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Revisiting constraints on asymmetric dark matter from collapse in white dwarf stars

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The runaway collapse phase of a small dark matter cluster inside a white dwarf star encompasses a reversible stage, where heat can be transferred back and forth between nuclear and dark matter. Induced nuclear burning phases are stable and early carbon depletion undermines previous claims of type Ia supernova ignition. Instead, mini black holes are formed at the center of the star that either evaporate or accrete stellar material until a macroscopic sub-Chandrasekhar-mass black hole is formed. In the latter case, a 0.1 to 1 second lasting electromagnetic transient signal can be detected upon ejection of the white dwarf’s potential magnetic field. Binary systems that transmute to black holes and subsequently merge emit gravitational waves. Advanced LIGO should detect one such sub-Chandrasekhar binary black hole inspiral per year, while future Einstein telescope-like facilities will detect thousands per year. The effective spin parameter distribution is peaked at 0.2 and permits to disentangle from primordial sub-Chandrasekhar black holes. Such signatures are compatible with current direct detection constraints, as well as with neutron star constraints in the case of bosonic dark matter, even though they remain in conflict with the fermionic case for part of the parameter space.

I. INTRODUCTION

Dark matter (DM) collapse under self-gravity has first been studied by Goldman and Nussinov [1] in the context of neutron stars (NSs). The authors show that a critical number N_{sg} of particles has to accumulate for collapse under self-gravity to initiate. This criterion was thence popularized as the “collapse criterion” and applied to explore DM phenomenology in NSs [2–5], white dwarf (WD) stars [2, 6–9], and main sequence stars [10, 11]. Yet, little attention has been devoted to the dynamics of collapse inside the star itself.

The process is intrinsically iterative: scattering between DM and stellar matter (SM) particles reduces the total energy of DM particles, while gravitational self-attraction and the resulting orbital hardening increases the DM kinetic energy. The runaway nature of the process lasts as long as heat imparted to the scattered SM particles is efficiently evacuated to the rest of the star. The process is also reversible, as long as scatterings are elastic. If SM particles became more energetic, on average, than DM particles, the process would reverse and the DM cluster expand.

The question of heat evacuation in the baryonic component has been addressed for NSs [1], but so far, to our knowledge, never in the context of non-degenerate stars (including WD stars, which are electron-degenerate, but not nucleon-degenerate).

Here, we seek an answer by deriving a system of first-order differential equations that permit to follow DM and nuclear macroscopic properties along the elastic collapse phase. We are particularly interested in previously investigated astrophysical phenomenology, including claims of type Ia supernova (SN Ia) ignition [6–9] and collapse to a black hole (BH) [2, 9].

The present-day understanding is that ‘normal’ SNe Ia originate either from deflagrations with transition to detonation in Chandrasekhar-mass ($\sim 1.4 M_{\odot}$) WDs [12, 13], or from pure detonations in sub-Chandrasekhar-mass WDs [14–18]. While the former channel unlikely produces all events [19], the latter is still lacking a convincing ignition mechanism [20], see, however, Ref. [21]. Additionally, pure deflagrations can reproduce certain types of ‘peculiar’ SNe Ia [22–25], and their ignition from DM collapse has been studied from thermonuclear ignition [6–9], pycnonuclear ignition [26], and Hawking radiation ignition [7, 9]. It has also been questioned if the observed correlation between SN Ia magnitudes and host galaxy masses has its origin in the local DM environment [27].

The formation of a mini BH inside a WD can lead to the implosion of the latter. In principle, this can generate BHs of mass 0.3–1.4 M_{\odot} , which may interact with other BHs, NSs, or WDs, and generate detectable gravitational waves (GWs) [5]. While generally, the observation of a BH with mass $< 1.4 M_{\odot}$ is considered a smoking gun of exotic new physics [28], BHs in this mass range could be attributed to either DM collapse inside a NS [4], to a primordial BH from the QCD phase transition (produced with 0.7 M_{\odot} and taking into account accretion of gas [29–32]), to the capture of a primordial BH with asteroid mass by a NS and subsequent transmutation to a macro-

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scopic BH ([33], even though constraints on these primordial BHs [34, 35] were largely overestimated [36, 37]), or to BH formation through atomic DM [38]. As we shall see, given that WDs account for the final evolutionary state of 97% of main sequence stars [39], GWs from the coalescence of binary BHs from transmuted binary WDs should already be observed with the advanced Laser Interferometer Gravitational-Wave Observatory (aLIGO) [40, 41]. In addition, the effective inspiral spin parameter distribution is inherited from the progenitor WD angular momentum distribution and permits to disentangle this sub-Chandrasekhar BH formation channel from others.

The present work investigates collapse of non self-annihilating DM, such as, for example, asymmetric DM [1, 2, 42–44]. Asymmetric DM is motivated as it can explain the matter-antimatter asymmetry in the Universe (see, e.g., Ref. [43], for a review). For the sake of generality, we also include a Yukawa-type non-gravitational attractive self-interaction with potential $V(r) = \alpha \exp(-\mu r)/r$, where μ is the mediator mass and α a coupling constant.

We use natural units, $c = k_B = \hbar = 1$, while keeping G explicit. Stellar quantities are indexed with an asterisk (*), while DM quantities are left without. The infinity symbol (∞) indicates stellar core quantities far from the DM cluster. We use the term stellar matter (SM) to designate initially ions, but later, as collapse proceeds and locally heats the center of the star, these are crushed to nucleons and then to partons (quarks and gluons). Therefore, we stick to the generic term SM particles keeping its meaning in mind.

The present Letter is organized as follows. In Sec. II we derive the general set of ‘elastic’ collapse equations. In Sec. III we explore phenomenology in WD stars. We present our conclusions in Sec. IV.

II. GENERAL EQUATIONS

A. DM capture and accumulation

Let $N(t)$ be the number of particles of a DM cluster at the center of a star with age t . The number of captured particles during an interval dt is $dt \Gamma_{\text{cap}}$, where Γ_{cap} is the capture rate [45]¹

$$\Gamma_{\text{cap}} = \frac{\sqrt{54\pi} R_*^2 v_{\text{esc}}^2 \rho_{\text{gal}}}{m v_{\text{gal}}} \times \sum_{j=1}^{\infty} p_j(\tau) \left[1 + \delta - (\gamma_j + \delta) e^{-(\gamma_j - 1)/\delta} \right], \quad (1)$$

where $v_{\text{esc}} \equiv (2 G M_*/R_*)^{1/2}$ is the escape velocity of the star, R_* and M_* are radius and mass of the star, respectively, $\delta \equiv 2 v_{\text{gal}}^2/3 v_{\text{esc}}^2$, $\gamma_j \equiv (1 - \beta_+/2)^{-j}$, $\beta_{\pm} \equiv 4 m m_*/(m \pm m_*)^2$, m and m_* are DM and SM particle masses, ρ_{gal} and v_{gal} are the galactic DM density and velocity dispersion, respectively, and

$$p_j(\tau) = 2 \int_0^1 \frac{y e^{-y\tau} (y\tau)^j dy}{j!} \quad (2)$$

is a Poisson weighting that gives the probability of j scatters for the optical depth $\tau \equiv 3\sigma_*/2 \sigma_{\text{sat}}$, where y is a kinematical quantity, σ_* is the DM-SM scattering cross-section and $\sigma_{\text{sat}} = R_*^2/N_*$ is the saturation cross-section, where $N_* = M_*/m_*$ is the total number of SM particles. Once captured, DM particles settle at the center of the star, where they thermalize after a timescale (see App. B for a derivation)

$$t_{\text{th}} = \frac{3m}{\rho_* \sigma_* v_*} \left[\frac{3\sqrt{2}\pi}{16} \frac{v_*}{v_{\text{gal}}} + \frac{1}{2} + \ln\left(\frac{m}{m_*}\right) \right], \quad (3)$$

where ρ_* is the central density of the star and v_* the mean velocity of SM particles. The number of captured and thermalized particles during an interval dt is dN . Since $dt \Gamma_{\text{cap}}$ must be equal to $(dt + dt_{\text{th}})(dN/dt)$, where $dt_{\text{th}} \equiv t_{\text{th}}(t + dt) - t_{\text{th}}(t)$ is the increase of thermalization time during dt , the number increase rate of DM particles is

$$\frac{dN}{dt} = \Gamma_{\text{cap}} \left(1 + \frac{dt_{\text{th}}}{dt} \right)^{-1}, \quad (t \geq t_{\text{th},0}) \quad (4)$$

where $t_{\text{th},0}$ is the initial thermalization time, $dN/dt = 0$ for $t < t_{\text{th},0}$, and $t_{\text{th},0}$ is the larger solution of $t = t_{\text{th}}[T_{*\infty}(t)]$. The time derivative of the thermalization time is

$$\frac{dt_{\text{th}}}{dt} = -\frac{3m}{2\rho_* \sigma_* v_* T_{*\infty}} \left[\frac{1}{2} + \ln\left(\frac{m}{m_*}\right) \right] \frac{dT_{*\infty}}{dt}, \quad (5)$$

where $T_{*\infty}$ is the stellar core background temperature and $dT_{*\infty}/dt$ its time derivative. If the stellar core temperature is constant, we have $dN/dt = \Gamma_{\text{cap}}$ and, integrating, $N(t) = \Gamma_{\text{cap}} t$.

B. Self-attraction and collapse

The mean potential energy per DM particle at the center of a star at $r = 0$ is (see, e.g., Ref. [4] and App. A)

$$U = -\frac{4\pi G \rho_* m R^2}{5} - \frac{3GNm^2}{5R} - \frac{3\alpha N e^{-\mu R_0}}{2\mu^2 R^3} (3 + 3\mu R_0 + \mu^2 R_0^2), \quad (6)$$

where R is the radius of the DM cluster and $R_0 = R(4\pi/3N)^{1/3}$ is the mean inter-particle distance. The terms on the right hand side of eq. (6) account for

¹ Eq. (1) is identical to eqs. (18) and (22) of Ref. [45], maybe slightly easier to handle in a numerical code. See also [46] for useful approximations and [47] for some improvements.

contributions coming from (1) gravitational attraction due to the density of the star, (2) gravitational self-attraction, and potentially (3) non-gravitational self-attraction². It is easy to verify that the accumulation timescale $(dN/dt)^{-1}$ is much longer than the orbital timescale of DM particles $(R^3/GM)^{1/2}$, where $M = Nm$ is the total thermalized DM mass, hence the DM cluster energy repartition is given by the virial theorem, with the mean kinetic energy per particle $K = -U/2$, and the mean total energy per particle is $E = K + U = U/2 = -K$.

