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Bulk viscosity of two-color superconducting quark matter in neutron star mergers

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We study the bulk viscosity of moderately hot and dense, neutrino-transparent color superconducting quark matter arising from weak-interaction-driven direct Urca processes. The quark matter is modeled using Nambu–Jona-Lasinio (NJL) model improved to account for vector and t’Hooft interactions as well as antisymmetric pairing among the red/green up and down quarks. The unpaired excitations are the strange quarks and the blue up or down quarks. We compute the relaxation rates associated with d and s -quark decay and electron capture processes on u quark for blue color. The resulting bulk viscosity for density oscillations in the 1 to 10 kHz range shows a resonant peak at $T \sim 5$ MeV, and the damping time may drop below 10 ms. This is short enough to affect the post-merger evolution and is very similar to the damping predicted in nuclear matter.

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I. INTRODUCTION

The recent discovery of gravitational waves followed by electromagnetic counterparts originating from binary neutron-star mergers, as observed by the LIGO-Virgo collaboration [1], has spurred interest in the transport characteristics of hot and dense matter. Numerical simulations of binary neutron star mergers typically conducted within the framework of non-dissipative hydrodynamics, have predicted significant density oscillations and robust gravitational wave emissions in the initial tens of milliseconds following the merger [2–7]. These oscillations are expected to eventually dissipate due to various dissipative processes within the post-merger matter, ultimately impacting the gravitational wave signal. Among these processes, bulk viscous dissipation facilitated by weak interactions appears to be the most efficacious mechanism in damping these oscillations as demonstrated by the initial estimates [8], first-principle computations of rates and associated relaxation times [9–12] and dissipative numerical simulations [13–15].

In the cold regime pertinent to mature compact stars, extensive studies on bulk viscosity have been conducted, largely building upon the seminal work of Ref. [16]. The case of cold quark matter in hybrid and strange stars was studied in Refs. [17–25] in the context of damping of r -mode oscillations of cold stars. In particular, two flavor (“2SC”) pairing was studied in Ref. [19] including finite temperatures within a model defined in terms of the 2SC gap and chemical potentials (without electrical

neutrality) while the numerical results were focused on non-leptonic processes [35].

The study of bulk viscosity of quark matter in the context of binary neutron star (BNS) mergers is still in its early stages [26, 27] and has focused on hot, unpaired quark matter. In this work, we present the first computation of the bulk viscosity of a BNS post-merger object that incorporates color superconductivity at intermediate densities and non-zero temperatures. We also consider conditions specific to BNS mergers, such as charge and color neutrality, and the density and temperature dependence of gaps and chemical potentials, which are accounted for self-consistently within our model.

We focus on the weak leptonic quark Urca processes within the phase structure of color superconducting matter relevant to compact stars. At intermediate densities, quark matter is expected to be in the two-color superconducting (2SC) phase, where red-green light quarks pair while light blue quarks, as well as s quarks, remain unpaired [28]. Our study, using the vector-interaction-improved Nambu–Jona-Lasinio model [29], examines bulk viscosity primarily from the Urca processes involving these unpaired blue quarks.

II. BULK VISCOSITY OF $udse$ MATTER

We consider neutrino-transparent matter composed of u, d, s quarks and electrons in the core of the compact star. The simplest (semi-leptonic) beta equilibration processes in this system are the d and s -quark decay and the electron capture processes of the direct Urca type:

$$d \rightarrow u + e^- + \bar{\nu}_e, \quad (1)$$

$$u + e^- \rightarrow d + \nu_e, \quad (2)$$

$$s \rightarrow u + e^- + \bar{\nu}_e, \quad (3)$$

$$u + e^- \rightarrow s + \nu_e, \quad (4)$$

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where ν_e and $\bar{\nu}_e$ are the electron neutrino and antineutrino, respectively. The processes (1)–(4) proceed only in the direction from left to right because in neutrino-transparent matter neutrinos/antineutrinos can appear only in final states.

The rates of the Urca processes (1)–(4) can be written in full analogy with the nucleonic counterparts, see for example, Eqs. (8)–(10) of [11] (for quark matter one simply needs to replace $g_A \rightarrow 1$ in the matrix element and also $\cos \theta_c \rightarrow \sin \theta_c$ for processes including s -quarks, where $\theta_c = 13.04^\circ$ is the Cabibbo angle).

