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Rajendra Prasad Bhatt, Sumanta Chakraborty, and Sukanta Bose

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# Rotating black holes experience dynamical tides

Rajendra Prasad Bhatt<sup>1,\*</sup>, Sumanta Chakraborty<sup>2,†</sup> and Sukanta Bose<sup>3,‡</sup>

<sup>1</sup>*Inter-University Centre for Astronomy and Astrophysics, Pune 411007, India*

<sup>2</sup>*School of Physical Sciences, Indian Association for the Cultivation of Science, Kolkata-700032, India*

<sup>3</sup>*Department of Physics and Astronomy, Washington State University,  
1245 Webster, Pullman, Washington 99164-2814, USA*

We find the *dynamical* tidal response of a Kerr black hole (BH) and demonstrate its tidal Love numbers to be non-vanishing when present in a non-axisymmetric external tidal field. To leading order, they depend quadratically on the black hole spin and linearly on the mode frequency. This implies that Kerr BHs are deformable under certain external, time-dependent perturbations. Since non-vanishing tidal Love numbers have been used in compact binary coalescences observed by the LIGO-Virgo-KAGRA (LVK) collaboration to infer their non-BH nature, our findings have important implications on such inferences from future gravitational wave observations.

**Introduction.**— Recent observations of gravitational waves (GWs) from binary black hole mergers [1, 2] and images of shadows cast by supermassive black holes (BHs) [3, 4] have provided interesting tests of the strong gravity regime in an unprecedented manner. They have also given a significant boost to the theoretical study of compact objects by ushering in new ways of probing their properties with increasing precision. The response of a compact object to an external (tidal) field is one such property. Here, we focus our attention on rotating BHs in General Relativity, i.e., Kerr BHs. This work is an extension of our previous work [5], where we studied the tidal response of Schwarzschild as well as slowly rotating Kerr BHs.

Tidal response of a self-gravitating body can be divided into two parts. The first part is related to its deformation due to a tidal environment and is conservative. The second part is the effect of tidal dissipation [6]. One can associate dimensionless numbers with both parts, respectively termed as the tidal Love numbers (TLNs) and dissipation numbers [5–10]. The TLNs of the Schwarzschild black hole have been found to be zero in various studies [5, 6, 9–19]. Similar results have also been found for slowly rotating Kerr BHs [5, 6, 20, 21]. Non-linearities [22] and stability [23] of the tidal response of non-rotating BHs have been studied too.

Contrastingly, Refs. [14, 15] found non-zero and purely imaginary static TLNs for rotating BHs in non-axisymmetric tidal backgrounds. They did so by using the Teukolsky equation that describes the behavior of gravitational wave perturbations of the Kerr BH. The perturbation was modeled by an appropriate Newman-Penrose scalar. Reference [6] extended those calculations and noted that the purely imaginary tidal Love numbers found in [14, 15] are not associated with tidal deformation but with tidal dissipation. Recently, we showed [5] that some terms are missing in the analysis of Ref. [6], and calculated the tidal response function of Schwarzschild and slowly rotating Kerr BHs. In this letter, we deduce the tidal response of an arbitrarily rotating BH and related quantities. In particular, we show

that in a non-axisymmetric external tidal field it possesses non-vanishing dynamical TLNs.

With the advent of a new generation of GW detectors, it becomes imperative to study the properties of compact objects carefully since they may bear an influence on the GWs emitted by their binaries. While the study of the tidal deformability of neutron stars is useful in inferring their equation of state [24], it also assumes importance for BHs in regards to tests of the presence of horizons. One can presumably use it for comparing the black holes of General Relativity with exotic compact objects [25, 26] and black holes in alternative theories of gravity [25, 27, 28], which have been found to have non-zero TLNs in some cases [25–27]. Non-zero TLNs have also been reported for black holes in higher dimensions (e.g., braneworld BH [29], Myers-Perry BH [30, 31], higher dimensional Schwarzschild BHs [17]), lower-dimensional BHs [32, 33], Schwarzschild-de Sitter BH [34], and for area quantized BHs [35]. Therefore, the study of TLNs can help us understand the various types of compact objects and might also be useful in studies of quantum effects in gravity. Below, we use  $G = 1 = c$  and the positive signature metric.

