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Simulating merging binary black holes with nearly extremal spins

Geoffrey Lovelace,[1] Mark A. Scheel,[2] and Béla Szilágyi[2]

[1] *Center for Radiophysics and Space Research, Cornell University, Ithaca, New York, 14853*
[2] *Theoretical Astrophysics 350-17, California Institute of Technology, Pasadena, CA 91125*

Astrophysically realistic black holes may have spins that are nearly extremal (i.e., close to 1 in dimensionless units). Numerical simulations of binary black holes are important tools both for calibrating analytical templates for gravitational-wave detection and for exploring the nonlinear dynamics of curved spacetime. However, all previous simulations of binary-black-hole inspiral, merger, and ringdown have been limited by an apparently insurmountable barrier: the merging holes' spins could not exceed 0.93, which is still a long way from the maximum possible value in terms of the physical effects of the spin. In this paper, we surpass this limit for the first time, opening the way to explore numerically the behavior of merging, nearly extremal black holes. Specifically, using an improved initial-data method suitable for binary black holes with nearly extremal spins, we simulate the inspiral (through 12.5 orbits), merger and ringdown of two equal-mass black holes with equal spins of magnitude 0.95 antialigned with the orbital angular momentum.

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Although there is considerable uncertainty, it is possible that astrophysical black holes exist with nearly extremal spins (i.e., in dimensionless units spins close to 1, the theoretical upper limit for a stationary black hole). Binary black hole (BBH) mergers in vacuum typically lead to remnant holes with dimensionless spins $\chi \sim 0.7 - 0.8$ [1–3], although if the merging holes are surrounded by matter the remnant's spin typically could be higher than $\chi \sim 0.9$ [1, 3]. Black holes can reach higher spins via prolonged accretion [4, 5]: thin accretion disks [with magnetohydrodynamic (MHD) effects neglected] lead to spins as large as $\chi \sim 0.998$ [6], while thick-disk accretion with MHD effects included can lead to spins as large as $\chi \sim 0.95$ [7, 8]. Even without accretion, at very high mass ratios with spins aligned with the orbital angular momentum, binary black hole mergers can also lead to holes with nearly extremal spins [9–11]. There is observational evidence suggesting the existence of black holes with nearly extremal spins in quasars [12], and some efforts to infer the spin of the black hole in microquasar GRS 1915+105 from its x-ray spectra suggest a spin larger than 0.98, though other analyses suggest the spin may be much lower [13–15].

Merging BBHs—possibly with nearly extremal spins—are among the most promising sources of gravitational waves for current and future detectors. Numerical simulations of BBHs are important tools both for predicting the gravitational waves that detectors will observe and for exploring the behavior of nonlinear, highly dynamical spacetimes. Following Pretorius' breakthrough in 2005 [16], several groups have successfully simulated the inspiral, merger, and ringdown of two coalescing black holes in a variety of initial configurations; however, all prior BBH simulations have been limited to spins of 0.93 or less, which is quite far from extremal. The parameter χ is a poor measure of how close a black hole is to extremality in terms of physical effects: a black hole with spin 0.93 has less than 60% of the rotational energy of an extremal hole with the same mass (Fig. 1).

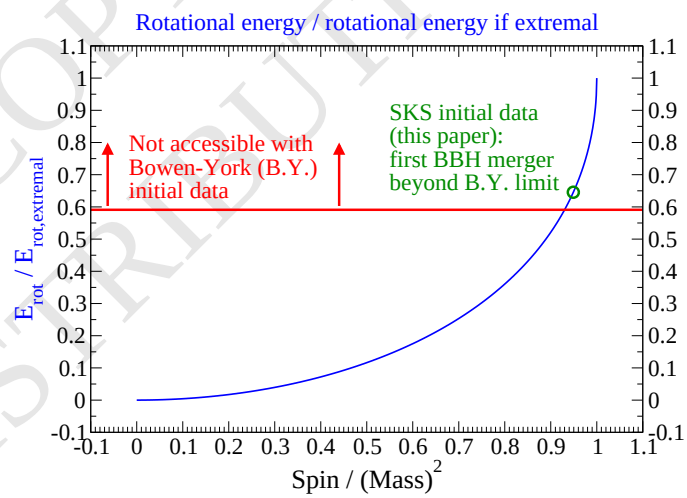

FIG. 1. The rotational energy of a Kerr black hole as a function of the hole's dimensionless spin parameter $\chi := \text{Spin}/(\text{Mass})^2$. The thick red line indicates the Bowen-York limit: standard Bowen-York puncture initial data—used in almost all numerical binary-black-hole calculations to date—cannot yield rotational energies more than 60% of the way to extremality. By using instead initial data based on two superposed Kerr-Schild holes ("SKS initial data"), in this paper we surpass the Bowen-York limit (green circle), opening the way for numerical studies of merging, nearly extremal black holes.