We will show below that, as has previously been noted [19, 26, 27], the non-leptonic processes

$$u + d \leftrightarrow u + s \quad (5)$$

proceed much faster than the Urca processes. Consequently, in the regime where Urca-process-driven bulk viscosity is at its maximum the non-leptonic equilibration effectively takes place immediately, and we can assume $\mu_s = \mu_d$ at all times (see [19], Appendix A). In this regime, there is only a single equilibrating quantity $\mu_\Delta \equiv \mu_d - \mu_u - \mu_e$, which is the relevant measure of how much the system is driven out of β -equilibrium state by a cycle of compression and rarefaction.

The bulk viscosity coefficient of neutrino-transparent $udse$ matter arising from the Urca processes (1)–(4) for small-amplitude density oscillations with a frequency ω is given by

$$\zeta = \frac{C^2}{A} \frac{\gamma}{\omega^2 + \gamma^2}, \quad (6)$$

which has the classic resonant form depending on two quantities: the susceptibility prefactor C^2/A which depends only on the equation of state, and the relaxation rate $\gamma = \lambda_d A$ which depends additionally on the microscopic interaction rates via the equilibration coefficient

$$\lambda \equiv \frac{d\Gamma}{d\mu_\Delta}, \quad (7)$$

where Γ is the total rate of Urca process. Analogous quantities for non-leptonic processes are defined in a similar fashion, e.g., $\lambda' \equiv d\Gamma'/d\mu'_\Delta$ where Γ' is the rate of non-leptonic process and μ'_Δ is the deviation from chemical equilibrium due to non-leptonic processes. The susceptibilities are defined as

$$A = -\frac{1}{n_B} \frac{\partial \mu_\Delta}{\partial x_u} \Big|_{n_B}, \quad C = n_B \frac{\partial \mu_\Delta}{\partial n_B} \Big|_{x_u}, \quad (8)$$

where n_B is the baryon density and x_u is the fraction of blue up quarks. The derivatives in Eq. (8) and the relaxation rate γ should be computed in β -equilibrium. (In the regime of interest the density oscillations are supposed to be adiabatic, however, the difference between adiabatic and isothermal susceptibilities is negligible in this case [12, 33].)

To describe the properties of 2SC quark matter we adopt an NJL-type Lagrangian which is given by

$$\begin{aligned} \mathcal{L}_Q = & \bar{\psi}(i\gamma^\mu \partial_\mu - \hat{m})\psi + G_V(\bar{\psi}i\gamma^\mu\psi)^2 \\ & + G_S \sum_{a=0}^8 [(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}i\gamma_5\lambda_a\psi)^2] \\ & + G_D \sum_{\gamma,c} [\bar{\psi}_\alpha^a i\gamma_5 \epsilon^{\alpha\beta\gamma} \epsilon_{abc} (\psi_C)_\beta^b] [(\bar{\psi}_C)_\rho^r i\gamma_5 \epsilon^{\rho\sigma\gamma} \epsilon_{rsc} \psi_\sigma^s] \\ & - K \{ \det_f [\bar{\psi}(1 + \gamma_5)\psi] + \det_f [\bar{\psi}(1 - \gamma_5)\psi] \}, \quad (9) \end{aligned}$$

where the quark spinor fields ψ_α^a carry color $a = r, g, b$ and flavor ($\alpha = u, d, s$) indices, the matrix of quark current masses is given by $\hat{m} = \text{diag}_f(m_u, m_d, m_s)$, λ_a with $a = 1, \dots, 8$ are the Gell-Mann matrices in the color space, and $\lambda_0 = (2/3)\mathbf{1}_f$. The charge conjugate spinors are defined as $\psi_C = C\bar{\psi}^T$ and $\bar{\psi}_C = \psi^T C$, where $C = i\gamma^2\gamma^0$ is the charge conjugation matrix. The original NJL model, which contains the scalar coupling term with coupling G_S has been extended here to include the t'Hooft interaction with coupling K and the vector interactions with coupling G_V ; this model is referred to as the vector-NJL model. The pairing gap in the quasiparticle spectrum is $\Delta_c \propto (\bar{\psi}_C)_\alpha^a i\gamma_5 \epsilon^{\alpha\beta\gamma} \epsilon_{abc} \psi_\beta^b$. The numerical values of the parameters of the Lagrangian are the same as in Ref. [29] and we fix the strength of the di-quark interaction at $G_D/G_S = 1.1$, this ratio being somewhat larger than the value 0.75 suggested by the Fierz-transformation. Note that the larger the gap, the stiffer the equation of state and the larger the maximum mass of a hybrid star.