**Executive summary.**— The main results of this letter are summarized below: (a) A Schwarzschild BH has vanishing static as well as dynamical TLNs; (b) A Kerr BH, both in the non-extremal and extremal limit, has vanishing static TLNs, but non-zero dynamical TLNs. The exact expressions for the non-zero dynamical TLNs have also been presented, for the first time. The dynamical TLNs vanish for axis-symmetric perturbations for both non-extremal and extremal BHs, and also in the slow-rotation limit. Thus, our results address all possible TLNs for BHs in vacuum GR, within the linear-in-frequency approximation scheme.

Implications of this result for GW astrophysics and tests of gravity are potentially far-reaching. All binary BH waveform models assume zero TLNs for BHs, while estimating BH parameters and in testing deviations from the predictions of General Relativity. Our results imply the presence of a systematic deviation in the waveform

phase, which will make its presence known as more binary BHs are detected and the power of the above tests improves. If this deviation is not accounted for, it will result in incorrect deduction of astrophysical parameters, e.g., the mass and spin demographics of black holes, and impact the tests of general relativity.

**Tidal signals in gravitational waves.**— Originally the tidal response was defined in Newtonian gravity, e.g., when studying the multipolar deformation of a static, spherically symmetric, non-rotating body of mass  $M$  and radius  $\mathcal{R}$  subjected to a tidal environment. Here, the gravitational potential at a position  $r$  from the center of mass of the object is the sum of (a) the undeformed object's potential, (b) the potential of the multipolar deformation, which is caused by the object's response to the tidal field, and (c) the potential of the ambient tidal field [5, 15, 36] (see Supplemental Material [37] for further details). In the frequency domain, this potential is

$$U = \frac{M}{r} - \sum_{lm} \frac{(l-2)!}{l!} \mathcal{E}_{lm} r^l \left[ 1 + F_{lm}(\omega) \left( \frac{\mathcal{R}}{r} \right)^{2l+1} \right] Y_{lm}, \quad (1)$$

where the sum is over  $l \in [2, \infty)$  and  $m \in [-l, l]$ ,  $\omega$  is the angular mode frequency, and

$$F_{lm} = 2k_{lm} + i\omega\tau_0\nu_{lm} + \mathcal{O}(\omega^2) \quad (2)$$

is the tidal response function.  $\mathcal{O}(\omega^2)$  terms are neglected since we are working with the linear response and have taken the tidal field to be varying weakly with time. The above expression introduces three physical quantities — (a) the TLNs  $k_{lm}$ , (b) the tidal dissipation numbers  $\nu_{lm}$ , and (c) the viscosity induced time-delay  $\tau_0$  [5, 6]. This formalism can also be applied to slowly rotating self-gravitating bodies, with  $\omega$  replaced by  $\omega'$ , which is the mode frequency in the body's co-rotating frame of reference [16, 36].

So far the discussion was purely Newtonian and hence the results are prone to gauge ambiguities when directly applied to relativistic systems [38]. In order to determine the TLNs and hence the tidal response function in a gauge invariant manner, as argued in Refs. [5, 15], the Newman-Penrose scalar  $\Psi_4$  associated with the gravitational perturbation is an appropriate quantity to analyze. In particular,  $\Psi_4$  is directly related to the Newtonian potential in the Newtonian limit [5, 15]. Therefore, to deduce the tidal response function in the relativistic context, one should determine  $\Psi_4$  by solving the Teukolsky equation, and then examine its large  $r$  behavior, which will have contributions from positive as well as negative powers of  $r$ . The coefficient of the  $r^{-2l-1}$  term then determines the tidal response function  $F_{lm}$ , whose real part is the TLN associated with a given  $(l, m)$  mode. While determining the TLNs, we will relax the domain of  $l$  and allow it to be any complex number in order to unambiguously identify the contributions from tidal deformation and dissipation [15]. However, since in the end we

are interested in the gravitational perturbation, the final answer will be gotten by restricting  $l$  at the end of the calculation to obey  $l \in \mathbb{Z}_{\geq 2}$ , which defines the set of positive integers greater than or equal to 2. A variant of the above procedure will be used to calculate the tidal response function of an extremal Kerr black hole in an external tidal field.

