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Baryon Magnetic Moments in Alternate $1/N_c$ Expansions

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Abstract

Recent work shows not only the necessity of a $1/N_c$ expansion to explain the observed mass spectrum of the lightest baryons, but also that at least two distinct large N_c expansions, in which quarks transform under either the color fundamental or the two-index antisymmetric representation of $SU(N_c)$, work comparably well. Here we show that the baryon magnetic moments do not support this ambivalence; they strongly prefer the color-fundamental $1/N_c$ expansion, providing experimental evidence that nature decisively distinguishes among $1/N_c$ expansions for this observable.

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I. INTRODUCTION

The $1/N_c$ expansion of QCD [1] associated with the strong interaction gauge group $SU(N_c)$ has in the past few decades produced a remarkable number of insights into both the qualitative and semi-quantitative aspects of fundamental strong interaction physics. The existence of numerous narrow meson states, the preference of scattering processes for channels with a minimum number of intermediate hadronic states, the suppression of glueball-meson mixing, the heaviness of baryons compared to mesons, the baryon mass spectrum, and the existence of baryon resonance multiplets are findings that now belong to the litany of standard large N_c hadronic lore.

The number N_c of distinct color charges [or alternately, the rank of $SU(N_c)$] is intrinsically linked to the number of valence quarks in the baryon, which was the key observation in the original proposal of the color quantum number [2]. One may naturally suppose that quarks, being the fundamental matter constituents of the baryons, each carry a unit of color charge in the fundamental (F) representation of $SU(N_c)$, in the same way that spin- $\frac{1}{2}$ fermions, being the most elementary matter particles in the Standard Model, transform under the smallest nontrivial representation of the Lorentz group. However, even after imposing this assignment at $N_c = 3$ (as is abundantly verified by the agreement of innumerable experiments with innumerable calculations in the asymptotic freedom regime of QCD), other generalizations are possible for $N_c > 3$. Note, for instance, that for $N_c = 3$ the (anti)fundamental and two-index antisymmetric (AS) representations of $SU(N_c)$ are equivalent, which is clearly seen from the identification of \bar{F} and AS quark fields, $q_i = \frac{1}{2}\epsilon_{ijk}q^{jk}$. In other words, at $N_c = 3$ an antigreen quark is formally equivalent to a red-blue quark. Gluons are assumed as usual to carry one color and one anticolor index in the color adjoint representation. For $N_c > 3$ the \bar{F} and AS color representations are distinct, meaning that although both theories possess an $SU(N_c)$ gauge group, they differ starkly in the details of their dynamics. Each theory has a distinct large N_c limit and hence a distinct $1/N_c$ expansion (denoted here as $1/N_c^F$ and $1/N_c^{AS}$, respectively), as can be seen for instance by noting that quark lines carry only one color index in the N_c^F theory but two color indices in the N_c^{AS} theory, and therefore internal quark loops are suppressed by $1/N_c^1$ compared to gluon loops in $1/N_c^F$ but not in $1/N_c^{AS}$.

While the $1/N_c^{AS}$ expansion is only one of many possible generalizations of 3-color QCD, it carries a particular theoretical distinction. As shown in Ref. [3], an orientifold equiva-

lence relates a large class of observables between the large N_c^{AS} and large N_c^{Adj} limits, where QCD_{Adj} is the corresponding generalization of QCD to a Yang-Mills theory in which Majorana quarks carry color-adjoint representation charges. In turn, the theory QCD_{Adj} with a single massless flavor of quark is supersymmetric, allowing the application of powerful SUSY theorems to its analysis. To what extent this chain of correspondences conveys valuable phenomenological insights into the physical case of $N_c=3$ with several flavors of massive quark remains an open and interesting question, but the step of analyzing whether the $1/N_c^{\text{AS}}$ expansion *per se* is supported or denied by phenomenology can be pursued independently of these lofty aspirations.

We investigate in this paper the viability of the $1/N_c^{\text{AS}}$ expansion for a class of baryon observables, namely, their magnetic moments. Baryon wave functions of course depend upon the color structure of the quarks. Using traditional F quarks, the construction of a color-singlet state from N_c quarks is straightforward [4]:

$$B_{\text{F}} \sim \epsilon_{i_1, i_2, \dots, i_{N_c}} q^{i_1} q^{i_2} \dots q^{i_{N_c}}. \quad (1.1)$$

The construction of color-singlet baryon states from AS quarks, on the other hand, is neither obviously unique nor simple to express in closed form. Nevertheless, a construction exists [5] for any value of N_c that combines $N_c(N_c-1)/2$ AS quarks into a form fully antisymmetric under the exchange of any two of them. For $N_c=3$, this expression reads

$$B_{\text{AS}} \sim (\epsilon_{i_2, j_2, i_1} \epsilon_{i_3, j_3, j_1} - \epsilon_{i_3, j_3, i_1} \epsilon_{i_2, j_2, j_1}) q^{i_1, j_1} q^{i_2, j_2} q^{i_3, j_3}, \quad (1.2)$$

and for all N_c the corresponding expression contains two Levi-Civita tensors. In both the F and AS cases, each allowed quark color combination appears precisely once in the wave function; these numbers are ${}_N C_1 = N_c$ for F and ${}_N C_2 = N_c(N_c-1)/2$ for AS. Moreover, baryon masses scale with the number of quarks: $\sim N_c$ for large N_c^{F} and $\sim N_c^2$ for large N_c^{AS} . The issue of N_c scaling of interactions among the quarks in the AS case [6, 7] is somewhat more subtle than in the F case; as an example, while the exchange of a single gluon between two F quarks introduces a factor of $g_s^2 \propto 1/N_c$ [1], in order to maintain the color neutrality of the full baryon state in the interaction of two AS quarks, in a typical case a gluon must be exchanged between each of the quarks' two color lines, leading to a factor of $g_s^4 \propto 1/N_c^2$.

Since the quark fields comprising the fermionic baryons are completely antisymmetrized under color in both cases, the baryon spin-flavor-space wave functions are completely symmetric. Under the assumption that baryons in the ground-state multiplet have spatially

symmetric wave functions, their spin-flavor wave functions are also completely symmetric. Indeed, in either large N_c limit, an emergent spin-flavor symmetry [8–11] collects baryon states into spin-flavor multiplets, and the baryon observables satisfy relations that hold to various orders in $1/N_c$. The ground-state multiplet for the three lightest (u, d, s) quark flavors is the large N_c generalization of the old SU(6) **56**-plet containing the familiar spin- $\frac{1}{2}$ octet and spin- $\frac{3}{2}$ decuplet baryons. The full analysis of the mass spectrum of this multiplet in the $1/N_c^F$ expansion appeared many years ago in Refs. [12], with the result that each operator expected to contribute to the observed baryon masses at a given order $1/N_c^n$ does indeed produce an effect equal to $1/3^n$ times a coefficient of order unity. Ignoring the $N_c = 3$ suppressions imposed by the $1/N_c^F$ expansion leads to a far worse accounting of the experimental mass spectrum.

Recently, the same techniques as in Refs. [12] were applied to the ground-state baryon masses using the $1/N_c^{AS}$ expansion [13]. Following the scaling and counting arguments discussed above, one might naively expect that changing from the $1/N_c^F$ to the $1/N_c^{AS}$ expansion would simply induce the modification $1/N_c^n \rightarrow 1/N_c^{2n}$, rescaling operator coefficients by powers of 3 for $N_c = 3$, and thus spoil the agreement. Remarkably, the $1/N_c^{AS}$ fits turn out to be of comparable quality to those for $1/N_c^F$, and both are far superior to fits with no $1/N_c$ suppressions at all; one concludes that nature requires *some* $1/N_c$ expansion, but does not—at least from the baryon mass spectrum—indicate which one is preferred.