Despite the sporadic nature of individual scatterings, the average *per particle* energies can be treated as differential functions, and we can write, for example, E as a total differential of N and R ,

$$dE = \frac{\partial E}{\partial N} dN + \frac{\partial E}{\partial R} dR, \quad (7)$$

where from equation (6) and using the virial theorem,

$$\frac{\partial E}{\partial R} = -\frac{4\pi G\rho_* m R}{5} + \frac{3GNm^2}{10R^2} + \frac{3\alpha N}{4R^4\mu^2} f(\mu R_0), \quad (8)$$

$$\frac{\partial E}{\partial N} = -\frac{3Gm^2}{10R} - \frac{\alpha}{4R^3\mu^2} f(\mu R_0), \quad (9)$$

where we have defined $f(y) \equiv e^{-y}(9 + 9y + 4y^2 + y^3)$. As long as $\partial E/\partial R > 0$, DM particles stay in thermal equilibrium with the star ($K = K_*$) and settle roughly inside the ‘thermal radius’ [1]³

$$R_{\text{th}} = \left(\frac{15T_{*\infty}}{4\pi G\rho_* m} \right)^{1/2}. \quad (10)$$

The actual radius during the adiabatic phase decreases slightly over time and has to be computed numerically in the general case, but in the absence of non-gravitational self-attraction (i.e. $\alpha = 0$), it is the larger positive solution of the cubic equation (6)

$$\frac{R}{R_{\text{th}}} = \left[-\frac{\eta}{2} + \left(\frac{\eta^2}{2} - \frac{1}{27} \right)^{1/2} \right]^{1/3} + \frac{1}{3} \left[-\frac{\eta}{2} + \left(\frac{\eta^2}{2} - \frac{1}{27} \right)^{1/2} \right]^{-1/3}, \quad (11)$$

where $\eta \equiv N/N_{\text{sg}}$, and where $N_{\text{sg}} \equiv 4\pi\rho_* R_{\text{th}}^3/3m$ is the critical number for self-gravitation of Ref. [1]. From eq. (11), it is evident that the *exact* collapse criterion for the case of vanishing non-gravitation attraction is $N/N_{\text{sg}} \geq 2/(3\sqrt{3}) \approx 0.3849$, which is slightly lower than

the criterion $N/N_{\text{sg}} \geq 1$ of Ref. [1]. In the general case, the runaway collapse criterion is

$$\frac{\partial E}{\partial R} \leq 0. \quad (12)$$

When this condition is satisfied, not only the DM cluster but also the SM enclosed by the DM cluster drop out of local thermodynamic equilibrium.

C. From kinematics to dynamics

Assuming elastic scatterings with non-degenerate SM, the mean variation of the total DM energy per DM-SM scatter is

$$\Delta E = \frac{1}{(2\pi)^2} \int_0^{2\pi} \int_0^{2\pi} \Delta E(\theta, \theta_*) d\theta d\theta_* \quad (13)$$

where the integrals are taken over the incidental angles θ and θ_* with respect to the line of centers and where (see App. C1 for a derivation)

$$\begin{aligned} \Delta E(\theta, \theta_*) = & 2 \left[\mathcal{E} p_*^2 \cos^2 \theta_* - \mathcal{E}_* p^2 \cos^2 \theta \right. \\ & \left. + (\mathcal{E} - \mathcal{E}_*) p p_* \cos \theta \cos \theta_* \right] \\ & \times \left[(\mathcal{E} + \mathcal{E}_*)^2 - (p \cos \theta + p_* \cos \theta_*)^2 \right]^{-1} \end{aligned} \quad (14)$$

is the energy gain of DM particles colliding with nuclei for given initial total relativistic energies $\mathcal{E} = K + m$ and $\mathcal{E}_* = K_* + m_*$ (including rest mass but not potential energy), and momenta $p = (\mathcal{E}^2 - m^2)^{1/2}$ and $p_* = (\mathcal{E}_*^2 - m_*^2)^{1/2}$. In the non-relativistic limit, equation (13) reduces to

$$\Delta E = -\frac{\beta_+}{2} (K - K_*) \quad (15)$$

where $\beta_+ \equiv 4mm_*/(m+m_*)^2$. Eq. (15) reduces to the usual formula $\Delta E = \beta_+ K/2$ [e.g. 48], valid whenever the temperature of the star can be neglected, e.g. during capture and the initial phase of thermalization. Note that in stars with degenerate nuclear matter (e.g. NSs), considerations are fundamentally different, because post-collision energy states $< E_F$ are Fermi blocked for nuclear particles [1].

The mean timescale between DM-SM scatters is

$$\Delta t = \frac{1}{n_* \sigma_* v_{\text{rel}}}, \quad (16)$$

where $n_* = \rho_*/m_*$ is the number density of SM, as seen by a non-relativistic observer, and v_{rel} is the mean relative velocity. For the present purpose, DM particles can be considered as non-relativistic. The mean relative velocity in eq. (16) can be shown to be (see, e.g., Ref. [49], and App. D)

$$v_{\text{rel}} = \frac{2[(1 + \zeta)^2 K_3(\xi) - (\zeta^2 - 1) K_1(\xi)]}{\xi K_2(x) K_2(x_*)}, \quad (17)$$

² For simplicity, we have assumed a constant distribution of DM particles inside R (a step function), the differences with a Maxwell-Boltzmann distribution are minor (see App. A). In Ref. [4], the numerical factor in term (1) is 8π instead of 4π .

³ Note that the numerical factor on the right hand side of eq. (10) is $9/8\pi$ in Ref. [1], $15/8\pi$ in Ref. [4], while it is $9/4\pi$ in Ref. [6].

where $\xi \equiv x + x_*$ and $\zeta \equiv (x^2 + x_*^2)/2xx_*$ are auxiliary variables and $x \equiv m/T$ and $x_* \equiv m_*/T_*$ are the standard thermal variables, $K_i(x)$ is the modified (or hyperbolic) Bessel function of the second kind (not to confuse with the kinetic energy that we denote K as well). In the non-relativistic limit, $v_{\text{rel}} = [8(mT_* + m_*T)/\pi mm_*]^{1/2}$.

Assembling the pieces, we obtain the time variation of the total DM mean *per particle* energy from the ratio of eqs. (13) and (16)

$$\frac{dE}{dt} = \frac{\Delta E}{\Delta t}. \quad (18)$$

On the other hand, dividing eq. (7) by dt , we obtain the time derivative of the collapse scale (for $\partial E/\partial R \neq 0$)

$$\frac{dR}{dt} = \left(\frac{\partial E}{\partial R}\right)^{-1} \left(\frac{dE}{dt} - \frac{\partial E}{\partial N} \frac{dN}{dt}\right), \quad (19)$$

where the terms on the right hand side are given by eqs. (8), (18), (9) and (4), respectively.

D. Heat diffusion

Starting from the diffusion equation of stellar specific thermal energy $e_* = K_*/m_*$ (Fick's second law)

$$\frac{\partial e_*}{\partial t} - D \nabla^2 e_* = [\text{heat sources}], \quad (20)$$

where $D = \kappa/c_p \rho_*$ is the thermal diffusivity, where κ is the thermal conductivity and c_p is the specific heat capacity at constant pressure. On the right hand side of eq. (20) we have heat release from DM-SM scattering and, potentially, nuclear reactions' heat release. DM-SM scatterings liberate N times the energy rate $-dE/dt$, given by eq. (18), per stellar mass $4\pi\rho_*R^3/3$.

Using the finite difference approximation, the Laplacian at $r = 0$ can be written as $\nabla^2 e_*|_{r=0} \simeq -6(e_* - e_{*\infty})/R^2$ [50, p.149].⁴ With this approximation, we obtain a closed form for the time derivative at $r = 0$ of the stellar specific energy

$$\frac{\partial e_*}{\partial t} = -\frac{6D(e_* - e_{*\infty})}{R^2} - \frac{3N}{4\pi\rho_*R^3} \frac{dE}{dt} + \sum_i \dot{q}_i, \quad (21)$$

where the terms on the right hand side account for contributions from (1) diffusion cooling, (2) DM scattering heating remembering that $dE/dt < 0$ during runaway collapse, and (3) nuclear reactions' heating, where \dot{q}_i is the specific energy generation rate due to nuclear reactions of species i .

The system is closed with the specification of an adequate equation of state $f(\rho_*, T_*, P_*, e_*) = 0$. Thus, equations (4), (19), and (21) constitute a closed set of first order differential equations, that determine the evolution of DM collapse in the elastic regime, i.e. the cluster particle number $N(t)$ and its radius $R(t)$, as well as the evolution of specific energy of non-degenerate SM at finite heat diffusion $e_*(t)$.

In this analysis, we have neglected the effect of pressure increase due to heating of stellar matter. This assumption is valid for WD stars where pressure is dominated by electron degeneracy, but might not be valid in main sequence stars, and density modifications can be computed with the help of TOV equation (see, e.g., Ref. [10] for a study of DM collapse in main sequence stars).

We note that it is also possible to determine the nuclear specific energy at each integration step using Fick's first law (see, e.g., Ref. [51]), however, with the cost of solving, at each integration step, a non-linear find root procedure which can be quite time consuming, specially in the relativistic regime where find root coefficients (ΔE) are numerical integrals.

E. Final stages

1. Fireball evolution

The reversible collapse process presented so far is valid as long as scatterings are predominantly elastic. Once the post-collision energy of stellar particles exceeds 200 MeV (Hagedorn limit, $T_H \sim 1.7 \times 10^{12}\text{K}$), baryonic particle creation is favored over further heating of stellar matter, and a region of quark-gluon plasma (QGP), also called fireball, is created [52].

We expect that collapse then enters an irreversible ('inelastic') stage that quickly leads to the formation of a BH. This assumption is justified unless partons (quarks and gluons) become degenerate and their Fermi sea filled up to mean DM kinetic energy. Since at fireball formation, DM particles are already extremely energetic, we argue that this situation does not occur, though care should be taken. A thorough investigation of the fireball regime exceeds by far the scope of the present article and should be treated in a dedicated study.

