Backgrounds in gravitational effective field theory

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Effective field theories describing gravity coupled to matter are investigated, allowing for operators of arbitrary mass dimension. Terms violating local Lorentz and diffeomorphism invariance while preserving internal gauge symmetries are included. The theoretical framework for violations of local Lorentz and diffeomorphism invariance and associated conceptual issues are discussed, including transformations in curved and approximately flat spacetimes, the treatment of various types of backgrounds, the implications of symmetry breaking, and the no-go constraints for explicit violation in Riemann geometry. Techniques are presented for the construction of effective operators, and the possible terms in the gravity, gauge, fermion, and scalar sectors are classified and enumerated. Explicit expressions are obtained for terms containing operators of mass dimension six or less in the effective action for General Relativity coupled to the Standard Model of particle physics. Special cases considered include Einstein-Maxwell effective field theories and the limit with only scalar coupling constants.

I. INTRODUCTION

The theory of General Relativity (GR) coupled to the Standard Model (SM) of particle physics provides an impressive description of many features of the Universe over a wide range of distance scales. Obtaining a fully satisfactory combination of gravity with quantum physics remains an open challenge, however, and a deeper underlying unified theory is expected to emerge at the Planck scale. This theory could be founded on Riemann geometry or one of its extensions, on a non-Riemann geometry, or on nongeometric mathematics. In any scenario, observable effects from the Planck scale at low energies can be expected to include small deviations from the known physics described by GR coupled to the SM, and their detection would offer guidance in the construction of the underlying theory. It is thus of central interest to ask how key properties of the underlying theory could in principle manifest themselves in experiments and observations performed using current and near-future technology.

A powerful tool for investigating prospective signals from the Planck scale in a model-independent way is provided by effective field theory [1]. Tiny remnant couplings from the underlying theory can be expected to emerge at the level of effective field theory as terms correcting the action of GR coupled to the SM, with the size of the physical effects governed by small coefficients reflecting features of the underlying theory. An individual term in the effective Lagrange density is the product of an operator constructed from fields observed in nature with a controlling coupling coefficient. In typical applications of effective field theory, the coupling coefficients are assumed to be constant scalars and are called coupling constants. However, the solutions of the underlying theory describing our Universe may include nontrivial backgrounds, which would then be reflected at the level of the effective field theory as coupling coefficients that could vary with spacetime position and that could be tensorial rather than scalar. Tensorial couplings could arise directly from features of the underlying theory, but

even if the underlying theory generates only a nonconstant scalar background then the ensuing effective field theory can contain vector and tensor coupling coefficients determined by the derivatives of the scalar.

For the above reasons, a comprehensive investigation of the effective description of the underlying theory requires the inclusion of nonconstant tensor backgrounds as couplings in the effective field theory. Backgrounds of this type have anomalous symmetry properties under the spacetime transformations of GR and the SM. In particular, their existence implies that the effective field theory can contain apparent violations of local Lorentz invariance and of diffeomorphism symmetry. It is therefore of interest to construct the general effective field theory describing gravity coupled to matter while allowing for terms violating these symmetries.

The framework for the general effective field theory based on GR coupled to the SM is presented in Ref. [2]. This framework allows for violations of local Lorentz and diffeomorphism invariances. Terms in the corresponding effective field theory can be organized according to their mass dimension d in natural units, with terms of larger d expected to have smaller effects at low energies. All terms with $d \leq 4$, called minimal terms, are presented in Ref. [2], and they have been the subject of numerous investigations [3, 4]. In the pure-gravity sector, phenomenological studies [5–21] and experiments [22–34] have constrained most minimal terms to impressive sensitivities. The minimal matter-gravity sector has also been widely explored both phenomenologically [35–42] and experimentally [43–49], with the focus to date being primarily on spin-independent effects. A subset of terms in the pure-gravity sector with operators of larger mass dimensions $d \geq 5$ has been constructed and some experimental constraints obtained [50–73].

One goal of this work is to extend the analysis of terms in the effective field theory to include nonminimal terms involving both gravity and matter as well as nonminimal terms in the pure-gravity sector. We present a systematic methodology for constructing all these terms, and we obtain explicit results for $d \leq 6$ for a generic theory, for GR

coupled to the SM, and for its limits including Einstein-Maxwell theories and the restriction to scalar coupling constants. To achieve this, we develop further the framework presented in Ref. [2], combining it with techniques recently developed for the construction of terms of arbitrary d in nonabelian effective field theories in Minkowski spacetime [74]. We discuss the relevant spacetime transformations, establishing the properties of various types of backgrounds and their symmetry violations, and we characterize the relationships between different types of effective terms and their linearizations to approximately flat spacetimes. The analysis in Ref. [2] revealed unexpected constraints on the form of the effective field theory, arising from compatibility requirements between the variational procedure and the Bianchi identities, which have recently been the subject of extensive study [75–78]. Here, we revisit these no-go constraints to clarify their impact in the context of perturbative corrections to known physics, showing that they can determine whether the underlying theory is based on Riemann geometry or instead emerges from an alternative geometry or nongeometric mathematics.

The outline of this work is as follows. In Sec. II, we study the framework for the gravitational effective field theory. Essential definitions and conventions are presented in Sec. II A, while Sec. II B discusses key concepts about spacetime transformations in both curved and approximately flat spacetimes. The treatment of backgrounds is initiated in Sec. II C, and the implications for violations of symmetries in curved spacetimes are described in Sec. II D. The relationships between broken symmetries in curved and flat spacetimes are elucidated in Sec. II E. Our discussion of the framework concludes with a treatment of the no-go constraints in Sec. II F.

The methodology for the construction of terms in an effective Lagrange density built within this framework is presented in Sec. III, along with a compact notation for various types of backgrounds. The procedure to obtain gauge-covariant spacetime operators and related results is described in Sec. III A. Using these results, in Sec. III B we enumerate and classify operators involving pure-gravity fields and backgrounds. We turn attention to matter fields in Sec. III C, which presents the explicit form of operators with $d \leq 6$ for gauge fields, Dirac fermions, and scalars. The application of all these results to realistic effective field theories is considered in Sec. IV. Terms with $d \leq 6$ in the Lagrange density for various Einstein-Maxwell theories are tabulated in Sec. IV A. Section IV B enumerates the explicit form of terms in effective field theories based on GR coupled to the SM. The limit with backgrounds acting only as scalar coupling constants is discussed in Sec. IV C. We conclude with a summary in Sec. V.

