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Determining the Hubble Constant without the Sound Horizon:
A 3.6% Constraint on H_0 from Galaxy Surveys, CMB Lensing and Supernovae

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Many theoretical resolutions to the so-called "Hubble tension" rely on modifying the sound horizon at recombination, r_s, and thus the acoustic scale used as a standard ruler in the cosmic microwave background (CMB) and large scale structure (LSS) datasets. As shown in a number of recent works, these observables can also be used to compute r_s-independent constraints on H_0 by making use of the horizon scale at matter-radiation equality, $k_{\rm eq}$, which has different sensitivity to high redshift physics than r_s. As such, r_s- and $k_{\rm eq}$-based measurements of H_0 (within a ΛCDM framework) may differ if there is new physics present pre-recombination. In this work, we present the tightest constraints on the latter from current data, finding $H_0 = 64.8^{+2.2}_{-2.5}\,\mathrm{km\,s^{-1}Mpc^{-1}}$ at 68% CL from a combination of BOSS galaxy power spectra, *Planck* CMB lensing, and the newly released Pantheon+ supernova constraints, as well as physical priors on the baryon density, neutrino mass, and spectral index. The BOSS and *Planck* measurements have different degeneracy directions, leading to the improved combined constraints, with a bound of $H_0 = 67.1^{+2.5}_{-2.9}$ ($63.6^{+2.9}_{-3.6}$) from BOSS (*Planck*) alone in $\mathrm{km\,s^{-1}Mpc^{-1}}$ units. The results show some dependence on the neutrino mass bounds, with the constraint broadening to $H_0 = 68.0^{+2.9}_{-3.2}\,\mathrm{km\,s^{-1}Mpc^{-1}}$ if we instead impose a weak prior on $\sum m_\nu$ from terrestrial experiments rather than assuming $\sum m_\nu < 0.26\,\mathrm{eV}$, or shifting to $H_0 = 64.6 \pm 2.4\,\mathrm{km\,s^{-1}Mpc^{-1}}$ if the neutrino mass is fixed to its minimal value. Even without dependence on the sound horizon, our results are in $\approx 3\sigma$ tension with those obtained from the Cepheid-calibrated distance ladder, which begins to cause problems for new physics models that vary H_0 by changing acoustic physics or the expansion history immediately prior to recombination.

For better or for worse, the "Hubble tension" has become one of the key research areas of twenty-first century cosmology. The problem is straightforward to define: the expansion rate, H_0, measured from the local distance ladder calibrated from geometric distances to Cepheid variables stars and type Ia supernovae [e.g., 1], is not in agreement with that extracted from the cosmic microwave background (CMB) assuming the standard cosmological model ($\nu\Lambda$CDM) [2, 3]. These measurements depend on different physics: the former relies on the late-time expansion history and certain astrophysical assumptions, whilst the latter is primarily sourced by the baryon acoustic oscillation (BAO) feature in the CMB power spectrum, which depends on the "sound-horizon" scale, r_s. Despite a wealth of effort, both theoretical and experimental, the problem persists, and, moreover, has become a multidimensional one, with the introduction of a number of new data-sets. These allow alternative probes of the expansion rate, and proceed via a wide number of mechanisms, such as alternative calibration of the distance ladder using tip of the red giant branch (TRGB) methods [e.g., 4–8] or megamasers [9], gravitational wave observations [e.g., 10] and time delay cosmography from strongly lensed sources [e.g., 11, 12]. For a brief time, H_0 constraints appeared to fall in one of two camps: measurements depending on the full $\nu\Lambda$CDM model preferred lower H_0, whilst those depending only on local physics tended towards higher values [e.g., 13]. However, this division has become much less clear (at least in the latter category) with the publication of new TRGB and strong lensing results [4, 6–8, 12].

A particularly interesting probe of the expansion rate is that of large scale structure. In the last decade this has generally been analyzed by way of the BAO feature, extracted from the oscillatory part of the galaxy power spectrum [e.g., 14–19]. In combination with external information on the Universe's composition (usually constraints on $\omega_{\rm cdm}$ and ω_b from the CMB), this feature can be used as a standard ruler to constrain H_0, and results in values consistent with those of *Planck* and more recent CMB experiments.

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It has long been known that the matter power spectrum contains information beyond the BAO feature and hence beyond the sound horizon scale. Indeed, a second key physical scale leaves a characteristic imprint on the shape of the matter power spectrum and contains a significant amount of cosmological information: the wavenumber corresponding to the horizon size at matter-radiation equality ($z \approx 3500$), $k_{\rm eq}$. This "equality scale" not only sets the scale of the peak of the matter power spectrum but is important for determining the broadband shape of the linear power spectrum at $k \gtrsim k_{\rm eq}$.[1]

Since many models hoping to resolve the "H_0 tension" proceed by modifying the sound horizon r_s (which sets the BAO scale) [e.g., 22, 23], extracting H_0 constraints by using the equality scale as a standard ruler instead of the sound horizon scale becomes highly desirable. However, it can be challenging to extract Hubble constant information only from $k_{\rm eq}$, because standard analyses of matter power spectrum observables are typically dominated by BAO constraints and hence by sound horizon information.

Several approaches have hence been developed to remove sound horizon information and thus measure the Hubble constant from only the equality scale (within a ΛCDM context). The first approach was to use the CMB lensing power spectrum [24]; since this observable is given by a projection of the matter power spectrum, the BAO oscillations average out such that only equality scale information remains. Later approaches [25, 26] improved upon these constraints with novel analyses of the full 3D galaxy power spectrum, building on recent advances in modeling and analyzing the galaxy power spectrum, beyond just its oscillatory component [e.g., 27–35]. Whilst constraints from standard full-shape analyses remain BAO-dominated, [26] successfully removed sound horizon information with a suitable choice of priors (omitting the baryonic information usually provided by Big Bang Nucleosynthesis (BBN) constraints [e.g., 15, 30, 36, 37]); subsequently, [25] proposed and validated a new method to "integrate out" the sound horizon even with BBN priors, resulting in the tightest equality scale H_0 constraints to date.

The existence of both $k_{\rm eq}$- and r_s-derived constraints on H_0 allows for interesting consistency checks: in particular, new physics occurring at redshifts of a few thousand will likely cause a discrepancy between the H_0 values inferred by the two ΛCDM analyses. As an example, [25] showed that, in the context of the Euclid spectroscopic survey, the early dark energy models preferred by *Planck* [38] and ACT data [39] (see also [40–43]), would induce shifts between the r_s- and $k_{\rm eq}$-derived values of $\Delta H_0 = 2.6\,{\rm km\,s^{-1}Mpc^{-1}}$ and $7.8\,{\rm km\,s^{-1}Mpc^{-1}}$ respectively (within a ΛCDM analysis framework), with neither measurement correctly reproducing the input value.[2] So far, the equality and sound horizon measurements are in agreement; however, it remains to be seen whether this holds true with the advent of higher precision data.

Motivated by the above, the goal of this work is to place the tightest indirect constraints on the expansion rate from large scale structure observables (galaxy clustering and CMB lensing) within $\nu\Lambda$CDM, but without dependence on the sound horizon. Although previous constraints have been presented in [25, 26], this work extends beyond the former in a number of ways: (a) we utilize the newest constraints on the matter density from PANTHEON+ [44], significantly reducing parameter degeneracies, (b) we include marginalization over the sound horizon following [25], both for the power spectrum alone and in combination with lensing (unlike [26]), (c) we add bounds on the neutrino mass following *Planck* and terrestrial experiments [2, 45], (d) we analyze the latest galaxy power spectra from BOSS, corrected for previous systematic errors, with an updated theoretical model. As shown below, this yields competitive r_s-independent constraints on H_0 and leads to an interesting cosmological interpretation.

I. DATASETS

We begin by discussing the datasets used in this work. Our analysis makes use of four sources of information:

- **CMB Lensing**: We use the *Planck* CMB-marginalized lensing likelihood discussed in [46, 47]. This constrains cosmological parameters via the integrated matter density over a broad redshift range from decoupling until today. As discussed in [24], this does not capture information from the sound horizon, due to the smoothing effects of projection integrals. This is implemented in MONTEPYTHON [48], using version R3.10 of the public CLIK likelihood.

- **Galaxy Power Spectra**: As shown in [25, 26], galaxy power spectra can be used to obtain an r_s-independent constraint on H_0 either by performing the analysis without a prior on the baryon density or by explicit marginalization over r_s within the likelihood, via a rescaling of the oscillatory component. Here, we adopt the latter strategy. We use the most up-to-date version of the BOSS DR12 galaxy survey dataset [14, 49–51], using the

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[1] In fact, the equality scale was used as the original source of cosmological information from galaxy surveys [e.g, 20, 21]).

