3E}{{\rm d}t\ {\rm d}\omega_0\ {\rm d}\Omega_{\rm s}} \tag{71}$$

where $\Omega_{\rm s}$ denotes solid-angle, then our results can be summarized as follows:

$$F_{\rm cusp} \approx \frac{(G\mu)^2}{L}(\omega_0 L)^{2/3}, \quad \Omega_{\rm s} < (\omega_0 L)^{-2/3} \tag{72}$$

$$F_{\rm kink} \approx \frac{(G\mu)^2}{L}(\omega_0 L)^{1/3}, \quad \theta < (\omega_0 L)^{-1/3} \tag{73}$$

$$F_{\rm k-k} \approx \frac{(G\mu)^2}{L} \ . \tag{74}$$

The cusp emits a beam within a solid angle, the kink emits along a curve, while the kink-kink emission is in all directions. The duration of the cusp and kink beams is given by Eq. (53), while the duration of the kink-kink radiation is given by the wavelength at which the emission is observed. Also, the cusp radiates at frequencies $\omega_0 < M\sqrt{ML}$ whereas the kink and kink-kink collisions radiate for $\omega_0 < M$, where M is the string scale.

Our results so far provide the emission characteristics from certain features on strings. Now we briefly discuss the cumulative effect of having many such features on a given loop of string. The cusp and kink emissions are beamed and this makes the analysis more involved. However, the emission due to kink-kink collisions is not beamed and is easier to estimate.

Eq. (69) gives the energy emitted from a single kink-kink collision. To get the energy emitted from a string segment, we need to sum over all the kink-kink collisions occurring on that string segment of length Δl in an interval of time Δt

$$\frac{dE}{d\omega} \sim (G\mu)^2 \int d\psi_a\ d\psi_b\ \psi_a \psi_b \frac{dn_a}{d\psi_a}\frac{dn_b}{d\psi_b}\ \Delta l\ \Delta t \tag{75}$$

where $n_a(\psi_a, t)$ is the number of kinks of sharpness ψ_a at time t per unit length of string, and similarly for n_b.

We will consider emission from a loop of string that formed at time t_f by breaking off a long string. The loop inherits a large number of (shallow) kinks from the long string and from Ref. [11] we can write

$$\int d\psi_a \psi_a \frac{dn_a}{d\psi_a} \sim \frac{1}{t_f}\left(\frac{t_f}{t_*}\right)^\alpha \ . \tag{76}$$

The exponent α is ~ 0.7 in the radiation-dominated epoch. In the matter-dominated epoch, strings contain all the kinks accumulated until the epoch of matter-radiation equality, $t_{\rm eq}$, and from then on the scaling in (76) has $\alpha \sim 0.4$ [12]. The time t_* denotes the epoch at which frictional effects on strings became unimportant. Hence Eq. (75) can be written as

$$\frac{dP_\omega}{dl d\omega} \sim \frac{(G\mu)^2}{t_f^2}\left(\frac{t_f}{t_*}\right)^{2\alpha} \tag{77}$$

where P_ω denotes the power emitted at frequency ω.

We now obtain some numerical estimates, leaving a detailed analysis for future work. The photons emitted from loops deep into the radiation epoch will get thermalized. Hence the emission from loops in the post-recombination era is most relevant for direct observation. Loops at the epoch of recombination could have been produced in the radiation epoch and for simplicity we consider a loop that was formed at the epoch of radiation-matter equality, $t_f = t_{\rm eq} \approx 10^{11}$s. With $G\mu \sim 10^{-8}$, $t_* \sim t_P/(G\mu)^2 \sim 10^{-27}$s [1], where $t_P \approx 10^{-43}$ s is the Planck time, and $\alpha = 0.7$, we get

$$\frac{dP_\omega}{dl d\omega} = \frac{(G\mu)^{2+4\alpha}}{t_f^{2-2\alpha} t_P^{2\alpha}} \approx 10^{-22}\ \frac{\rm ergs}{\rm cm} \ . \tag{78}$$

Detectors on Earth observe a flux of photons and it is more relevant to calculate the number of photons arriving at the detector. This follows from $E = N_\omega \omega$ where N_ω is the number of photons of frequency ω emitted by the string,

$$\frac{dN_\omega}{dtdl} \approx 10^{-22} \ \frac{\text{ergs}}{\text{cm}} \ \frac{d\omega}{\omega} \ . \tag{79}$$

If the loop of length $t_{\rm eq} \sim 10^{21}$cm is at a distance comparable to the present horizon, $r \sim 10^{27}$cm, then the flux of photons at the detector is obtained by multiplying (79) by $t_{\rm eq}/r^2$,

$$\left.\frac{d\mathcal{N}_\omega}{dtdA}\right|_{1 \text{ loop}} \approx \frac{10^{-10}}{\text{km}^2 - \text{yr}} \frac{d\omega}{\omega} \left(\frac{G\mu}{10^{-8}}\right)^{2(1+2\alpha)} \tag{80}$$

where $\mathcal{N}_\omega$ denotes the number of photons arriving at the detector with collecting area dA. If at $t_{\rm eq}$ there was one loop of length $t_{\rm eq}$ per horizon, the number of loops that can contribute to the flux at the detector is given by the number of horizons at $t_{\rm eq}$ that fit within a comoving volume equal to our present horizon volume: $t_0^3/(t_{\rm eq} z_{\rm eq})^3$, where $t_0 \sim 10^{17}$s and $z_{\rm eq} \approx (t_0/t_{\rm eq})^{2/3}$. Therefore the number of contributing loops is $\sim t_0/t_{\rm eq} \sim 10^6$ and the photon flux due to all of these loops is

$$\left.\frac{d\mathcal{N}_\omega}{dtdA}\right|_{\text{loops at rec.}} \approx \frac{10^{-4}}{\text{km}^2 - \text{yr}} \frac{d\omega}{\omega} \left(\frac{G\mu}{10^{-8}}\right)^{2(1+2\alpha)} \tag{81}$$

This estimate suggests that a detector with collecting area $(100 \text{ km})^2$ – comparable to the Auger observatory – will detect ~ 1 photon/year in every logarithmic frequency interval emitted by string loops from the recombination era. The estimate (81) holds for frequencies all the way up to the string scale $\sim 10^{15}$ GeV but it does not take into account any propagation effects. Neither does it take into account the network of long strings and the spatial and length distribution of loops. Note that the emission falls steeply with decreasing string tension. The effect can only be useful for small $G\mu$ if the amount of string in a horizon volume is inversely proportional to some high power of $G\mu$.

The pattern of photon emission from a string is lineal and this may be helpful to distinguish it from conventional sources. It is also possible that the emission from the string will be polarized (though our analysis so far has summed over polarizations and hence erases the polarization information). The beamed emission from cusps and kinks may provide distinctive events that can signal the presence of strings. We plan to explore these signatures in future work.

We would like to close with a cautionary note. The emission rate from kink-kink collisions is greatly enhanced by the factor $(t_f/t_*)^\alpha$ in Eq. (76). This factor is due to the accumulation of kinks on strings from the time, t_*, when their dynamics became undamped. The exponent, α, depends on dynamical factors, such as Hubble expansion, that tend to straighten out the kinks, but the estimate does not take radiation backreaction into account and this will have a tendency to reduce the emission rate. However, it is possible that emission at frequencies much lower than the string scale remain relatively unaffected by the backreaction. We can be more confident of our estimates of light from cosmic strings only once this issue is satisfactorily resolved.

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