Throughout this work, we adopt the conventions of Ref. [2]. We assume vanishing torsion and nonmetricity except where otherwise indicated, so the definitions and results in Appendix A of Ref. [2] apply with the torsion and contortion set to zero. In particular, Greek indices

are used for tensorial components on the spacetime manifold, and Latin ones for ones on the tangent space. The Minkowski metric η_{ab} has positive signature $+2$, and the Levi-Civita tensor ϵ_{abcd} is fixed by $\epsilon_{0123} = +1$. The Dirac matrices satisfy $\{\gamma^a, \gamma^b\} = -2\eta^{ab}$, with σ^{ab} defined as $\sigma^{ab} = i[\gamma^a, \gamma^b]/2$.

II. FRAMEWORK

In this section, we describe the concepts and framework appropriate for the construction of the general effective field theory based on GR coupled to the SM. Individual subsections treat the basic setup, spacetime transformations, backgrounds, symmetry violations, linearization, and the no-go constraints.

A. Metric, vierbein, and covariant derivative

The geometric underpinning for GR coupled to the SM is a four-dimensional smooth manifold called spacetime that contains a dynamical four-dimensional metric $g_{\mu\nu}$. While the underlying unified theory may be nongeometric, its low-energy approximation must reduce to known physics. The corresponding effective field theory extending GR coupled to the SM can therefore reasonably be taken as based on a manifold with a metric.

The SM incorporates fermions as spinor fields, which are conveniently described using the vierbein formalism [79]. In this approach, spinor fields at each spacetime point and related objects including the Dirac gamma matrices are all defined in a coordinate frame in the tangent space, called a local frame. In contrast, a coordinate frame on the manifold is called a spacetime frame. The vierbein $e_\mu{}^a(x)$ connects local frames with spacetime frames.

Since tangent spaces are flat, the vierbein is related to the metric by

$$g_{\mu\nu} = e_\mu{}^a e_\nu{}^b \eta_{ab}. \tag{1}$$

For simplicity, we assume the connection is both metric and torsion free, so the metric $g_{\mu\nu}$, the spacetime connection $\Gamma^\lambda{}_{\mu\nu}$, and the spin connection $\omega_\mu{}^{ab}$ are all fixed by the vierbein $e_\mu{}^a$. The vierbein can therefore be taken as the sole field describing the gravitational dynamics. We adopt a covariant derivative satisfying

$$D_\mu e_\nu{}^a \equiv \partial_\mu e_\nu{}^a - \Gamma^\lambda{}_{\mu\nu} e_\lambda{}^a + \omega_\mu{}^a{}_b e_\nu{}^b = 0, \tag{2}$$

which then also implies $D_\lambda g_{\mu\nu} = 0$. Note, however, that the theoretical framework used in this work can readily be extended to include nonzero nonmetricity and torsion [2]. In particular, key implications such as the no-go constraints also hold when the requirements of zero torsion and nonmetricity are relaxed.

The SM incorporates bosons in gauge field theory. The corresponding covariant derivative in curved spacetime

then incorporates a gauge connection A_μ along with the spacetime connection $\Gamma^\lambda{}_{\mu\nu}$ and the spin connection $\omega_\mu{}^{ab}$. The gauge connection is the connection for the internal gauge degrees of freedom and acts on objects with gauge indices, the spacetime connection is the connection on the spacetime manifold and acts on spacetime indices, and the spin connection is the connection in local frames and acts on local indices.

Following standard convention, the explicit use of gauge and spinor indices is avoided in this work. For example, the action of the covariant derivative on a fermion field ψ with suppressed spinor and gauge indices can be written in this convention as

$$D_\mu\psi = \partial_\mu\psi + \tfrac{1}{4}i\omega_\mu{}^{ab}\sigma_{ab}\psi - igA_\mu\psi, \qquad (3)$$

where g is the gauge coupling constant. As another example, the gauge field strength $F_{\mu\nu}$ contains both spacetime and gauge indices, and its covariant derivative can be written as

$$D_\lambda F_{\mu\nu} = \partial_\lambda F_{\mu\nu} - \Gamma^\rho{}_{\lambda\mu}F_{\rho\nu} - \Gamma^\rho{}_{\lambda\nu}F_{\mu\rho} - ig[A_\lambda, F_{\mu\nu}]. \quad (4)$$

Note that the 16 Dirac gamma matrices $\Gamma^A \in \{I, i\gamma_5, \gamma^a, \gamma_5\gamma^a, \sigma^{ab}/2\}$ forming a complete basis for spinor matrices typically appear in fermion bilinears containing local-frame indices. We define $D_\mu\Gamma^A$ by imposing the product rule

$$D_\mu(\overline{\psi}\Gamma^A\psi) = (\overline{D_\mu\psi})\Gamma^A\psi + \overline{\psi}(D_\mu\Gamma^A)\psi + \overline{\psi}\Gamma^A(D_\mu\psi), \tag{5}$$

which can be shown to imply that $D_\mu\Gamma^A = \partial_\mu\Gamma^A = 0$.

B. Spacetime transformations

Spacetime transformations play a key role in GR [80] and hence also in the construction of the general effective field theory based on GR coupled to the SM. In this subsection, we discuss some essential concepts for transformations in curved spacetime and in approximately flat spacetime, focusing on requisite aspects for the present work.

1. Particle versus observer

It is convenient and useful to distinguish two notions of transformations, called particle and observer transformations [2, 81]. Particle transformations change dynamical particles and fields, while observer transformations change the observer frame. In the absence of backgrounds, the component forms of the two transformations are inverses of each other and in that context are sometimes called active and passive. However, this equivalence fails in the presence of backgrounds. A particle transformation affects dynamical particles and fields but leaves any backgrounds invariant, which can modify the physics associated with couplings between the dynamical

variables and the background. In contrast, an observer transformation amounts to a coordinate transformation, which changes the components of fields and backgrounds but is assumed to leave invariant the physics. A physical symmetry associated with a given particle transformation can therefore be violated in the presence of backgrounds, even though the physics remains invariant under the corresponding observer transformation.

