Revealing intrinsic flat math xmlns="http://www.w3.org/1998/Math/MathML" display="inline">mrow>mi mathvariant="normal">Λ/mi>mi>CDM/mi>/mrow>/math> biases with standardizable candles

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Revealing Intrinsic Flat ΛCDM Biases with Standardizable Candles

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Emerging high redshift cosmological probes, in particular quasars (QSOs), show a preference for larger matter densities, $\Omega_m \approx 1$, within the flat ΛCDM framework. Here, using the Risaliti-Lusso relation for standardizable QSOs, we demonstrate that the QSOs recover the *same* Planck-ΛCDM Universe as Type Ia supernovae (SN), $\Omega_m \approx 0.3$ at lower redshifts $0 < z \lesssim 0.7$, before transitioning to an Einstein-de Sitter Universe ($\Omega_m = 1$) at higher redshifts $z \gtrsim 1$. We illustrate the same trend, namely increasing Ω_m and decreasing H_0 with redshift, in SN but poor statistics prevent a definitive statement. We explain physically why the trend may be expected and show the intrinsic bias through non-Gaussian tails with mock SN data. Our results highlight an intrinsic bias in the flat ΛCDM Universe, whereby Ω_m increases, H_0 decreases and S_8 increases with effective redshift, thus providing a new perspective on ΛCDM tensions; even in a Planck-ΛCDM Universe the current tensions may be expected.

INTRODUCTION

Our current understanding of the Universe, as described by the flat ΛCDM model, largely rests upon three pillars: Type Ia supernovae (SN) cosmology [1, 2], the cosmic microwave background (CMB) [3] and baryon acoustic oscillations (BAO) [4]. While these observables show perfect agreement on $\Omega_m \approx 0.3$, recent direct checks of the flat ΛCDM model in the late Universe challenge the current paradigm [5–7]. The crux of this letter is that cosmological probes already point to evolution of matter density Ω_m, and consequently H_0, within the flat ΛCDM model.

Risaliti & Lusso have introduced a relation between X-ray and UV QSO luminosities, respectively L_X, L_{UV}, for cosmological purposes [8, 9]:

$$\log_{10} L_X = \beta + \gamma \log_{10} L_{UV}, \tag{1}$$

where β and γ are fitting constants. This relation follows from an empirical relation between the corresponding fluxes and it has been shown that it is robust to selection biases and redshift evolution [10], so the relation appears intrinsic to QSOs. It has been shown that the slope $\gamma \approx 0.6$ is robust across luminosities and redshifts [11–15]. In contrast to other QSO standardization methods [16–19], equation (1) represents an approach that is extremely powerful, as it can be applied across extended redshifts and luminosities.

Here, we largely highlight synergies between QSOs and SN within flat ΛCDM. First, we show that the Risaliti-Lusso QSOs recover Planck-ΛCDM in a lower redshift range where SN are numerous. Nevertheless, as higher redshift QSOs are added, QSOs gradually return larger values of Ω_m until

one enters an Einstein-de Sitter Universe (EdS) (spatially flat FLRW with only pressureless matter) when $z \gtrsim 1$. Taken at face value, QSOs transition from a dark energy (DE) dominated Universe to a matter dominated Universe, which may partially explain the preference of QSO data for larger values of Ω_m, and consequently less DE [20, 21].

Next, we show hints of the same evolution of Ω_m with redshift, but in Type Ia SN [22]. Concretely, we show that as Ω_m increases, then H_0 decreases, at least within the flat ΛCDM model. This trend is simply recovering earlier results in the literature [23–25]. The main point is that we see the *same* trend independently in both QSOs and SN, both of which have distinct strengths and weaknesses. On one hand, QSOs are plentiful at higher redshifts and have good statistics, but are relatively new cosmological probes (see Ref. [26] for a review) and suffer from greater intrinsic scatter. On the flip side, SN represent a cornerstone of modern cosmology, but become sparse at higher redshifts, thereby preventing us from confirming that $\Omega_m > 0.3$. Nevertheless, combining both probes, not only does one recover a Planck-ΛCDM Universe in a similar redshift range $z \lesssim 0.7$, but one sees hints of a deviation from the Planck-ΛCDM at $z \sim 1$.