We caution again that DM collapse might follow very different rules in the centers of NSs where matter is expected to be in a degenerate QGP state from the beginning (see, e.g., [53]).

2. BH evolution

A BH is formed when $R/N < 2Gm$ and $N > N_{\text{Ch}}$, where $N_{\text{Ch}} \simeq (M_{\text{Pl}}/m)^d$ is the Chandrasekhar number where $d = 2$ for bosons [54] and $d = 3$ for fermions [2], respectively, and where $M_{\text{Pl}} = 1/\sqrt{G}$ is the Planck mass [55]. For all considered DM models (see Sec. III)

⁴ To convince oneself, one can naturally assume that the specific energy profile is $e_* \propto \exp(-r^2/2R^2)$ for a Gaussian source, then the Laplacian at $r = 0$ is $\nabla^2 e_*|_{r=0} = -e_*/R^2$.

and assuming no substructure (see, e.g., Ref. [38] for a counter example), we find that the DM cluster does not become degenerate.

The BH's initial mass is $M = Nm$ (neglecting the tiny amount of stellar matter engulfed in the process), and its temporal evolution is given by

$$\frac{dM}{dt} = \frac{4\pi\rho_*G^2M^2}{c_*^3} - \frac{1}{15360\pi G^2M^2} + m\frac{dN}{dt}, \quad (22)$$

where the terms on the right hand side account for contributions from (1) Bondi accretion, where c_* is the local sound speed of the star, (2) Hawking radiation, and (3) DM capture and thermalization, where dN/dt is given by equation (4). If the second term dominates, the BH evaporates ($M \rightarrow 0$) and a new DM collapse cycle begins. Otherwise, the star is swallowed by the BH ($M \rightarrow M_*$).

We anticipate here noting that in WD stars, for $m \gtrsim 10^{11}\text{GeV}$, the mean free path between ions, $(\rho_*/m_*)^{-1/3}$, exceeds the BH's sound horizon, $2GNm/c_s^2$, and Bondi accretion in eq. (22) is no longer valid. In this regime, accretion is either nearly-collisionless or quantum (see, e.g., Ref. [56], and references therein). However, we have checked that these effects have little impact on the implosion/evaporation limit in Fig. 2.

III. PHENOMENOLOGY WITH WD STARS

A. Input physics

1. Nuclear structure

We assume WDs composed of equal parts of carbon and oxygen, with mean mass number $A = 14$. The cross-section between DM and SM particles, σ_* , depends on the scattering momentum transfer, Δp , and the de Broglie wavelength of nuclei, $\Delta r = A^{1/3}r_n$, where $r_n = 1.25\text{fm}$ is mean separation between nucleons (protons or neutrons) in the nucleus. If $\Delta p \Delta r \geq 1/2$, scattering is coherently enhanced, $\sigma_A \simeq A^2[3j_1(y)/y]^2 \exp(-y^2/3) \sigma_n$, where σ_n is the DM-nucleon cross-section, $y \equiv 2\Delta p \Delta r$ and j_1 is the Bessel function of the first kind [57, 58]. Otherwise, $\sigma_A = A\sigma_n$. We neglect additional form factors (see, e.g., Ref. [9], for a very detailed discussion).

2. Nuclear reactions

At the time of collapse, WDs have cooled down and crystallization has started from the center [59]. We model the background core temperature by a simple fitting formula $T_{*\infty} \simeq \min[10^8, 3 \times 10^6 (t/\text{Gyr})^{-1}] \text{K}$ (see, for example, [60]). The radial temperature profile of the heated WD material drops quickly to the background core temperature $T_{*\infty}$ on a scale of the characteristic radius R , such that elemental diffusion is negligible between the

heated region and the outside.⁵ Therefore, the concentration X_i of species i drops with time according to

$$\frac{dX_i}{dt} = -\rho_* \frac{\lambda_i}{\bar{M}_i}, \quad (23)$$

where λ_i and \bar{M}_i are reaction rate and mean molar mass of species i , respectively.

In the present analysis, we limit our investigation to the $^{12}\text{C}(\gamma, \alpha)^{12}\text{C}$ reaction, since it is the most interesting for SN Ia phenomenology. We use the reaction rate of Ref. [61], and assume initially $X_C = 0.5$. The specific nuclear energy generation rate can be written as [61]

$$\dot{q}_i = f_i \rho_* \frac{N_A \bar{Q}_i}{2} \frac{X_i^2}{\bar{M}_i^2} \lambda_i, \quad (24)$$

where N_A is Avogadro's constant, \bar{Q}_i is the mean energy liberated per reaction, and f_i is a factor accounting for electron screening. For carbon fusion, $\bar{Q}_C \simeq 3\text{MeV}$, $\bar{M}_C = 12\text{g mol}^{-1}$ and $f_C \simeq \exp[3.5(\rho_*/10^9\text{g cm}^{-3})^{1/3}(T_*/10^9\text{K})^{-1}]$ [14].

3. Heat diffusion

WDs have thermal diffusion dominated by relativistic electrons when $\rho_* \gtrsim 10^6\text{g cm}^{-3}$ [62]. We use an interpolation of the results of [63]⁶ for the thermal conductivity, finding that $\kappa \simeq 2.4 \times 10^{17} \text{erg cm}^{-1}\text{s}^{-1}\text{K}^{-1}(\rho_*/10^8\text{g cm}^{-3})^{1/2}(T_*/10^7\text{K})^{1/2}$, strictly valid for $T_* \in [10^3, 10^9]\text{K}$ and $\rho_* \in [10^{-6}, 10^9]\text{g cm}^{-3}$. In the absence of predictions for higher temperatures, we extrapolate their results, finding, reassuringly, agreement with $\kappa_{\text{QGP}} \simeq 10^{20} \text{erg cm}^{-1}\text{s}^{-1}\text{K}^{-1}$ [e.g. 64] at the Hagedorn temperature. The equation of state of ideal gas ions is simply $e_* = 3T_*/2m_*$ while the specific heat capacity at constant pressure is $c_p = 5/2A$ [62].

4. Numerical integration

We integrate the system (N , R , e_* and X_C) according to the previously derived equations (4), (19), (21) and (23) using a 4th order Runge-Kutta method with adaptive time step. Initial conditions are $(0, R_{\text{th}}, e_{*\infty}, 0.5)$. We investigate three WD masses: 0.6, 1.0 and 1.4 solar masses.

In Fig. 1, we illustrate the time evolution of the collapse scale for some selected DM models highlighting the previously debated collapse phases. In Fig. 2 we show

⁵ It is easy to show that the temperature profile drops with $\propto 1/r$ outside the region where heat is released. Also, if the star is at a temperature $\sim 10^7$, nuclei are stuck in a crystal lattice.

⁶ <http://www.ioffe.ru/astro/conduct/>

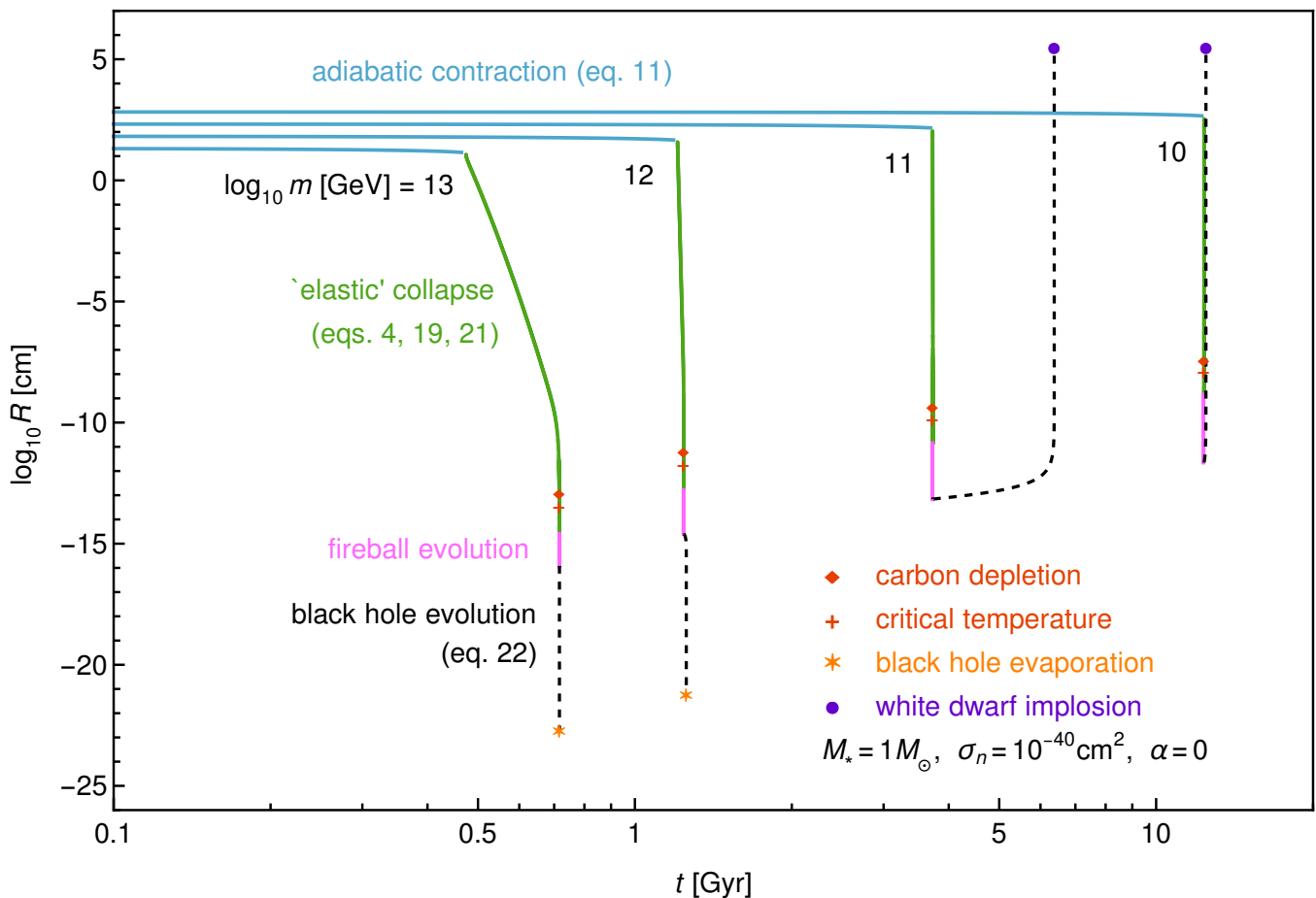


FIG. 1. Radius R of the collapsing DM sphere versus age t of the host star, a $1 M_{\odot}$ WD, for various DM masses $\log_{10}(m/\text{GeV})$ as indicated by numbers and DM-nucleon cross section $\sigma_n = 10^{-40} \text{ cm}^2$. Lines represent adiabatic contraction (cyan), runaway elastic (green) and inelastic (pink) collapse, and BH evolution (black dashed). Carbon depletion and the critical temperature $4.3 \times 10^9 \text{ K}$ are shown as red diamond and red plus sign, respectively. The destiny of the system is either WD implosion (purple disk) or BH evaporation (yellow star). In the latter case, the process is cyclic (omitted for clarity).

the parameter regions where collapse leads to runaway (black), WD implosion (purple) and Hawking evaporation (orange) in less than 0.1 Gyr (dotted), 1.0 Gyr (dashed) and 10 Gyr (full) for a $1.4 M_{\odot}$ WD (top panel) and a $1.0 M_{\odot}$. Evidently, the most stringent constraints can be obtained with heavy and old WDs.