Mathematically, particle transformations involve mappings of the spacetime manifold and its tangent and cotangent bundles, whereas observer transformations are implemented on the atlas of the manifold. Since physics is independent of the coordinate frames used for the atlas but can depend on the manifold mappings, discussions of symmetry violations are best conducted in the language of particle transformations without invoking frame changes. For this reason, we focus here primarily on particle transformations, often omitting the word particle for simplicity. Unless indicated otherwise, every transformation in this work should be understood as a particle transformation.

Note that distinguishing between the two types of transformations can be subtle in practice. For instance, special relativity is typically introduced in textbooks from the perspective of observer Lorentz transformations. This approach works well for Lorentz-invariant theories, where the component forms of particle and observer Lorentz transformations are essentially equivalent. However, analyzing generic violations of Lorentz invariance in the context of observer Lorentz transformations is challenging at best. Physical Lorentz-violating effects are features of experimental configurations of particles and fields rather than features of the observer, so the general treatment of Lorentz violation cannot readily be described using modified observer transformations [82].

2. Transformations in curved spacetime

In Minkowski spacetime, the central spacetime transformations are global transformations that include spatial rotations, Lorentz boosts, and translations. The rotations and Lorentz boosts form the group of Lorentz transformations, which is enlarged by translations to the Poincaré group. All these Minkowski-spacetime transformations are isometries of the Minkowski metric $\eta_{\mu\nu}$, and they move spacetime points. For example, a global rotation about a point P in the spacetime maps all points other than P into different points.

In contrast, the metric $g_{\mu\nu}$ in a generic curved spacetime typically has no isometries, and so the usual notions of global Lorentz transformations and translations play no particular role. Instead, it is useful to study local Lorentz transformations and diffeomorphisms.

Local Lorentz transformations are Lorentz transformations in the tangent space at each spacetime point, leaving the spacetime point unmoved. Under a local Lorentz

transformation, the vierbein and metric transform as

$$e_\mu{}^a(x) \to \Lambda^a{}_b(x) e_\mu{}^b(x), \quad g_{\mu\nu}(x) \to g_{\mu\nu}(x), \quad (6)$$

where $\Lambda^a{}_b(x)$ are the components of the matrix $\Lambda(x)$ for the local Lorentz transformation at the point x. Other dynamical boson fields transform similarly, with spacetime indices unchanged and local indices acted on by the components of $\Lambda(x)$. Fermion fields are transformed by the corresponding matrices $S(\Lambda(x))$ in the appropriate spinor representation of the local Lorentz group.

Note that local Lorentz transformations at different spacetime points are typically different. However, an associated global transformation can be defined in any curved spacetime by requiring that the same local Lorentz transformation is performed simultaneously at every spacetime point. This can be termed a global local Lorentz transformation, and it is the analogue of a global gauge transformation constructed from a local gauge transformation in a gauge field theory. Global local Lorentz transformations leave spacetime points fixed, so they cannot be the analogues of global Lorentz transformations in Minkowski spacetime. Instead, the analogues can be taken to be certain types of Lorentz transformations defined in approximately flat spacetimes, as described in Sec. II B 3 below.

Diffeomorphisms in a curved spacetime capture the idea of moving spacetime points. Under a diffeomorphism, a spacetime point at position x is mapped to another point at x' according to

$$x^\mu \to x'^\mu = x^\mu + \xi^\mu(x), \quad (7)$$

where $\xi^\mu(x)$ is smooth and the mapping is assumed invertible. Here, x'^μ denotes the components of the new position in the original coordinates, which remain unchanged by the transformation. Dynamical fields on the manifold transform according to the pushforward or pullback induced by the diffeomorphism. For example, the vierbein and metric transform as

$$e_\mu{}^a(x) \to e'_\mu{}^a(x') = \frac{\partial x^\rho}{\partial x'^\mu} e_\rho{}^a(x),$$
$$g_{\mu\nu}(x) \to g'_{\mu\nu}(x') = \frac{\partial x^\rho}{\partial x'^\mu} \frac{\partial x^\sigma}{\partial x'^\nu} g_{\rho\sigma}(x), \quad (8)$$

where $e'_\mu{}^a(x')$ and $g'_{\mu\nu}(x')$ are the new vierbein and metric at the point x' after the diffeomorphism. In contrast, dynamical fields valued in local frames, including spinor fields, transform like scalar fields under a diffeomorphism.

Although the expressions (8) appear similar to those for a general coordinate transformation, the physical interpretation is different. Only the coordinates change under general coordinate transformations, leaving physical particles and fields invariant. General coordinate transformations can thus be identified as observer diffeomorphisms. In contrast, the particle diffeomorphisms of interest here change physical particles and fields while leaving the coordinate system unchanged.

Fields can be valued at any position on the manifold. When valued at the same position, dynamical fields undergoing a diffeomorphism (7) with infinitesimal $\xi^\mu(x)$ change by the corresponding Lie derivative. For example, under an infinitesimal diffeomorphism the vierbein and metric transform as

$$\begin{aligned} e'_\mu{}^a(x) &= e_\mu{}^a(x) - \mathcal{L}_\xi e_\mu{}^a(x) \\ &= e_\mu{}^a - e_\rho{}^a \partial_\mu \xi^\rho - \xi^\lambda \partial_\lambda e_\mu{}^a, \\ g'_{\mu\nu}(x) &= g_{\mu\nu}(x) - \mathcal{L}_\xi g_{\mu\nu}(x) \\ &= g_{\mu\nu} - g_{\rho\nu} \partial_\mu \xi^\rho - g_{\mu\sigma} \partial_\nu \xi^\sigma - \xi^\lambda \partial_\lambda g_{\mu\nu}. \end{aligned} \quad (9)$$

These results can be derived directly from Eqs. (7) and (8). Equations where dynamical fields are valued at the same position are often used in calculations involving diffeomorphisms, and they are distinct from ones like Eq. (8) in which the fields are valued at different positions.

3. Transformations in approximately flat spacetime

As discussed above, a local Lorentz transformation in a curved spacetime leaves spacetime points fixed while acting on local frames, and a diffeomorphism moves spacetime points while leaving local frames unchanged. Next, we show that suitable combinations of local Lorentz transformations and diffeomorphisms in approximately flat spacetimes can mimic the roles of global Lorentz transformations and translations in Minkowski spacetime.