The purpose of the current work is to determine whether the $1/N_c^{AS}$ expansion provides a viable explanation of the ground-state baryon magnetic moment spectrum. After the masses, the magnetic moments provide the largest set of precisely measured baryon static observables. They also provide a nearly orthogonal set of information to the masses, since they are strongly dependent upon the charges of the component quarks: Even baryons in a single isospin multiplet such as the proton and neutron, which differ chiefly by the substitution of a single quark $u \leftrightarrow d$, carry rather different values of magnetic moment, approximately related by the famous SU(6) result $\mu_n = -2\mu_p/3$.

Baryon magnetic moments in the $1/N_c^F$ expansion were first considered in Refs. [14–16]. In each case, a set of operators considered physically most significant to the observables were included while others were neglected. Further work focused on the $\Delta \rightarrow N\gamma$ transition [17]. A complete basis in the $1/N_c^F$ expansion was not produced until much later, in Ref. [18]. This work explored two $1/N_c^F$ expansion variants: one in which the sole parameter organizing

the expansion is $1/N_c$, and one a more physical *single-photon ansatz*, in which the flavor structure respects that magnetic moments involve couplings of a baryon to a single photon at lowest order, and thus each quark should couple proportionally to its electric charge.

In this paper we work entirely within the single-photon ansatz and compute matrix elements of a set of operators truncated at a consistent order in both the $1/N_c^F$ and $1/N_c^{AS}$ expansions. While Ref. [18] shows that one may compute a complete set of operator matrix elements, a full analysis such as performed for the masses is not currently possible since many of the baryon magnetic moments (particularly for the decuplet and strange decuplet-octet transitions) remain unmeasured. As a result, we produce a fit to coefficients at as high of an order in the $1/N_c$ expansions as possible, given current data. As shown below, the fits in the $1/N_c^F$ and $1/N_c^{AS}$ expansions are not both consistent with data to comparable confidence; in particular, the $1/N_c^{AS}$ fit generates effects too large to be consistent with the $1/N_c^{AS}$ expansion, while the corresponding quantities in the $1/N_c^F$ expansion are all of a natural size, and using no $1/N_c$ expansion at all would predict these quantities to be anomalously small. That the spectrum of baryon magnetic moments appears to require and prefer one particular $1/N_c$ expansion strongly is the conclusion of this work.

This paper is organized as follows: In Sec. II we reprise the operator basis for the $1/N_c^F$ expansion used in Ref. [18], both for the pure and single-photon ansatz $1/N_c$ expansions. In Sec. III we detail the modifications necessary to carry out the analogous analysis in the $1/N_c^{AS}$ expansion. Results of fits to magnetic moment observables and a discussion of their significance appear in Sec. IV, and we summarize in Sec. V.

II. OPERATOR BASES

The enumeration of independent baryon magnetic moment operators and the calculation of their matrix elements in the $1/N_c^F$ expansion are discussed in detail in Secs. II and III of Ref. [18]. We summarize here the essential points, inasmuch as they are germane to providing a point of comparison to the calculation in the $1/N_c^{AS}$ case to be described in the next section.

The method by which observables classified by their spin-flavor properties can be calculated for the ground-state multiplet of baryons in the $1/N_c$ expansion has been understood for many years, in the current context dating back to Ref. [19], with similar approaches ap-

pearing in Refs. [11, 20]. One defines a complete set of spin, flavor, and spin-flavor operators that act upon the quarks comprising the baryon using the building blocks:

$$\begin{aligned} J^i &\equiv \sum_{\alpha} q_{\alpha}^{\dagger} \left(\frac{\sigma^i}{2} \otimes \mathbb{1} \right) q_{\alpha}, \\ T^a &\equiv \sum_{\alpha} q_{\alpha}^{\dagger} \left(\mathbb{1} \otimes \frac{\lambda^a}{2} \right) q_{\alpha}, \\ G^{ia} &\equiv \sum_{\alpha} q_{\alpha}^{\dagger} \left(\frac{\sigma^i}{2} \otimes \frac{\lambda^a}{2} \right) q_{\alpha}, \end{aligned} \tag{2.1}$$

where the index α sums over all the quarks in the baryon, σ^i are the Pauli spin matrices, and λ^a are the Gell-Mann flavor matrices. This set of operators spans all possible spin-flavor actions upon a single quark (*one-body operators*); in a baryon containing M quarks [$M=N_c$ in the $1/N_c^F$ expansion, $M=N_c(N_c-1)/2$ in the $1/N_c^{AS}$ expansion], the most general operator linearly independent from those appearing at lower orders requires a polynomial in one-body operators no higher than degree M (*i.e.*, $0 \leq n \leq M$ -body operators). For the magnetic moments of the ground-state multiplet baryons, one forms all independent operators transforming as T odd, $\Delta J=1$, $\Delta J_3=0$, $\Delta Y=0$, and $\Delta I_3=0$ [28]; their number must precisely equal that of the distinct observables carrying these quantum numbers.

Many of the operators produced in this fashion are linearly dependent, particularly when acting upon the completely symmetric ground-state multiplet (For example, any operator acting antisymmetrically on two quarks annihilates such states). The full *operator reduction rule* for removing all such superfluous operators acting upon the ground-state multiplet appears in Ref. [19], and has already been applied to the set of operators appearing here.

Once the basis is established, one must take into account both explicit and implicit factors of N_c . The explicit powers arise as overall scaling due to the 't Hooft power counting; an n -body operator requires at minimum the exchange of $(n-1)$ [$2(n-1)$] gluons between the n quark lines in the $1/N_c^F$ ($1/N_c^{AS}$) expansion, leading to a scaling coefficient of $1/N_c^n$ for the $1/N_c^F$ expansion, $1/N_c^{2n}$ for the $1/N_c^{AS}$ expansion. Implicit factors of N_c arise due to combinatorics; contributions from the M quarks can add coherently for many of the operators to give factors of M^n in their matrix elements.

Finally, since operator matrix elements can contain both leading and subleading contributions in N_c , it is possible for a set of operators to be linearly independent when all terms are included but linearly dependent when only the leading terms are retained. As an example, the operators $\mathbb{1}$ and T^8 both have $O(M^1)$ matrix elements, but their linear com-

TABLE I: From Ref. [18]: The 27 linearly independent operators contributing to the magnetic moments of the spin- $\frac{1}{2}$ and spin- $\frac{3}{2}$ ground-state baryons, organized according to the leading N_c counting of their matrix elements in the $1/N_c^F$ expansion.