**The near-zone Teukolsky equation.**— We introduce the re-scaled radial coordinate  $z \equiv (r - r_+)/ (r_+ - r_-)$ , which is dimensionless and has the event horizon at its origin,<sup>1</sup> and  $R$  as the function that determines the radial dependence of  $\Psi_4$ . In terms of  $z$ , applying the near-zone condition ( $M\omega z \ll 1$ ) and small frequency approximation ( $M\omega \ll 1$ ), the Teukolsky equation for  $R$  in the in-going Kerr coordinate simplifies to [5, 26] (see Supplemental Material [37] for further details):

$$\begin{aligned} \frac{d^2 R}{dz^2} + \left[ \frac{2iP_+ - 1}{z} - \frac{2iP_1 + 1}{1+z} \right] \frac{dR}{dz} \\ + \left[ -\frac{4iP_+}{z^2} + \frac{4iP_2}{(z+1)^2} - \frac{l(l+1)-2}{z(1+z)} \right. \\ \left. + \frac{2am\omega}{z(1+z)} \left\{ 1 + \frac{4}{l(l+1)} \right\} - \frac{2i\omega r_+}{z(1+z)} \right] R = 0, \end{aligned} \quad (3)$$

where  $P_{\pm} = (am - 2r_{\pm}M\omega)/(r_+ - r_-)$ ,  $P_1 = P_+ + 2\omega r_+$ , and  $P_2 = P_+ + (\omega/2)(r_+ + 3r_-)$ . We next solve the above master equation for an arbitrarily rotating black hole and obtain its tidal response function from the fall-off of the Weyl scalar in the intermediate region.

**Tidal response of a spinning black hole.**— Since Eq. (3) is a second order differential equation, and has three regular singular points at  $-z = 0, 1$ , and  $\infty$ , its solution can be written in terms of the Gauss hypergeometric function as

$$R(z) = c_1 z^2 (z+1)^{2-N_3} \times {}_2F_1(3+l-N_2, 2-l-N_1; 3+2iP_+; -z), \quad (4)$$

where  $c_1$  is a constant of integration.<sup>2</sup> The arguments of the hypergeometric function and the power of  $(1+z)$  are written upto linear order in  $M\omega$ . For notational simplicity, the radial Teukolsky function is given in terms of three quantities,  $N_1$ ,  $N_2$ , and  $N_3$ , all of which are linear functions of  $\omega$ , and are described in the Supplemental Material [37].<sup>3</sup> The above result reduces to the one for non-rotating and slowly rotating BHs [5].

<sup>1</sup> The extremal Kerr BH obeys  $r_+ = r_-$  and will be addressed separately below since  $z$  diverges in this case.

<sup>2</sup> The second integration constant is fixed by invoking purely in-going boundary condition at the event horizon.

<sup>3</sup> In the limit of non-rotating and slowly rotating BHs, they are identical to  $Q_1$ ,  $Q_2$ , and  $Q_3$ , respectively, defined in Ref. [5].

Given the radial Teukolsky function, which is purely ingoing at the BH horizon, in order to determine tidal effects one computes the radial part of  $\Psi_4$  in the intermediate regime. This is obtained by taking the large  $r$  (or, equivalently, large  $z$ ) limit (see Supplemental Material [37] for further details). We note the existence of the  $z^{-2l-1}$  term in the large  $r$  limit of radial  $\Psi_4$ , albeit with a frequency-dependent correction such that, upto linear order in  $M\omega$ , the fall-off behavior of the tidal response with  $z$  becomes  $z^{-2l-1}\{1 + (N_2 - N_1) \ln z\}$ . Thus, in the dynamical context, the leading behavior is as desired, with an additional  $\ln z$  correction in the frequency. Following [39, 40] one can either retain the  $\ln z$  term in the response function or define the response function without the  $\ln z$  term [28]. We choose the latter option<sup>4</sup> and obtain the tidal response function to be

$$F_{lm} = \frac{\Gamma(-2l - N_1 + N_2 - 1) \Gamma(l + 2iP_+ + N_1 + 1)}{\Gamma(2 - l - N_1) \Gamma(-l + 2iP_+ + N_2)} \times \frac{\Gamma(3 + l - N_2)}{\Gamma(2l + N_1 - N_2 + 1)}. \quad (5)$$

The above expression differs from [6] by the presence of terms involving  $N_1$  and  $N_2$ , both of which are linear order in  $M\omega$ . Therefore, if we take the limit of both  $N_1$  and  $N_2$  going to zero, it can be verified that the above response function correctly reproduces the result presented in Ref. [6]. It should be emphasized that the terms  $N_1$  and  $N_2$  must be included in the above expression for consistency since they are of  $\mathcal{O}(M\omega)$ . Hence, it is the form given in Eq. (5) that correctly depicts the tidal response function of an arbitrarily rotating BH when one limits it to linear order terms in the frequency.