In a bid to assign a statistical significance to SN observations, we note that fits of higher redshift mock ΛCDM data lead to distributions with non-Gaussian tails towards larger Ω_m and smaller H_0 values. We explain this feature as an inherent bias in flat ΛCDM, which makes it more likely that early Universe determinations of H_0 and S_8 are smaller and larger, respectively, than late Universe counterparts. Interestingly, strong lensing time delay also reports a descending trend in H_0 with lens redshift [27, 28], prompting Ref. [29] to investi-

gate the same trend in other cosmological probes. We also encounter some intriguing trends in Ω_m with BAO observations, which we present in supplemental material. In some sense, ΛCDM may be a smart model that predicts its own demise, including a decreasing H_0 with redshift (see Refs. [30, 31] for related comments). Given the universal confidence in SN cosmology, the outlined trends can be confirmed or refuted by simply increasing the number of high redshift ($z \sim 1$) SN. This is expected to happen soon [32].

QSOS

Standardizable QSOs represent a game changer for cosmology. They are plentiful, in contrast to gamma-ray bursts (GRBs) [33–39], but like GRBs, promise to open up the redshift range beyond SN. Based on an empirical relation between X-ray and UV QSO fluxes, Risaliti and Lusso have proposed a relation intrinsic to QSO luminosities for cosmological purposes (1) (see [16–19] for other methods). The constant γ is directly inherited from the flux relation through the standard luminosity-flux relation, $L = 4\pi D_L(z)^2 F$, where $D_L(z)$ denotes the luminosity distance. The robustness of $\gamma \approx 0.6$ to redshift evolution has been demonstrated over both orders of magnitude in luminosity and extended redshifts [11–15].

However, in contrast to SN, there is considerable intrinsic scatter in QSO fluxes. As with SN in the 1990s [40], before corrections for color, shape and host galaxy mass [41, 42] (however, see [43–46] for ongoing debate), this scatter necessitates an additional intrinsic dispersion parameter δ. Given corrections made to SN since the 1990s, it is worth bearing in mind that (1) is a working proposal and future corrections may be necessary, especially in light of criticisms [47–49]. Moreover, one cannot rule out the possibility going forward that better data selection criteria could also reduce the scatter. Nevertheless, we adopt the Risaliti-Lusso relation (1) and obtain best-fit parameters by marginalizing or maximizing the likelihood function [8, 9],

$$\mathcal{L} = -\frac{1}{2} \sum_{i=1}^{N} \left[\frac{\left(\log_{10} F_{X,i}^{\text{obs}} - \log_{10} F_{X,i}^{\text{model}}\right)^2}{s_i^2} + \ln(2\pi s_i^2) \right], \quad (2)$$

where the cosmological model enters through the flux relation that follows from (1):

$$\log_{10} F_X = \beta + \gamma \log_{10} F_{UV} + (\gamma - 1) \log_{10}(4\pi D_L^2). \quad (3)$$

Here, $s_i^2 = \sigma_i^2 + \delta^2$ in (2) contains the measurement error on the observed flux $\log_{10} F_{X,i}^{\text{obs}}$. The F_{UV} errors are ignored [8]. Note that F_X and F_{UV} errors are considerably smaller than δ.

While it is customary in the literature to calibrate QSOs with SN to identify β [8, 9], this risks hiding physics that is intrinsic to QSOs, since QSOs simply track SN, so here we work with uncalibrated QSOs and flat ΛCDM with nuisance parameters (β, γ, δ). Since β is degenerate with H_0, one cannot determine both, so we fix $H_0 = 70$ km/s/Mpc. Our first goal