$O(N_c^1)$	G^{33}
$O(N_c^0)$	$J^3, G^{38}, \frac{1}{N_c}T^3G^{33}, \frac{1}{N_c}N_sG^{33}, \frac{1}{N_c^2}\frac{1}{2}\{J^iG^{i3}, G^{33}\}$
$O(N_c^{-1})$	$\frac{1}{N_c}T^3J^3, \frac{1}{N_c}N_sJ^3, \frac{1}{N_c}T^3G^{38}, \frac{1}{N_c}N_sG^{38}, \frac{1}{N_c^2}\frac{1}{2}\{J^2, G^{33}\}, \frac{1}{N_c^2}(T^3)^2G^{33}, \frac{1}{N_c^2}N_s^2G^{33},$ $\frac{1}{N_c^2}T^3N_sG^{33}, \frac{1}{N_c^2}J^iG^{i3}J^3, \frac{1}{N_c^2}\frac{1}{2}\{J^iG^{i8}, G^{33}\}, \frac{1}{N_c^2}\frac{1}{2}\{J^iG^{i3}, G^{38}\}$
$O(N_c^{-2})$	$\frac{1}{N_c^2}J^2J^3, \frac{1}{N_c^2}N_s^2J^3, \frac{1}{N_c^2}(T^3)^2J^3, \frac{1}{N_c^2}T^3N_sJ^3, \frac{1}{N_c^2}\frac{1}{2}\{J^2, G^{38}\}, \frac{1}{N_c^2}(T^3)^2G^{38}, \frac{1}{N_c^2}N_s^2G^{38},$ $\frac{1}{N_c^2}T^3N_sG^{38}, \frac{1}{N_c^2}J^iG^{i8}J^3, \frac{1}{N_c^2}\frac{1}{2}\{J^iG^{i8}, G^{38}\}$

bination $N_s \equiv \frac{1}{3}(\mathbb{1} - 2\sqrt{3}T^8)$ simply counts the number of strange quarks in a baryon state, and hence has matrix elements of $O(M^0)$ when applied to the familiar ground-state baryon states. Such suppressed linear combinations are called *demoted operators* [21]. A proper enumeration of operators in either $1/N_c$ expansion includes the effect of all demotions.

Even a list of operators satisfying all of these conditions is unnecessary; since one ultimately compares the results of the calculation to $N_c = 3$ baryon states, only an expansion up to and including 3-body operators is required for a full accounting of data. A complete set of operators in the $1/N_c^F$ expansion up to 3-body level, after accounting for all scaling and combinatoric powers of N_c , and imposing the restrictions of operator reduction rules and demotions, appears as Table I of Ref. [18], and is reproduced for convenience as Table I here. One notes that the list contains precisely 27 operators, which matches the number of baryon magnetic moment observables in the ground-state multiplet of spin- $\frac{1}{2}$ and spin- $\frac{3}{2}$ baryons: A magnetic moment for each of the octet and decuplet baryons, and transitions between the 9 pairs of states with the same values of electric charge and strangeness (*i.e.*, $\Sigma^0\Lambda, \Delta^+p$, *etc.*). The full set of matrix elements for these 27 operators evaluated for the 27 observables occupies Tables IV–IX in Ref. [18].

However, as indicated above we adopt a different organization of the operator basis, founded on the physical assumption (the single-photon ansatz) that the quarks in any magnetic moment operator couple proportionally to their electric charges. That is, one assumes the magnetic moment operator couples to a single photon through the flavor combinations

$$Q = T^Q \equiv T^3 + \frac{1}{\sqrt{3}}T^8,$$

$$G^{iQ} \equiv G^{i3} + \frac{1}{\sqrt{3}} G^{i8}. \quad (2.2)$$

Under this assumption, the only operators appearing up to 3-body level (obtained as linear combinations of those in Table I) with no other source of SU(3) flavor breaking are given by [18]

$$\mathcal{O}_1 \equiv G^{3Q}, \quad \mathcal{O}_2 \equiv \frac{1}{N_c} Q J^3, \quad \tilde{\mathcal{O}}_3 \equiv \frac{1}{N_c^2} \frac{1}{2} \{J^2, G^{3Q}\}, \quad \mathcal{O}_4 \equiv \frac{1}{N_c^2} J^i G^{iQ} J^3, \quad (2.3)$$

which give matrix elements of $O(N_c^1)$, $O(N_c^0)$, $O(N_c^{-1})$, and $O(N_c^{-1})$, respectively. The operator combination

$$\mathcal{O}_3 \equiv (\tilde{\mathcal{O}}_3 - \mathcal{O}_4) \quad (2.4)$$

has the interesting property that it vanishes for all diagonal magnetic moments and therefore provides particularly incisive information about the transition moments. The list of leading-order operators in the $1/N_c^F$ single-photon ansatz is therefore given by $\mathcal{O}_{1,2,3,4}$.

Additional sources of SU(3) flavor breaking include negligibly small effects due to the presence of a second (loop) photon (proportional to $\alpha/4\pi$) or to the difference $(m_u - m_d)$. However, the dominant additional SU(3)-flavor breaking effects occur due to the distinction of the strange quark, $m_s \gg m_{u,d}$, and are indicated by the presence of an SU(3)-breaking parameter ε expected to be ~ 0.3 . Such phenomena are manifested either through the strangeness-counting operator N_s or the strange quark spin operator,

$$J_s^i \equiv \frac{1}{3} (J^i - 2\sqrt{3} G^{i8}). \quad (2.5)$$

Even for the operators in this category, the couplings are still assumed to follow the single-photon ansatz and therefore either include one Q or G^{3Q} operator, or one power of the strange quark charge q_s . At $O(\varepsilon^1 N_c^0)$, one then obtains the additional operators:

$$\varepsilon \mathcal{O}_5 \equiv \varepsilon q_s J_s^3, \quad \varepsilon \mathcal{O}_6 \equiv \frac{\varepsilon}{N_c} N_s G^{3Q}, \quad \varepsilon \mathcal{O}_7 \equiv \frac{\varepsilon}{N_c} Q J_s^3, \quad (2.6)$$

and at $O(\varepsilon^1 N_c^{-1})$, one finds the operators:

$$\begin{aligned} \varepsilon \mathcal{O}_8 &\equiv \varepsilon q_s \frac{N_s}{N_c} J^3, \quad \varepsilon \mathcal{O}_9 \equiv \varepsilon \frac{N_s}{N_c^2} Q J^3, \quad \varepsilon \mathcal{O}_{10} \equiv \frac{\varepsilon}{N_c^2} \frac{1}{2} \{\mathbf{J} \cdot \mathbf{J}_s, G^{3Q}\}, \\ \varepsilon \mathcal{O}_{11} &\equiv \frac{\varepsilon}{N_c^2} J_s^j G^{jQ} J^3, \quad \varepsilon \mathcal{O}_{12} \equiv \frac{\varepsilon}{N_c^2} \frac{1}{2} \{J^j G^{jQ}, J_s^3\}. \end{aligned} \quad (2.7)$$

Beyond this point, the next operators would have matrix elements of $O(\varepsilon^2 N_c^{-1})$ or $O(\varepsilon^1 N_c^{-2})$. However, none appear at $O(\varepsilon^0 N_c^{-2})$ because the list of operators in Eq. (2.3) is exhaustive

to the 3-body level, and none appear at $O(\varepsilon^2 N_c^0)$ because ε^2 implies at least a 2-body operator, whose matrix elements on the physical baryon states are at most $O(1/N_c^1)$. These observations have important consequences for choosing consistent truncation points in the combined expansion in ε and $1/N_c$, which we discuss in detail in the next section.