The static limit ( $\omega = 0$ ) of the tidal response function can be simplified to

$$F_{lm}^{\text{static}} = - \left( \frac{iam}{r_+ - r_-} \right) \frac{(l-2)!(l+2)!}{(2l+1)!(2l)!} \times \prod_{j=1}^l \left[ j^2 + \left( \frac{2am}{r_+ - r_-} \right)^2 \right]. \quad (6)$$

The above correctly reproduces the results of [14, 15]. Moreover, the static tidal response function is purely imaginary and hence the static TLNs of an arbitrarily rotating BH identically vanish. This shows that BHs in vacuum General Relativity have vanishing static TLNs.

Since our analysis is valid only upto linear order in  $M\omega$ , we should expand the Gamma functions in powers of  $M\omega$ . Using the prescription given in Eq. (2), we deduce the TLNs for the general Kerr to be

$$k_{lm} = (am)^2 k_{lm}^{(0)} + am\omega k_{lm}^{(1)} + \mathcal{O}(M^2\omega^2), \quad (7)$$

where the explicit forms of  $k_{lm}^{(0)}$  and  $k_{lm}^{(1)}$  are given in Supplemental Material [37] and both of them are independent of frequency  $\omega$ . The first thing to notice from the above expression is that they are non-zero, signaling dynamical deformation for Kerr BHs under external tidal field. Moreover, from the above expression it is clear that the dynamical TLNs vanish for a Schwarzschild black hole ( $a = 0$ ), slowly rotating Kerr BH and for axisymmetric tidal perturbation ( $m = 0$ ). These results are consistent with the earlier findings in the literature [5, 6]. Even though it is tempting to take  $\omega \rightarrow 0$  limit of the above equation and argue that Kerr BHs have non-zero static TLNs, that is not a correct statement. This is because the above expression was derived on the sole premise that  $\omega \neq 0$ , and for strictly static tidal fields the real part of the response function indeed vanishes, as we have already demonstrated above. Hence Eq. (7) is applicable only when  $\omega \neq 0$ , yielding non-zero dynamical TLNs. The singular behavior of the  $\omega \rightarrow 0$ , or, the static limit, in connection with the TLNs, has also appeared earlier in various contexts, e.g., in the case of magnetic TLNs [41], as well as for ultra-compact objects [26]. To the best of our knowledge, the above result demonstrates for the first time that rotating BHs have non-zero dynamical TLNs. Since all physical processes associated with binary BH coalescence are intrinsically dynamical in nature and all astrophysical BHs are expected to have some spin, the importance of the above result cannot be overstated.

For a clearer depiction of the above result, we have presented the variation of the TLN associated with the  $l = 2 = m$  mode ( $k_{22}$ ) with the dimensionless mode frequency  $M\omega$  and the dimensionless rotation parameter ( $a/M$ ) in Fig. 1. Evidently, for certain values of  $M\omega$  and ( $a/M$ ),  $k_{22}$  vanishes identically. Moreover, for larger values of ( $a/M$ ),  $k_{22}$  is negative, while for smaller rotation and larger values of the frequency  $M\omega$ ,  $k_{22}$  becomes positive. Hence, rotation and frequency act oppositely in determining the sign of the TLN. Thus, for smaller mode frequency,  $k_{22}$  of a rapidly rotating Kerr BH is negative, while for higher mode frequency,  $k_{22}$  of a slowly rotating Kerr BH is positive.

#### Tidal response of extremal Kerr black hole.—

While one might hope for using the results of previous section to also obtain the TLNs for extremal Kerr – by taking  $a = M$  – this simplistic idea does not work here: The TLNs there explicitly depend on  $(r_+ - r_-)^{-1}$ , which diverges in the extremal limit. This is simply due to a bad choice of coordinates. For this reason, we introduce

<sup>4</sup> Note that the Logarithmic term is often considered as an ambiguity term arising from differentiating the response of the body with the source of tidal perturbation. In this work, following [28], we have ignored the Logarithmic term. Had we kept the term following the world-like effective field theory formalism, the results would be dependent on the cutoff scale. Connecting the TLNs derived here with GW observables will leverage such a scale dependence of the TLNs.