We assume 2SC pairing by adopting a diquark-antidiquark interaction (4th term in Eq. (9)) that has color and flavor antisymmetric pairing of u and d quarks while s -quarks remain unpaired.

The 2SC pairing gap depends weakly on the density: it varies in the range $130 \leq \Delta \leq 150$ MeV in the number density range $0.6 \leq n_B \leq 1.14 \text{ fm}^{-3}$. It also depends weakly on the temperature in the range of interest $\Delta \gg T \sim (1 - 10) \text{ MeV}$. The masses of quarks in the same density interval vary in the range $10 \leq m_{u,d} \leq 30 \text{ MeV}$ and $300 \leq m_s \leq 400 \text{ MeV}$ and are weakly dependent on temperature in the range $1 \leq T \leq 10 \text{ MeV}$. Further input characteristics of the unperturbed quark matter model forming the background equilibrium are given in Supplemental Material [30].

The equilibration coefficients for the Urca processes (1)–(4) as well as for the non-leptonic processes (5) are plotted in Fig. 1 for fixed baryon density $n_B/n_0 = 7$. The non-leptonic processes have a much higher rate than the Urca processes, as mentioned above, whereas the rates of the d -Urca and s -Urca processes are comparable in the temperature range $0.1 \leq T \leq 10 \text{ MeV}$. This implies that the bulk viscosity produced by the non-leptonic processes will, for typical BNS merger oscillation frequencies in the kHz range, be peaked at much lower temperatures which lie out of the temperature range of interest of BNS mergers. This argument justifies our assumption that the bulk viscosity at temperatures $1 \leq T \leq 10 \text{ MeV}$

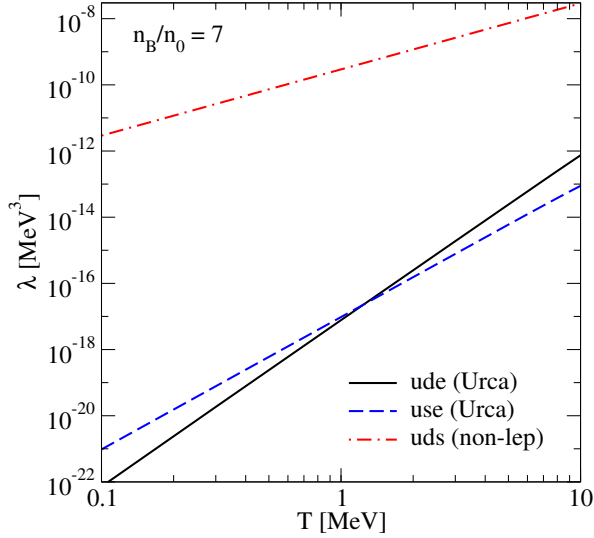


FIG. 1: The total equilibration coefficients λ for processes (1) and (2) (solid line), for processes (3) and (4) (dashed line) and for the non-leptonic processes (dash-dotted line) as functions of the temperature for baryon density fixed at $n_B/n_0 = 7$.

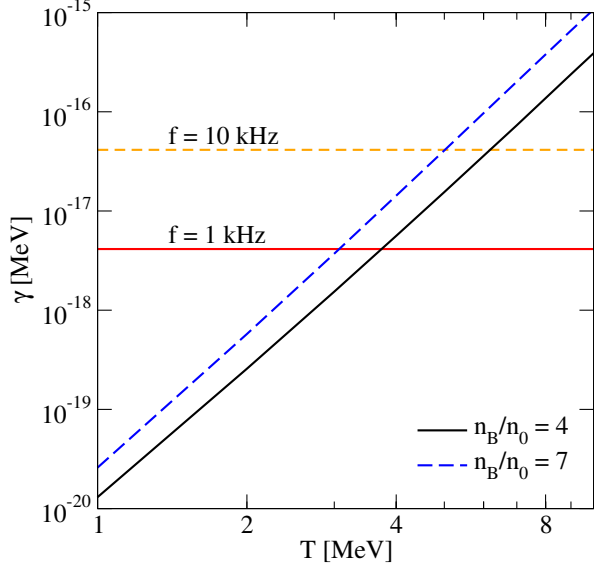


FIG. 2: The β -relaxation rate γ as a function of the temperature for two values of the baryon density. The horizontal lines show where $\gamma = 2\pi f$ for selected values of oscillation frequency $f = 1$ kHz (solid line) and $f = 10$ kHz (dashed line).

can be computed from Urca processes only, with the additional constraint $\mu_d = \mu_s$.