is to restrict the maximum redshift $z_{\max}$ of the latest sample of 2421 QSOs [50] in order to demonstrate that QSOs at lower redshifts, where SN are numerous, inhabit a Planck-ΛCDM Universe with $\Omega_m \approx 0.3$. In Fig. 1, we confirm that matter density is peaked close to the Planck value $\Omega_m \approx 0.3$ when $z_{\max} = 0.7$ and there are 398 QSOs in the range. Therefore, in the redshifts where they overlap well, both SN and QSOs agree on DE, in contrast to findings [20, 21] over extended redshift ranges. Note, in contrast to Refs. [8, 9], here the QSOs are uncalibrated, so they recover DE without guidance from SN. This is easy to take for granted, but it is a valid consistency test for the QSOs.
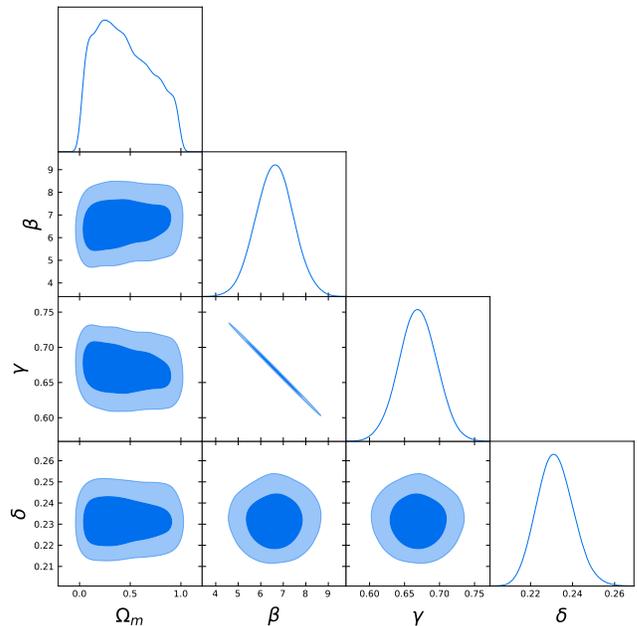

FIG. 1: Marginalized parameters for the QSO sample [50] with cutoff $z_{\max} = 0.7$.

$z_{\max}$	Ω_m	β	γ
0.7 (398 QSOs)	0.266	6.601	0.670
	$0.411^{+0.342}_{-0.259}$	$6.620^{+0.814}_{-0.841}$	$0.669^{+0.027}_{-0.027}$
0.8 (543 QSOs)	0.418	7.162	0.652
	$0.511^{+0.305}_{-0.275}$	$7.162^{+0.715}_{-0.712}$	$0.651^{+0.023}_{-0.023}$
0.9 (678 QSOs)	0.592	7.736	0.633
	$0.601^{+0.248}_{-0.250}$	$7.709^{+0.662}_{-0.679}$	$0.633^{+0.022}_{-0.021}$
1 (826 QSOs)	0.953	7.921	0.626
	$0.717^{+0.184}_{-0.231}$	$7.792^{+0.571}_{-0.571}$	$0.631^{+0.019}_{-0.019}$

TABLE I: Best fit and marginalized inferences of (Ω_m, β, γ) for QSOs below a maximum redshift $z_{\max}$.

Now comes a remarkable observation. Namely, as the maximum redshift ticks up towards $z = 1$, the best and marginalized values of Ω_m also increase towards $\Omega_m \approx 1$ in the flat

ΛCDM model. This can be seen from Table I, where we have omitted δ as it is consistent with $\delta \sim 0.23$ throughout. Since we have imposed the flat prior $0 < \Omega_m < 1$, our marginalized results are impacted by the bounds, but we have checked that the best-fit values for Ω_m agree with the peaks of the Ω_m distribution. It should be noted that we have made use of few inputs, merely that (1) holds and we marginalize or maximize the likelihood (2) following the Risaliti-Lusso prescription [8, 9]. Nevertheless, we recover a Planck-ΛCDM Universe, where it is expected, in more or less the same redshift range as SN, however QSOs transition to an EdS Universe ($\Omega_m = 1$) with larger z_{max}. Concretely, at $z_{max} \approx 1.3$, the QSOs inhabit an EdS Universe. Throughout, we find that $\gamma \gtrsim 0.6$ even as z_{max} is increased beyond $z_{max} \approx 1.3$.