B. Observational signatures

1. Thermonuclear SN Ia ignition?

According to Ref. [66], thermonuclear runaway fusion (deflagration) can proceed if a mass of carbon

$$m_C \simeq \frac{4}{3} \pi R^3 \rho_* X_C \in [10^{-5}, 10^{15}] \text{ g} \quad (25)$$

is heated to the critical temperature

$$\left(\frac{T_*}{4.3 \times 10^9 \text{ K}} \right)^{70/3} \gtrsim \left(\frac{\rho_*}{10^8 \text{ g cm}^{-3}} \right)^{1/2} \left(\frac{m_C}{1 \text{ g}} \right)^{-1}, \quad (26)$$

where in the first equality of eq. (25) we have assumed that the whole region enclosing the DM cluster is heated to temperature T_* . Checking the runaway criteria of eqs. (25) and (26) at each integration step, we find that for all investigated WD masses and DM models (including a Yukawa type self-interacting), stable nuclear burning exhausts the combustible before runaway criteria can be met. The explanation resides in the reversible nature of DM collapse based on elastic scatterings, mathematically best appreciable in eq. (15): if $K_* > K$, then $\Delta E > 0$, and DM particles gain energy. In practice, from our numerical simulations, we find that once nuclei become almost as hot as DM particles, the collapse process slows down or interrupts momentarily until exothermic nuclear reactions are over.

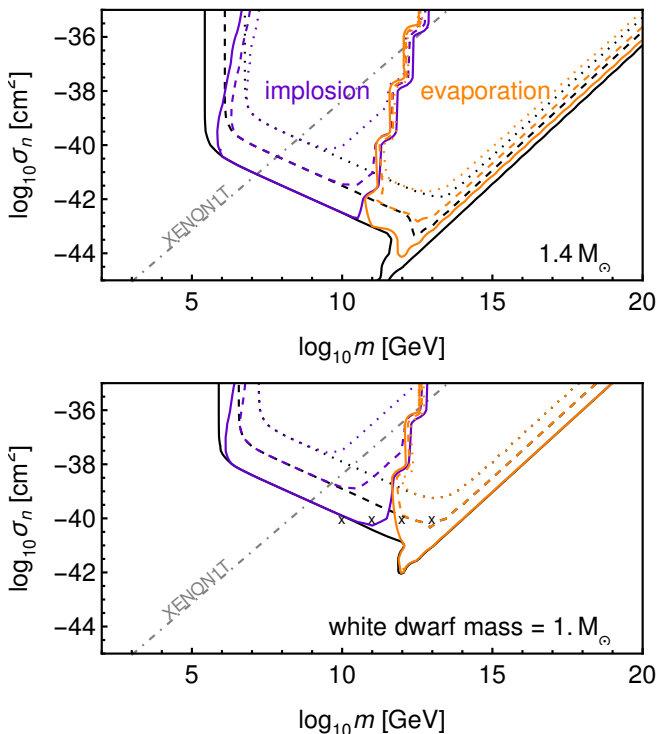


FIG. 2. Top panel: Parameter regions where DM collapse leads to a mini BH (black) inside a WD with $M_* = 1.4 M_\odot$, $\alpha = 0$, $\mu = 0$, and either evaporates (orange) or causes the star to implode (purple) in less than 0.1 Gyr (dotted), 1.0 Gyr (dashed) and 10 Gyr (full). Including a self-interaction with amplitude $\alpha = 10^{-3}$ and range $\mu = 1$ MeV, does not change these parameter regions. The region above the gray dot-dashed line is excluded from XENON1T spin-independent 2σ bound on DM-nucleon scattering [65]. Bottom panel: Same as top panel but for $M_* = 1.0 M_\odot$. The small ‘x’s correspond to the evolutionary paths traced in Fig. 1.

2. Pycnonuclear SN Ia ignition?

Pycnonuclear (density driven) carbon reactions start if the nuclear density exceeds $\rho_* \gtrsim 3 \times 10^9 \text{ g cm}^{-3}$. Solving the TOV equation with the addition of a top hat DM density profile and assuming zero temperature equation of state, we find that sizable density increase never occurs before thermonuclear reactions. However, after the passage of the thermonuclear flame, the central region of the WD is carbon depleted, while the outer shells of the core are crystallized and elemental diffusion suppressed. Since density increase at later stages encloses a smaller region than the carbon depleted one, pycnonuclear ignition remains illusive. This is a conservative estimate, since assuming the hot equation of state, additional ideal gas pressure counters the gravitational pull, and it is questionable if density is increased at all.

Name	mass	cooling age	reference
WD 0346	$0.77 M_\odot$	11.0 Gyr	[67]
WD 1832+089	$1.33 M_\odot$	330 Myr	[68]
SDSS J2322+2528	$1.13 M_\odot$	4.58 Gyr	[69–71]

TABLE I. Solar neighborhood WDs that constrain asymmetric DM models, see Fig. 3 (only the two most constraining are shown).

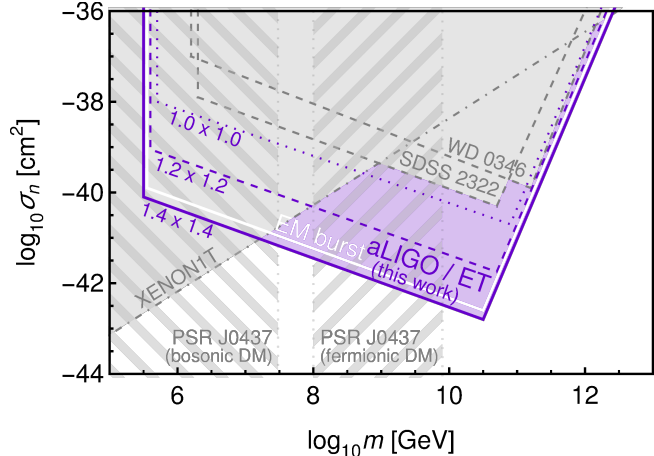


FIG. 3. Constraints on asymmetric DM. The gray shaded regions are excluded from non-detections with the XENON1T experiment (dot-dashed contour; [65]), from the existence of various old WDs in the solar neighborhood (dashed contour, see Table I for details). The diagonal (antidiagonal) hatched region is excluded from the existence of pulsar PSR J0437-4715 in case of bosonic (fermionic) DM [5]. The purple shaded region is marginally excluded from the non-detection of binary BHs by aLIGO, originating from DM collapse-induced implosion of binary WDs with component masses as indicated by numbers and ~ 10 Gyr merger delay times; future facilities like the Einstein telescope (ET) can confirm these constraints, see Table II. The parameter region above the white full line is excluded due to the non-observation of specific electromagnetic (EM) bursts following the implosion of magnetic WDs (see Sec. III B 4 for details). Mean Galactic DM parameters ($\rho_{\text{gal}} \sim 0.4 \text{ GeV/cm}^3$ and $v_{\text{gal}} \sim 200 \text{ km/s}$) have been assumed.

3. Constraints from existing WDs

With current telescopes, WDs can only be observed directly in the neighborhood of the Sun. These experience a DM density $\rho_{\text{gal},0} \simeq 0.4 \text{ GeV/cm}^{-3}$ and a velocity dispersion $v_{\text{gal},0} \simeq 200 \text{ km s}^{-1}$. The best constraints come from heavy and old observed WDs (see table I).

Interpolating the previously obtained results, we show in Fig. 3 the DM models that are excluded simply because these WDs did not implode to a BH. As can be seen, these constraints are competitive with current direct detection experiments in the mass range $10^9 - 10^{12} \text{ GeV}$ where they roughly exclude DM models with DM-nucleon cross-section $\sigma_n \gtrsim 10^{-40} \text{ cm}^2$.

SDSS J2322+25328 is the most constraining. Heavier

and, at the same time, older have not been found so far, but are expected to exist, especially in the thick disc and stellar halo components of the Galaxy which were formed before approximately 11 Gyr. It is possible that these have imploded under the pull of DM in the green parameter region (fig. 3).

4. Electromagnetic bursts

The recent 20 pc volume-limited survey has revealed a high incidence (22%) of magnetic WDs [72]. Most (85%) of these concern single WDs, and the field strength distribution is logarithmically uniform in the range $\sim 4 \times 10^4 \text{G} - 10^9 \text{G}$ [72]. Besides, no evidence for a correlation between field strength and WD mass, neither any sign for field strength decay with time have been found [72].