The operators $\mathcal{O}_{1,2,\dots,12}$ were defined in Ref. [18]. While their matrix elements evaluated on the 27 observables for $N_c=3$ form a rank-12 matrix (and hence are independent), they were not explicitly tabulated in Ref. [18], and it was not noticed at the time that two combinations of them are demotable. To wit, $\frac{1}{2}\varepsilon\mathcal{O}_8 + \varepsilon\mathcal{O}_9$ has matrix elements of $O(\varepsilon^1 N_c^2)$ and hence should be neglected in this analysis (*i.e.*, $\varepsilon\mathcal{O}_9$ can be eliminated in favor of $\varepsilon\mathcal{O}_8$ at this order), and the combination

$$\varepsilon\mathcal{O}_{13} \equiv \frac{1}{2}\varepsilon\mathcal{O}_5 + \varepsilon\mathcal{O}_7 \quad (2.8)$$

has matrix elements of $O(\varepsilon^1 N_c^{-1})$ while each of $\varepsilon\mathcal{O}_5$ and $\varepsilon\mathcal{O}_7$ have matrix elements of $O(\varepsilon^1 N_c^0)$, and hence $\varepsilon\mathcal{O}_{13}$ belongs to the same list as Eq. (2.6), while $\varepsilon\mathcal{O}_7$ can be eliminated in favor of $\varepsilon\mathcal{O}_5$. The full set of matrix elements for all 27 observables for all of $\mathcal{O}_{1,\dots,13}$ in the $1/N_c^F$ expansion are presented in Tables II–IV.

III. THE $1/N_c^{\text{AS}}$ EXPANSION

Calculating the corresponding matrix elements in the $1/N_c^{\text{AS}}$ expansion is remarkably straightforward once one possesses their values in the $1/N_c$ expansion. Since baryons are fermions and thus have wave functions completely antisymmetric under the exchange of any two quarks, and the baryon wave functions constructed from both F and AS quarks are completely antisymmetrized under the exchange of any two quarks in color space, the spin-flavor-space wave functions are completely symmetric. In the ground-state multiplet, which by assumption is completely symmetric in spatial coordinates, the spin-flavor wave functions must also be completely symmetric. Precisely the same symmetrization condition holds for baryons built from either F or AS quarks; the operators in spin-flavor space carry precisely all the same indices in either case, and all of the results on the definition and interpretation of various operators, what operator reduction rules they obey, which combinations are demoted, *etc.*, carry over *mutatis mutandis*.

In fact, only two changes need to be made in order to apply the results of the previous

TABLE II: Matrix elements of magnetic moment operators defined in Eqs. (2.3)–(2.4).

State	$\langle \mathcal{O}_1 \rangle$	$\langle \mathcal{O}_2 \rangle$	$\langle \mathcal{O}_3 \rangle$	$\langle \mathcal{O}_4 \rangle$
Δ^{++}	$\frac{1}{20}(3N_c + 11)$	$\frac{1}{4N_c}(N_c + 9)$	0	$\frac{3}{16N_c^2}(3N_c + 11)$
Δ^+	$\frac{1}{20}(N_c + 7)$	$\frac{1}{4N_c}(N_c + 3)$	0	$\frac{3}{16N_c^2}(N_c + 7)$
Δ^0	$-\frac{1}{20}(N_c - 3)$	$\frac{1}{4N_c}(N_c - 3)$	0	$-\frac{3}{16N_c^2}(N_c - 3)$
Δ^-	$-\frac{1}{20}(3N_c + 1)$	$\frac{1}{4N_c}(N_c - 9)$	0	$-\frac{3}{16N_c^2}(3N_c + 1)$
Σ^{*+}	$\frac{1}{8}(N_c + 1)$	$\frac{1}{4N_c}(N_c + 3)$	0	$\frac{15}{32N_c^2}(N_c + 1)$
Σ^{*0}	0	$\frac{1}{4N_c}(N_c - 3)$	0	0
Σ^{*-}	$-\frac{1}{8}(N_c + 1)$	$\frac{1}{4N_c}(N_c - 9)$	0	$-\frac{15}{32N_c^2}(N_c + 1)$
Ξ^{*0}	$\frac{1}{12}(N_c - 3)$	$\frac{1}{4N_c}(N_c - 3)$	0	$\frac{5}{16N_c^2}(N_c - 3)$
Ξ^{*-}	$-\frac{1}{12}(N_c + 3)$	$\frac{1}{4N_c}(N_c - 9)$	0	$-\frac{5}{16N_c^2}(N_c + 3)$
Ω^-	$-\frac{1}{2}$	$\frac{1}{4N_c}(N_c - 9)$	0	$-\frac{15}{8N_c^2}$
p	$\frac{1}{12}(N_c + 3)$	$\frac{1}{12N_c}(N_c + 3)$	0	$\frac{1}{16N_c^2}(N_c + 3)$
n	$-\frac{1}{12}(N_c + 1)$	$\frac{1}{12N_c}(N_c - 3)$	0	$-\frac{1}{16N_c^2}(N_c + 1)$
Σ^+	$\frac{1}{12}(N_c + 3)$	$\frac{1}{12N_c}(N_c + 3)$	0	$\frac{1}{16N_c^2}(N_c + 3)$
Σ^0	$\frac{1}{6}$	$\frac{1}{12N_c}(N_c - 3)$	0	$\frac{1}{8N_c^2}$
Λ	$-\frac{1}{6}$	$\frac{1}{12N_c}(N_c - 3)$	0	$-\frac{1}{8N_c^2}$
$\Sigma^0\Lambda$	$-\frac{1}{12}\sqrt{(N_c - 1)(N_c + 3)}$	0	0	$-\frac{1}{16N_c^2}\sqrt{(N_c - 1)(N_c + 3)}$
Σ^-	$-\frac{1}{12}(N_c - 1)$	$\frac{1}{12N_c}(N_c - 9)$	0	$-\frac{1}{16N_c^2}(N_c - 1)$
Ξ^0	$-\frac{1}{36}(N_c + 9)$	$\frac{1}{12N_c}(N_c - 3)$	0	$-\frac{1}{48N_c^2}(N_c + 9)$
Ξ^-	$\frac{1}{36}(N_c - 9)$	$\frac{1}{12N_c}(N_c - 9)$	0	$\frac{1}{48N_c^2}(N_c - 9)$
Δ^+p	$\frac{1}{6\sqrt{2}}\sqrt{(N_c - 1)(N_c + 5)}$	0	$\frac{3}{8\sqrt{2}N_c^2}\sqrt{(N_c - 1)(N_c + 5)}$	0
Δ^0n	$\frac{1}{6\sqrt{2}}\sqrt{(N_c - 1)(N_c + 5)}$	0	$\frac{3}{8\sqrt{2}N_c^2}\sqrt{(N_c - 1)(N_c + 5)}$	0
$\Sigma^{*0}\Lambda$	$\frac{1}{6\sqrt{2}}\sqrt{(N_c - 1)(N_c + 3)}$	0	$\frac{3}{8\sqrt{2}N_c^2}\sqrt{(N_c - 1)(N_c + 3)}$	0
$\Sigma^{*0}\Sigma^0$	$\frac{1}{3\sqrt{2}}$	0	$\frac{3}{4\sqrt{2}N_c^2}$	0
$\Sigma^{*+}\Sigma^+$	$\frac{1}{12\sqrt{2}}(N_c + 5)$	0	$\frac{3}{16\sqrt{2}N_c^2}(N_c + 5)$	0
$\Sigma^{*-}\Sigma^-$	$-\frac{1}{12\sqrt{2}}(N_c - 3)$	0	$-\frac{3}{16\sqrt{2}N_c^2}(N_c - 3)$	0
$\Xi^{*0}\Xi^0$	$\frac{1}{9\sqrt{2}}(N_c + 3)$	0	$\frac{1}{4\sqrt{2}N_c^2}(N_c + 3)$	0
$\Xi^{*-}\Xi^-$	$-\frac{1}{9\sqrt{2}}(N_c - 3)$	0	$-\frac{1}{4\sqrt{2}N_c^2}(N_c - 3)$	0

TABLE III: Matrix elements of magnetic moment operators defined in Eqs. (2.6)–(2.7).