Note, as explained in [50], there is concern that some of the UV fluxes have been extrapolated from the optical below $z = 0.7$, however, as we have seen, QSOs still recover DE. Moreover, as is evident from Table I, there is evolution in (β, γ) as the redshift range is extended. One could seize upon this fact and immediately jump to the conclusion that QSOs are not standardizable, but there is a kicker; SN show the same evolution in the central value of Ω_m. Moreover, as we will argue later, evolution in Ω_m with redshift may be fundamental to the flat ΛCDM Hubble diagram, and the remaining parameters simply compensate. Thus, if Ω_m evolves, so too must β or γ (cf. comments in [47–49]). We will see the same with SN, where H_0 compensates evolution in Ω_m. In supplemental material, we discuss the robustness of the QSO results to subsample restrictions.

Finally, our analysis here can be contrasted with the methodology in Ref. [10], where a fiducial cosmology and corresponding Hubble diagram are assumed, while the luminosities are corrected for redshift evolution. Here we are conversely interested in extracting the cosmology, in particular Ω_m, so the results in Table I assume the Risaliti-Lusso relation (1). For this reason, some differences in the values of (β, γ) are expected, especially here since (β, γ) compensate for evolution in Ω_m, as explained above.

PANTHEON SN

We now switch gears to Pantheon SN [22], where it is already documented that H_0 descends [23, 25] and Ω_m increases with redshift binning [24] (see their Fig. 6). Here we simply confirm these results by imposing a low redshift cutoff z_{min}, which allows us to decouple SN below a given redshift. For concreteness, we fix the absolute magnitude to $M_B = -19.35$, which is consistent with a nominal $H_0 \approx 70$ km/s/Mpc value, while fitting H_0 and Ω_m within the flat ΛCDM model in intervals of $\Delta z = 0.05$ in the redshift range $0.1 \leq z \leq 1$. We show the results of this exercise in Fig. 2, where we include 1σ confidence intervals and interpolate between the values of cosmological parameters using a cubic spline. Note, our analysis includes both statistical and systematic uncertainties through the full Pantheon covariance matrix, which we crop
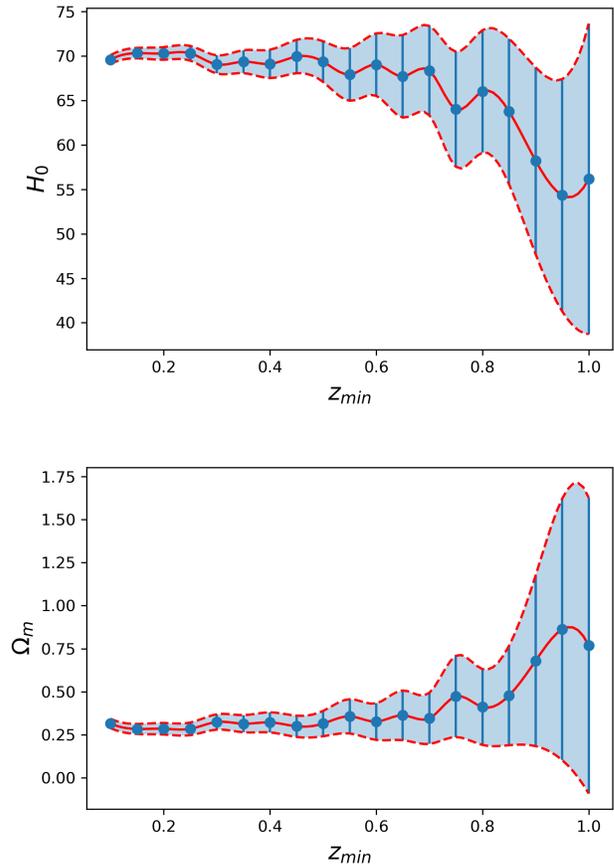
appropriately when we remove SN.

FIG. 2: Variations of best fit cosmological parameters (H_0, Ω_m) as low redshift SN below z_{min} are removed. The central values of H_0 and Ω_m favour lower and higher values at higher redshifts, respectively. The error bars denote the variance in fitted Ω_m values at each z_{min} taken before performing a cubic interpolation..