Most experiments are performed in weak gravitational fields such as those found in the solar system. The corresponding spacetimes are therefore approximately flat. The vierbein and metric can then be decomposed as

$$\begin{aligned} e_{\mu a}(x) &= \eta_{\mu a} + \epsilon_{\mu a}(x) \approx \eta_{\mu a} + \tfrac{1}{2} h_{\mu a}(x) + \chi_{\mu a}(x), \\ g_{\mu\nu}(x) &= \eta_{\mu\nu} + h_{\mu\nu}(x), \end{aligned} \quad (10)$$

where $\epsilon_{\mu a} \ll 1$, $h_{\mu\nu} \ll 1$, and $\chi_{\mu a} \ll 1$ with $h_{\mu\nu}$ symmetric and $\chi_{\mu a}$ antisymmetric. Under the diffeomorphism (7), the vierbein and metric transform according to Eq. (8). We can assign the resulting changes in the vierbein and metric to the fluctuations $h_{\mu\nu}(x)$ and $\chi_{\mu a}(x)$, with the Minkowski metric defined as invariant. For infinitesimal diffeomorphisms and at lowest order in the fluctuations, we then find

$$\begin{aligned} h_{\mu\nu}(x) &\to h'_{\mu\nu}(x') = h_{\mu\nu}(x) - \eta_{\rho\nu}\partial_\mu \xi^\rho(x) - \eta_{\mu\sigma}\partial_\nu \xi^\sigma(x), \\ \chi_{\mu a}(x) &\to \chi'_{\mu a}(x') = \chi_{\mu a}(x) - \tfrac{1}{2}\eta_{\rho a}\partial_\mu \xi^\rho(x) \\ &\qquad\qquad + \tfrac{1}{2}\eta^\rho{}_a \eta_{\mu\sigma}\partial_\rho \xi^\sigma(x). \end{aligned} \quad (11)$$

These transformations are called linearized diffeomorphisms in approximately Minkowski spacetime.

Since the spacetime is assumed to be approximately flat, one might anticipate the existence of notions similar to the global Lorentz transformations and translations in Minkowski spacetime. Indeed, when the displacements $\xi^\mu(x)$ are independent of spacetime position, the

linearized diffeomorphisms (11) take the same form as Minkowski-spacetime translations. It is therefore natural to define translations in an approximately Minkowski spacetime as linearized diffeomorphisms with constant ξ^μ. This definition applies to all dynamical quantities, including matter fields.

The linearized diffeomorphisms (11) can alternatively be expressed as transformations of the vierbein and metric fluctuations valued at the same position,

$$h_{\mu\nu}(x) \to h'_{\mu\nu}(x) \approx h_{\mu\nu} - \eta_{\rho\nu}\partial_\mu\xi^\rho - \eta_{\mu\sigma}\partial_\nu\xi^\sigma$$
$$-\xi^\lambda\partial_\lambda h_{\mu\nu},$$
$$\chi_{\mu a}(x) \to \chi'_{\mu a}(x) \approx \chi_{\mu a} - \tfrac{1}{2}\eta_{\rho a}\partial_\mu\xi^\rho + \tfrac{1}{2}\eta^\rho{}_a\eta_{\mu\sigma}\partial_\rho\xi^\sigma$$
$$-\xi^\lambda\partial_\lambda\chi_{\mu a}, \quad (12)$$

in parallel with the result (9). If we make the further approximation of keeping only terms at leading order in small quantities, the linearized diffeomorphisms (12) reduce to the simpler form

$$h_{\mu\nu}(x) \to h_{\mu\nu} - \eta_{\rho\nu}\partial_\mu\xi^\rho - \eta_{\mu\sigma}\partial_\nu\xi^\sigma,$$
$$\chi_{\mu a}(x) \to \chi_{\mu a} - \tfrac{1}{2}\eta_{\rho a}\partial_\mu\xi^\rho + \tfrac{1}{2}\eta^\rho{}_a\eta_{\mu\sigma}\partial_\rho\xi^\sigma.$$
$$(13)$$

These simplified transformations in approximately Minkowski spacetime are called gravitational gauge transformations, or gauge transformations if there is no risk of confusion with internal gauge transformations of matter fields. For $h_{\mu\nu}$ and $\chi_{\mu a}$, gauge transformations and linearized diffeomorphisms valued at the same position differ by contributions involving the operator $\xi^\lambda\partial_\lambda$ that originates from the Lie derivative (9). However, nongravitational fields are unaffected by the linearization procedure, and so for consistency the corresponding contributions must be kept when expanding nongravitational expressions at leading order in small quantities. A gauge transformation of a nongravitational operator $\mathcal{O}(x)$ in the Lagrange density therefore can be defined as

$$\mathcal{O}(x) \to \mathcal{O}(x) - \mathcal{L}_\xi\mathcal{O}(x), \quad (14)$$

which retains the contribution $-\xi^\lambda\partial_\lambda\mathcal{O}(x)$.

To identify the analogues of Minkowski-spacetime Lorentz transformations in an approximately flat spacetime, it is useful to introduce a special set of transformations on the curved manifold called manifold Lorentz transformations. These transformations are distinct both from the usual local Lorentz transformations and also from the global local Lorentz transformations described in the previous subsection. By definition, manifold Lorentz transformations act both on spacetime points and on local frames. Under a transformation of this type specified by a fixed element Λ of the Lorentz group, every spacetime point at position x is mapped to another point x' according to the special diffeomorphism

$$x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu, \quad (15)$$

where x'^μ is the new position expressed in the original coordinate system and $\Lambda^\mu{}_\nu$ are the components of Λ. In addition, the vierbein and the metric on the manifold are defined to transform as

$$e_\mu{}^a(x) \to e'_\mu{}^a(x') = (\Lambda^{-1})^\rho{}_\mu\Lambda^a{}_b e_\rho{}^b(x),$$
$$g_{\mu\nu}(x) \to g'_{\mu\nu}(x') = (\Lambda^{-1})^\rho{}_\mu(\Lambda^{-1})^\sigma{}_\nu g_{\rho\sigma}(x), \quad (16)$$

where the new vierbein $e'_\mu{}^a(x')$ and the new metric $g'_{\mu\nu}(x')$ are at the new position x' after the diffeomorphism, and where $(\Lambda^{-1})^\rho{}_\mu = \Lambda_\mu{}^\rho$ are the components of the inverse of the matrix Λ. Other dynamical boson fields are defined to transform similarly under manifold Lorentz transformations, with both spacetime and local indices transforming according to Λ and its inverse, while fermion fields transform according to the corresponding spinor transformation $S(\Lambda)$.