[2] Although these results were obtained from a forecast using Euclid survey parameters, they are roughly independent of the experimental precision, and we expect them to have only weak dependence on the redshift-binning strategies, though they may be influenced by prior volume effects. To assess this, we have repeated the analysis of [25] for the *Planck* EDE model, finding $\Delta H_0 = 2.4\,{\rm km\,s^{-1}Mpc^{-1}}$, when using the BOSS experimental set-up and prior choices.

power spectrum measured in [29] via the window-free estimators of [52, 53]. We use BOSS data from two redshift bins (centered at $z = 0.38$ and $z = 0.61$) in two regions of the sky (NGC and SGC), analyzing the unreconstructed power spectrum multipoles (monopole, quadrupole, and hexadecapole) up to $k_{\max} = 0.2h^{-1}\mathrm{Mpc}$, as well as the real-space extension, Q_0, up to $k_{\max} = 0.4h^{-1}\mathrm{Mpc}$ [54]. Data are analyzed using publicly available likelihoods, which implement a theoretical model based on the Effective Field Theory of Large Scale Structure, and marginalize over all necessary nuisance parameters, in addition to the sound horizon.[3] Note that the galaxy dataset differs slightly from that used in influenced by [25, 26], and corrects a previous error in the normalization. The addition of the large-scale galaxy bispectrum [29, 55] was not found to appreciably improve the parameter constraints.

- **Supernovae (SNe)**: This work constrains H_0 by measuring the angular scale of the cosmological horizon at matter-radiation-equality, *i.e.* $k_{\mathrm{eq}}D_A(z) \propto k_{\mathrm{eq}}h$. Within ΛCDM, the equality scale is proportional to $\omega_{cb} \equiv \omega_{cdm} + \omega_b$ [56], thus our measurements are necessarily degenerate with the matter density, and can be improved by the addition of Ω_m priors. Here, we adopt the Gaussian prior $\Omega_m = 0.338 \pm 0.018$ from the recent PANTHEON+ analysis [44]. Notably, the central value is higher than that of the original PANTHEON constraint: 0.298 ± 0.022 [57], which will have implications for the H_0 constraints. The origin of this shift in Ω_m is discussed in [44, Sec. 5]. These constraints do not fix the supernova absolute magnitude calibration (such as from the local distance ladder), and are thus largely independent of the local distance ladder H_0. We note that, fixing the dark energy of state and ignoring any additional calibration data, the SNe sample primarily measures Ω_m, thus there is little to be gained by utilizing the full PANTHEON+ posterior rather than just an Ω_m prior.

- **Big Bang Nucleosynthesis (BBN)**: To maximize the information that can be extracted from the equality scale, we impose a prior on the physical baryon density $\omega_b = 0.02268 \pm 0.00036$ following [30]. As shown in [25], this does not add sound-horizon-dependence (due to our r_s-marginalization), and is additionally not reliant on *Planck*. In the absence of a BBN prior, there is a degeneracy between the equality-based H_0 measurements and ω_b [25]; however, the dependence of k_{eq} on ω_b is comparatively shallow, thus for this degeneracy to appreciably affect H_0, we would require a large ($\gg 5\sigma$) change in ω_b. This is strongly disfavored given the consistency of BBN and *Planck* ω_b constraints.

H_0 constraints may be tightened by imposing additional physically-motivated parameter constraints, allowing the breaking of important degeneracies. In this work, we utilize the following set of priors (which are carefully chosen so as not to fold in information from the sound horizon):

- **Neutrino Mass**: Equality-based measurements of H_0 from CMB lensing are degenerate with the neutrino mass, $\sum m_\nu$ (though those from galaxy analyses are less so). Analysis of the *Planck* dataset excluding lensing information found $\sum m_\nu < 0.26\,\mathrm{meV}$ at 95% CL [2]: this motivates the flat prior $m_\nu \in [0, 0.26]\,\mathrm{eV}$. We additionally consider a weaker prior $m_\nu \in [0, 0.52]\,\mathrm{eV}$ (roughly at the *Planck* 4σ level), as well as the pessimistic but uninformative choice $m_\nu \in [0, 1]\,\mathrm{eV}$. CMB neutrino mass constraints at sub-eV levels primarily arise from lensing effects in the power spectrum, in particular the smoothing of the power spectrum peaks. Since the degree of smoothing of the power spectrum peaks is physically distinct from the peak location, we expect very broad, conservative priors on the neutrino mass (at the several σ level) to have only a negligible dependence on the precise value of r_s. An alternative choice is to obtain the priors from non-CMB sources, for example terrestrial experiments. Recent results from the KATRIN analysis give $\tilde{m}_\nu^2 \lesssim 0.73\,\mathrm{eV}^2$ at 90% CL in a Bayesian context for the effective electron anti-neutrino mass $\tilde{m}_\nu$ [45]. Below, we use the KATRIN posterior as a prior on $\sum m_\nu$, showing this to yield similar results to the *Planck*-derived priors above. Finally, we will also consider fixing the neutrino mass to its minimal $\nu\Lambda$CDM value of $0.06\,\mathrm{eV}$ (following a number of previous analyses). In all cases, we assume three degenerate massive neutrinos.

- **Spectral Index**: Following the *Planck* lensing-only analyses [47], we adopt a weak Gaussian prior of $n_s = 0.96 \pm 0.02$ on the primordial spectral slope. This is measured from a comparison of the amplitude of large-scale CMB modes with small-scale ones; since the overall tilt of the spectrum is not expected to be significantly degenerate with the position of the acoustic oscillation features (set by r_s), the value of n_s is also not expected to be strongly influenced by the sound horizon. Our constraint is also $\approx 3\times$ weaker than that of the *Planck* primary analysis [2] and may thus be considered conservative. Furthermore, it is well motivated by theoretical considerations, since slow-roll inflation requires n_s slightly below unity. To test the impact of this, we also perform analyses without the n_s prior, and with the n_s prior replaced by a weak 8% prior on the primordial amplitude A_s, following [24]. Consideration of the *Planck* posterior shows the latter to be r_s-independent [2].

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[3] Available at github.com/oliverphilcox/full_shape_likelihoods.

TABLE I. H_0 constraints from the analysis of *Planck* lensing and the BOSS DR12 galaxy power spectra, as shown in Fig. 1. Here, we show the results from various data-set combinations, with and without a prior on Ω_m. In all cases, we assume BBN priors on ω_b, weak priors on n_s and a flat prior on the neutrino mass sum with $\sum m_\nu < 0.26\,\mathrm{eV}$. The H_0 posterior from *Planck* lensing without PANTHEON+ is unconstraining. The bold entry is the main primary result of this work, and all values are quoted in $\mathrm{km\,s^{-1}Mpc^{-1}}$ units at 68% CL.

	Fiducial	Without PANTHEON+
Planck Lensing	$63.6^{+2.9}_{-3.6}$	71^{+20}_{-20}
BOSS Galaxies	$67.1^{+2.5}_{-2.9}$	$69.6^{+4.1}_{-5.4}$
Planck Lensing & BOSS Galaxies	$\mathbf{64.8^{+2.2}_{-2.5}}$	$65.0^{+3.9}_{-4.3}$

- **Sound Horizon Rescaling**: The r_s-marginalization procedure discussed in [25] integrates over a rescaling of the sound horizon using a free parameter α_{r_s}. This naturally requires a prior: here, we assume a Gaussian prior of 1.0 ± 0.5 (with $\alpha_{r_s} = 1$ giving no rescaling). As can be seen from the previous work [25], this is not informative.

In this work, parameter constraints are derived by sampling a multivariate likelihood via Markov Chain Monte Carlo (MCMC) methods. This is implemented in MONTEPYTHON [58, 59], and we sample over the following set of cosmological parameters:

$$\{H_0, \omega_\mathrm{b}, \omega_\mathrm{cdm}, \log 10^{10} A_s, n_s, \sum m_\nu\}. \tag{1}$$

We additionally fix the optical depth of reionization, τ_reio, to 0.055, following the *Planck* lensing analyses [47].[4] To account for various galaxy formation and non-linear effects, the BOSS likelihood also includes a number of nuisance parameters for each subsample; discussion of these can be found in [29] and they are marginalized over in all cases. For each analysis, we run a number of MCMC chains in parallel, assuming them to converge when the Gelman-Rubin diagnostic satisfies $|R-1| < 0.05$. Finally, we note that all analyses are performed using a ΛCDM theory model, to facilitate robust null tests.