While the result is expected [23–25], it is interesting to note that Pantheon+ shows a similar Ω_m trend with z_{min} through to $z_{min} = 0.3$ [51] (see their Fig. 16). The descending H_0 trend is also reminiscent of similar trends in strong lensing time delay with statistical significance 1.9σ [27] and 1.7σ [28], respectively.[1] We performed approximately 2500 simulations of mock data based on Planck values, where we kept track of the sum of the discrepancy with Planck [3] in Ω_m evaluated at each z_{min} we sampled,

$$\sigma := \sum_{z_{min}} (\Omega_m^{z_{min}} - \Omega_m^{Planck}). \tag{4}$$

One could define an analogous sum for H_0, but since (H_0, Ω_m) are anti-correlated, it suffices to focus on one parameter. For

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[1] In strong lensing time delay, one is less sensitive to Ω_m.

the real data, this sum is positive, $\sigma = 2.14$, as is evident from Fig. 2. We present the simulations in Fig. 3, where we find that larger positive sums arise by chance with probability $p = 0.16$ ($\sim 1\sigma$), which is consistent with the 1σ deviation from Planck-ΛCDM evident with real data in Fig. 2.

Interestingly, we find that the median and 1σ confidence intervals are all shifted to larger σ values. In particular, we find that the median is $\sigma = 0.19$, while the 1σ confidence interval is $-1.06 < \sigma < 2.16$. We will argue in the next section that this is an intrinsic feature of the flat ΛCDM model, which arises at higher redshifts. However, here it is not clear how much of this effect is attributable to observations and how much to the ΛCDM model. Either way, there is a problem. That point aside, the goal here is simply to point out that SN are expected to follow QSOs, if QSOs are *bona fide* standardizable candles.
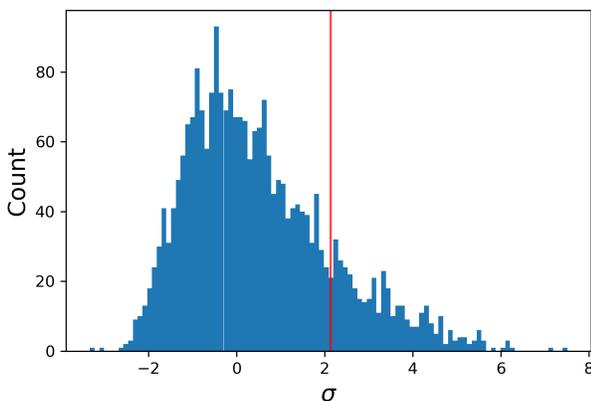

FIG. 3: ~ 2500 mock realisations of the Pantheon SN sample and the corresponding sum (4). The value corresponding to real data is denoted by the red line.

ΛCDM DIGRESSION

As is evident from Fig. 3, the sum distribution is not Gaussian and has developed some non-Gaussian tails. Here we will argue that these tails are a generic feature of the flat ΛCDM model that arise at higher redshifts. See [52] where these ideas are further developed. To begin, recall the flat ΛCDM model:

$$H(z)^2 = H_0^2(1 - \Omega_m) + H_0^2 \Omega_m (1 + z)^3. \tag{5}$$

Here, the Hubble constant H_0 is an integration constant from the perspective of the Friedmann equations, while Ω_m is the matter density today. The latter is bounded in a physical regime, $0 < \Omega_m \leq 1$; $\Omega_m = 0$ is ruled out by the mere fact that $H(z)$ is not a constant and $\Omega_m = 1$ corresponds to the EdS Universe.

At low redshifts, $z \ll 1$, expanding (5), one has $H(z) = H_0\left(1 + \frac{3}{2}\Omega_m z + O(z^2)\right)$. Thus low redshift data first constrains

H_0 and then Ω_m, which is subleading in $z < 1$. Within the prevailing Planck-ΛCDM Universe [3], one expects $\Omega_m \approx 0.3$. However, as is clear from (5), the high redshift behaviour of the Hubble parameter is $H(z) \sim H_0 \sqrt{\Omega_m}(1 + z)^{\frac{3}{2}}$, which only depends on a single parameter $H_0^2 \Omega_m$. Thus, high redshift observational data ensures that H_0 and Ω_m are anti-correlated; as H_0 increases, Ω_m decreases, and *vice versa*. Observe that neglecting galaxy BAO, the anti-correlation between H_0 and Ω_m is pretty generic [53] (see their Fig. 1). Note, we have dropped the $(1 - \Omega_m)$ term as despite being relevant at lower redshifts, it becomes less relevant at higher redshifts. As explained in [52], there is an inevitable spreading in the $H_0^2(1 - \Omega_m)$ distribution of best fit values within the flat ΛCDM model in high redshift bins, which pushes best fit Ω_m values away from the Planck value $\Omega_m \sim 0.3$ and towards the boundary $\Omega_m = 1$. This is a direct consequence of the irrelevance of DE density at higher redshifts.