The manifold Lorentz transformations can be defined on any single coordinate chart in the atlas of a curved spacetime, but the dependence on the coordinate chart limits the value of the definition in the generic case. However, approximately flat spacetimes permit natural choices of coordinate systems in which the vierbein and metric are given approximately by $e_\mu{}^a \approx \delta_\mu{}^a$ and $g_{\mu\nu} \approx \eta_{\mu\nu}$. These natural coordinate systems are not strictly unique, but all manifold Lorentz transformations defined on them are closely related. So for practical applications and in particular in the context of experimental analyses, we can select any one of them without loss of physical generality. Moreover, the existence of these natural coordinates in approximately flat spacetimes insures that manifold Lorentz transformations are the natural analogues of Minkowski-spacetime Lorentz transformations. This follows because Minkowski spacetime is a special manifold on which a coordinate system can be chosen such that the vierbein and the metric take the form $e_\mu{}^a = \delta_\mu{}^a$ and $g_{\mu\nu} = \eta_{\mu\nu}$. A Minkowski-spacetime Lorentz transformation can then be viewed as a manifold Lorentz transformation defined in this chosen coordinate system for Minkowski spacetime.

Within this setup, we can verify that manifold Lorentz transformations are combinations of local Lorentz transformations and diffeomorphisms. Assume first that the local Lorentz transformations are global local Lorentz transformations, defined to have components $\Lambda^a{}_b(x)$ in Eq. (6) independent of spacetime position. Assume further that the diffeomorphisms are the special transformations $x'^\mu = \Lambda^\mu{}_\nu x^\nu$, which preserve the Minkowski metric. The transformations in Eq. (16) can then be identified as combinations of those in Eqs. (6) and (8) in this limit. Other dynamical boson fields transform appropriately, as do the spinor fields. It is therefore natural to identify global Lorentz transformations in an approximately Minkowski spacetime as manifold Lorentz transformations. The global Lorentz transformations can thus be understood as suitable combinations of global local Lorentz transformations and special diffeomorphisms that preserve the Minkowski metric.

Table I summarizes the various transformations introduced above. The first column identifies the type of manifold. An entry in the second column specifies the

TABLE I. Some transformations in curved and approximately flat spacetimes.

Manifold	Transformation	Definition
General spacetime	Local Lorentz transformation	Eq. (6)
	Diffeomorphism	Eqs. (7) and (8)
	Infinitesimal diffeomorphism	Eq. (9)
	Global local Lorentz transformation	Local Lorentz transformation with $\Lambda^a{}_b$ constant
	Manifold Lorentz transformation	Diffeomorphism $x^\mu \to \Lambda^\mu{}_\nu x^\nu$, $\Lambda^\mu{}_\nu$ constant
		and global local Lorentz transformation
	Translation	Diffeomorphism with ξ^μ constant
Approximately flat spacetime	Lorentz transformation	Manifold Lorentz transformation
	Linearized diffeomorphism	Eqs. (11) and (12)
	Gauge transformation	Eqs. (13) and (14)
	Translation	Linearized diffeomorphism with ξ^μ constant

transformation of interest, while one in the third column provides its definition, either via a brief descriptive statement or as a reference to defining equations in the text. All these transformations play a key role in the present work.

C. Backgrounds

A given term in the Lagrange density $\mathcal{L}$ of the general effective field theory extending GR coupled to the SM is the product of a field operator $\mathcal{O}(x)$ with a coupling coefficient $k(x)$ or its derivatives. Since it plays the role of a coupling, k can be viewed as a background in the theory or, equivalently, as a nonzero vacuum value of a field [2]. This perspective holds irrespective of the detailed origin or nature of the coefficient in the context of the underlying theory.

Since the field operator $\mathcal{O}$ may behave nontrivially under spacetime transformations and since the Lagrange density is a scalar density under general coordinate transformations, the background k can carry spacetime and local indices. For example, backgrounds with tensorial indices may arise in string theory [83]. In effective field theory, the operator $\mathcal{O}$ is bosonic and so contains spinor fields only as combinations of fermion bilinears. The background k therefore carries no spinor indices. For definiteness and simplicity, we assume here that k carries no indices associated with any internal gauge degrees of freedom. Backgrounds carrying gauge indices are possible in principle and would appear in an effective field theory that violates internal gauge invariance, but investigating this possibility lies outside our scope. For present purposes, we can therefore treat k as a tensor under general coordinate transformations and under observer local Lorentz transformations. Note that k must remain invariant under all particle transformations, including both diffeomorphisms and local Lorentz transformations, because it is nondynamical by construction.

The distinction between any upper and lower local indices carried by k is physically irrelevant because the two types of indices can be interconverted using the Minkowski metric, which by definition is a nondynamical quantity. However, upper spacetime indices on k can represent physically different effects from lower spacetime indices because the two are connected by the metric $g_{\mu\nu}$, which is a dynamical field. As an example, when k is a nondynamical background then a term in $\mathcal{L}$ of the form $k_\mu \mathcal{O}^\mu$ generates different contributions to the equations of motion than does the term $k^\mu g_{\mu\nu} \mathcal{O}^\nu$. These considerations imply that we can limit attention to three types of indices on k without loss of generality: upper spacetime indices, lower spacetime indices, and local indices in any position. An arbitrary background can thus be denoted as $k^{\mu\cdots}{}_{\nu\cdots}{}^{a\cdots}(x)$. Note that adopting a nonstandard definition of the vierbein, which could lead to local frames with a nondynamical local metric and nontrivial local curvature, cannot introduce new physical effects because the nonstandard and conventional vierbeins are related via nondynamical algebraic equations. Modulo possible derivatives acting on the background, the general structure of a term in the Lagrange density $\mathcal{L}$ can therefore be written in the form

$$\mathcal{L} \supset k^{\mu\cdots}{}_{\nu\cdots}{}^{a\cdots}(x)\mathcal{O}_{\mu\cdots}{}^{\nu\cdots}{}_{a\cdots}(x), \tag{17}$$

where $\mathcal{O}$ contains all dynamical fields including any factors involving the vierbein $e_\mu{}^a$ and metric $g_{\mu\nu}$. If derivatives acting on the background are present, their indices must also be contracted to insure that $\mathcal{L}$ remains a scalar density.