To gain physical insight, we quote here approximate analytical low-temperature results for the equilibration coefficients $\lambda_d \simeq 0.2 G_F^2 \cos^2 \theta_c p_{Fd}^2 T^5$, and $\lambda_s \simeq 0.03 G_F^2 \sin^2 \theta_c \mu_s^* m_s^2 T^4$, where G_F is the Fermi coupling constant and p_{Ff} is the Fermi momentum of quark of

flavor $f = u, d, s$. These formulas were derived under the assumptions $T \ll \mu_i, m_d, m_s \rightarrow 0$ and $\omega_0 - \phi_0 \ll m_s$ and are accurate to a few percent when compared to numerical results that do not assume these approximations. The rates of d -Urca processes include an extra power of T because the scattering matrix element contains the four-product of the momenta of the u -quark and electron, which are nearly collinear at chemical equilibrium. This near collinearity occurs because the light quarks are almost massless, with a small angle deviation allowed only by the thermal smearing of these momenta around the Fermi spheres. Consequently, the scattering matrix element adds an extra power of T to the process rates, in addition to the standard T^5 scaling for massive particles, which arises from the available scattering phase space. This difference in temperature scaling leads to the intersection of the $\lambda_d(T)$ and $\lambda_s(T)$ curves at a temperature in the MeV range, as seen in Fig. 1. Two more aspects are worth noticing: (i) $\lambda_s(T)$ is suppressed by a factor of $\sin^2 \theta_c = 0.05$ compared to $\lambda_d(T)$ which has $\cos^2 \theta_c = 0.95$ and has a smaller numerical pre-factor arising from the phase-space integration. The remaining factors such as m_s, p_{Fd} , and μ_s^* are all of the same order of magnitude. (ii) Because the interactions modify the chemical potentials of light quarks via the iso-scalar ω -field, $p_{Fd} = p_{fu} + p_{fe}$ in β -equilibrium for massless quarks, which in turn implies that the direct Urca channel is only thermally open. In contrast, s -Urca processes are widely open with $p_{Fu} + p_{Fe} - p_{Fs} \geq 70$ MeV as a result of large s -quark mass [31, 32].

The temperature-dependence of the total β -relaxation rate γ due to the Urca processes (1)–(4) is shown in Fig. 2. It follows a power-law scaling $\gamma \propto T^{4.5}$ in the temperature range $1 \leq T \leq 10$ MeV, and increases weakly with the density. We see that γ matches the constant angular frequency $\omega = 2\pi f$ at a temperature around $T \simeq 3 - 4$ MeV for $f = 1$ kHz and around $T \simeq 5 - 6$ MeV for $f = 10$ kHz, the exact values of the temperature being dependent on the baryon density.

Figure 3 shows the temperature dependence of the bulk viscosity of 2SC quark matter for the density oscillation frequency fixed at a characteristic to BNS mergers value $f = 1$ kHz. The bulk viscosity has a maximum at the temperature where $\omega = \gamma$. The point of the maximum shifts slowly to lower temperatures with increasing density. The maximum value and the overall magnitude of the bulk viscosity is a sensitive function of density through its dependence on the susceptibility factor C^2/A , see Eq. (6). It decreases with the density, and, *e.g.*, at $n_B/n_0 = 7$ the maximum bulk viscosity is more than an order of magnitude smaller than at $n_B/n_0 = 4$. For comparison, we show also the bulk viscosity of *ude* matter (*i.e.*, when the processes (3) and (4) are not taken into account) with short-dashed lines at $n_B/n_0 = 4$. We see that neglecting the s -quark contribution underestimates the bulk viscosity at low temperatures and overestimates it at higher temperatures. However, the location of the maximum does not change significantly, because at these