Previous simulations have been unable to reach higher spins because of the way they construct their initial data. Just as initial data for Maxwell's equations must satisfy constraints (the electric and magnetic fields must have vanishing divergence in vacuum), initial data for the Einstein equations must satisfy constraint equations. Most BBH simulations begin with puncture initial data [17], which assumes that the initial spatial metric is *conformally flat* (i.e. proportional to the metric of flat space). With this assumption, 3 of the 4 constraint equations can

M_i/M	0.5000	$M_{\rm ADM}/M$	0.9933	d_0/M	15.366
χ_i^z	-0.9498	$J^z_{\rm ADM}/M^2$	0.6845	$\Omega_0 M$	0.014508
				$\dot{a}_0$	-0.0007139

TABLE I. Properties of initial data evolved in this paper. The quantity M denotes the sum of the holes' Christodoulou masses at $t = 0$. Hole i (where $i = A$ or B) has Christodoulou mass M_i and dimensionless spin χ_i^z along the z axis (i.e., in the direction of the orbital angular momentum). Also listed is the Arnowitt-Deser-Misner (ADM) mass $M_{\rm ADM}$ and angular momentum $J^z_{\rm ADM}$ (e.g., Eqs. (25)–(26) of Ref. [35]). The initial angular velocity Ω_0, radial velocity $\dot{a}_0$, and coordinate separation d_0 were tuned to reduce the orbital eccentricity.

be solved analytically using the solutions of Bowen and York [18, 19]; however, conformally flat initial data cannot describe single or binary black holes that are both in equilibrium and possess linear [20] or angular [21, 22] momentum. Bowen-York puncture data *can* yield solutions of binary black holes with spins as large as $\chi = 0.984$ initially, but when such initial data are evolved, the holes quickly relax to spins of about $\chi = 0.93$ or less [23–25]. Several groups have evolved BBH puncture data with spins near but below this *Bowen-York limit* [9, 26, 27], with the simulation by Dain, Lousto, and Zlochower [28] coming the closest with spins of 0.967 at time $t = 0$ quickly falling to 0.924.

To reach spins beyond the Bowen-York limit, one must begin with initial data that is *conformally curved*. Recently, Liu and collaborators [29] have constructed and evolved conformally curved initial data based on that of Brandt and Seidel [30, 31] for a single black hole with spins as high as $\chi = 0.99$. Hannam and collaborators [32] have constructed and evolved conformally curved BBH initial data [33, 34] for head-on mergers of black holes with spins as large as $\chi = 0.9$. In Ref. [35], conformally curved BBH data with spins of $\chi = 0.93$ were constructed and evolved through the first 1.9 orbits of an inspiral, but no attempt was made to simulate the complete inspiral, merger, and ringdown.

In this paper, we demonstrate that conformally curved initial data is suitable for simulations with nearly extremal spins by using it to compute the first inspiral, merger, and ringdown of two black holes with spins larger than the Bowen-York limit. By surpassing this limit, our results open the way for numerical exploration of the gravitational waveforms and nonlinear dynamics of black holes that are nearly extremal.

Initial data—We evolve a low-eccentricity initial data set: a BBH where the holes have equal masses and equal spins of magnitude 0.95 antialigned with the orbital angular momentum. Some properties of the initial data used in this paper are listed in Table I.

Following Ref. [35] and the references therein, we construct constraint-satisfying initial data by solving the extended conformal thin sandwich equations with quasiequilibrium boundary conditions [36–41] using a spectral elliptic solver [42]. The initial spatial metric is proportional to a weighted superposition of the metrics of two boosted, spinning Kerr-Schild black holes.

We measure the quasilocal spin $S_{\rm AKV}$ of each hole in the initial data using the approximate-Killing-vector method summarized in Appendix A of Ref. [35], which is very similar to the prescription previously published by Cook and Whiting [43]. The dimensionless spin of each hole χ is then related to $S_{\rm AKV}$ by the formula $\chi := S_{\rm AKV}/M_{\rm chr}^2$, where $M_{\rm chr} := \sqrt{M_{\rm irr}^2 + S^2/4M_{\rm irr}^2}$ is the Christodoulou mass, $M_{\rm irr} := \sqrt{A/16\pi}$ is the irreducible mass, and A is the area of the horizon. (For a single Kerr black hole, $M_{\rm chr}$ reduces to the usual Kerr mass parameter.)