Two classes of backgrounds k can conveniently be identified, according to whether they are spontaneous or explicit. As the two classes have different physical implications, for clarity in much of what follows we denote spontaneous backgrounds by $\langle k \rangle$ and explicit ones by $\overline{k}$. Spontaneous backgrounds $\langle k \rangle$ arise as solutions of the equations of motion in the underlying theory and hence are vacuum expectation values of underlying fields. They

satisfy the equations of motion and are thus on-shell quantities. Fluctuations of the underlying fields about $\langle k \rangle$ then exist and can represent additional modes in the effective theory [84, 85], including Nambu-Goldstone [86] and massive modes. In contrast, explicit background fields $\overline{k}$ are specified by fiat and so are nondynamical. They are unconstrained by equations of motion and hence can be off shell. Moreover, no dynamical fluctuations about them exist. Intuitively, a spontaneous background $\langle k \rangle$ can be viewed as a special nondynamical background $\overline{k}$ that must be on shell and that has accompanying dynamical fluctuations. The on-shell restriction and the presence of dynamical fluctuations imply that the backgrounds $\langle k \rangle$ and $\overline{k}$ are associated with distinct physics.

Among the set of possible general background fields k is a subset consisting of background vierbeins and background metrics. In realistic applications, the usual vierbein and metric have nonzero values in the vacuum, which insures nonzero distances between points. We can view these quantities as a background vierbein and a background metric. In many theories, including GR, they emerge spontaneously as solutions of the equations of motion in otherwise empty regions of spacetime, where the energy-momentum tensor and other relevant sources vanish. Like other background fields, they are invariant under particle transformations. In the present context, it is thus appropriate to denote the usual background vierbein by $\langle e \rangle$ and the usual background metric by $\langle g \rangle$. For example, the Minkowski solution to the GR field equations has $\langle e \rangle_\mu{}^a = \eta_\mu{}^a$ and $\langle g \rangle_{\mu\nu} = \eta_{\mu\nu}$, and for the background the basic relation (1) reduces to the identity [84, 85]

$$\langle g \rangle_{\mu\nu} = \langle e \rangle_\mu{}^a \langle e \rangle_\nu{}^b \eta_{ab}. \tag{18}$$

It follows that the general effective field theory based on GR coupled to the SM must contain at least one spontaneous background vierbein and background metric. Note that one or more explicit backgrounds $\overline{k}$ in the theory may also have the same index structure as the usual vierbein and metric and may therefore be identified as one or more explicit background vierbeins and metrics. The notion of an explicit background vierbein that relates explicit backgrounds $\overline{k}$ with local and spacetime indices was introduced and investigated in Refs. [77, 78]. In what follows, any explicit background vierbeins and background metrics are denoted as $\overline{e}$ and $\overline{g}$. They are nondynamical and by definition cannot arise from a dynamical vierbein or metric on the manifold, so they can be treated in the same way as other explicit backgrounds $\overline{k}$. An effective field theory based on GR containing an explicit background vierbein and background metric must therefore have at least two metrics and two vierbeins.

To illustrate some implications of these various results, consider a background k carrying a single index. The above discussion reveals that six versions of this k can usefully be distinguished, with the two classes of spontaneous and explicit k being further subdivided according

to the three possible index types,

$$k \in \{\langle k \rangle^\mu, \langle k \rangle_\mu, \langle k \rangle^a, \overline{k}^\mu, \overline{k}_\mu, \overline{k}^a\}. \tag{19}$$

Consider first the spontaneous case. Since any spontaneous background $\langle k \rangle$ arises as the solution of dynamical equations of motion, the three types of spontaneous $\langle k \rangle$ are related by the spontaneous background vierbein and background metric [84],

$$\langle k \rangle_\mu = \langle e \rangle_{\mu a} \langle k \rangle^a = \langle g \rangle_{\mu\nu} \langle k \rangle^\nu. \tag{20}$$

All three spontaneous backgrounds $\langle k \rangle$ thus represent the same physics. In contrast, the three types of explicit backgrounds $\overline{k}$ correspond to different physics because they are nondynamical and couple differently with the usual vierbein and metric. For instance, given an explicit background $\overline{k}^\mu$ with a contravariant index, we can take advantage of the existence of the usual metric $g_{\mu\nu}$ to form the product $g_{\mu\nu}\overline{k}^\nu$, which might naively seem to represent an explicit background with a covariant index. However, this product involves the dynamical operator $g_{\mu\nu}$ and hence cannot be treated as an explicit background in the variational procedure. Attempting instead to take advantage of the existence of the usual metric background $\langle g \rangle_{\mu\nu}$ to write the product $\langle g \rangle_{\mu\nu}\overline{k}^\nu$ also fails to generate a satisfactory explicit background with a covariant index because the product mixes on-shell and off-shell quantities. The situation for explicit backgrounds is further complicated in scenarios with an explict background vierbein $\overline{e}_\mu{}^a$ and background metric $\overline{g}_{\mu\nu}$ in addition to the usual background vierbein $\langle e \rangle_\mu{}^a$ and background metric $\langle g \rangle_{\mu\nu}$ [77, 78]. Formal equations such as $\overline{k}_\mu \equiv \overline{e}_{\mu a}\overline{k}^a$ or $\overline{k}_\mu \equiv \overline{g}_{\mu\nu}\overline{k}^\nu$ can then be introduced, but these must be understood as definitions instead of physical relations. Since generic theories lack an explicit background vierbein and background metric, the three explicit backgrounds $\overline{k}$ typically cannot be related even by definitions of this type. All these examples generalize to backgrounds k with more complicated index structures.

D. Symmetry violations

The presence of a background can violate spacetime symmetries because backgrounds behave differently from dynamical fields under particle spacetime transformations. Both backgrounds and dynamical fields behave covariantly under observer transformations, which insures invariance of the physics under coordinate changes. For instance, physical invariance under general coordinate transformations, which are observer diffeomorphisms, is assumed to be a property of a realistic theory. However, backgrounds are invariant under particle transformations, while dynamical fields transform covariantly. This difference can lead to physical symmetry violations in observables that involve dynamical fields coupled to a background.

TABLE II. Examples of terms in the Lagrange density with different transformation properties.