II. RESULTS

Fig. 1 and Tab. I show the main results of our analysis: sound-horizon-independent constraints on H_0 from CMB lensing and galaxy power spectra, supplemented by priors on Ω_m from PANTHEON+ and ω_b from BBN. In all cases, the upper limit of $\sum m_\nu = 0.26\,\mathrm{eV}$ (the *Planck* 2σ limit) is assumed. Considering first the CMB lensing results, we find the constraint $H_0 = 63.6^{+2.9}_{-3.6}\,\mathrm{km\,s^{-1}Mpc^{-1}}$ at 68% CL, or, using the approximate ΛCDM relation between k_eq and ω_m/h given in [56], $k_\mathrm{eq} = (1.60^{+0.07}_{-0.08}) \times 10^{-2} h\,\mathrm{Mpc}^{-1}$. If the PANTHEON+ dataset is removed, the constraining power reduces to almost zero, with the figure showing a strong $\Omega_m - H_0$ degeneracy, as expected from a k_eq-based measurement. This result is markedly different to that quoted in [24]: $73.5 \pm 5.3\,\mathrm{km\,s^{-1}Mpc^{-1}}$, both with a lower expansion rate and a significantly tighter errorbar. This arises from two factors: (a) we assume a tighter prior on the neutrino mass that promotes smaller H_0 and reduces the posterior width (see below), and (b) the previous study used Ω_m priors from PANTHEON, rather than PANTHEON+. As remarked above, the updated supernovae measurements yield a $\approx 1.5\sigma$ higher mean value of Ω_m and a 20% tighter errorbar (which is more consistent with other recent measurements, such as those of DES Y3 [60]). Due to the negative $\Omega_m - H_0$ correlation seen in Fig. 1, shifts the result to smaller H_0.[5] Although the posterior is somewhat non-Gaussian, this CMB+SNe measurement is in some tension with the most recent Cepheid H_0 results of [1].

The r_s-marginalized BOSS power spectra constrain H_0 to $67.1^{+2.5}_{-2.9}\,\mathrm{km\,s^{-1}Mpc^{-1}}$ and $k_\mathrm{eq} = (1.66 \pm 0.05) \times 10^{-2} h\,\mathrm{Mpc}^{-1}$ with a factor of ≈ 2 degradation in H_0 when the PANTHEON+ prior is removed. Note that LSS power spectra can measure Ω_m internally (via Alcock-Paczynski distortions, [61]), thus the dataset still retains some constraining power. Previous work found $69.5^{+3.0}_{-3.5}\,\mathrm{km\,s^{-1}Mpc^{-1}}$ [25], which is somewhat weaker. In this case the improvements are due to the tighter Ω_m prior, as well as updates to the new BOSS likelihoods, which include more small-scale data and an improved treatment of the survey window function [29], as well as the addition of BBN information relative to [26]. We additionally note that the σ_8 posterior from BOSS is somewhat below that of the Planck

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[4] This is of minimal importance since we do not use the CMB primary anisotropies.

[5] At fixed k_eq, $\Omega_m H_0$ is constant, thus $\Delta H_0/H_0 \approx -\Delta\Omega_m/\Omega_m$. The shift in the central value of Ω_m moving from PANTHEON to PANTHEON+ is thus expected to induce changes in H_0 of approximately -12%, which is consistent with that found in our CMB lensing analyses. When spectroscopic data is included, the shift is expected to reduce, since galaxy power spectra also constrain Ω_m.

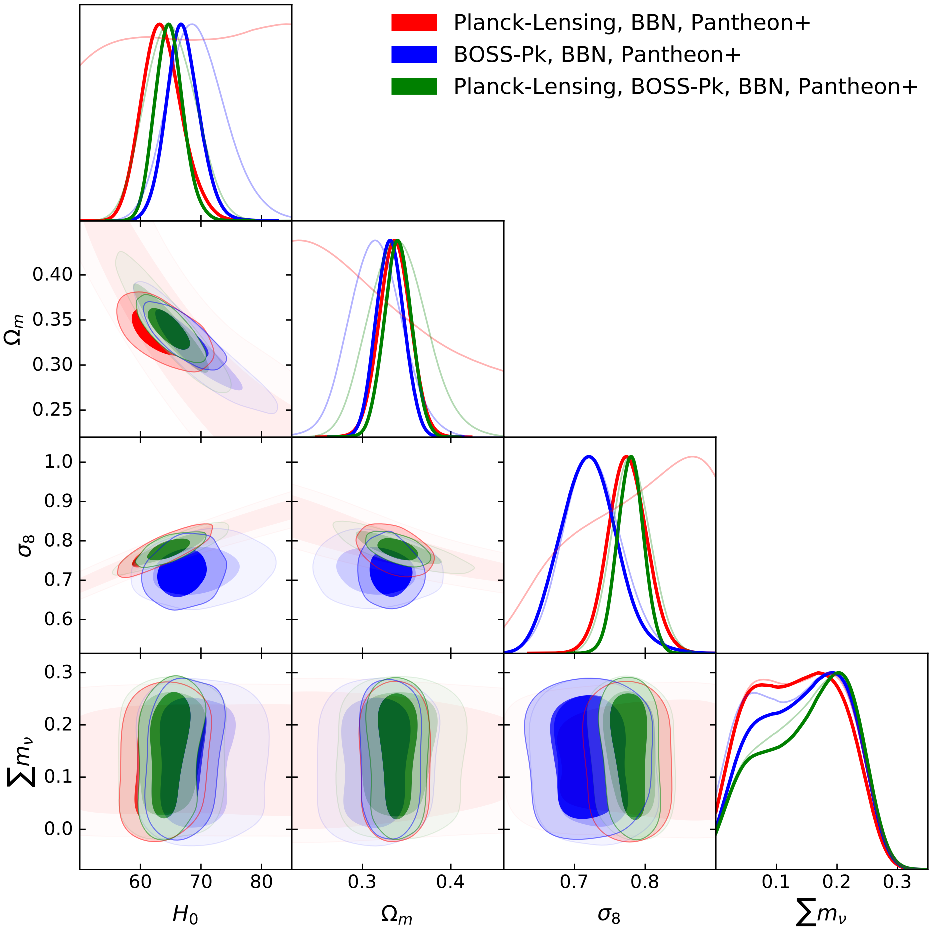


FIG. 1. Sound-horizon-independent constraints on the Hubble parameter from CMB lensing (red), galaxy surveys (blue) and their combination (green). All analyses include constraints on Ω_m from the PANTHEON+ sample, a prior on ω_b from BBN, a weak *Planck*-inspired prior on n_s, and the neutrino-mass constraint $\sum m_\nu < 0.26\,\mathrm{eV}$, matching the *Planck* 95% limit. The faint lines show the results without a PANTHEON+ prior on Ω_m; we find clear degradation, particularly for the lensing-only case, which is unable to constrain H_0. Assuming the approximate $k_{\rm eq} - \omega_m/h$ relation from [56], the three datasets constrain $k_{\rm eq} = 1.60^{+0.07}_{-0.08}$, 1.66 ± 0.05, 1.64 ± 0.05, respectively, in $10^{-2} h\,\mathrm{Mpc}^{-1}$ units. Corresponding H_0 values are given in Tab. I. The dark green constraint (combining *Planck*, BOSS and PANTHEON+) is the main result of this work.

lensing; this could be a manifestation of the claimed "S_8 tension", though the individual posteriors remain largely in agreement. Finally, we note that these constraints are only around a factor of $(2-3)\times$ weaker than those obtained *including* the sound horizon [29]. This highlights the utility of full-shape information, the importance of which will progressively grow in the next decade [25].

The final dataset in Fig. 1 is that obtained from the combination of CMB lensing and LSS power spectra. Here, we find a 1σ constraint of $H_0 = 64.8^{+2.2}_{-2.5}\,\mathrm{km\,s^{-1}Mpc^{-1}}$, or $k_{\rm eq} = (1.64 \pm 0.05) \times 10^{-2} h\,\mathrm{Mpc}^{-1}$. Naïvely, one might have expected a $\sqrt{2}$ improvement from combining two independent data-sets with similar H_0 posteriors; in practice, this does not occur since both results discussed above use the same set of priors. That said, the inclusion of *Planck* lensing tightens the BOSS posterior by $\approx 15\%$ (or $\approx 30\%$ in terms of survey volume), mostly driven by the somewhat different degeneracy directions of the two in Fig. 1. Although both datasets extract information from the same physical feature, the shape in the $\Omega_m - H_0$ plane is expected to differ (cf. the ω_m/h contours in Fig. 3a) due to the differences in the angular diameter distance scaling at high- and low-redshifts. We may compare this constraint to that of [26], which finds $70.6^{+3.7}_{-5.0}\,\mathrm{km\,s^{-1}Mpc^{-1}}$. Our results are significantly tighter due to the combination of the reasons above, or in short: better power spectrum modelling including r_s-marginalization, more ambitious neutrino mass priors, and new supernova data. The downwards shift in the posterior occurs due to the shift in Ω_m, which also serves to increase the datasets' compatibility, since the lensing analyses now use an Ω_m posterior consistent with that found from the galaxy power spectrum.