State	$\langle \mathcal{O}_5 \rangle$	$\langle \mathcal{O}_6 \rangle$	$\langle \mathcal{O}_7 \rangle$	$\langle \mathcal{O}_8 \rangle$	$\langle \mathcal{O}_9 \rangle$
Δ^{++}	0	0	0	0	0
Δ^+	0	0	0	0	0
Δ^0	0	0	0	0	0
Δ^-	0	0	0	0	0
Σ^{*+}	$-\frac{1}{6}$	$\frac{1}{8N_c}(N_c + 1)$	$\frac{1}{12N_c}(N_c + 3)$	$-\frac{1}{2N_c}$	$\frac{1}{4N_c^2}(N_c + 3)$
Σ^{*0}	$-\frac{1}{6}$	0	$\frac{1}{12N_c}(N_c - 3)$	$-\frac{1}{2N_c}$	$\frac{1}{4N_c^2}(N_c - 3)$
Σ^{*-}	$-\frac{1}{6}$	$-\frac{1}{8N_c}(N_c + 1)$	$\frac{1}{12N_c}(N_c - 9)$	$-\frac{1}{2N_c}$	$\frac{1}{4N_c^2}(N_c - 9)$
Ξ^{*0}	$-\frac{1}{3}$	$\frac{1}{6N_c}(N_c - 3)$	$\frac{1}{6N_c}(N_c - 3)$	$-\frac{1}{N_c}$	$\frac{1}{2N_c^2}(N_c - 3)$
Ξ^{*-}	$-\frac{1}{3}$	$-\frac{1}{6N_c}(N_c + 3)$	$\frac{1}{6N_c}(N_c - 9)$	$-\frac{1}{N_c}$	$\frac{1}{2N_c^2}(N_c - 9)$
Ω^-	$-\frac{1}{2}$	$-\frac{3}{2N_c}$	$\frac{1}{4N_c}(N_c - 9)$	$-\frac{3}{2N_c}$	$\frac{3}{4N_c^2}(N_c - 9)$
p	0	0	0	0	0
n	0	0	0	0	0
Σ^+	$\frac{1}{18}$	$\frac{1}{12N_c}(N_c + 3)$	$-\frac{1}{36N_c}(N_c + 3)$	$-\frac{1}{6N_c}$	$\frac{1}{12N_c^2}(N_c + 3)$
Σ^0	$\frac{1}{18}$	$\frac{1}{6N_c}$	$-\frac{1}{36N_c}(N_c - 3)$	$-\frac{1}{6N_c}$	$\frac{1}{12N_c^2}(N_c - 3)$
Λ	$-\frac{1}{6}$	$-\frac{1}{6N_c}$	$\frac{1}{12N_c}(N_c - 3)$	$-\frac{1}{6N_c}$	$\frac{1}{12N_c^2}(N_c - 3)$
$\Sigma^0\Lambda$	0	$-\frac{1}{12N_c}\sqrt{(N_c - 1)(N_c + 3)}$	0	0	0
Σ^-	$\frac{1}{18}$	$-\frac{1}{12N_c}(N_c - 1)$	$-\frac{1}{36N_c}(N_c - 9)$	$-\frac{1}{6N_c}$	$\frac{1}{12N_c^2}(N_c - 9)$
Ξ^0	$-\frac{2}{9}$	$-\frac{1}{18N_c}(N_c + 9)$	$\frac{1}{9N_c}(N_c - 3)$	$-\frac{1}{3N_c}$	$\frac{1}{6N_c^2}(N_c - 3)$
Ξ^-	$-\frac{2}{9}$	$\frac{1}{18N_c}(N_c - 9)$	$\frac{1}{9N_c}(N_c - 9)$	$-\frac{1}{3N_c}$	$\frac{1}{6N_c^2}(N_c - 9)$
Δ^+p	0	0	0	0	0
Δ^0n	0	0	0	0	0
$\Sigma^{*0}\Lambda$	0	$\frac{1}{6\sqrt{2}N_c}\sqrt{(N_c - 1)(N_c + 3)}$	0	0	0
$\Sigma^{*0}\Sigma^0$	$\frac{\sqrt{2}}{9}$	$\frac{1}{3\sqrt{2}N_c}$	$-\frac{1}{9\sqrt{2}N_c}(N_c - 3)$	0	0
$\Sigma^{*+}\Sigma^+$	$\frac{\sqrt{2}}{9}$	$\frac{1}{12\sqrt{2}N_c}(N_c + 5)$	$-\frac{1}{9\sqrt{2}N_c}(N_c + 3)$	0	0
$\Sigma^{*-}\Sigma^-$	$\frac{\sqrt{2}}{9}$	$-\frac{1}{12\sqrt{2}N_c}(N_c - 3)$	$-\frac{1}{9\sqrt{2}N_c}(N_c - 9)$	0	0
$\Xi^{*0}\Xi^0$	$\frac{\sqrt{2}}{9}$	$\frac{\sqrt{2}}{9N_c}(N_c + 3)$	$-\frac{1}{9\sqrt{2}N_c}(N_c - 3)$	0	0
$\Xi^{*-}\Xi^-$	$\frac{\sqrt{2}}{9}$	$-\frac{\sqrt{2}}{9N_c}(N_c - 3)$	$-\frac{1}{9\sqrt{2}N_c}(N_c - 9)$	0	0

TABLE IV: Matrix elements of magnetic moment operators defined in Eqs. (2.7)–(2.8).