The pertinent question now is, how strong is this bias and when does it become a concern? In particular, could it explain the effect that we see in Fig. 2? Once again, we turn to SN mocks, but now instead of summing, we simply work with the full sample of 1048 SN and and a subsample of 124 SN above $z = 0.7$. The effective redshifts are $z_{\text{eff}} \approx 0.28$ and $z_{\text{eff}} \approx 0.9$, respectively, where we have weighted by the uncertainty in apparent magnitude m_B. Here we have chosen values that lead to an exaggerated effect, but for values of z_{eff} in between, one still notices some effect. For both the full sample and subsample, we mock up SN data with canonical values $H_0 = 70$ km/s/Mpc and $\Omega_m = 0.3$. In total, we produce 2000 mock realisations of the data and fit the flat ΛCDM model back to each mock and record the best-fit values of the cosmological parameters. As can be seen from Fig. 4, the distribution of best-fit values of Ω_m develops a long tail for larger Ω_m values at higher redshifts. Although we omit the plot, it is a given that the H_0 distribution shows a similar tail towards smaller values of H_0 (see [52]). That being said, we have checked that both the mean and median are consistent with the input values for H_0 and Ω_m, which simply underscores that one is analyzing mock data. The real story here is the high redshift tails.

It is an easy deduction to see that the non-Gaussian distribution in our sum in Fig. 3 is coming from the higher redshift contributions to the sum. As teased out in [52], the non-Gaussian tails at higher redshifts arise from the spreading of $H_0^2(1 - \Omega_m)$ distribution of best fits until one encounters the boundary at $\Omega_m = 1$. This boundary precludes negative DE densities in the flat ΛCDM model. Thus, as one bins data by redshift and confronts with the flat ΛCDM model, non-Gaussian tails in the direction of larger Ω_m and lower H_0 values arise. This feature, which is evident in mocks, and therefore inherent to the flat ΛCDM model, suggests that observations of decreasing H_0 values [23–25, 27–29] with redshift are physical and can be expected within flat ΛCDM. Note, our analysis here has been model dependent within ΛCDM, but there are diagnostics allowing one to track trends model independently [30, 54].
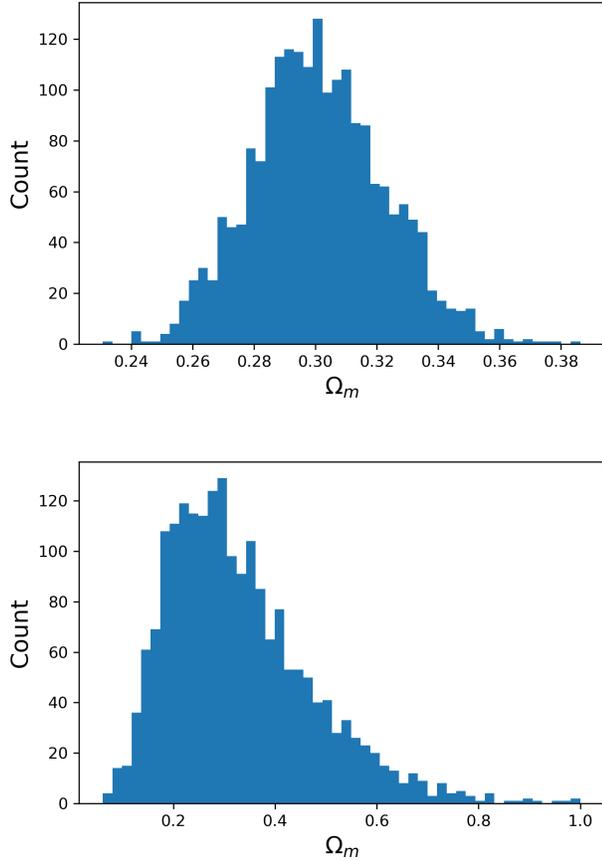
FIG. 4: The distribution of best-fit Ω_m for 2000 mocks of the full Pantheon SN sample with redshifts $0 < z \leq 2.26$ (above) and SN subsample with redshifts $0.7 < z \leq 2.26$ (below). The Ω_m distribution becomes non-Gaussian at higher redshifts.