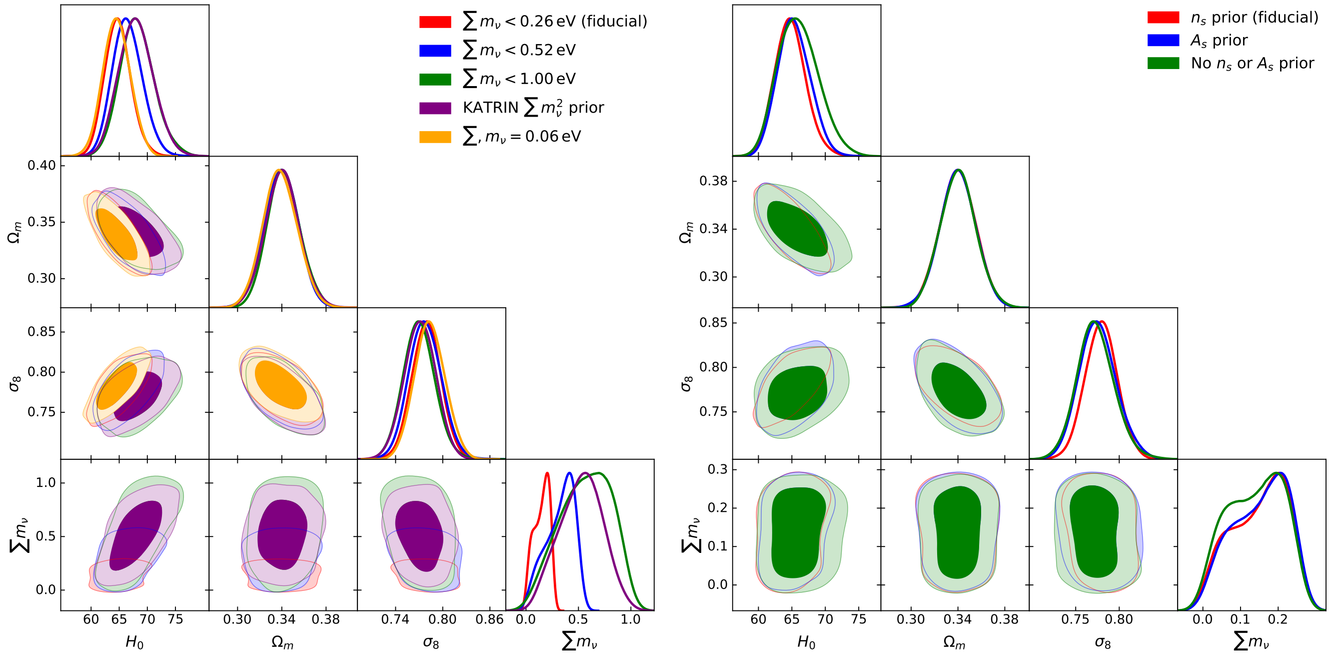


FIG. 2. **Left panel**: As Fig. 1, but examining the dependence on the neutrino mass priors. We show results for an upper bound of 0.26, 0.52 and 1.0 eV on the neutrino mass sum in red, blue, and green respectively, and include priors on Ω_m, ω_b and n_s in all cases. The other contours show analyses including a physical prior on $\sum m_\nu^2$ from ground-based experiments (purple) or fixing the neutrino mass sum to its minimal value, 0.06 eV. Corresponding H_0 constraints are given in Tab. II. **Right panel**: Dependence of the posterior on the spectral slope prior. We show results from the fiducial analysis (red, with $n_s = 0.96 \pm 0.02$), an analysis with an 8% prior on A_s (blue), and one with neither an n_s nor A_s prior (green). The three cases have $H_0 = 64.8^{+2.2}_{-2.5}, 65.3^{+2.3}_{-2.6}$ and $66.0^{+2.7}_{-3.4}\,\mathrm{km\,s^{-1}Mpc^{-1}}$ respectively.

TABLE II. H_0 constraints from the combination of *Planck* lensing and the BOSS DR12 galaxy power spectra (Fig. 2) with different choices of neutrino mass prior. We use a flat prior in all cases, except for the penultimate, which is an experimental prior on $\sum m_\nu^2$ from the ground-based KATRIN experiment, and the last, which fixes the neutrino mass sum to the minimal value of 0.06 eV. All results are given in $\mathrm{km\,s^{-1}Mpc^{-1}}$ units.

$\sum m_\nu < 0.26\,\mathrm{eV}$	$\sum m_\nu < 0.52\,\mathrm{eV}$	$\sum m_\nu < 1.00\,\mathrm{eV}$	KATRIN	$\sum m_\nu = 0.06\,\mathrm{eV}$
$\mathbf{64.8^{+2.2}_{-2.5}}$	$66.5^{+2.5}_{-2.8}$	$68.2^{+2.8}_{-3.2}$	$68.0^{+2.9}_{-3.2}$	64.6 ± 2.4

As noted above, our results have significant dependence on the neutrino mass prior. To explore this, we have re-run the joint analysis with two- and four-times wider neutrino mass priors. The corresponding results are displayed in the left panel of Fig. 2 and Tab. II and demonstrate a clear correlation between $\sum m_\nu$ and H_0 (and a slight preference for $\sum m_\nu > 0$ due to prior-volume effects). Increasing the width of the flat prior by a factor of four shifts H_0 upwards by $3.4\,\mathrm{km\,s^{-1}Mpc^{-1}}$, and increases the width by 30%. Some degeneracy is expected, since the effects of neutrino suppression are slightly degenerate with the turnover (and later decline) of the linear power spectrum at $k \gtrsim k_\mathrm{eq}$, and further, they degrade any ω_m information contained within internal ω_{cdm} constraints. As discussed in [26], this effect is significantly stronger for the CMB lensing scenario than for LSS power spectra, and reduces our constraining power. For the reasons described above, however, we consider the $\sum m_\nu < 0.26\,\mathrm{eV}$ prior to be conservative within $\nu\Lambda$CDM, and thus regard those measurements as robust. As an additional check, we adopt the CMB-independent prior from KATRIN. This yields $H_0 = 68.0^{+2.9}_{-3.2}\,\mathrm{km\,s^{-1}Mpc^{-1}}$ from the combination of datasets, with a similar errorbar to the $\sum m_\nu < 1\,\mathrm{eV}$ analysis. This result is expected, since the experiment yields an effective 2σ constraint on the squared mass sum of $0.73\,\mathrm{eV}^2$. To probe the opposite (and less conservative) limit, we consider fixing the neutrino mass sum to its minimal $\nu\Lambda$CDM value; $\sum m_\nu = 0.06\,\mathrm{eV}$. This is a common approximation for analyses with limited dependence on the neutrino mass [e.g., 2, 25, 29], and leads to the constraint $H_0 = 64.6 \pm 2.4\,\mathrm{km\,s^{-1}Mpc^{-1}}$. This is similar to the fiducial ($\sum m_\nu < 0.26\,\mathrm{eV}$) case and is primarily dominated by the galaxy survey data, which do not show a strong degeneracy with $\sum m_\nu$ [26].

Finally, we consider the impact of the prior on the spectral slope n_s. This is expected to improve the measurements

TABLE III. Forecasted uncertainties on H_0 from a Fisher forecast matching our joint analysis of *Planck* lensing and the BOSS DR12 galaxy power spectra. These should be compared to the full MCMC results given in Tab. I, recapitulated below in parentheses. We show the results from various data-set combinations, with and without the Pantheon+ constraints on Ω_m. Matching our fiducial analysis, we assume BBN priors on ω_b, and weak priors on n_s. The forecast also includes exact marginalization over the sound horizon via an Eisenstein-Hu (EH) model for the BOSS data. We note that we expect our constraints to be marginally tighter than those found in Tab. I since the EH model does not include the effect of massive neutrinos. All values are quoted in $\mathrm{km\,s^{-1}Mpc^{-1}}$ units at 68% CL.

	Fiducial	Without Pantheon+
Planck Lensing	± 2.8 ($^{+2.9}_{-3.6}$)	-
BOSS Galaxies	± 2.4 ($^{+2.5}_{-2.9}$)	± 3.9 ($^{+4.1}_{-5.4}$)
Planck Lensing & BOSS Galaxies	± 2.1 ($^{+2.2}_{-2.5}$)	± 3.2 ($^{+3.9}_{-4.3}$)

of $k_{\rm eq}$ by fixing the large-scale power spectrum shape; however, it is extracted from a joint fit to the *Planck* primary CMB spectra (but significantly inflated) and thus could, in principle, contain some r_s dependence. Rerunning the joint *Planck*, BOSS and Pantheon+ analysis without the n_s prior, we find $H_0 = 66.0^{+2.7}_{-3.4}\,\mathrm{km\,s^{-1}Mpc^{-1}}$, which is 30% broader than the fiducial constraint of $64.8^{+2.2}_{-2.5}\,\mathrm{km\,s^{-1}Mpc^{-1}}$, and a little higher, yet still fully consistent (at 0.6σ, using the approach of [62]). As in [24], we may also replace the n_s prior by a prior on the primordial amplitude A_s: using a weak 8% prior centered on the *Planck* best-fit (but encompassing both sets of values suggested by the "S_8 tension"), we find $H_0 = 65.3^{+2.3}_{-2.6}\,\mathrm{km\,s^{-1}Mpc^{-1}}$, in good agreement with the fiducial result, as shown in the right panel of Fig. 2. From these results, we conclude that our measurement of the expansion rate is robust to changes in the n_s prior, since these do not lead to sound-horizon-induced biases in H_0.