State	$\langle \mathcal{O}_{10} \rangle$	$\langle \mathcal{O}_{11} \rangle$	$\langle \mathcal{O}_{12} \rangle$	$\langle \mathcal{O}_{13} \rangle$
Δ^{++}	0	0	0	0
Δ^+	0	0	0	0
Δ^0	0	0	0	0
Δ^-	0	0	0	0
Σ^{*+}	$\frac{5}{32N_c^2}(N_c + 1)$	$\frac{1}{32N_c^2}(3N_c - 5)$	$\frac{5}{32N_c^2}(N_c + 1)$	$\frac{1}{4N_c}$
Σ^{*0}	0	$-\frac{1}{4N_c^2}$	0	$-\frac{1}{4N_c}$
Σ^{*-}	$-\frac{5}{32N_c^2}(N_c + 1)$	$-\frac{1}{32N_c^2}(3N_c + 11)$	$-\frac{5}{32N_c^2}(N_c + 1)$	$-\frac{3}{4N_c}$
Ξ^{*0}	$\frac{5}{24N_c^2}(N_c - 3)$	$\frac{1}{8N_c^2}(N_c - 7)$	$\frac{5}{24N_c^2}(N_c - 3)$	$-\frac{1}{2N_c}$
Ξ^{*-}	$-\frac{5}{24N_c^2}(N_c + 3)$	$-\frac{1}{8N_c^2}(N_c + 7)$	$-\frac{5}{24N_c^2}(N_c + 3)$	$-\frac{3}{2N_c}$
Ω^-	$-\frac{15}{8N_c^2}$	$-\frac{15}{8N_c^2}$	$-\frac{15}{8N_c^2}$	$-\frac{9}{4N_c}$
p	0	0	0	0
n	0	0	0	0
Σ^+	$-\frac{1}{48N_c^2}(N_c + 3)$	$-\frac{1}{48N_c^2}(3N_c + 13)$	$-\frac{1}{48N_c^2}(N_c + 3)$	$-\frac{1}{12N_c}$
Σ^0	$-\frac{1}{24N_c^2}$	$-\frac{5}{24N_c^2}$	$-\frac{1}{24N_c^2}$	$\frac{1}{12N_c}$
Λ	$-\frac{1}{8N_c^2}$	$-\frac{1}{8N_c^2}$	$-\frac{1}{8N_c^2}$	$-\frac{1}{4N_c}$
$\Sigma^0\Lambda$	$-\frac{1}{48N_c^2}\sqrt{(N_c - 1)(N_c + 3)}$	$-\frac{1}{16N_c^2}\sqrt{(N_c - 1)(N_c + 3)}$	$-\frac{1}{48N_c^2}\sqrt{(N_c - 1)(N_c + 3)}$	0
Σ^-	$\frac{1}{48N_c^2}(N_c - 1)$	$\frac{1}{48N_c^2}(3N_c - 7)$	$\frac{1}{48N_c^2}(N_c - 1)$	$\frac{1}{4N_c}$
Ξ^0	$-\frac{1}{36N_c^2}(N_c + 9)$	$-\frac{1}{12N_c^2}(N_c + 5)$	$-\frac{1}{36N_c^2}(N_c + 9)$	$-\frac{1}{3N_c}$
Ξ^-	$\frac{1}{36N_c^2}(N_c - 9)$	$\frac{1}{12N_c^2}(N_c - 5)$	$\frac{1}{36N_c^2}(N_c - 9)$	$-\frac{1}{N_c}$
Δ^+p	0	0	0	0
Δ^0n	0	0	0	0
$\Sigma^{*0}\Lambda$	$\frac{1}{6\sqrt{2}N_c^2}\sqrt{(N_c - 1)(N_c + 3)}$	0	$\frac{1}{24\sqrt{2}N_c^2}\sqrt{(N_c - 1)(N_c + 3)}$	0
$\Sigma^{*0}\Sigma^0$	$\frac{1}{6\sqrt{2}N_c^2}$	0	$-\frac{1}{12\sqrt{2}N_c^2}$	$\frac{1}{3\sqrt{2}N_c}$
$\Sigma^{*+}\Sigma^+$	$\frac{1}{24\sqrt{2}N_c^2}(N_c + 5)$	0	$-\frac{1}{48\sqrt{2}N_c^2}(7N_c + 11)$	$-\frac{1}{3\sqrt{2}N_c}$
$\Sigma^{*-}\Sigma^-$	$-\frac{1}{24\sqrt{2}N_c^2}(N_c - 3)$	0	$\frac{1}{48\sqrt{2}N_c^2}(7N_c + 3)$	$\frac{1}{\sqrt{2}N_c}$
$\Xi^{*0}\Xi^0$	$\frac{7}{36\sqrt{2}N_c^2}(N_c + 3)$	0	$-\frac{1}{18\sqrt{2}N_c^2}(N_c - 6)$	$\frac{1}{3\sqrt{2}N_c}$
$\Xi^{*-}\Xi^-$	$-\frac{7}{36\sqrt{2}N_c^2}(N_c - 3)$	0	$\frac{1}{18\sqrt{2}N_c^2}(N_c + 6)$	$\frac{1}{\sqrt{2}N_c}$

section and the results of Tables II–IV to the $1/N_c^{\text{AS}}$ expansion. As mentioned there, the powers of N_c due to 't Hooft scaling are changed from $1/N_c^n$ in the $1/N_c^{\text{F}}$ expansion to $1/N_c^{2n}$ in the $1/N_c^{\text{AS}}$ expansion, and the combinatoric factors due to the number of quarks (called M in the previous section) are changed from N_c^1 in the $1/N_c^{\text{F}}$ expansion to $N_c(N_c-1)/2$ in the $1/N_c^{\text{AS}}$ expansion. Since the former and the latter factors are clearly segregated in Tables II–IV, a simple substitution generalizes their application from QCD_{F} to QCD_{AS} .

One may now address the question of consistent truncation of the operator basis in powers of ε and $1/N_c$ in the two expansions. In order to proceed, one must determine the relative parametric size of ε compared to $1/N_c$. Traditionally, ε is estimated numerically from the relative size of strangeness mass splittings in a baryon multiplet or the size of departures of strange hadron couplings from their $\text{SU}(3)$ symmetric values, or in chiral perturbation theory as effects of $O(m_s/\Lambda_\chi)$, where Λ_χ is the chiral symmetry-breaking scale. Such effects are estimated to be no more than about 30%, *i.e.*, parametrically equal to $1/N_c$. As discussed in Ref. [18], however, chiral perturbation theory also contains $\text{SU}(3)$ -violating loop corrections of $O(m_s^{1/2})$, suggesting that, in some cases, $\varepsilon \sim 1/N_c^{1/2}$. Reference [18] then showed that using either scaling of ε , the operator expansion for the $1/N_c^{\text{F}}$ expansion may be consistently truncated either including $\mathcal{O}_{1,\dots,7}$ [including effects up through $O(\varepsilon^1 N_c^0)$ and $O(\varepsilon^0 N_c^{-1})$] or including $\mathcal{O}_{1,\dots,13}$ [including up through $O(\varepsilon^1 N_c^{-1})$ while neglecting $O(\varepsilon^2 N_c^{-1})$ and $O(\varepsilon^1 N_c^{-2})$]. Remarkably, for either scaling of ε the $1/N_c^{\text{AS}}$ expansion may be truncated after the same sets of operators: The set $\mathcal{O}_{1,\dots,7}$ includes effects up through $O(\varepsilon^1 N_c^0)$ and $O(\varepsilon^0 N_c^{-2})$, while the set $\mathcal{O}_{1,\dots,13}$ includes effects up through $O(\varepsilon^1 N_c^{-2})$ while neglecting $O(\varepsilon^2 N_c^{-2})$ and $O(\varepsilon^1 N_c^{-4})$.

IV. RESULTS OF FITS TO MEASURED MOMENTS

While the ground-state baryon multiplet contains 27 magnetic moment observables, many of them have never been measured chiefly due to the fact that most of the decuplet states are strongly decaying resonances, for which detecting electromagnetic processes is extremely difficult. 9 are tabulated in the *Review of Particle Physics* [22]: magnetic moments of 7 of the 8 octet baryons (μ_{Σ^0} is unknown), the Ω^- , and the $\Sigma^0\Lambda$ transition moment. The Δ^+p transition moment can be extracted from the $\Delta \rightarrow N\gamma$ helicity amplitudes and is found to be $\mu_{\Delta^+p} = 3.51 \pm 0.09 \mu_N$. We also use the extracted value $\mu_{\Delta^{++}} = 6.14 \pm 0.51 \mu_N$ [23], which is not the sole value used in Ref. [22] and is obtained from an analysis of data that has some model

dependence, but the extraction performed respects both gauge invariance and the finite Δ^{++} width. The tabulated Δ^+ moment value [24], $\mu_{\Delta^+} = 2.7_{-1.3}^{+1.0} (\text{stat}) \pm 1.5 (\text{syst}) \pm 3 (\text{theor}) \mu_N$ has such a large theoretical uncertainty that we do not use it in our fits.