To reduce eccentricity, we follow the iterative method of Ref. [44], which is an improvement of the earlier method of Ref. [45]. For each iteration, we construct an initial data set and evolve it for approximately 3 orbits. Then, the initial angular and radial motion of the holes are adjusted to minimize oscillations in the orbital frequency. Using this method, we reduce the orbital eccentricity to approximately 10^{-3}.

Evolution—We evolve our initial data using the Spectral Einstein Code `SpEC` [46]. Building on the methods of Ref. [47] and the references therein, we have made several technical improvements to our code which both enable us to evolve our $\chi = 0.95$ initial data through merger and make our code more robust in general. Here we briefly summarize some of the most important improvements; full details of these techniques will be described in a future paper.

We use a computational domain with the singularities inside the horizons excised, and we use a time-dependent coordinate mapping to keep the excision boundaries inside the individual apparent horizons as the horizons orbit and slowly approach each other [48]. Our coordinate mapping also ensures that the excision surfaces' shapes conform to those of the horizons which enclose them. One important ingredient of our improved binary black hole evolutions is that the coordinate mapping is adjusted *adaptively* throughout the evolution, which is helpful because the horizons' dynamics change from slow to fast during the simulation.

Because we apply no boundary condition on the excision surfaces, these surfaces must be pure-outflow boundaries (i.e., must have no incoming characteristic fields) in order for the evolution to be well-posed. A second improvement to our code is that we now can adjust the velocity of each excision surface to keep the characteristic fields outgoing there. For the $\chi = 0.95$ simulation considered here, this characteristic speed control is necessary only during the last orbit before merger; earlier, it is sufficient to control the size of the excision surface using the method of Ref. [47].

A third element which we have recently added to `SpEC` is spectral adaptive mesh refinement (AMR). During the evolution, we monitor the truncation error of each evolved field, the resolution requirements of the apparent horizons, and the local magnitude of constraint vi-

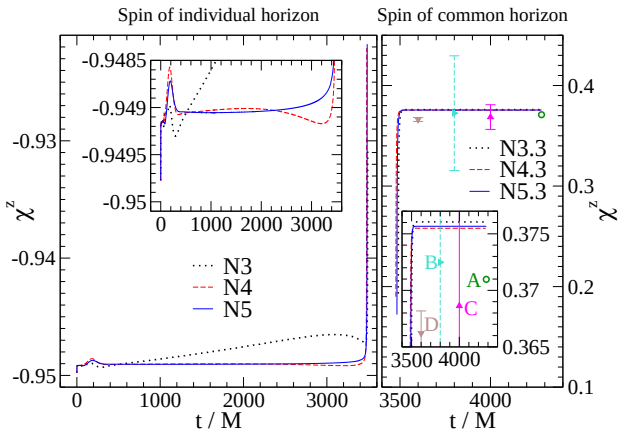

FIG. 2. *Color online.* Left panel: The z component χ^z of the dimensionless quasilocal spin of one individual horizon vs time t. (The individual holes' spins are equal within numerical error.) Right panel: spin of the common horizon vs. time and the final spins predicted by the fitting formulae in Ref. [49] ("A"), Ref. [1] ("B"), Ref. [50] ("C"), and Ref. [9] ("D"), and (for "B" and "C", and "D") the error bars corresponding to the fitting formulae's listed uncertainties. (Note that the horizontal positions of points "A"–"D" on the figure are arbitrary.) Our results are shown for several resolutions (labeled Nx or $Nx.y$, where $x \in 3,4,5$ and $y \in 1,2,3$ label the resolution used before and after merger, respectively).