DISCUSSION

Although we have glossed over a host of interesting details, let us revisit the facts. Risaliti & Lusso have a proposal for standardizable QSOs [8, 9], based on the relation (1), which one can argue is intrinsic to QSOs [10]. In turn, QSOs recover the Planck-ΛCDM Universe at lower redshits $z \lesssim 0.7$, in line with the expectations of SN cosmology. Nevertheless, SN and QSOs are very different beasts and while SN are weighted towards low reshifts, $z_{\text{eff}} \approx 0$, the Risaliti-Lusso QSO samples are more numerous at higher redshift, $z_{\text{eff}} \approx 1$. We have demonstrated that within the Risaliti-Lusso assumptions QSOs transition from a Planck-ΛCDM Universe to an EdS Universe as one increases the redshift range.

One could write off this behaviour simply on the grounds that QSOs are not standardizable, but what then if SN show similar trends? As we have shown, there is an increasing Ω_m, decreasing H_0 trend, in Pantheon SN [32] as the low redshift SN anchoring the sample in the DE dominated regime are decoupled (see also [23–25]). Note, while the QSOs become

more numerous at higher redshifts, the SN become less numerous, and statistics currently prevent a definitive statement. This will change in coming years and WFIRST [55, 56] is expected to lead to $\times 1000$ improvement in $z > 1$ SN statistics. This will allow us to confirm if both QSOs and SN are following the same trend. It is worth stressing that any evolution in H_0 within SN is equivalent to evolution in absolute magnitude M_B, so if confirmed, it represents a stark choice between SN cosmology and the flat ΛCDM model.

Finally, we have explained why this trend to be expected in the flat ΛCDM model. The ideas are further developed in [52]. In short, there is no guarantee that Ω_m is not increasing and H_0 is not decreasing at higher redshifts as one bins the data. Indeed, it is possible that our real SN sample in Fig. 2 is somewhere in the tails of Fig. 4. The non-Gaussian tail highlights the ease at which one could perform an experiment and get higher values of Ω_m and lower values of H_0. Ultimately, this suggests that documented trends in H_0 [23–25, 27–29] in the literature may be physical. Moreover, as we discuss in supplemental material, an increasing Ω_m with z_{eff} may be supported by BAO observations [62–64], but this requires further investigation.

Observe that this also gives a new perspective on cosmological or ΛCDM tensions. All things being equal, one is more likely to find that H_0 is lower at higher redshifts, thereby seemingly explaining why early Universe determinations of H_0 (and Ω_m) are indeed smaller (and larger) when one interprets the physical Universe through the ΛCDM model (see [57] for related comments). Moreover, as Ω_m increases, so too does S_8 within the flat ΛCDM model (see Fig. 1 of [58]). Once again, this trend could explain why Planck measures larger values of S_8.

Going further, there is a lensing anomaly in the CMB and it is well documented that one infers a lower H_0 and higher Ω_m from higher multipoles [59]. Could this too be explained as some artifact of viewing CMB through the prism of flat ΛCDM? In addition, could any preference in data sets for interacting DE models [60, 61] be explained by this trend? Regardless, there is an inherent bias in the flat ΛCDM model, as the non-Gaussian tails in SN mocks demonstrate, and whether larger Ω_m values come from this bias or the physical data is less relevant. Evidently, SN (and perhaps BAO) have the potential to shore up Risaliti-Lusso QSOs [8, 9] as standardizable candles while ruling out the Planck-ΛCDM Universe. On the flip side, if Ω_m does not increase with redshift in SN and BAO, then the intrinsic scatter in QSOs is presumably problematic. Attention must then focus on reducing the scatter or turning to other approaches for standardizable QSOs [16–19].

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