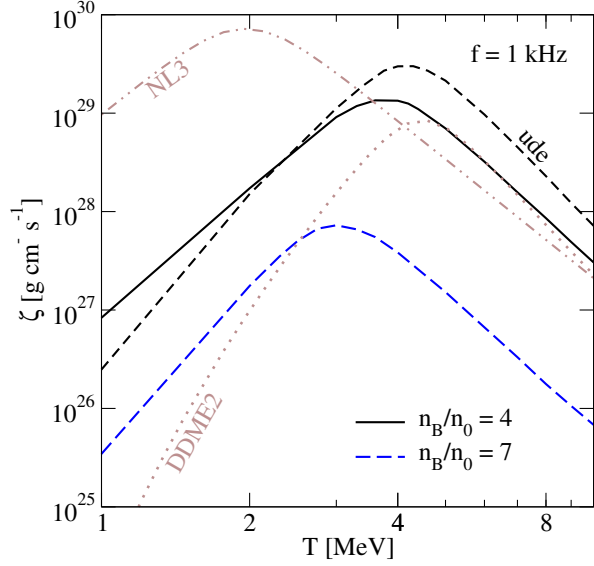


FIG. 3: The bulk viscosity of $udse$ 2SC quark matter as a function of temperature for two values of baryon density. The frequency of density oscillations is fixed at $f = 1$ kHz. The bulk viscosity of ude matter is shown by short-dashed lines at $n_B/n_0 = 4$. In addition, the bulk viscosity of npe matter from Ref. [12] is shown for comparison by dotted (model DDME2) and dashed-double-dotted (model NL3) lines at density $n_B/n_0 = 4$.

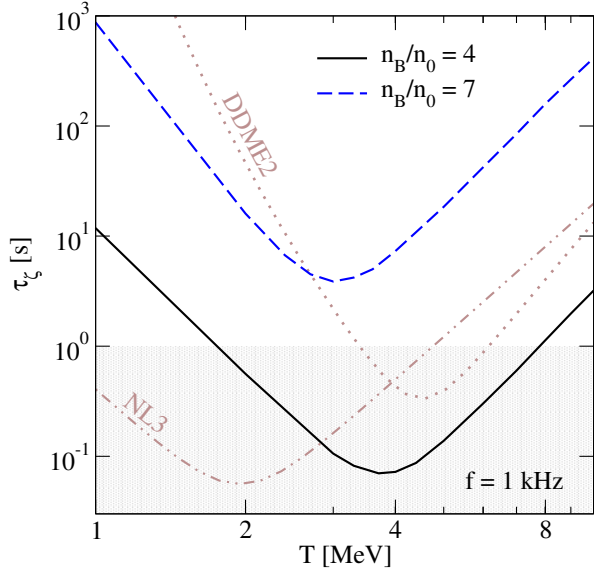


FIG. 4: The damping timescale of density oscillations for 2SC quark matter as a function of temperature for $f = 1$ kHz. The results for nucleonic matter from Ref. [12] are shown for comparison by dotted (model DDME2) and dashed-double-dotted (model NL3) lines at density $n_B/n_0 = 4$.

temperatures the d -Urca and s -Urca processes have comparable rates, and the inclusion of s -quarks increases the relaxation rate by a factor between 1 and 2, leading to a

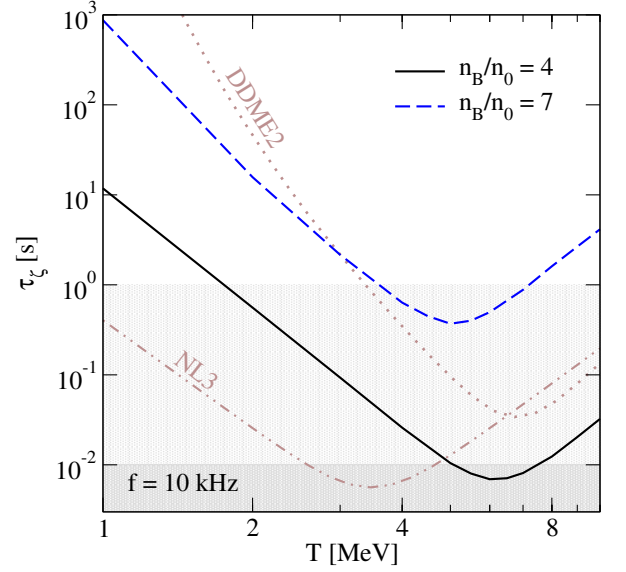


FIG. 5: Same as Fig. 4 but for $f = 10$ kHz.

slight shift of the maximum to lower temperatures.

For comparison, we show also the bulk viscosity of neutrino-transparent nucleonic matter as was computed in Ref. [12] for models DDME2 (dotted line) and NL3 (dashed line) for $n_B/n_0 = 4$. The main difference between these two models is that the low-temperature direct Urca channel is allowed for NL3 and is blocked for DDME2 at the density value considered. As a result, the bulk viscosities for the two models differ significantly at temperatures $T \leq 4$ MeV, but are rather similar at $T \geq 4$ MeV. At these temperatures, the bulk viscosity of 2SC quark matter is not very different from the bulk viscosity of nucleonic matter. In the low-temperature regime, 2SC quark matter's bulk viscosity has intermediate values compared to the two nucleonic models. The reason for this is the fact that the u -quark fraction is below, but very close to the direct Urca threshold for the reactions (1) and (2): the difference between the Fermi momenta of initial and final state particles $p_{Fd} - p_{Fu} - p_{Fe}$ is less than 1 MeV for densities $4 \leq n_B/n_0 \leq 7$. This implies that already at temperature $T \geq 1$ MeV thermal smearing is sufficient to open up phase space for light-quark Urca processes.