According to the no-hair theorem, which prevents magnetic field lines from puncturing the event horizon, the newly formed BH must expel its magnetic field [73], liberating the energy contained in the magnetosphere, at least [e.g. 74]

$$E_B \sim \frac{B^2}{8\pi} \frac{4\pi}{3} R_*^3 \simeq 6 \times 10^{44} \text{ erg} \left(\frac{B}{10^8 \text{G}} \right)^2 \left(\frac{R_*}{4 \times 10^8 \text{cm}} \right)^3, \quad (27)$$

where B is the surface magnetic dipole field strength of the WD and R_* its radius (in eq. 27, the radius of a $1.2 M_\odot$ WD is shown). The relevant timescale of the final stage of implosion is the free-fall time [74, 75]

$$\Delta t_{\text{ff}} \sim \left(\frac{R_*^3}{8GM_*} \right)^{1/2} \simeq 225 \text{ ms} \left(\frac{M_*}{1.2 M_\odot} \right)^{-1/2} \left(\frac{R_*}{4 \times 10^8 \text{cm}} \right)^{3/2}, \quad (28)$$

and we assume that this sets the duration of the energy emission. Magneto-hydrodynamics simulations of non-rotating magnetic NSs show that about 5% of the available energy is emitted in the main burst [73]. We assume that a similar fraction is emitted in the case of WDs, leading to a luminosity of

$$L_B \sim \eta \frac{E_B}{\Delta t_{\text{ff}}} \simeq 1.3 \times 10^{44} \text{ erg s}^{-1} \left(\frac{\eta}{0.05} \right) \left(\frac{B}{10^8 \text{G}} \right)^2 \times \left(\frac{M_*}{1.2 M_\odot} \right)^{1/2} \left(\frac{R_*}{4 \times 10^8 \text{cm}} \right)^{3/2}, \quad (29)$$

where η is an efficiency factor.

According to eq. (27), the most energetic bursts disrupt in WDs with the strongest magnetic fields. Assuming 10^{10} WDs in the Galaxy [76], we estimate that 6×10^6 single WDs have masses heavier than $1.2 M_\odot$ and magnetic dipole fields stronger than 10^8G [72, 77]. Adopting the Galactic stellar structure model of Ref. [78], the fraction of WDs exposed to a DM density larger than the

local DM density and taking into account only thick disc and stellar halo WDs (which are 10 Gyr old) is 52%. Assuming that the initial star burst lasted $\sim 1 \text{Gyr}$ [79], and considering that the number density of MW-like galaxies is 10^{-2}Mpc^{-3} , we find a volumetric burst rate of $1.3 \times 10^{-4} \text{Mpc}^{-3} \text{yr}^{-3}$.

The details of how this energy is converted into radiation is uncertain (see, for example, Refs. [74, 80] for options ranging from gamma-ray to radio bursts). For instance, the bulk of fast radio bursts (FRBs) emit a luminosity in the range $10^{41} - 10^{44} \text{erg s}^{-1}$ [e.g. 81] and their volumetric rate is $10^{-4} \text{Mpc yr}^{-3}$ [82]. Thus, the rate of the most energetic ($> 10^8 \text{G}$) bursts from WD implosions alone equals the total rate of all known FRBs taken together. Clearly, these WD implosions should have been noticed. When it comes to durations, typical FRBs last 1–10 ms with none longer than 100 ms detected so far, and non-repeating are typically shorter than repeating [83]. According to eq. (28), WD implosions expedite longer (50–1000 ms) lasting bursts, roughly 2 orders of magnitude longer. If their emission occurs in radio wave lengths, then the non-detection of long FRBs stringently constrains asymmetric DM models (see Fig. 3).

Gamma ray bursts (GRBs) have much wider spread durations ranging from 10 ms to several hours. The particular class of short ($< 2 \text{s}$) GRBs accounts for 30% of the total rate, and is associated with regions of little or no star formation, such as large elliptical galaxies and the central regions of galaxy clusters [84]. This rules out a link to massive stars, but makes them eligible for emission from the transmutation of old magnetic WDs. The commonly accepted mechanism of short GRBs is the merger of two NS [85] or the merger of a NS with a BH, which is consistent with minutes to hours lasting afterglows, caused by fragments of tidally disrupted material remaining in orbit while inspiraling into the BH over a longer period of time. On the other hand, these afterglows are difficult to explain with WD implosions.

In sum, neither FRBs nor GRBs match with the expected properties of DM collapse induced WD implosions. We note that the field energy estimation in eq. 27 is an absolute lower bound, because in ideal magneto-hydrodynamics, the field is “frozen-in” with the fluid and increases linearly with density (while eq. (27) assumes constant field strength); analytical and numerical calculations in Newtonian gravity and General Relativity show that internal magnetic field strengths of up to 10^{12-16}G are possible (see [86] and references therein). However, these simple energy arguments should be tested in magneto-hydrodynamics simulations of imploding WDs.

5. Detection of GWs from sub-Chandrasekhar BHs

The GW signature of binary WD mergers is very different from that of binary BH mergers with the same mass. Typically, the secondary (lighter and larger) is

M_1	M_2	aLIGO	aLIGO-DS	ET
$[M_\odot]$	$[M_\odot]$	$[\text{yr}^{-1}]$	$[\text{yr}^{-1}]$	$[\text{yr}^{-1}]$
[1.0, 1.2]	[1.0, 1.2]	0.6	16	3 025
[1.2, 1.4]	[1.0, 1.2]	0.3	7	1 323
[1.2, 1.4]	[1.2, 1.4]	0.1	3	579
[1.0, 1.4]	[1.0, 1.4]	1.2	33	6 250

TABLE II. GW detection rates per year (columns 3 to 5) from transmuted binary BH mergers with component masses comprised in the specified bins (columns 1 and 2). The last line is the total rate. These rates assume the following detector ranges for $\mathcal{O}(1) M_\odot$ binary BH mergers: 110 Mpc for aLIGO [40], 330 Mpc for aLIGO at design sensitivity (DS; [41]), and 1.9 Gpc for ET [88].

spaguetified by tidal forces of the primary (heavier and more compact) prior to coalescence. The orbital motion is expected to be observed at sub-Hz frequencies by future space-born laser interferometric detectors of gravitational waves [87], while super-Hz emission of binary BH mergers is already detectable by aLIGO [40].

In Table II, we compare the detection rates per year of current and future GW detectors for different combinations of component masses. In order to obtain strong constraints (Figure 3), both components must be heavy and have long (~ 10 Gyr) formation-to-merger delays to allow for DM accumulation and subsequent transmutation of both WDs to BHs. Since heavier WDs are also rarer we consider two mass bins, $[1.0, 1.2] M_\odot$ and $[1.2, 1.4] M_\odot$ (Table II).

We estimate the rates as follows. The total number of WDs in a MW-like galaxy is 10^{10} [76], and the Galactic merger rate per WD is 10^{-11}yr^{-1} [89]. Assuming the 100 pc volume-limited mass function in the SDSS footprint [77], the fraction of WDs with mass greater than $1.0 M_\odot$ is 4.4%; since both binary companions must satisfy this, we have a fraction $(0.044)^2$. Since 30–50% of high-mass WDs have a merger history [90], we have to multiply by an additional factor of $\sim (1 - 0.4)^2$ (neglecting systems of higher multiplicity than 2). Results an expected binary WD merger rate of $7 \times 10^{-5} \text{yr}^{-1}$ for component masses in $[1.0, 1.4] M_\odot$ per MW-like galaxy.

This rough estimate is consistent with detailed binary population synthesis calculations of Ref. [91], who compute the rate of binary mergers with primary mass between 0.85 and $1.05 M_\odot$ and a mass ratio $0.9 \leq M_2/M_1 \leq 1.0$ to be 2–11% of the type Ia supernova rate (see supplementary information of [91]). For the type Ia supernova rate from the LOSS survey, $(5.4 \pm 0.1) \times 10^{-3}$ [92], the resulting double WD merger rate is $(3.5 \pm 2.4) \times 10^{-4} \text{yr}^{-1}$, while our crude estimate for this mass range yields $7 \times 10^{-4} \text{yr}^{-1}$. The remaining discrepancy of a factor of 2 could be due to the adopted mass function; if we assume the binary population synthesis mass function of Ref. [90], we find a rate $3 \times 10^{-4} \text{yr}^{-1}$.

Adopting the Galactic stellar structure model of Ref. [78], we estimate the fraction of WDs exposed to a DM density larger than the local DM density and tak-

ing only into account thick disc and stellar halo WDs (which are 10 Gyr old), to 52%. Since the main channel for high-mass double degenerates is a single common envelope phase, we estimate that 60% of these have long formation-to-merger delay times (~ 10 Gyr, see Appendix E for details). Taking into these cuts, and considering that the number density of MW-like galaxies is 10^{-2}Mpc^{-3} , we finally find the detection rates specified in Table II. The non-observation of sub-Chandrasekhar binary BH mergers over a period of time δt would bound their rate to $\leq 2.3/\delta t$ at 90% confidence level.

The constraints on DM models from capture in binary systems are slightly more stringent than those from single WDs, due to enhanced DM capture rates in binary systems [93]. A maximum enhancement factor of ~ 4.3 has been found for orbital periods of 8 h [93], attributed to the energy loss by DM particles resulting from their gravitational scattering off moving companions. Binary WDs that merge in a Hubble time due to gravitational radiation have initial orbital periods of at most 13.5 h [94]. Based on the results in table 1 of Ref. [93], and since the binary spends most of its evolution at large orbital periods, we estimate that the integrated amplification factor is ~ 3 –4. This means 3–4 times earlier collapse and the parameter region of constraints from binary systems is correspondingly larger than it would from single stellar systems (see Figure 3).

6. Disentangling transmuted from primordial BH inspirals

As the two BHs merge, the morphology of the resulting gravitational waveform depends on the phenomenological effective inspiral spin parameter [95]

$$\chi_{\text{eff}} \equiv \frac{M_1 \chi_1 \cos \theta_1 + M_2 \chi_2 \cos \theta_2}{M_1 + M_2}, \quad (30)$$

where θ_1 and θ_2 are the misalignment angles between the component spins and the orbital angular momentum, M_1 and M_2 are the component masses, and χ_1 and χ_2 are the dimensionless component spins, defined by $\chi \equiv J/M^2$ and limited to values $\in [0, 1]$, where $J = 2\pi I/P$ is the angular momentum, I the moment of inertia and P the rotation period. Since the angular momentum J is conserved during WD transmutation to a BH, we can estimate χ from typical values of the progenitor WD star.

The primary mechanism for producing tight binary systems is a common-envelope evolution [96, 97], where each component results from single stellar evolution. The rotation periods of typical low mass WDs originating from single stellar evolution are of the order of 1 d, as inferred from rotational broadening of spectral lines [98] and asteroseismology [99, 100]. For high mass WDs, the rotation periods tend to be much shorter [100], consistent with the tendency for faster rotating stellar progenitors to produce heavier cores [101]. Lacking number statistics for heavy WDs, we assume here a representative value for the rotation period of 1 h (roughly extrapolating data in

the right panel of fig. 8 of Ref. [100]), yielding dimensionless spin values

$$\chi \simeq 0.2 \left(\frac{P}{1 \text{ h}} \right)^{-1} \left(\frac{M_*}{1 M_\odot} \right)^{-2}, \quad (31)$$

where we have assumed conservatively the moment of inertia of non-rotating WDs, $I = 10^{50} \text{ g cm}^2$, which has little dependence on mass [102].