We have seen in Sec. III that the full set of operators up to and including $O(\varepsilon^1 N_c^{-1})$ in the $1/N_c^F$ expansion, or $O(\varepsilon^1 N_c^{-2})$ in the $1/N_c^{AS}$ expansion, consists only of the 11 operators $\mathcal{O}_{1,2,3,4,5,6,8,10,11,12,13}$ defined in Sec. II. In fact, when restricted to the 11 observed moments, two more combinations among these operators are demoted, as may be verified through a quick check of Table II: Each of $\varepsilon \mathcal{O}_{10,11,12}$ has matrix elements of $O(\varepsilon^1 N_c^{-1})$ in the $1/N_c^F$ expansion [$O(\varepsilon^1 N_c^{-2})$ in the $1/N_c^{AS}$ expansion], but the combinations $(-\frac{1}{3}\mathcal{O}_{11} + \mathcal{O}_{12})$ and $(\mathcal{O}_{10} - \mathcal{O}_{12})$ have matrix elements of $O(\varepsilon^1 N_c^{-2})$ [$1/N_c^F$] or $O(\varepsilon^1 N_c^{-4})$ [$1/N_c^{AS}$], which should be neglected in our consistent-order expansion. Thus, \mathcal{O}_{11} and \mathcal{O}_{12} may be eliminated in favor of \mathcal{O}_{10} , leaving a basis of only 9 operators, $\mathcal{O}_{1,2,3,4,5,6,8,10,13}$.

One may ask whether it is appropriate to include terms both leading and subleading in N_c in the matrix elements. We argue that such terms are essential to reproduce the complete physical nature of electromagnetic interactions such as magnetic moments. Consider, for example, the operator Q . Since the physical ground-state baryons contain M quarks [either N_c or $N_c(N_c - 1)/2$] but differ in no more than the 3 valence quarks, the leading $O(M^1)$ contribution from Q is the same for all of the physical ground-state baryons; *i.e.*, all of the observed baryons have the same electric charge at leading order in N_c . This peculiar result arises from the independence of quark electric charges $q_u = +\frac{2}{3}$, $q_d = q_s = -\frac{1}{3}$ (a manifestly electromagnetic effect) from N_c scaling (a strong interaction effect). Since we insist that our results extrapolate in a physically meaningful way from $N_c = 3$, we retain subleading terms in the matrix elements for our fits.

The 11 observed moments are fit to a set of 9 $O(N_c^0)$ operator coefficients $d_{i_n} = d_{1,2,3,4,5,6,8,10,13}$ used to define the full magnetic moment operator:

$$\mu_z = \mu_0 \sum_{n=1}^9 d_{i_n} \varepsilon^{k_{i_n}} \mathcal{O}_{i_n}, \quad (4.1)$$

where, as indicated by Eqs. (2.3)–(2.8), $k_{i_n} = 0$ for $i_n = 1, \dots, 4$ and $k_{i_n} = 1$ for $i_n = 5, \dots, 13$. As in Ref. [18], we set the overall scale μ_0 to equal $2\mu_p$ in order to make the coefficient d_1 of the sole leading-order operator, \mathcal{O}_1 , of order unity for $N_c = 3$. Since Eq. (4.1) neglects all $O(\varepsilon^2/M)$ and $O(\varepsilon/M^2)$ contributions to the magnetic moments, one must combine the statistical uncertainty of each moment with a “theoretical uncertainty” of magnitude given

TABLE V: Best fit values for the coefficients in the $1/N_c^F$ expansion using Eq. (4.1).

$d_1 = +0.992 \pm 0.044$	$d_2 = -0.078 \pm 0.148$	$d_3 = +1.363 \pm 0.272$
$d_4 = +0.461 \pm 0.489$	$d_5 = -1.652 \pm 0.566$	$d_6 = -0.288 \pm 0.438$
$d_8 = +1.588 \pm 0.865$	$d_{10} = -3.727 \pm 2.852$	$d_{13} = +0.499 \pm 0.438$

by the larger of $O(\mu_p \varepsilon^2/M)$ and $O(\mu_p \varepsilon/M^2)$.

Using $N_c=3$ and $\varepsilon=\frac{1}{3}$, one obtains the coefficients for the $1/N_c^F$ expansion in Table V and those for the $1/N_c^{\text{AS}}$ expansion in Table VI. The $\chi^2/\text{d.o.f.}$ for the $1/N_c^F$ expansion is 0.31, and that for the $1/N_c^{\text{AS}}$ is 1.55. If the scale of the theoretical uncertainty uses $2\mu_p$ rather than μ_p , these numbers drop to 0.09 and 0.61, respectively, meaning that the quality of the fit is good in either case, and therefore the values obtained for the operator coefficients are reliably determined. A glance at Table V shows that every coefficient in the $1/N_c^F$ neatly assumes a value of $O(1)$ or less, thus following the dictates of either $1/N_c$ expansion. Furthermore, unlike the fit to only 7 operators in Ref. [18] in which most of the coefficients are anomalously suppressed, only d_2 appears to have an especially small coefficient. On the other hand, the coefficients d_3 , d_4 , d_8 , and d_{10} in the $1/N_c^{\text{AS}}$ fit are substantially larger than $O(1)$, indicating the failure of the $1/N_c^{\text{AS}}$ expansion for the magnetic moments.

One can also check that the obvious candidates for rescuing the $1/N_c^{\text{AS}}$ expansion are inadequate to the task. Using, as discussed above, either the $1/N_c^F$ or $1/N_c^{\text{AS}}$ expansion without including subleading terms leads to the prediction of some coefficients of unnaturally large size, while using the alternate parameter choice $\varepsilon \sim N_c^{-1/2}$ rather than $\varepsilon \sim N_c^{-1}$ has relatively little effect on the pattern of coefficient magnitudes. Meanwhile, neglecting the $1/N_c$ expansion entirely—achieved by deleting all scaling powers of $1/N_c$ from Eqs. (2.3)–(2.8) while setting combinatoric factors of N_c to 3—leads to a fit given in Table VII [here, $\chi^2/\text{d.o.f.}$ is only 0.039 because the theoretical uncertainty is now $O(\mu_p \varepsilon^2)$] with several coefficients d_{i_k} substantially less than $O(1)$ (d_2 , d_3 , d_4 , and likely others when uncertainties are taken into account): A $1/N_c$ expansion is clearly needed to explain their natural sizes. The $1/N_c^F$ expansion in the single-photon ansatz including subleading terms appears to have a “Goldilocks” quality: While the other $1/N_c$ expansions produce coefficients too large and ignoring the $1/N_c$ expansion entirely produces coefficients too small, the $1/N_c^F$ expansion thus far is unique in producing uniformly “just right” $O(1)$ coefficients.

TABLE VI: Best fit values for the coefficients in the $1/N_c^{\text{AS}}$ expansion using Eq. (4.1).

$d_1 = +0.976 \pm 0.023$	$d_2 = -0.188 \pm 0.176$	$d_3 = +12.846 \pm 1.553$
$d_4 = +5.289 \pm 2.743$	$d_5 = -1.474 \pm 0.223$	$d_6 = -1.147 \pm 0.491$
$d_8 = +4.841 \pm 1.046$	$d_{10} = -36.332 \pm 12.322$	$d_{13} = +1.218 \pm 0.490$

TABLE VII: Best fit values for the coefficients using no $1/N_c$ expansion in Eq. (4.1).