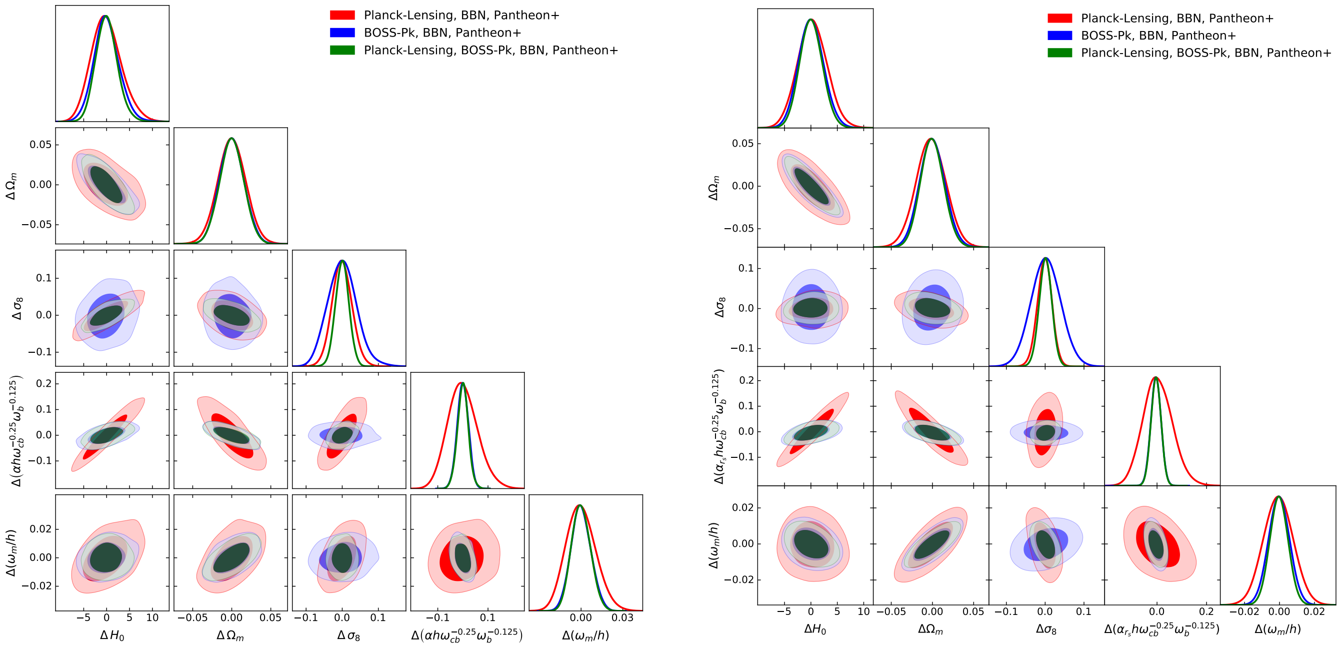
III. r_s INDEPENDENCE OF H_0 CONSTRAINTS

Previous works have demonstrated that, in the era of Euclid and DESI, analysis of the galaxy power spectra with explicit marginalisation over the sound horizon scale can yield r_s-independent constraints on H_0 even when information on the baryon density is included via a BBN-derived prior on ω_b [25]. The former work also argued that this independence should extend to an analysis of the BOSS dataset with an identical model, *i.e.* the scenario considered in this work. Nevertheless, we here demonstrate this explicitly, applying tests devised in [24–26] to the joint analysis of this work, incorporating the new datasets and prior choices.

In Fig. 3 we show a comparison between our fiducial MCMC analysis and a Fisher forecast conducted using an Eisenstein-Hu (EH) model for the linear power spectrum [56]. Within the EH model we are able to explicitly and exactly marginalize over the sound horizon as described in [25]; this stands in contrast to full Boltzmann computations of the linear power spectrum, in which r_s is an emergent quantity. Notably, we observe a very close match between the full results in Fig. 3a (which include our heuristic r_s-marginalization procedure, via a rescaling of the BAO feature) and the EH Fisher forecast with exact marginalization in Fig. 3b. To make this comparison, we use the restrictive prior on the neutrino mass ($\sum m_\nu < 0.26$ eV) for the full analysis since the EH model does not include the effect of neutrinos. Forecast uncertainties on H_0 for a range of different dataset combinations with and without the Pantheon+ prior on Ω_m are summarised in Tab. III which should be compared to Tab. I.

For the *Planck* lensing forecasts shown in Fig. 3, we do not include explicit marginalization over the sound horizon; initial testing with Fisher forecasts found this to have no impact on H_0, as predicted in [24]. This is additionally consistent with the poor constraints found on the parameter combination $h\omega_{cb}^{-0.25}\omega_b^{-0.125}$ (proportional to the sound horizon in h^{-1}Mpc-units) from lensing alone. For the BOSS analysis alone and that joint with *Planck*, this sound-horizon proxy is more well constrained, but we find it to have negligible degeneracy with H_0 (see leftmost panel in the second to last row in Fig. 3a), again illustrating r_s-independence.

We have also considered the importance of the r_s-dependent baryon suppression scale, which we do not marginalize over in either the lensing or spectroscopic analysis. Similarly to [25], forecasts show that including this marginalization has only a negligible impact on the Hubble parameter ($\Delta\sigma_{H_0} = 0.2\,\mathrm{km\,s^{-1}Mpc^{-1}}$, $\Delta\sigma_{H_0} = 0.1\,\mathrm{km\,s^{-1}Mpc^{-1}}$, and $\Delta\sigma_{H_0} = 0.03\,\mathrm{km\,s^{-1}Mpc^{-1}}$ comparing the forecast uncertainty without any r_s marginalisation and with marginalisation over the power spectrum suppression scale for the lensing-only, galaxy-only and joint analyses respectively). All in all, these checks – alongside the extensive battery of tests applied to both CMB lensing and galaxy survey probes individually [24–26] – support our claim that the information on H_0 is not coming from the sound horizon and our analysis thus represents a physically distinct probe of the early Universe.



(a) Full power spectrum

(b) Eisenstein-Hu with r_s marginalization

FIG. 3. Comparison between the fiducial *Planck*, BOSS and joint MCMC analyses (left) to Fisher forecasts run with the same set of parameters and priors, but with explicit marginalization over the sound horizon via an Eisenstein-Hu model (right). In all cases, PANTHEON+ constraints on Ω_m are included. including a prior on ω_b from BBN and the To simplify the visual comparison, we show only parameter posteriors relative to their mean. We observe an excellent match between the results and forecasts supporting out claim that the sound horizon marginalization procedure is working as expected, and that our constraints are informed primarily by the equality scale. The "full power spectrum" results adopt the restrictive prior on the neutrino mass sum ($\sum m_\nu < 0.26$ eV) while the forecasts neglect neutrinos given that the Eisenstein-Hu model does not include the effect of neutrinos. In addition to H_0, Ω_m and σ_8 we also show the parameter combinations $\alpha_{r_s} h\omega_{cb}^{-0.25}\omega_b^{-0.125}$ (α_{r_s} is the sound horizon rescaling parameter) and ω_m/h; these are roughly proportional to the sound horizon in h^{-1}Mpc-units and the equality scale in $h\,$Mpc^{-1}-units respectively.

IV. DISCUSSION

The main result of this work is the following ΛCDM constraint on H_0 from the equality scale alone: $H_0 = 64.8^{+2.2}_{-2.5}\,\mathrm{km\,s^{-1}Mpc^{-1}}$, using data from BOSS, *Planck* lensing, and PANTHEON+ supernova constraints, with a weakly restrictive prior on the neutrino mass density. This does not depend on sound horizon physics, though will be affected by any physical changes to the expansion rate. In contrast, the most recent Cepheid-calibrated local distance ladder measurement from SH0ES found $73.04 \pm 1.04\,\mathrm{km\,s^{-1}Mpc^{-1}}$ [1], or, when using TRGB calibrators,[6] $69.8 \pm 1.9\,\mathrm{km\,s^{-1}Mpc^{-1}}$ [4], though there is some disagreement on the TRGB calibration [e.g., 6–8, 63]. Assuming independence, our main result is inconsistent with that of SH0ES at 3.2σ, but agrees with the CCHP-calibrated TRGB results at 1.7σ. Furthermore, we find consistency with the main (r_s-dominated) *Planck* H_0 constraints, $H_0 = 67.4 \pm 0.5\,\mathrm{km\,s^{-1}Mpc^{-1}}$ [2] at 1.1σ, or the r_s-dominated full-shape BOSS constraints, $H_0 = 68.3 \pm 0.8$ [29] at 1.4σ (though this is not fully independent). Strictly, our constraints are not completely independent from those of previous works, since the same supernovae are used in both the distance ladder and our Ω_m priors. However, the SH0ES H_0 posterior shows very weak dependence on Ω_m [44, Fig. 9], implying that any correlation of our results with those of SH0ES is weak.