Local Lorentz transformations	Diffeomorphisms	Examples
invariance (LLI)	invariance (DI)	GR
	spontaneous violation (SDV)	$\langle k \rangle^{ab} \mathcal{O}_{ab}, \ \langle k \rangle^{ab} = \eta^{ab}$
	explicit violation (EDV)	$\overline{k}^\mu \mathcal{O}_\mu$
spontaneous violation (SLLV)	invariance (DI)	none
	spontaneous violation (SDV)	$\langle k \rangle^\mu \mathcal{O}_\mu$
	explicit violation (EDV)	$k^{\mu a} \mathcal{O}_{\mu a}, \ k^{\mu a} \equiv \overline{k}^\mu \langle k \rangle^a$
explicit violation (ELLV)	invariance (DI)	$\overline{k}^a \mathcal{O}_a, \ \overline{k}^a$ constant
	spontaneous violation (SDV)	$k^{\mu a} \mathcal{O}_{\mu a}, \ k^{\mu a} \equiv \langle k \rangle^\mu \overline{k}^a$
	explicit violation (EDV)	$\overline{k}^a \mathcal{O}_a, \ \overline{k}^a$ nonconstant

Consider, for example, a generic background $k^{a\cdots}$ in a local frame. This can be viewed as specifying an orientation in the frame, sometimes called a preferred direction, which is invariant under local Lorentz transformations. Unless $k^{a\cdots}$ happens to have no indices and is independent of position, or unless it is proportional to combinations of the Lorentz-group invariants η_{ab} and ϵ_{abcd}, the coupling of a dynamical field to $k^{a\cdots}$ can produce changes of physical observables under local rotations or local Lorentz boosts. These are violations of local Lorentz invariance, which can thus be traced to a direction-dependent background in a local frame [2]. Note that even a scalar background $k(x)$ without indices but varying with spacetime position can introduce violations of local Lorentz invariance because the derivatives of $k(x)$ specify an orientation in a local frame [87]. Similarly, a generic background $k^{\mu\cdots}{}_{\nu\cdots}$ on the spacetime manifold can lead to violations of diffeomorphism invariance unless it has no indices and is independent of spacetime position. Only a background serving as a scalar coupling constant, such as the expectation value of the Higgs field in the SM, can preserve local Lorentz invariance and diffeomorphisms.

For explicit backgrounds, the above results hold without further subtleties. An explicit background $\overline{k}^{\mu\cdots}{}_{\nu\cdots}{}^{a\cdots}(x)$ defined both on the manifold and in local frames violates local Lorentz and diffeomorphism invariance in ways determined directly by its index structure and by its nonvanishing derivatives. For spontaneous backgrounds, however, conditions like Eq. (20) relate the different types of indices. A spontaneous background $\langle k \rangle^{\mu\cdots}{}_{\nu\cdots}{}^{a\cdots}(x)$ can therefore be viewed equivalently as defined entirely on the manifold, entirely in local frames, or as a mixture of the two. Consequently, we recover the result obtained in Ref. [84]: a generic theory contains spontaneous local Lorentz violation (SLLV) if and only if it contains spontaneous diffeomorphism violation (SDV),

$$\text{SLLV} \iff \text{SDV} \quad \text{(generic theories).} \qquad (21)$$

Two exceptions to this result exist, one due to an accidental symmetry and the other to convention. The first exception arises when the spontaneous background happens to be proportional to combinations of the Lorentz-group invariants η_{ab} and ϵ_{abcd}, in which case it has accidental local Lorentz invariance but can still violate diffeomorphism invariance. The other is specific to the usual background vierbein $\langle e \rangle_\mu{}^a = \eta_\mu{}^a$ and metric $\langle g \rangle_{\mu\nu} = \eta_{\mu\nu}$ in an approximately Minkowski spacetime. As described in the previous subsection, these quantities are taken by convention to be invariant under both local Lorentz transformations and diffeomorphisms, with special transformation rules (11) assigned to the fluctuations around them to compensate for this defined invariance.

The action of the effective field theory is defined as usual via integration over the spacetime manifold,

$$S = \int d^4x \ e\mathcal{L}, \qquad (22)$$

and is assumed invariant by construction under general coordinate transformations, which can be understood as observer diffeomorphisms as described in Sec. II B 2. A generic term in $\mathcal{L}$ involving a background takes the form (17) or its generalization incorporating background derivatives. The properties of the term under local Lorentz transformations and diffeomorphisms are determined by the index structure and spacetime dependence of the background k.

Table II shows some examples of terms in $\mathcal{L}$ and their properties under local Lorentz transformations and diffeomorphisms: invariance, spontaneous violation, or explicit violation. In principle, this yields nine possible classes of terms identified according to the transformation properties displayed in the first two columns of the table, which we denote by the abbreviations LLI, SLLV, ELLV, DI, SDV, EDV as shown in the parentheses. However, the generic result (21) insures that the SLLV-DI class is empty, while the LLI-SDV class contains only the exceptions to the result mentioned above. Note that

terms in the SLLV-EDV and ELLV-SDV classes must involve mixed backgrounds arising partly from spontaneous violation and partly from explicit violation, so in this sense the corresponding examples are more complicated. More generally, a given model may contain several terms lying in distinct classes and can therefore be expected to exhibit multiple features associated with different types of symmetry violations.

The third column of Table II provides examples of individual terms in each of the eight nonempty classes. Other than the exceptions in the LLI-SDV class, for which the backgrounds must be formed from the Minkowski metric or the Levi-Civita tensor, the chosen examples all involve the comparatively simple backgrounds with a single index taken from the set (19). The operators $\mathcal{O}$ are understood to have transformation properties determined by their index structure and to enter the action (22) as dynamical fields on the manifold, which can include the usual vierbein $e_\mu{}^a$ and metric $g_{\mu\nu}$ along with matter fields. Any local indices on an operator $\mathcal{O}$ therefore arise from the presence of vierbeins and fermion bilinears involving Dirac matrices rather than from spacetime fields expressed in local coordinates. For instance, the choice $\mathcal{O}_{ab} = R_{ab}$ is excluded to avoid the spurious appearance of the usual GR combination $\eta^{ab}R_{ab} = g^{\mu\nu}R_{\mu\nu} = R$ in the LLI-SDV class. Some classes also contain additional simple examples beyond those shown in the table. As an illustration, the term $\overline{k}_\mu \mathcal{O}^\mu$ lies in the LLI-EDV class and is distinct from the term $\overline{k}^\mu \mathcal{O}_\mu$ listed in the table. Also, the three possible SLLV-SDV terms $\langle k \rangle^\mu \mathcal{O}_\mu$, $\langle k \rangle_\mu \mathcal{O}^\mu$, and $\langle k \rangle^a \mathcal{O}_a$ are related by virtue of the equivalence (20).