$d_1 = +0.995 \pm 0.116$	$d_2 = -0.029 \pm 0.138$	$d_3 = +0.150 \pm 0.075$
$d_4 = +0.051 \pm 0.121$	$d_5 = -1.708 \pm 1.593$	$d_6 = -0.085 \pm 0.420$
$d_8 = +0.535 \pm 0.829$	$d_{10} = -0.420 \pm 0.845$	$d_{13} = +0.178 \pm 0.420$

The fit values for the coefficients given in Table V may be used to predict all 16 unknown magnetic moment observables, as done in Ref. [18]; the results are compiled in Table VIII [29]. The difference is that the operator basis has been fit here including effects of $O(\varepsilon^1 N_c^{-1})$, but only including $O(\varepsilon^1 N_c^0)$ and $O(\varepsilon^0 N_c^{-1})$ in Ref. [18], and so the theoretical uncertainty for the best-determined moments here is $O(\mu_p \varepsilon^1 N_c^{-2})$ or $O(\mu_p \varepsilon^2 N_c^{-1})$. For the strange decuplet moments or strange decuplet-octet transitions, however, at least one of the combinations $(-\frac{1}{3}\mathcal{O}_{11} + \mathcal{O}_{12})$ and $(\mathcal{O}_{10} - \mathcal{O}_{12})$ is no longer demoted, so that the fit values for those moments must include the larger theoretical uncertainty of $O(\mu_p \varepsilon^1 N_c^{-1})$. The results presented in Table VIII improve upon, and in almost all cases agree within 1σ with, the results in Table XI of Ref. [18].

Since we fit 9 operator coefficients using 11 observables, one can also investigate the two magnetic moment combinations satisfied by all the operators. One of them has been known

TABLE VIII: Best fit values for the 16 unknown magnetic moments in units of μ_N using the $1/N_c^{\text{F}}$ expansion.

$\mu_{\Delta^+} = +3.09 \pm 0.16$	$\mu_{\Delta^0} = +0.00 \pm 0.10$	$\mu_{\Delta^-} = -3.09 \pm 0.16$	$\mu_{\Sigma^{*+}} = +2.62 \pm 0.35$
$\mu_{\Sigma^{*0}} = -0.06 \pm 0.32$	$\mu_{\Sigma^{*-}} = -2.73 \pm 0.35$	$\mu_{\Xi^{*0}} = -0.12 \pm 0.33$	$\mu_{\Xi^{*-}} = -2.37 \pm 0.39$
$\mu_{\Sigma^0} = +0.65 \pm 0.11$	$\mu_{\Delta^0 n} = +3.51 \pm 0.11$	$\mu_{\Sigma^{*0}\Lambda} = +2.65 \pm 0.32$	$\mu_{\Sigma^{*0}\Sigma^0} = +1.21 \pm 0.31$
$\mu_{\Sigma^{*+}\Sigma^+} = +2.69 \pm 0.32$	$\mu_{\Sigma^{*-}\Sigma^-} = -0.26 \pm 0.31$	$\mu_{\Xi^{*0}\Xi^0} = +2.30 \pm 0.33$	$\mu_{\Xi^{*-}\Xi^-} = -0.26 \pm 0.31$

since the early days of heavy baryon chiral perturbation theory [25]:

$$\mu_n - \frac{1}{4}(\mu_{\Sigma^+} + \mu_{\Sigma^-}) - \frac{3}{2}\mu_\Lambda - \sqrt{3}\mu_{\Sigma^0\Lambda} + \mu_{\Xi^0} = O(\mu_p \varepsilon^2 M^{-1}), O(\mu_p \varepsilon^1 M^{-2}). \quad (4.2)$$

Experimentally, this combination is $(0.22 \pm 0.14)\mu_N$. The best scale-independent measure for this relation is obtained by dividing it by the average of the same combination with all negative values replaced by positive ones; such a combination is $O(\mu_p M^1)$, and their ratio is therefore predicted to be the larger of $O(\varepsilon^2 M^{-2})$ and $O(\varepsilon^1 M^{-3})$. One obtains 0.057 ± 0.036 *vs.* 0.012 ($1/N_c^F$) or 0.0014 ($1/N_c^{\text{AS}}$). While it is tempting to ascribe a superior agreement to the $1/N_c^F$ expansion, in fact the central experimental value is only 1.6σ from zero (due almost entirely to the large $\mu_{\Sigma^0\Lambda}$ uncertainty), so that neither expansion is particularly favored for this single result. This conclusion is even more stark for the other relation:

$$\mu_{\Delta^{++}} + 2\mu_{\Omega^-} + 8(\mu_p + \mu_n) - 6\mu_{\Sigma^+} + 12\mu_\Lambda + 4\mu_{\Sigma^-} - 12\mu_{\Xi^0} - 2\mu_{\Xi^-} = O(\mu_p \varepsilon^2 M^{-1}), O(\mu_p \varepsilon^1 M^{-2}), \quad (4.3)$$

for which the experimental value $(-1.30 \pm 0.56)\mu_N$ converts to the ratio -0.029 ± 0.012 . In this case, the central value is about 2.3σ from zero, and the $1/N_c^F$ expansion is somewhat preferred; reducing the large $\mu_{\Delta^{++}}$ uncertainty would sharpen this conclusion.

V. CONCLUSIONS

The remarkable fact that the baryon mass spectrum not only requires a $1/N_c$ expansion in order to explain the size of its suppressed combinations but works about as well for more than one such possible expansion— $1/N_c^F$ and $1/N_c^{\text{AS}}$ —does not survive the scrutiny of results from baryon magnetic moments. We have shown with fits to the observed data that the $1/N_c^F$ expansion produces no unnaturally large coefficients in the $1/N_c$ expansion of the magnetic moment operator, while such large coefficients are unavoidable in the $1/N_c^{\text{AS}}$ expansion. Moreover, numerous coefficients in a fit that entirely ignores the $1/N_c$ expansion are highly suppressed, mandating a $1/N_c$ expansion in order to satisfy the naturalness criterion.

Such a result might seem surprising; after all, the baryon mass data set is more complete than that of the magnetic moments. However, as remarked above, the mass spectrum and magnetic moments probe largely orthogonal physical effects. The sole leading-order [$O(M^1)$, with $M \sim N_c^1$ for $1/N_c^F$, N_c^2 for $1/N_c^{\text{AS}}$] operator $\mathbb{1}$ for the mass spectrum simply gives the same universal mass to all baryons, while the sole leading-order operator G^{3Q} for

the magnetic moments gives contributions that vary from state to state. Thus it is perhaps not so surprising that a more incisive result arises from the magnetic moment sector.

In addition, this work advances the analysis of the baryon magnetic moments to one order higher in the $1/N_c$ expansion [now including all $O(\varepsilon^1 N_c^{-1})$ effects] than was previously known. Issues about numerous potentially small coefficients have largely evaporated in light of the new analysis. Should the measured but less well-known moments, particularly $\mu_{\Sigma^0\Lambda}$, $\mu_{\Delta^{++}}$, and μ_{Δ^+} , receive renewed experimental scrutiny, and should more radiative decays of strange decuplet baryons be observed [26] and analyzed [27], the $1/N_c$ expansion will be subject to ever more precise tests of its applicability.

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