Binary WDs formed via ‘isolated’ evolution (the dominant formation channel) have spin vectors which are likely to be closely aligned with the orbital angular momentum, hence $\theta_1 \simeq \theta_2 \simeq 0$. We assume $\chi_1 \simeq \chi_2$ and $M_1 \simeq M_2$ for simplicity. Then, eq. (30) reduces to $\chi_{\text{eff}} \simeq \chi$. Using eq. (31), we find a representative value $\chi_{\text{eff}} \simeq 0.2$, a value that is already measurable with current 90% credible intervals, $\simeq 0.15$, for the default model [e.g. 103].

In particular, transmuted binary BH inspirals can be disentangled from primordial binary BH binary inspirals. In standard cosmology, the QCD phase transition is expected during a radiation-dominated cosmological epoch and hence these primordial BHs are expected to have very low intrinsic spin magnitude, roughly distributed as a Gaussian peaked at $\chi_{\text{eff}} = 0$ and with variance $\sigma_{\chi_{\text{eff}}} \simeq 0.35$ [31]. We note that while current sensitivity is sufficient to discriminate between these peak values, a certain number of events must be observed to overcome the intrinsic dispersion of χ_{eff} to disentangle between the different solar mass BH production channels. This further motivates upcoming GW detectors like the Einstein telescope.

IV. SUMMARY AND DISCUSSION

Runaway collapse of a DM cluster at the center of a star at finite temperature is governed by a system of differential equations. We have derived these equations for the relatively simple situation where nuclei are non-degenerate, pressure feedback is small, and DM-nuclear collisions are elastic. Nevertheless, these ‘elastic’ collapse equations, i.e. eq. (4), eq. (19) and eq. (21), are valid for most of the collapse evolution in WD stars (see Fig. 1). In the presence of nuclear reactions, the system can be coupled with a set of equations governing elemental concentrations, i.e. eq. (23).

Local heating of nuclear matter from scattering with DM is controlled by finite heat diffusion. When carbon reactions dominate the heat release, further collapse is interrupted until reactions are over. Consequently, when the critical temperature for thermonuclear runaway is reached, carbon is already depleted (Fig. 1). Thus, in crystallized WDs where elemental diffusion is suppressed,

type Ia supernova ignition from DM collapse remains illusive. Subsequent ignition mechanisms, i.e. when the DM cluster has collapsed to a smaller radius, face a situation where heat release occurs in a region much smaller than the previously carbon depleted.

Instead, a mini BH is formed at the center of the star, and the stellar matter is accreted leaving behind a macroscopic BH. In case of total accretion of the star onto the mini BH, several observational signals are detectable with current technology. First, the mere existence of old and heavy WDs in the solar neighborhood imposes weak but solid constraints on asymmetric DM (see Fig. 3). Second, the non-detection of 50–1000 ms lasting electromagnetic bursts from the ejection of the WD magnetic field upon transmutation to a BH places stringent constraints (see Fig. 3); these restrictions are uncertain as the exact details of the burst are model dependent (with possibilities ranging from FRBs to GRBs).

Third, the most stringent and solid constraints result from the non-detection of GW signals from binary BH coalescences with sub-Chandrasekhar component mass. We find that aLIGO should detect ~ 1 event per year (~ 30 per year at design sensitivity), while future gravitational wave facilities like the Einstein telescope [88] would detect ~ 6000 per year. Their exclusion limits are competitive with current direct detection experiments [65] and pulsar constraints [5] in case of bosonic DM; in case of fermionic DM, pulsar constraints are currently more stringent for part of the parameter space (see Fig. 3).

GWs emitted by transmuted BH mergers can be disentangled from those of primordial BHs with the same mass due to different effective inspiral spin parameter distributions. We have predicted a peak value of $\chi_{\text{eff}} \simeq 0.2$ for transmuted origin, while for primordial origin $\chi_{\text{eff}} \simeq 0$ is expected. We note that our prediction depends on the relatively uncertain rotation periods of solar mass WDs that originate from single stellar evolution. Ongoing space-based short-cadence photometric missions like TESS and CHEOPS will greatly improve asteroseismic studies [104].

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Appendix A: Potential energy

Consider a system composed of N particles with mass m arranged in a spherically symmetric mass distribution $\rho(r)$. The total potential energy due to its own gravity is

$$U = -4\pi G \int_{r=0}^{\infty} M(r) \rho(r) r dr, \quad (\text{A1})$$

where $M(r)$ is the integrated mass (mass inside r) is

$$M(r) = 4\pi \int_{r'=0}^r \rho(r') r'^2 dr'. \quad (\text{A2})$$

For a top hat distribution ($\rho(r) = \rho$ for $r \leq R$ and $\rho(r) = 0$ for $r > R$), we have

$$U = -\frac{3GM^2}{5R}. \quad (\text{A3})$$

For a Maxwell-Boltzmann mass distribution, with

$$\rho(r) = \rho_0 \exp\left(-\frac{r^2}{2R^2}\right) = \sqrt{\frac{2}{\pi}} \frac{M}{4\pi R^3} \exp\left(-\frac{r^2}{2R^2}\right), \quad (\text{A4})$$

the total potential energy due to its own gravity is

$$U = -\frac{GM^2}{2\sqrt{\pi}R}. \quad (\text{A5})$$

Now consider the gravitational potential of the system due to an external mass distribution $\rho_*(r)$,

$$U_* = -4\pi G \int_{r=0}^{\infty} M_*(r) \rho(r) r dr, \quad (\text{A6})$$

where $M_*(r)$ is given by equation (A2) with stars added to M and ρ . If the external mass density is constant $\rho_*(r) = \rho_*$ (which is a good approximation for the center of a star), and the system has a top-hat distribution (unrealistic), we have

$$U_* = -\frac{4\pi G \rho_* M R^2}{5}. \quad (\text{A7})$$

If the system has a Maxwell-Boltzmann density distribution (and $\rho_*(r)$ still constant), we have

$$U_* = -4\pi G \rho_* M R^2. \quad (\text{A8})$$

The mean potential energy *per particle* of a system composed by N particles with mass m and a top-hat

mass distribution ρ in an external constant distribution ρ_* is (dividing by N and replacing $M = Nm$)

$$\frac{1}{N}(U + U_*) = -\frac{3GNm^2}{5R} - \frac{4\pi G\rho_* mR^2}{5}. \quad (\text{A9})$$

The mean potential energy *per particle* of a Maxwell-Boltzmann mass distribution in an external constant distribution ρ_* is

$$\frac{1}{N}(U + U_*) = -\frac{GNm^2}{2\sqrt{\pi}R} - 4\pi G\rho_* mR^2. \quad (\text{A10})$$

In the main article we use the letter U for the total mean potential energy *per particle*.

Appendix B: Thermalization timescale

In this Section, we derive the formula (3). A derivation of part of the formula has been given previously by [2]. The total thermalization time can be divided into three stages (1) orbital decrease crossing the star twice every orbital period, (2) orbital decrease completely inside the star with $v > v_*$, and (3) orbital decrease completely inside the WD with $v < v_*$.

1. First stage

During the first stage, the DM particle has a chance to loose kinetic energy twice each orbital period $P = 2\pi\sqrt{a^3/(GM)}$, where a is the semi-major axis and M the mass of the star, such that the timescale between collisions is

$$\langle\Delta t\rangle = \frac{1}{2}P\frac{\sigma_{\text{sat}}}{\sigma_{\chi A}} = \pi\left(\frac{a^3}{GM}\right)^{1/2}\frac{\sigma_{\text{sat}}}{\sigma_{\chi A}} \quad (\text{B1})$$

where $\sigma_{\text{sat}} = R^2 m/M$ is the saturation cross-section. The total energy of the DM particle with semi-major axis a is

$$E = -\frac{GMm}{a}. \quad (\text{B2})$$

Assuming radial orbits and constant density star, the potential energy at a radial position r inside the star is

$$U = -\frac{GMm}{R}\left(\frac{3}{2} - \frac{r^2}{2R^2}\right), \quad (r < R) \quad (\text{B3})$$

where R is the radius of the star. The (instantaneous) kinetic energy is

$$K = E - U = \frac{GMm}{R}\left(\frac{3}{2} - \frac{r^2}{2R^2} - \frac{R}{a}\right), \quad (r < R) \quad (\text{B4})$$

Averaging over radial positions, the mean kinetic energy is

$$\langle K \rangle = \frac{1}{R}\int_0^R K dr = \frac{GMm}{R}\left(\frac{4}{3} - \frac{R}{a}\right). \quad (\text{B5})$$

The mean variation per scatter is (assuming the non-relativistic limit, eq. (15), and neglecting K_*)

$$\langle\Delta E\rangle = -\frac{\beta_+}{2}\frac{GMm}{R}\left(\frac{4}{3} - \frac{R}{a}\right) \quad (\text{B6})$$

and, treating scattering as a continuous process, combining eq. (B1) and eq. (B6), we have

$$\frac{dE}{dt} = \frac{\langle\Delta E\rangle}{\langle\Delta t\rangle} \quad (\text{B7})$$

and, from (B2) we have

$$\frac{da}{dE} = \frac{GMm}{E^2} = \frac{a^2}{GMm} \quad (\text{B8})$$

Assembling these equations

$$\frac{da}{dt} = \frac{da}{dE}\frac{dE}{dt} = -\frac{A_1}{\sqrt{a}}(B_1 a - 1) \quad (\text{B9})$$

where

$$A_1 = \frac{\beta_+\sqrt{GM}\sigma_A}{2\pi\sigma_{\text{sat}}}, \quad (\text{B10})$$

$$B_1 = \frac{4}{3R} \quad (\text{B11})$$

Integrating, we have

$$t_1 = \int dt = \frac{1}{A_1}\int_{a_0}^R \frac{\sqrt{a} da}{(B_1 a - 1)} \quad (\text{B12})$$