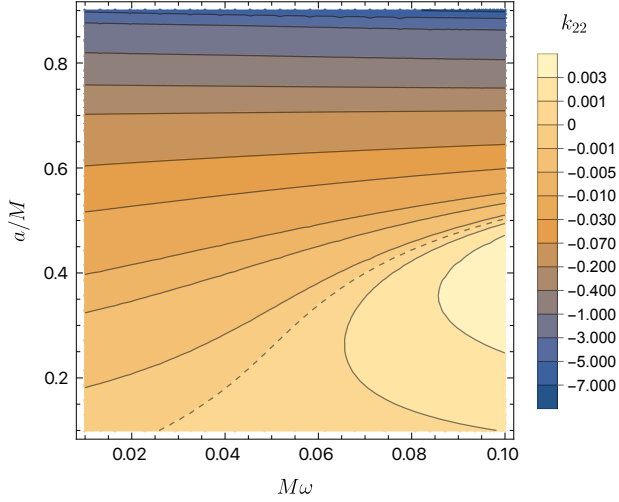


FIG. 1. Contour plot of the tidal Love number  $k_{22}$ , as a function of  $M\omega$  and  $(a/M)$ . As evident for larger values of the rotation parameter, the TLN becomes negative, while for smaller rotation and larger frequencies the TLN is positive. Intriguingly, for certain choices of frequency and rotation parameter, the TLN can be zero as well (shown by dashed contour). Note that our analysis is valid to linear order in  $M\omega$ ; therefore, the  $x$ -axis has been plotted upto  $M\omega = 0.1$ .

the new coordinate  $\bar{z} \equiv (r - r_+)/r_+$ , which merely shifts the origin of the radial coordinate to the position of the event horizon. Furthermore, since for an extremal Kerr BH  $r_+ = r_- = M$ , one can express  $\Delta = (r - M)^2$ , such that the source-free Teukolsky equation in the new coordinate,  $\bar{z}$ , is solvable in the near zone (see Supplemental Material [37] for further details). The dynamical tidal response function can be deduced from its solution as before, and is found to be:

$$F_{lm} = -\{2i(m - 2M\omega)\}^{2l+1-2N} \times \frac{\Gamma(-2l+2N)\Gamma(3+l-2iM\omega-N)}{\Gamma(2-l-2iM\omega+N)\Gamma(2l+2-2N)} \quad (8)$$

for an extremal Kerr BH under external tidal perturbation. Here,  $N \equiv 2mM\omega(l^2 + l + 4)/\{l(l+1)(2l+1)\}$  is a real quantity. Note that in the static limit ( $\omega = 0$ ), we obtain  $N = 0$  and, hence, the above dynamical tidal response function becomes

$$F_{lm}^{\text{static}} = (-1)^{-1-l} im(2im)^{2l} \frac{\Gamma(3+l)\Gamma(-1+l)}{\Gamma(2l+1)\Gamma(2l+2)}, \quad (9)$$

which is finite and purely imaginary, implying vanishing static TLNs for extremal Kerr BHs as well.

Given the fact that  $N$  is real, one can read off the real part of the dynamical tidal response function in Eq. (8) to obtain the dynamical TLNs,

$$k_{lm} = (2m)^{2l-2N-2} \left( m^2 \tilde{k}_{lm}^{(0)} + mM\omega \tilde{k}_{lm}^{(1)} \right) + \mathcal{O}(M^2\omega^2), \quad (10)$$

where the explicit forms of  $\tilde{k}_{lm}^{(0)}$  and  $\tilde{k}_{lm}^{(1)}$  are given in Supplemental Material [37] and show that they are non-zero for an extremal Kerr BH.

It is also interesting to note that for an axis-symmetric tidal field ( $m = 0$ ), the above dynamical TLNs identically vanish. This can be verified from the  $m \rightarrow 0$  limit of the dynamical tidal response function as well, which we have elaborated in the Supplemental Material [37]. To summarize, for extremal Kerr BHs, there are non-trivial TLNs, which vanish in the static limit but yield non-zero result in the dynamical scenario, akin to that of non-extremal Kerr BHs.

**Discussion.**— The tidal deformations of a compact object under an external gravitational field is a remarkable probe of its structure and composition. They are encoded in the TLNs, which are imprinted on GWs radiated by it, e.g., when inspiraling in a binary. The measurement of the TLNs of the neutron stars in GW170817 constrained the equation of state of nuclear matter at extreme densities. It is now turning out that non-zero TLNs may not be exclusive to neutron stars; rather, the TLNs of ultra-compact objects depict a characteristic logarithmic behavior and BHs in higher dimensions have non-zero TLNs [17, 25, 26, 29]. However, BHs in General Relativity have vanishing static TLNs. It is an exciting prospect that at least some of these predictions can be tested in GW observations.