The disagreement between our results and those of SH0ES could be resolved in one of two ways: (1) unknown systematics in one or both measurements, or (2), extensions to the standard νΛCDM model. Importantly, any such extension cannot just alter the sound horizon at recombination, since this would alter the r_s-derived H_0 constraints but not those of this work. Recent work has largely ruled out the possibility of late-time solutions [22, 23, 64, 65], thus a theoretical explanation would likely involve new physics pre-recombination that also affected the equality scale,

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[6] Combining systematic and statistical errors in quadrature.

i.e. active at redshifts $z \approx 3500$. Furthermore, any new physics would have to affect the equality scale and the sound horizon in the same way; unless the modifications occur at very early times (which are themselves constrained by other observations, such as BBN), this seems unlikely, thus our result disfavors a range of resolutions [cf. 25], though is not yet precise enough to strongly constrain models such as the best-fit *Planck* EDE model. Of course, new physics that changes the sound-horizon and equality-based constraints in the same manner is not constrained: however, it remains to be seen if such models arise naturally.

Consistency of H_0 measurements from the equality and sound-horizon scale provide a useful null test of the $\nu\Lambda$CDM model. In practice, this can be considered by simply comparing our results to those of the main *Planck* analysis, since the latter is r_s-dominated. New physics operating at $z \gtrsim 1100$ could lead to a discrepancy in the two scales; in [25] it was shown that this approach could show strong signatures of phenomena such as early dark energy (EDE), with shifts of $\Delta H_0 = 2.6\,\mathrm{km\,s^{-1}Mpc^{-1}}$ ($7.8\,\mathrm{km\,s^{-1}Mpc^{-1}}$) expected from EDE models fit to *Planck* (ACT) data [39, 66] (see also [40–43]) in the context of the Euclid experiment, or $\Delta H_0 = 2.4\,\mathrm{km\,s^{-1}Mpc^{-1}}$ for BOSS with a *Planck* EDE model. In this work, we detect no statistically significant shifts between the r_s- and $k_{\rm eq}$-derived datasets using *Planck* and BOSS data. Whilst shifts of a few $\mathrm{km\,s^{-1}Mpc^{-1}}$ cannot be ruled out (given the combined errorbar of $\sigma(H_0) = 2.4\,\mathrm{km\,s^{-1}Mpc^{-1}}$), our results place constraints on the more extreme models, and disfavor the best-fit EDE model from ACT. A note of caution is in order: to obtain the strongest constraints on a specific new physics realization, one should perform a dedicated analysis with the relevant theoretical model ([e.g. 67, 68] for EDE). The null tests considered herein allow a broad class of resolutions to be tested simultaneously, albeit at slightly reduced sensitivity.

It is interesting to consider how these constraints will develop in the future. From current data, there is little room for improvement; whilst we can push to somewhat smaller scales in the galaxy power spectrum model, these are mostly shot-noise dominated, and do not add significant information about $k_{\rm eq}$. For CMB lensing, the theory is, in general, well understood, thus little improvements can be expected in this case. More promising is the progress expected within the next decade. For the CMB, lensing will be measured to significantly higher precision with AdvancedACT, SPT3G, the Simons Observatory, and CMB-S4 [e.g., 69, 70], sharpening the lensing-derived H_0 contours. This alone will not give particularly large improvements however, with [24] forecasting an asymptotic errorbar of $\sigma(H_0) = 3\,\mathrm{km\,s^{-1}Mpc^{-1}}$ from future data when using PANTHEON+ priors. However, the current methodology may be similarly applied to galaxy lensing, such as with the Rubin observatory. Galaxy surveys will also see a tremendous increase in survey volume, with early data releases from DESI and Euclid expected in the near future. Furthermore, the number of supernovae measured will continue to grow, with forthcoming results expected from surveys such as DES [71] and the Zwicky Transient Facility [72], which will sharpen the Ω_m prior by a considerable volume. Moreover, neutrino mass constraints will be greatly aided by upcoming CMB and terrestrial experiments, reducing the lensing degeneracies found herein and improving the H_0 precision. Whilst it is difficult to forecast exact values on the combination of future data-sets, it is clear that large improvements can be expected; from the Euclid survey alone (without external priors, except from BBN), [25] predicted $\sigma(H_0) = 0.7\,\mathrm{km\,s^{-1}Mpc^{-1}}$. Coupled with the strong constraints one can place on H_0 from the BAO feature, consistency of the two scales will provide a vital cross-check both of our analysis pipelines, and of models of new physics. In this aspect, the future looks exceedingly bright.

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[1] A. G. Riess, W. Yuan, L. M. Macri, D. Scolnic, D. Brout, S. Casertano, D. O. Jones, Y. Murakami, L. Breuval, T. G. Brink, A. V. Filippenko, S. Hoffmann, S. W. Jha, W. D. Kenworthy, J. Mackenty, B. E. Stahl, and W. Zheng, (2021), arXiv:2112.04510 [astro-ph.CO].

[2] N. Aghanim *et al.* (Planck), Astron. Astrophys. **641**, A6 (2020), [Erratum: Astron.Astrophys. 652, C4 (2021)], arXiv:1807.06209 [astro-ph.CO].

[3] G. Efstathiou, arXiv e-prints , arXiv:2007.10716 (2020), arXiv:2007.10716 [astro-ph.CO].

[4] W. L. Freedman, B. F. Madore, D. Hatt, T. J. Hoyt, I. S. Jang, R. L. Beaton, C. R. Burns, M. G. Lee, A. J. Monson, J. R. Neeley, M. M. Phillips, J. A. Rich, and M. Seibert, ApJ **882**, 34 (2019), arXiv:1907.05922 [astro-ph.CO].

[5] W. Yuan, A. G. Riess, L. M. Macri, S. Casertano, and D. M. Scolnic, ApJ **886**, 61 (2019), arXiv:1908.00993 [astro-ph.GA].

[6] S. Dhawan *et al.*, (2022), arXiv:2203.04241 [astro-ph.CO].

[7] S. Li, S. Casertano, and A. G. Riess, (2022), arXiv:2202.11110 [astro-ph.GA].

[8] W. L. Freedman, B. F. Madore, T. Hoyt, I. S. Jang, R. Beaton, M. G. Lee, A. Monson, J. Neeley, and J. Rich, (2020), 10.3847/1538-4357/ab7339, arXiv:2002.01550 [astro-ph.GA].

[9] S. S. Boruah, M. J. Hudson, and G. Lavaux, arXiv e-prints , arXiv:2010.01119 (2020), arXiv:2010.01119 [astro-ph.CO].

[10] B. P. Abbott, R. Abbott, T. D. Abbott, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, V. B. Adya, C. Affeldt, *et al.*, Nature **551**, 85 (2017), arXiv:1710.05835 [astro-ph.CO].

[11] K. C. Wong, S. H. Suyu, G. C. F. Chen, C. E. Rusu, M. Millon, D. Sluse, V. Bonvin, C. D. Fassnacht, S. Taubenberger, M. W. Auger, S. Birrer, J. H. H. Chan, F. Courbin, S. Hilbert, O. Tihhonova, T. Treu, A. Agnello, X. Ding, I. Jee, E. Komatsu, A. J. Shajib, A. Sonnenfeld, R. D. Bland ford, L. V. E. Koopmans, P. J. Marshall, and G. Meylan, MNRAS **498**, 1420 (2019), arXiv:1907.04869 [astro-ph.CO].

[12] S. Birrer, A. J. Shajib, A. Galan, M. Millon, T. Treu, A. Agnello, M. Auger, G. C. F. Chen, L. Christensen, T. Collett, F. Courbin, C. D. Fassnacht, L. V. E. Koopmans, P. J. Marshall, J. W. Park, C. E. Rusu, D. Sluse, C. Spiniello, S. H. Suyu, S. Wagner-Carena, K. C. Wong, M. Barnabè, A. S. Bolton, O. Czoske, X. Ding, J. A. Frieman, and L. Van de Vyvere, arXiv e-prints , arXiv:2007.02941 (2020), arXiv:2007.02941 [astro-ph.CO].

[13] L. Verde, T. Treu, and A. G. Riess, Nature Astronomy **3**, 891 (2019), arXiv:1907.10625 [astro-ph.CO].

[14] F. Beutler, H.-J. Seo, A. J. Ross, P. McDonald, S. Saito, A. S. Bolton, J. R. Brownstein, C.-H. Chuang, A. J. Cuesta, D. J. Eisenstein, A. Font-Ribera, J. N. Grieb, N. Hand, F.-S. Kitaura, C. Modi, R. C. Nichol, W. J. Percival, F. Prada, S. Rodriguez-Torres, N. A. Roe, N. P. Ross, S. Salazar-Albornoz, A. G. Sánchez, D. P. Schneider, A. Slosar, J. Tinker, R. Tojeiro, M. Vargas-Magaña, and J. A. Vazquez, MNRAS **464**, 3409 (2017), arXiv:1607.03149 [astro-ph.CO].