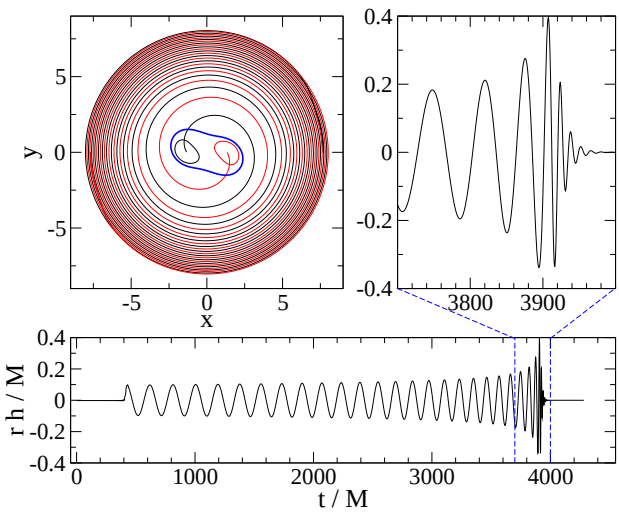

FIG. 3. *Color online.* The orbital trajectory of the centers of the individual horizons and the individual and common horizons at the end of the inspiral (top left) and the real part of the $\ell = 2, m = 2$ mode of the emitted gravitational waveform h extracted at radius $r = 405M$ (bottom). The holes travel through about 12.5 orbits before merging. All data is from resolution $N5.3$.

olation; to maintain a desired accuracy, we then add or remove spectral basis functions as needed. In the simulation presented in this paper, we use AMR only during the final quarter orbit before merger. Throughout the entire simulation, we also adaptively adjust the resolution of the apparent horizon finder as the horizon becomes more distorted.

Results—In Fig. 2, we plot the dimensionless quasilocal spin χ measured on one individual horizon and also on the common horizon. From $t = 0$ to $t = 50M$, there is a sharp, numerically-resolved drop in the magnitude of the dimensionless spin χ from 0.9498 to 0.9492. During the remainder of the inspiral, the spin drifts, with the amount of drift decreasing as resolution increases; at the highest resolution ($N5$), the spin remains $\chi = 0.949$ throughout the next 11.8 orbits, until just before merger, when the magnitude of the spin of each hole drops sharply. This result demonstrates that it is now possible to simulate BBHs where the holes retain spins beyond the Bowen-York limit throughout the inspiral; it also opens the way for future explorations of the strong-field dynamics of merging, nearly extremal holes—dynamics that can only be explored using numerical simulations.

During the ringdown, the spin χ of the common horizon quickly relaxes to its final value of $\chi = 0.3757 \pm 0.0002$ (where the uncertainty is estimated as the difference between the highest and second-highest resolutions). This is approximately consistent with but slightly larger than the predictions obtained by extrapolating fitting formulae from simulations with lower initial spins of Ref. [49] ($\chi_{\rm fit} \approx 0.371$), Ref. [1] ($\chi_{\rm fit} = 0.372 \pm 0.057$), Ref. [50] ($\chi_{\rm fit} = 0.369 \pm 0.012$), and Ref. [9] ($\chi_{\rm fit} = 0.366 \pm 0.002$). This result is a first step toward a better understanding of the relation between the properties of the remnant hole and those of the merging holes when the latter are nearly extremal. Numerical simulations that directly measure this relation (instead of extrapolating from lower-spin results) will yield greater understanding of the properties of black holes produced from merging extremal holes.

Figure 3 shows the individual horizon trajectories and the real part of the $(\ell, m) = (2, 2)$ spherical harmonic mode of the emitted gravitational waveform. We extract waves on a series of concentric spherical shells; the waveform shown was extracted on the outermost spherical shell (at radius $r = 405M$). Accurate gravitational waveforms obtained from this and future simulations with spins beyond the Bowen-York limit will be useful for calibrating analytic template banks for gravitational-wave searches.

The Christodoulou mass of the final black hole is $M_{\rm final}/M_{\rm relax} = 0.9683 \pm 0.0001$, where $M_{\rm relax} = 1.0003M$ is the sum of the masses of the individual holes after the initial relaxation and where the uncertainties of $M_{\rm final}$ and $M_{\rm relax}$ are estimated as the difference between the highest and second-highest resolution. Under the assumption that each hole has a constant mass $M_{\rm relax}$ throughout the inspiral (which holds within $O\left(10^{-5}\right)$ in our simulation after the holes have relaxed), the quantity $1 - M_{\rm final}/M_{\rm relax}$ represents the fraction of the initial mass that would have been radiated from $t = -\infty$ to $t = +\infty$, had our simulation contained the entire inspiral instead of just the final 12.5 orbits.

Our results demonstrate for the first time that it is possible to simulate merging black holes with spins larger than the Bowen-York limit of $\chi = 0.93$, the highest spin previously obtainable. Because astrophysical black holes may be nearly extremal, these simulations have astrophysical as well as physical relevance. In particular, this work opens the way to use numerical simulations to explore the strong-field dynamics of merging, nearly extremal black holes, to gain a better understanding of the properties of the remnant hole formed by a nearly extremal BBH merger, and to provide high-spin gravitational waveforms for data analysis.

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