Since the bulk viscosity comes from semi-leptonic weak interactions of the unpaired blue quarks, one would expect that calculations for unpaired quark matter would see a similar maximum in the bulk viscosity at MeV temperatures. This is visible in Ref. [22] (Fig. 3), and also in Ref. [27] (Fig. 4) though their maximum bulk viscosity is smaller because their strange quark is lighter.

Next, we estimate the bulk viscous damping timescales of density oscillations. The damping timescale is given by Refs. [8, 9, 33]

$$\tau_\zeta = \frac{1}{9} \frac{K n_B}{\omega^2 \zeta}, \quad (10)$$

where the incompressibility is $K = 9(\partial P/\partial n_B)$, and P is the pressure. The damping timescale exhibits a temperature dependence that is inverse to that of bulk viscosity, as K remains insensitive to the temperature within the range $1 \leq T \leq 10$ MeV. As seen from Figs. 4 and 5 the time-scale τ_ζ is shortest when the bulk viscosity is largest, at $T \simeq 3 - 4$ MeV for oscillations of frequency $f = 1$ kHz, and at $T \simeq 5 - 6$ MeV for $f = 10$ kHz. At this minimum, the bulk viscous damping timescales of 2SC quark matter are in the range from a few milliseconds to a few hundred milliseconds, which coincides with the typical short-term evolution timescales of the post-merger object. The light-shaded region in Figs. 4 and 5 corresponds $\tau_\zeta \leq 1$ s, whereas the heavy-shaded region in Fig. 5 corresponds to $\tau_\zeta \leq 10$ ms.

For a typical oscillation frequency of $f = 1$ kHz, our results predict that bulk viscous damping would be marginally relevant over the short BNS postmerger evolution timescales of approximately 10 ms, but noticeable for longer-lived remnants in the temperature range of 2 to 8 MeV. For higher frequencies, such as $f = 10$ kHz, bulk viscous damping significantly affects the short-term evolution timescale of approximately 10 ms within the temperature range of 5 to 8 MeV at relatively low densities. It is also noteworthy that the damping timescales for 2SC quark matter are quite similar to those found for nucleonic matter.

This implies that, according to our NJL model of quark matter, at temperatures in the 1 to 10 MeV range it will be difficult to distinguish 2SC quark matter from nuclear matter based on its bulk viscous dissipation properties.

III. CONCLUSIONS

In this work, we studied the bulk viscosity of neutrino-transparent 2SC quark matter driven by direct Urca processes in the temperature range $1 \leq T \leq 10$ MeV and density range $4n_0 \leq n_B \leq 7n_0$. This complements our recent study of relativistic $npe\mu$ matter [12]. Here, we focused on direct Urca rates, including the strange sector and the effects of color superconductivity in the red/green up

and down quarks. Blue quarks primarily drive the bulk viscosity, although gapless superconductivity may allow contributions from red-green pairs [34].

We found that bulk viscosity in *udse* matter peaks at $T = 3 - 6$ MeV, where Urca process rates resonate with the characteristic frequencies of density oscillations in BNS post-merger objects. The resulting damping timescales, down to tens of milliseconds, suggest that bulk viscosity may significantly contribute to energy dissipation in BNS mergers at these temperatures.

The bulk viscosity and damping time for 2SC quark matter closely resemble those of nucleonic matter [12], making it challenging to distinguish these states via their bulk viscous behavior. However, at lower temperatures, quark matter shows a second resonant peak in bulk viscosity at $T \sim 0.05$ MeV, absent in nuclear matter [19, 26, 27].

Interestingly, semileptonic rates for unpaired quark matter may be similar to those for 2SC matter, suggesting that unpaired quark matter at $T \approx 1$ to 10 MeV may also be difficult to distinguish from nuclear matter. A recent study of unpaired quark matter [27] found a peak in bulk viscosity in this temperature range, though with longer damping times, likely due to different susceptibilities in their model. Future studies should address these model-dependent uncertainties in quark matter phenomenology under BNS merger conditions.

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