[15] N. Schöneberg, J. Lesgourgues, and D. C. Hooper, JCAP **2019**, 029 (2019), arXiv:1907.11594 [astro-ph.CO].

[16] T. M. C. Abbott, F. B. Abdalla, J. Annis, K. Bechtol, J. Blazek, B. A. Benson, R. A. Bernstein, G. M. Bernstein, E. Bertin, D. Brooks, D. L. Burke, A. Carnero Rosell, M. Carrasco Kind, J. Carretero, F. J. Castander, C. L. Chang, T. M. Crawford, C. E. Cunha, C. B. D'Andrea, L. N. da Costa, C. Davis, J. DeRose, S. Desai, H. T. Diehl, J. P. Dietrich, P. Doel, A. Drlica-Wagner, A. E. Evrard, E. Fernand ez, B. Flaugher, P. Fosalba, J. Frieman, J. García-Bellido, E. Gaztanaga, D. W. Gerdes, T. Giannantonio, D. Gruen, R. A. Gruendl, J. Gschwend, G. Gutierrez, W. G. Hartley, J. W. Henning, K. Honscheid, B. Hoyle, D. Huterer, B. Jain, D. J. James, M. Jarvis, T. Jeltema, M. D. Johnson, M. W. G. Johnson, E. Krause, K. Kuehn, S. Kuhlmann, N. Kuropatkin, O. Lahav, A. R. Liddle, M. Lima, H. Lin, N. MacCrann, M. A. G. Maia, A. Manzotti, M. March, J. L. Marshall, R. Miquel, J. J. Mohr, T. Natoli, P. Nugent, R. L. C. Ogando, Y. Park, A. A. Plazas, C. L. Reichardt, K. Reil, A. Roodman, A. J. Ross, E. Rozo, E. S. Rykoff, E. Sanchez, V. Scarpine, M. Schubnell, D. Scolnic, I. Sevilla-Noarbe, E. Sheldon, M. Smith, R. C. Smith, M. Soares-Santos, F. Sobreira, E. Suchyta, G. Tarle, D. Thomas, M. A. Troxel, A. R. Walker, R. H. Wechsler, J. Weller, W. Wester, W. L. K. Wu, J. Zuntz, Dark Energy Survey Collaboration, and South Pole Telescope Collaboration, MNRAS **480**, 3879 (2018), arXiv:1711.00403 [astro-ph.CO].

[17] A. de Mattia, V. Ruhlmann-Kleider, A. Raichoor, A. J. Ross, A. Tamone, C. Zhao, S. Alam, S. Avila, E. Burtin, J. Bautista, F. Beutler, J. Brinkmann, J. R. Brownstein, M. J. Chapman, C.-H. Chuang, J. Comparat, H. du Mas des Bourboux, K. S. Dawson, A. de la Macorra, H. Gil-Marín, V. Gonzalez-Perez, C. Gorgoni, J. Hou, H. Kong, S. Lin, S. Nadathur, J. A. Newman, E.-M. Mueller, W. J. Percival, M. Rezaie, G. Rossi, D. P. Schneider, P. Tiwari, M. Vivek, Y. Wang, and G.-B. Zhao, MNRAS **501**, 5616 (2021), arXiv:2007.09008 [astro-ph.CO].

[18] G. E. Addison, D. J. Watts, C. L. Bennett, M. Halpern, G. Hinshaw, and J. L. Weiland, ApJ **853**, 119 (2018), arXiv:1707.06547 [astro-ph.CO].

[19] A. Cuceu, J. Farr, P. Lemos, and A. Font-Ribera, JCAP **2019**, 044 (2019), arXiv:1906.11628 [astro-ph.CO].

[20] M. Tegmark, Phys. Rev. Lett. **79**, 3806 (1997), arXiv:astro-ph/9706198 [astro-ph].

[21] W. J. Percival, C. M. Baugh, J. Bland-Hawthorn, T. Bridges, R. Cannon, S. Cole, M. Colless, C. Collins, W. Couch, G. Dalton, R. De Propris, S. P. Driver, G. Efstathiou, R. S. Ellis, C. S. Frenk, K. Glazebrook, C. Jackson, O. Lahav, I. Lewis, S. Lumsden, S. Maddox, S. Moody, P. Norberg, J. A. Peacock, B. A. Peterson, W. Sutherland, and K. Taylor, MNRAS **327**, 1297 (2001), arXiv:astro-ph/0105252 [astro-ph].

[22] E. Di Valentino, O. Mena, S. Pan, L. Visinelli, W. Yang, A. Melchiorri, D. F. Mota, A. G. Riess, and J. Silk, Classical and Quantum Gravity **38**, 153001 (2021).

[23] L. Knox and M. Millea, Phys. Rev. D **101**, 043533 (2020).

[24] E. J. Baxter and B. D. Sherwin, Monthly Notices of the Royal Astronomical Society **501**, 1823–1835 (2020).

[25] G. S. Farren, O. H. E. Philcox, and B. D. Sherwin, (2021), arXiv:2112.10749 [astro-ph.CO].

[26] O. H. E. Philcox, B. D. Sherwin, G. S. Farren, and E. J. Baxter, (2020), arXiv:2008.08084.

[27] M. M. Ivanov, M. Simonović, and M. Zaldarriaga, JCAP **2020**, 042 (2020), arXiv:1909.05277 [astro-ph.CO].

[28] M. M. Ivanov, arXiv e-prints , arXiv:2106.12580 (2021), arXiv:2106.12580 [astro-ph.CO].

[29] O. H. E. Philcox and M. M. Ivanov, (2021), arXiv:2112.04515 [astro-ph.CO].

[30] O. H. E. Philcox, M. M. Ivanov, M. Simonović, and M. Zaldarriaga, JCAP **2020**, 032 (2020), arXiv:2002.04035 [astro-ph.CO].

[31] G. d'Amico, J. Gleyzes, N. Kokron, K. Markovic, L. Senatore, P. Zhang, F. Beutler, and H. Gil-Marín, JCAP **2020**, 005 (2020), arXiv:1909.05271 [astro-ph.CO].

[32] P. Zhang, G. D'Amico, L. Senatore, C. Zhao, and Y. Cai, arXiv e-prints , arXiv:2110.07539 (2021), arXiv:2110.07539 [astro-ph.CO].

[33] S.-F. Chen, Z. Vlah, and M. White, arXiv e-prints , arXiv:2110.05530 (2021), arXiv:2110.05530 [astro-ph.CO].

[34] Y. Kobayashi, T. Nishimichi, M. Takada, and H. Miyatake, (2021), arXiv:2110.06969 [astro-ph.CO].

[35] R. C. Nunes, S. Vagnozzi, S. Kumar, E. Di Valentino, and O. Mena, (2022), arXiv:2203.08093 [astro-ph.CO].

[36] T. M. C. Abbott *et al.* (DES), Mon. Not. Roy. Astron. Soc. **480**, 3879 (2018), arXiv:1711.00403 [astro-ph.CO].

[37] G. E. Addison, D. J. Watts, C. L. Bennett, M. Halpern, G. Hinshaw, and J. L. Weiland, Astrophys. J. **853**, 119 (2018), arXiv:1707.06547 [astro-ph.CO].

[38] V. Poulin, T. L. Smith, T. Karwal, and M. Kamionkowski, Physical Review Letters **122** (2019), 10.1103/physrevlett.122.221301.

[39] J. C. Hill, E. Calabrese, S. Aiola, N. Battaglia, B. Bolliet, S. K. Choi, M. J. Devlin, A. J. Duivenvoorden, J. Dunkley, S. Ferraro, P. A. Gallardo, V. Gluscevic, M. Hasselfield, M. Hilton, A. D. Hincks, R. Hlozek, B. J. Koopman, A. Kosowsky, A. La Posta, T. Louis, M. S. Madhavacheril, J. McMahon, K. Moodley, S. Naess, U. Natale, F. Nati, L. Newburgh, M. D. Niemack, B. Partridge, F. J. Qu, M. Salatino, A. Schillaci, N. Sehgal, B. D. Sherwin, C. Sifon, D. N. Spergel, S. T. Staggs, E. R. Storer, A. van Engelen, E. M. Vavagiakis, E. J. Wollack, and Z. Xu, arXiv e-prints , arXiv:2109.04451 (2021), arXiv:2109.04451 [astro-ph.CO].

[40] V. Poulin, T. L. Smith, and A. Bartlett, (2021), arXiv:2109.06229 [astro-ph.CO].

[41] A. Moss, E. Copeland, S. Bamford, and T. Clarke, (2021), arXiv:2109.14848 [astro-ph.CO].

[42] A. La Posta, T. Louis, X. Garrido, and J. C. Hill, Phys. Rev. D **105**, 083519 (2022), arXiv:2112.10754 [astro-ph.CO].

[43] T. L. Smith, M. Lucca, V. Poulin, G. F. Abellan, L. Balkenhol, K. Benabed, S. Galli, and R. Murgia, (2022), arXiv:2202.09379 [astro-ph.CO].