All of the above results were derived in the context of a static external tidal field, which does not depict the reality of an inspiraling binary. This motivates the pursuit of a dynamical description of tidal deformability, preferably in a gauge invariant way. Both of these objectives have been recently achieved through the Newman-Penrose scalar in the Teukolsky formalism [6, 15]. The asymptotic expansion of the solution of the Teukolsky equation in the near-zone regime and under small-frequency approximation yields the dynamical tidal response function, whose real part yields the TLNs. As already pointed out in [5], and also explicitly demonstrated here, the small-frequency approximation of [6] had missed quite a few terms, all of which are linear in  $M\omega$ . In this work we have kept all of these  $\mathcal{O}(M\omega)$  terms and solved the Teukolsky equation to extract Gauss hypergeometric functions and, from their asymptotic limit, the dynamical tidal response function. Intriguingly, the static limit of this dynamical tidal response function yields a purely imaginary quantity, implying vanishing static TLNs for Kerr BHs. This holds true for extremal Kerr BHs as well. This leads to the first main result of this work, showing consistency of our formalism with previous literature:

1. *Schwarzschild, slowly-rotating Kerr, non-extremal Kerr and extremal Kerr BHs have vanishing static TLNs. In other words, black holes in vacuum General Relativity have vanishing static TLNs.*

Progressing to dynamical TLNs, we first proved that:

2. *Schwarzschild* ( $a = 0$ ) and *slowly-rotating Kerr BHs* (i.e., with negligible  $\mathcal{O}(a\omega)$  and  $\mathcal{O}[(a/M)^2]$  terms) have vanishing dynamical TLNs.
3. Both *non-extremal and extremal Kerr BHs* have vanishing dynamical TLNs for *axi-symmetric external tidal perturbations* (i.e., with  $m = 0$ ).

In all cases in items 2 and 3, the dynamical tidal response function was shown to be purely imaginary.

Finally, in the case of arbitrarily rotating BH, under non-axi-symmetric tidal perturbation, the dynamical tidal response function is non-zero and has a real part, which has contributions from terms  $\mathcal{O}(am\omega)$  as well as  $\mathcal{O}(a^2m^2)$ . Given these parameter dependencies of the dynamical TLNs, it is obvious that the limiting cases in items 2 and 3 are readily satisfied. A similar pattern of parameter dependencies holds for extremal Kerr BHs, with  $a$  set to  $M$ . Therefore, our final result is:

4. *Kerr BHs – both non-extremal and extremal – have non-vanishing dynamical TLNs, within the linear-in-frequency approximation scheme<sup>5</sup>, which depend on  $(am)^2$  and  $(am\omega)$ , apart from other functions of the BH parameters.*

Thus, the detection of a non-zero (dynamical) TLN via GWs from a binary with spinning components can not conclusively prove that the components are non-black holes, such as neutron stars.

We emphasize that our results are consistent with recent findings in the literature [16, 44], for Ref. [44] showed that the linear-in-frequency TLNs for Kerr BHs have Logarithmic running, suggesting non-zero dynamical TLNs, which we not only verified, but provided exact expressions for as well. The connection with GW waveform can also be achieved through matched asymptotic expansion of  $\Psi_4$  and then relating the flux at infinity with GW phasing formula [45, 46]. In particular, it would be interesting to see if there are non-trivial effects on the parameter estimation from BBH merger events, as the GW waveforms will be modified due to the non-zero dynamical TLNs, derived here.

There are several future directions of exploration. For example, the dependence of the non-zero dynamical TLNs on the spin of the perturbing field needs to be determined. In deriving the non-zero dynamical TLNs for a rotating BH, we employed perturbation theory. It would be interesting to see if the same result can be arrived at from other approaches, e.g., the ladder symmetry approach (we should be able to show that there is no ladder

symmetry for gravitational perturbation of the Kerr BH in the dynamical context), or, in effective field theory. Moreover, implications of the non-zero dynamical TLNs for extremal BH in the context of AdS/CFT correspondence will be worth exploring. We leave these issues for the future.

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\* [rajendra@iucaa.in](mailto:rajendra@iucaa.in)

† [tpsc@iacs.res.in](mailto:tpsc@iacs.res.in)

‡ [sukanta@wsu.edu](mailto:sukanta@wsu.edu)

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<sup>5</sup> However, we would like to emphasize that some other recent and ongoing works [18, 42, 43] use different approximation schemes than the one presented here, and yet find results that are consistent with our findings, except for an overall normalization factor.

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