Using $aB_1 = \cosh^2 x$, we obtain

$$t_1 = \frac{2}{A_1 B_1^{3/2}} \left\{ \sqrt{B_1 a_0} - \sqrt{B_1 R} + \frac{1}{2} \ln \left[\frac{R(B_1 a_0 - 1)}{a_0(B_1 R - 1)} \right] \right\} \quad (\text{B13})$$

neglecting the logarithm which is $\ln 2$, and assuming $m \ll m_*$, we have $\beta_+ \simeq 4m_*/m$, and

$$t_1 \simeq \frac{3\pi R\sigma_{\text{sat}} m}{4\sigma_A m_*} \left(\frac{a_0}{GM}\right)^{1/2} \quad (\text{B14})$$

Assuming $a_0 = Rv_e^2/v_{\text{gal}}^2$, and using $\sigma_{\text{sat}} = \pi R^2 m_*/M \simeq 3m_*/(4R\rho_*)$, we have

$$t_1 \simeq \frac{9\sqrt{2}\pi m}{16\rho_*\sigma_A v_{\text{gal}}} \quad (\text{B15})$$

2. Second stage

For orbits totally inside the star ($a < R$), and assuming constant density, the total energy for a DM particle with semi-major axis a is

$$E = -\frac{GMm}{R}\left(\frac{3}{2} - \frac{a^2}{2R^2}\right) \quad (a \leq R). \quad (\text{B16})$$

The (instantaneous) potential energy at a radial position $r \leq a$ is

$$U = -\frac{GMm}{R} \left(\frac{3}{2} - \frac{r^2}{2R^2} \right), \quad (r \leq a) \quad (\text{B17})$$

and the (instantaneous) kinetic energy at a radial position $r \leq a$ is

$$K = E - U = \frac{GMm}{2R^3} (a^2 - r^2). \quad (r \leq a) \quad (\text{B18})$$

Averaging over radial positions, the mean kinetic energy is

$$\langle K \rangle = \frac{1}{a} \int_0^a K dr = \frac{GMma^2}{3R^3} \quad (\text{B19})$$

As long as $v \geq v_*$, DM-nuclear encounters are dominated by DM movements, thus the mean scattering timescale is

$$\langle \Delta t \rangle = (n_* \sigma_A v)^{-1} \quad (\text{B20})$$

where $v = \sqrt{2\langle K \rangle/m}$. From equation (B16) we have

$$\frac{da}{dE} = \frac{R^3}{GMma} \quad (\text{B21})$$

and using treating scattering again as a continuous process

$$\frac{da}{dt} = \frac{da}{dE} \frac{dE}{dt} = -A_2 (B_2 a^2 - 1) \quad (\text{B22})$$

with

$$A_2 = \sqrt{\frac{2R^3}{3GM}} \frac{n_* \sigma_A \beta_+ \langle K_* \rangle}{4m}, \quad (\text{B23})$$

$$B_2 = \frac{GMm}{3R^3 \langle K_* \rangle} \quad (\text{B24})$$

Integrating, we have

$$t_2 = \int dt = -\frac{1}{A_2} \int_R^{R_2} \frac{da}{B_2 a^2 - 1} \quad (\text{B25})$$

where R_2 is given by $v = v_*$, or $\sqrt{2\langle K \rangle/m} = \sqrt{2\langle K_* \rangle/m_*}$. Using equation (B19) with $a = R_2$ and solving for R_2 , we have

$$R_2 = \left(\frac{9k_B T_{*\infty} R^3}{2GMm_*} \right)^{1/2} \quad (\text{B26})$$

If we pose $x = \sqrt{B_2} a$, then $x \in \sqrt{m/m_*} [1, R/R_2]$ is larger than 1 and we have

$$\int \frac{dx}{1-x^2} = \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right| = \text{arccoth } x, \quad (x > 1) \quad (\text{B27})$$

and

$$t_2 = \int dt = \frac{1}{A_2 \sqrt{B_2}} \left[\text{arccoth } x \right]_{\sqrt{B_2} R}^{\sqrt{B_2} R_2} \quad (\text{B28})$$

We have

$$t_2 = \frac{1}{2A_2 \sqrt{B_2}} \ln \left[\frac{(\sqrt{B_2} R_2 + 1)(\sqrt{B_2} R - 1)}{(\sqrt{B_2} R_2 - 1)(\sqrt{B_2} R + 1)} \right] \quad (\text{B29})$$

Using $\ln(1+x) \simeq x - x^2$ and $\ln(1-x) \simeq -x - x^2$ where $x \simeq 0$,

$$t_2 = \left(\frac{18m}{n_*^2 \sigma_A^2 \beta_+^2 \langle K_* \rangle} \right)^{1/2} \left[2\sqrt{\frac{m_*}{m}} - 2\sqrt{\frac{3R \langle K_* \rangle}{GMm}} \right] \quad (\text{B30})$$

or, assuming $m_* \ll m$,

$$t_2 \simeq \frac{3m}{2\rho_* \sigma_A v_*} \left(1 - \sqrt{3} \frac{v_*}{v_e} \right) \quad (\text{B31})$$

where $v_e = \sqrt{2GM/R}$.

3. Third stage

During this stage, scatterings are dominated by ion movements. Therefore, the mean scattering timescale is

$$\langle \Delta t \rangle = (n_* \sigma_A v_*)^{-1}, \quad (\text{B32})$$

And we have

$$t_3 = \int dt = \frac{1}{A_3} \int_{R_2}^{R_{\text{th}}} \frac{a da}{1 - B_3 a^2} \quad (\text{B33})$$

Les bornes sont $\sqrt{B_3} R_{\text{th}} = 1$ and $\sqrt{B_3} R_2 = \sqrt{m/m_*}$ and the time is parametrically infinite

$$t_3 = \frac{1}{A_3 B_3} \int_1^{\sqrt{m/m_*}} \frac{x dx}{x^2 - 1} = \frac{1}{A_3 B_3} \left[\ln(x^2 - 1) \right]_1^{\sqrt{m/m_*}} \quad (\text{B34})$$

However, we can consider that the DM particle is thermalized when it reaches the thermal energy of stellar particles within $\langle \delta \Delta E \rangle$, where δE is the root mean square energy transfer,

$$\langle \delta \Delta E \rangle = \sqrt{\langle \Delta E^2 \rangle - \langle \Delta E \rangle^2} = \frac{\beta_+}{4\sqrt{2}} \sqrt{K^2 + K_*^2} \quad (\text{B35})$$

Then we integrate t_3 to the radius R_3 , defined by $K = K_* + \delta \Delta K$, solving for K , we find

$$K = K_* \left[1 + \sqrt{1 - (1 - \beta_+/16\sqrt{2})^2} \left(1 - \beta_+^2/16\sqrt{2} \right)^{-1/2} \right] \quad (\text{B36})$$

Pour $\beta_+ \ll 1$,

$$K \simeq K_* \left(1 + 2^{5/4} \frac{m_*}{m} \right) \quad (\text{B37})$$

Then we have

$$R_3^2 = \frac{3R^3 \langle K_* \rangle}{GMm} \left(1 + 2^{5/4} \frac{m_*}{m} \right) \quad (\text{B38})$$

and

$$\sqrt{B_3} R_3 \simeq 1 + 2^{1/4} \frac{m_*}{m} \quad (\text{B39})$$

and the integral is finite

$$\begin{aligned} t_3 &= \frac{1}{A_3 B_3} \left[\ln \left(\frac{m}{m_*} - 1 \right) - \ln \left(1 + 2^{1/4} \frac{m_*}{m} - 1 \right) \right] \\ &\simeq \frac{2}{A_3 B_3} \ln \left(\frac{m}{m_*} \right) \end{aligned} \quad (\text{B40})$$

Finally

$$t_3 = \frac{3m}{n_* \sigma_A v_* m_*} \ln \frac{m}{m_*} \quad (\text{B41})$$

Finally, summing up, the total thermalization timescale is

$$t_{\text{th}} = t_1 + t_2 + t_3 = \frac{3m}{\rho_* \sigma_A v_*} \left[\frac{3\sqrt{2}\pi}{16} \frac{v_*}{v_{\text{gal}}} + \frac{1}{2} + \ln \left(\frac{m}{m_*} \right) \right] \quad (\text{B42})$$

Appendix C: Scattering energy transfer

1. Classical regime

Consider scattering of two rigid body spheres with mass m and m_* and initial scalar velocities v and v_* respectively. A general scattering event occurs in a plane. In full generality, we can choose a coordinate system such that the line of centers coincides with the x axis (in other words, the contact angle is zero). Momentum conservation along the line of centers and kinetic energy conservation imply

$$m v \cos \theta + m_* v_* \cos \theta_* = m v'_x + m_* v'_{*x}, \quad (\text{C1})$$

$$m v^2 + m_* v_*^2 = m v'^2 + m_* v_*'^2 \quad (\text{C2})$$

where θ and θ_* are the pre-collision movement angles (with respect to the x axis) of m and m_* , respectively, and primes indicate post-collision quantities. Momentum perpendicular to the line of centers is conserved for each mass, so we obtain immediately,

$$v'_y = v \sin \theta, \quad (\text{C3})$$

$$v'_{*y} = v_* \sin \theta_*, \quad (\text{C4})$$

With four unknowns ($v'_x, v'_y, v'_{*x}, v'_{*y}$) and four equations, the system has a unique non-trivial solution,

$$v'_x = \frac{m - m_*}{m + m_*} v \cos \theta + \frac{2m_*}{m + m_*} v_* \cos \theta_*, \quad (\text{C5})$$

Using $(m - m_*)^2 / (m + m_*)^2 = 1 - 4m m_* / (m + m_*)^2$ and defining $\beta_+ \equiv 4m m_* / (m + m_*)^2$, we can express the total post-shock velocity squared as

$$\begin{aligned} v'^2 &= v^2 + \beta_+ \left[\frac{m_*}{m} v_*^2 \cos^2 \theta_* - v^2 \cos^2 \theta \right. \\ &\quad \left. + \frac{m - m_*}{m} v v_* \cos \theta \cos \theta_* \right]. \end{aligned} \quad (\text{C6})$$

In terms of energy (for $E = K$)

$$\begin{aligned} \Delta E = E' - E &= \beta_+ \left[E_* \cos^2 \theta_* - E \cos^2 \theta \right. \\ &\quad \left. + (m - m_*) \sqrt{E E_* / m m_*} \cos \theta \cos \theta_* \right] \end{aligned} \quad (\text{C7})$$

Since all pre-collision movement angles are equally likely, the average energy transfer is

$$\langle \Delta K \rangle = \frac{1}{(2\pi)^2} \int_0^{2\pi} \int_0^{2\pi} \Delta E d\theta d\theta_* = \frac{\beta_+}{2} (K_* - K) \quad (\text{C8})$$

In the main text, we omit the brackets and average additionally over velocity distribution. Note that, whenever both species are at thermal equilibrium, (C8) implies that the energy transfer is zero.