[44] D. Brout *et al.*, (2022), arXiv:2202.04077 [astro-ph.CO].

[45] M. Aker *et al.*, (2021), arXiv:2105.08533 [hep-ex].

[46] Planck Collaboration, N. Aghanim, Y. Akrami, M. Ashdown, J. Aumont, C. Baccigalupi, M. Ballardini, A. J. Banday, R. B. Barreiro, N. Bartolo, S. Basak, K. Benabed, J. P. Bernard, M. Bersanelli, P. Bielewicz, J. J. Bock, J. R. Bond, J. Borrill, F. R. Bouchet, F. Boulanger, M. Bucher, C. Burigana, R. C. Butler, E. Calabrese, J. F. Cardoso, J. Carron, B. Casaponsa, A. Challinor, H. C. Chiang, L. P. L. Colombo, C. Combet, B. P. Crill, F. Cuttaia, P. de Bernardis, A. de Rosa, G. de Zotti, J. Delabrouille, J. M. Delouis, E. Di Valentino, J. M. Diego, O. Doré, M. Douspis, A. Ducout, X. Dupac, S. Dusini, G. Efstathiou, F. Elsner, T. A. Enßlin, H. K. Eriksen, Y. Fantaye, R. Fernand ez-Cobos, F. Finelli, M. Frailis, A. A. Fraisse, E. Franceschi, A. Frolov, S. Galeotta, S. Galli, K. Ganga, R. T. Génova-Santos, M. Gerbino, T. Ghosh, Y. Giraud-Héraud, J. González-Nuevo, K. M. Górski, S. Gratton, A. Gruppuso, J. E. Gudmundsson, J. Hamann, W. Handley, F. K. Hansen, D. Herranz, E. Hivon, Z. Huang, A. H. Jaffe, W. C. Jones, E. Keihänen, R. Keskitalo, K. Kiiveri, J. Kim, T. S. Kisner, N. Krachmalnicoff, M. Kunz, H. Kurki-Suonio, G. Lagache, J. M. Lamarre, A. Lasenby, M. Lattanzi, C. R. Lawrence, M. Le Jeune, F. Levrier, A. Lewis, M. Liguori, P. B. Lilje, M. Lilley, V. Lindholm, M. López-Caniego, P. M. Lubin, Y. Z. Ma, J. F. Macías-Pérez, G. Maggio, D. Maino, N. Mandolesi, A. Mangilli, A. Marcos-Caballero, M. Maris, P. G. Martin, E. Martínez-González, S. Matarrese, N. Mauri, J. D. McEwen, P. R. Meinhold, A. Melchiorri, A. Mennella, M. Migliaccio, M. Millea, M. A. Miville-Deschênes, D. Molinari, A. Moneti, L. Montier, G. Morgante, A. Moss, P. Natoli, H. U. Nørgaard-Nielsen, L. Pagano, D. Paoletti, B. Partridge, G. Patanchon, H. V. Peiris, F. Perrotta, V. Pettorino, F. Piacentini, G. Polenta, J. L. Puget, J. P. Rachen, M. Reinecke, M. Remazeilles, A. Renzi, G. Rocha, C. Rosset, G. Roudier, J. A. Rubiño-Martín, B. Ruiz-Granados, L. Salvati, M. Sandri, M. Savelainen, D. Scott, E. P. S. Shellard, C. Sirignano, G. Sirri, L. D. Spencer, R. Sunyaev, A. S. Suur-Uski, J. A. Tauber, D. Tavagnacco, M. Tenti, L. Toffolatti, M. Tomasi, T. Trombetti, J. Valiviita, B. Van Tent, P. Vielva, F. Villa, N. Vittorio, B. D. Wandelt, I. K. Wehus, A. Zacchei, and A. Zonca, A&A **641**, A5 (2020), arXiv:1907.12875 [astro-ph.CO].

[47] N. Aghanim *et al.* (Planck), Astron. Astrophys. **641**, A8 (2020), arXiv:1807.06210 [astro-ph.CO].

[48] T. Brinckmann and J. Lesgourgues, Phys. Dark Univ. **24**, 100260 (2019), arXiv:1804.07261 [astro-ph.CO].

[49] D. J. Eisenstein *et al.* (SDSS), Astron. J. **142**, 72 (2011), arXiv:1101.1529 [astro-ph.IM].

[50] S. Alam *et al.* (SDSS-III), Astrophys. J. Suppl. **219**, 12 (2015), arXiv:1501.00963 [astro-ph.IM].

[51] S. Alam *et al.* (BOSS), Mon. Not. Roy. Astron. Soc. **470**, 2617 (2017), arXiv:1607.03155 [astro-ph.CO].

[52] O. H. E. Philcox, Phys. Rev. D **103**, 103504 (2021), arXiv:2012.09389 [astro-ph.CO].

[53] O. H. E. Philcox, Phys. Rev. D **104**, 123529 (2021), arXiv:2107.06287 [astro-ph.CO].

[54] M. M. Ivanov, O. H. E. Philcox, M. Simonović, M. Zaldarriaga, T. Nishimichi, and M. Takada, (2021), arXiv:2110.00006 [astro-ph.CO].

[55] M. M. Ivanov, O. H. E. Philcox, T. Nishimichi, M. Simonović, M. Takada, and M. Zaldarriaga, (2021), arXiv:2110.10161 [astro-ph.CO].

[56] D. J. Eisenstein and W. Hu, ApJ **496**, 605 (1998), arXiv:astro-ph/9709112 [astro-ph].

[57] D. M. Scolnic, D. O. Jones, A. Rest, Y. C. Pan, R. Chornock, R. J. Foley, M. E. Huber, R. Kessler, G. Narayan, A. G. Riess, S. Rodney, E. Berger, D. J. Brout, P. J. Challis, M. Drout, D. Finkbeiner, R. Lunnan, R. P. Kirshner, N. E. Sanders, E. Schlafly, S. Smartt, C. W. Stubbs, J. Tonry, W. M. Wood-Vasey, M. Foley, J. Hand, E. Johnson, W. S. Burgett, K. C. Chambers, P. W. Draper, K. W. Hodapp, N. Kaiser, R. P. Kudritzki, E. A. Magnier, N. Metcalfe, F. Bresolin, E. Gall, R. Kotak, M. McCrum, and K. W. Smith, The Astrophysical Journal **859**, 101 (2018).

[58] B. Audren, J. Lesgourgues, K. Benabed, and S. Prunet, JCAP **2013**, 001 (2013), arXiv:1210.7183 [astro-ph.CO].

[59] T. Brinckmann and J. Lesgourgues, arXiv e-prints , arXiv:1804.07261 (2018), arXiv:1804.07261 [astro-ph.CO].

[60] T. M. C. Abbott *et al.* (DES), Phys. Rev. D **105**, 023520 (2022), arXiv:2105.13549 [astro-ph.CO].

[61] M. M. Ivanov, M. Simonović, and M. Zaldarriaga, JCAP **2020**, 042 (2020), arXiv:1909.05277 [astro-ph.CO].

[62] S. Gratton and A. Challinor, Monthly Notices of the Royal Astronomical Society **499**, 3410 (2020), https://academic.oup.com/mnras/article-pdf/499/3/3410/34052415/staa2996.pdf.

[63] W. Yuan, A. G. Riess, L. M. Macri, S. Casertano, and D. Scolnic, Astrophys. J. **886**, 61 (2019), arXiv:1908.00993 [astro-ph.GA].

[64] S. Dhawan, D. Brout, D. Scolnic, A. Goobar, A. G. Riess, and V. Miranda, Astrophys. J. **894**, 54 (2020), arXiv:2001.09260 [astro-ph.CO].

[65] G. Benevento, W. Hu, and M. Raveri, Phys. Rev. D **101**, 103517 (2020), arXiv:2002.11707 [astro-ph.CO].

[66] T. L. Smith, V. Poulin, and M. A. Amin, Phys. Rev. D **101**, 063523 (2020).

[67] T. L. Smith, V. Poulin, J. L. Bernal, K. K. Boddy, M. Kamionkowski, and R. Murgia, Phys. Rev. D **103**, 123542 (2021).

[68] M. M. Ivanov, E. McDonough, J. C. Hill, M. Simonović, M. W. Toomey, S. Alexander, and M. Zaldarriaga, Phys. Rev. D **102**, 103502 (2020).

[69] P. Ade *et al.* (Simons Observatory), JCAP **02**, 056 (2019), arXiv:1808.07445 [astro-ph.CO].

[70] K. Abazajian *et al.*, Bull. Am. Astron. Soc. **51**, 209 (2019), arXiv:1908.01062 [astro-ph.IM].

[71] A. Möller *et al.* (DES), (2022), arXiv:2201.11142 [astro-ph.CO].

[72] S. Dhawan *et al.*, (2021), 10.1093/mnras/stab3093, arXiv:2110.07256 [astro-ph.CO].