2. Relativistic regime

In the relativistic case, we have momentum conservation along the line of centers

$$p \cos \theta + p_* \cos \theta_* = p'_x + p'_{*x} \quad (\text{C9})$$

and energy conservation

$$E + E_* = E' + E'_* \quad (\text{C10})$$

Momentum conservation perpendicular to the line of centers for each mass

$$p'_y = p \sin \theta, \quad (\text{C11})$$

$$p'_{*y} = p_* \sin \theta_* \quad (\text{C12})$$

First, we eliminate p'_* . On the one hand, we isolate the x and y components using equations (C9) and (C12) and writing the sum of squares

$$\begin{aligned} p_*'^2 &= p_*'^2 + p_*'^2 = p^2 \cos^2 \theta + 2pp_* \cos \theta \cos \theta_* + p_*^2 + p_x'^2 \\ &\quad - 2(p \cos \theta + p_* \cos \theta_*) p'_x \end{aligned} \quad (\text{C13})$$

On the other hand, from equation (C10) we have

$$\begin{aligned} p_*'^2 &= E_*'^2 - m_*^2 = (E + E_* - E')^2 - m_*^2 \\ &\quad E^2 + p_*^2 + E'^2 + 2EE_* - 2(E + E_*)E' \end{aligned} \quad (\text{C14})$$

Equating (C13) and (C14) and using equation (C11) in the form $p_x'^2 = p'^2 - p_y'^2 = p'^2 - p^2 \sin^2 \theta$, we obtain after some algebra

$$\begin{aligned} &[(E + E_*)^2 - C^2] E'^2 - 2B(E + E_*)E' \\ &+ [B^2 + C^2(m^2 + p^2 \sin^2 \theta)] = 0 \end{aligned} \quad (\text{C15})$$

where we have defined

$$B \equiv m^2 + E E_* + p^2 \sin^2 \theta - p p_* \cos \theta \cos \theta_*, \quad (\text{C16})$$

$$C \equiv p \cos \theta + p_* \cos \theta_*. \quad (\text{C17})$$

Solving for E' , we have

$$(E')_{\pm} = \frac{B(E + E_*) \pm C \sqrt{|\Delta|}}{(E + E_*)^2 - C^2} \quad (\text{C18})$$

where the discriminant is

$$\Delta = B^2 - (m^2 + p^2 \sin^2 \theta)[(E + E_*)^2 - C^2] \quad (\text{C19})$$

The difference is

$$(\Delta E)_{\pm} = E' - E = \frac{B(E + E_*) - E[(E + E_*)^2 - C^2] \pm C \sqrt{|\Delta|}}{(E + E_*)^2 - C^2} v(x) = \left(\frac{\pi}{2x}\right)^{1/2} e^{-x} \left[1 + \frac{1-4\nu^2}{8x} + \frac{9-40\nu^2+16\nu^4}{128x^2} + \mathcal{O}(x^{-3})\right] \quad (\text{C20})$$

It can be shown, after long algebra, that

$$C \sqrt{|\Delta|} = B(E + E_*) - E[(E + E_*)^2 - C^2] \quad (\text{C21})$$

Therefore, the non-trivial solution is the plus solution. After some simplification, we obtain

$$\Delta E = \frac{2[Ep_*^2 \cos^2 \theta_* + (E - E_*)pp_* \cos \theta \cos \theta_* - E_* p^2 \cos^2 \theta]}{(E + E_*)^2 - (p \cos \theta + p_* \cos \theta_*)^2} \quad (\text{C22})$$

It is easy to verify that for $m \gg p$ and $m_* \gg p_*$, equation (C22) reduces to the non-relativistic equation (C7).

Appendix D: Collision rate

Here we assume that the cross section does not vary with the relative velocity. The characteristic collision time is

$$\Delta t = \frac{1}{n_* \sigma \langle v_{\text{rel}} \rangle} \quad (\text{D1})$$

where v_{rel} is the relative velocity between colliding particles [105]

$$v_{\text{rel}} = \frac{\sqrt{(\mathbf{v} - \mathbf{v}_*)^2 - (\mathbf{v} \times \mathbf{v}_*)^2}}{1 - \mathbf{v} \cdot \mathbf{v}_*} = \frac{\sqrt{(p \cdot p_*)^2 - m^2 m_*^2}}{p \cdot p_*} \quad (\text{D2})$$

and the mean $\langle \dots \rangle$ is taken over the Jüttlich distribution (relativistic generalization of Maxwell distribution)

$$f_{\text{J}}(\mathbf{p}) = (4\pi m^2 T K_2(x))^{-1} \exp[-\sqrt{\mathbf{p}^2 + m^2}/T] \quad (\text{D3})$$

The mean can be shown to be (see, for ex., [49])

$$\langle v_{\text{rel}} \rangle = \frac{2[(1 + \zeta)^2 K_3(\xi) - (\zeta^2 - 1) K_1(\xi)]}{\xi K_2(x) K_2(x_*)} \quad (\text{D4})$$

where $\xi = x + x_*$, $\zeta = (x^2 + x_*^2)/2x x_*$ are auxiliary variables and $x = m c^2/k_{\text{B}} T$ and $x_* = m_* c^2/k_{\text{B}} T_*$ are standard thermal variables, $K_i(x)$ is the modified (or hyperbolic) Bessel function of the second kind (not to confuse with the kinetic energy that we denote K as well).

For n an integer, the modified Bessel functions of the first and second kind are

$$I_n(x) = K_n(x) = \lim_{\alpha \rightarrow n} \frac{\pi I_{-\alpha}(x) - I_{\alpha}(x)}{\sin \alpha \pi} \quad (\text{D5})$$

where α is a non-integer.

The following asymptotic formula for large arguments is useful

For numerical purposes, it is useful to rewrite the following form

$$\langle v_{\text{rel}} \rangle = \left(\frac{8xx_*}{\pi\xi}\right)^{1/2} \frac{(1+\zeta)^2 \tilde{K}_3(\xi) - (\zeta^2-1) \tilde{K}_1(\xi)}{\xi \tilde{K}_2(x) \tilde{K}_2(x_*)} \quad (\text{D7})$$

$$\tilde{K}_i(y) = K_i(y) \sqrt{\frac{2y}{\pi}} e^y \quad (\text{D8})$$

The advantage is that this has a simple expansion in the non-relativistic limit, $y \rightarrow \infty$ (which we use as soon as $y > 100$)

$$\tilde{K}_i(y) = 1 + \frac{4i^2-1}{8y} + \frac{16i^4-40i^2+9}{128y^2} + \mathcal{O}(y^{-3}) \quad (\text{D9})$$

Another useful limit is when $\zeta \gg 1$ (typically $\zeta > 10^{12}$, or machine precision), corresponding to either $x \gg x_*$ or $x \ll x_*$. Then we have

$$\lim_{\zeta \rightarrow \infty} \langle v_{\text{rel}} \rangle \simeq \left(\frac{8xx_*}{\pi\xi}\right)^{1/2} \frac{2\zeta \tilde{K}_3(\xi) + \zeta^2 \Delta \tilde{K}_{31}(\xi)}{\xi \tilde{K}_2(x) \tilde{K}_2(x_*)} \quad (\text{D10})$$

where

$$\Delta \tilde{K}_{ij}(y) = \frac{4(i^2-j^2)}{8y} + \frac{16(i^4-j^4) - 40(i^2-j^2)}{128y^2} + \mathcal{O}(y^{-3}) \quad (\text{D11})$$

Note that in eq. (D1) assumes DM particles remain non-relativistic. Comparing equations (D7) and (D9), it is easy to verify that for both $x \rightarrow \infty$ and $x_* \rightarrow \infty$, the well-known non-relativistic expression is recovered

$$\langle v_{\text{rel}} \rangle = \left[\frac{8(x+x_*)}{\pi x x_*}\right]^{1/2} \quad (\text{D12})$$

Appendix E: Details on the merger delay

Using the binary population synthesis code StarTrack, Ref. [91] computes the rate of double degenerate mergers with primary mass between 0.85 and $1.05 M_{\odot}$ and secondary with mass ratio $0.9 < M_2/M_1 < 1.0$. These systems count on contributions from three distinct evolutionary channels:

(a) Prompt (< 0.1 Gyr) delay times originate from 6.0 – $7.5 M_{\odot}$ zero-age main sequence masses (rare) and close initial orbits ($a_0 < 200 R_{\odot}$)⁷, undergo two common envelopes, and comprise 25% of the channels (a) and (b) together.

(b) Intermediate (1–3 Gyr) delay times originate from 4.8 – $5.8 M_{\odot}$ zero-age main sequence masses and wider initial separations ($a_0 \sim 80$ – $1000 R_{\odot}$), undergo only one common envelope, and comprise 75% of channels (a) and (b) together.

(c) Very long (~ 10 Gyr) delay times consist of binaries with zero-age main sequence component masses in the range 3.8 – $4.5 M_{\odot}$ and large spread in initial separations $a_0 \sim 100$ – $2000 R_{\odot}$. These experience only one common envelope (when the primary has already evolved into a WD), and the mass-losing star is a bloated late-AGB star. The orbital separation upon ejection of the common envelope is $a \sim 3 R_{\odot}$ implying a multi-Gyr delay time.

For our purpose only in channel (c) is of interest but its incidence has not been specified by Ref. [91]. Assuming a Salpeter [106] initial mass function, $N(M) dM \propto M^{-2.35} dM$, and logarithmically uniform initial period distribution [e.g. 107], We find that the respective fractions of the channels (a), (b), and (c) are 10%, 30%, and 60%. We assume that these respective fractions persist the same when considering slightly higher component masses.

⁷ This must be an error. In order to obtain (b)/(a) = 3 as stated by the author, we must have $a_0 \sim 60$ – $200 R_{\odot}$ instead of $a_0 <$

$200 R_{\odot}$.