The transformations listed in Table II refer to properties of terms in the action rather than to physical observables. The relationship between properties of the action and experimental measurements can be subtle. Consider, for example, a term of the form $\overline{k}_\mu \mathcal{O}^\mu$, which exhibits local Lorentz invariance at the level of the action. Experiments searching for local Lorentz violation may nonetheless be sensitive to this term because the nonzero vierbein $\langle e \rangle_\mu{}^a$ implies that the combination $\overline{k}_\mu \langle e \rangle^\mu{}_a$ provides a definite orientation in the local experimental frame, which can yield observable local Lorentz violation. An example is the fermion-sector term [2]

$$\mathcal{L} \supset -\overline{b}_\mu e^\mu{}_a \overline{\psi} \gamma_5 \gamma^a \psi, \tag{23}$$

which causes orientation-dependent splittings in the fermion energy spectrum and has been studied in numerous experiments searching for Lorentz violation in quantum electrodynamics [4]. A kind of converse is also possible: some terms of the form $\overline{k}_\mu \mathcal{O}^\mu$ explicitly violate diffeomorphism invariance at the level of the action but are undetectable in experiments. This can often be confirmed directly for a given case by identifying a suitable field or coordinate redefinition that removes the term from the action and thereby demonstrates its physical irrelevance [2, 36, 81]. A well-known example is the term

$$\mathcal{L} \supset -\overline{a}_\mu e^\mu{}_a \overline{\psi} \gamma^a \psi, \tag{24}$$

for which one component of the background $\overline{a}_\mu(x)$ can be removed using the field redefinition $\psi = \exp[if(x)]\psi'$, representing a position-dependent change of phase [2].

Another subtlety arises in spontaneous symmetry violation, where the underlying theory is invariant under local Lorentz transformations and diffeomorphisms. In any spontaneous symmetry violation, the symmetry of the full theory remains unbroken but becomes hidden when the Lagrange density of the full theory is expressed in terms of field fluctuations about the background [88]. In the context of spontaneous breaking of spacetime symmetries, the field fluctuations transform in unconventional ways, which insure that the full theory retains the complete spacetime symmetry [85]. However, experiments cannot change the background by performing local Lorentz and diffeomorphism transformations, and they treat the fluctuations as conventional tensor fields. As a result, experiments can be sensitive to the existence of spontaneous backgrounds despite the hidden invariance of the underlying theory. Note, however, that in approximately Minkowski spacetime the usual spontaneous background vierbein $\langle e \rangle_\mu{}^a = \eta_\mu{}^a$ and spontaneous background metric $\langle g \rangle_{\mu\nu} = \eta_{\mu\nu}$ form an exception to this picture because the fluctuations (10) are conventionally assumed to transform so that the full vierbein and metric behave as tensor fields. In experimental analyses, the backgrounds $\langle e \rangle_\mu{}^a = \eta_\mu{}^a$ and $\langle g \rangle_{\mu\nu} = \eta_{\mu\nu}$ can therefore be viewed as preserving local Lorentz invariance and diffeomorphism invariance.

The experimental situation can be further complicated by dynamical fields from objects outside the control of the experimentalist that can mimic the effects of a background and hence play the role of one or more nonzero coefficients k in the effective field theory. A simple example is the gravitational field of the Earth, which acts as a nontrivial background and provides a preferred direction in the laboratory. Some coefficients k then depend on the local gravitational acceleration $\vec{g}$. This introduces apparent signals for local Lorentz and diffeomorphism violation even in a scenario with an invariant theory. The invariance would be manifestly evident under transformations of the experimental conditions only if the Earth could be transformed as well. Similarly, a background distribution of particles or a thermal bath establishes a preferred inertial frame and hence can also create apparent signals for Lorentz violation. For instance, a neutrino beam that travels through the body of the Earth interacts with the electrons in the Earth's material [89]. This acts as a background described by an a-type coefficient similar to that in Eq. (24), producing apparent Lorentz violation in neutrino flavor oscillations [90]. Note that mimic backgrounds can occur at various scales, including cosmological ones. For example, the cosmic microwave background fixes a rest frame throughout the Universe. This leads to apparent violations of Lorentz invariance, including subtle effects such as the observed dipole temperature anisotropy due to the velocity of the Earth relative to this frame [91]. Preferred spacetime directions

can also be expected from other mimic backgrounds at large scales, including the cosmic neutrino background and perhaps also dark matter and dark energy.

The effects of known mimic backgrounds must be removed in any experimental analysis searching for violations of spacetime symmetries arising in an underlying theory. Alternatively, since mimic backgrounds can play the role of the coefficients k, laboratory searches for violations of spacetime symmetries can be reinterpreted as providing constraints on unknown dynamical fields, even in theories that are invariant under local Lorentz transformations and diffeomorphisms. For instance, extensions of Riemann geometry to include spacetime torsion or nonmetricity tensors typically generate nontrivial backgrounds in nature, and matching these to the above framework permits sensitive experimental constraints on the components of both torsion [92] and nonmetricity [93] to be achieved by reinterpreting experimental bounds obtained in laboratory searches for Lorentz violation.

The frame dependence of the backgrounds implies that meaningful comparisons of results obtained in different experiments must be made in a specified frame. For this purpose, it is desirable to choose a standard frame that is approximately inertial over the time scale of typical measurements and that is experimentally accessible. No Earth-based frame is a suitable choice due to the rotation of the Earth about its axis and its revolution around the Sun, which imply consequent experimental effects such as sidereal variations of observables [94]. Instead, the canonical frame adopted in the literature is the Sun-centered frame [95], which uses a right-handed coordinate system determined by the Earth's rotational axis and the direction to the 2000 vernal equinox. This frame has been used to report results of numerous experimental investigations performed in the last two decades [4].

E.   Linearization

Experimental and observational tests of spacetime symmetries mostly involve weak gravitational fields in approximately flat spacetime, for which it is appropriate to adopt the linearized description (10) of the vierbein and metric introduced in Sec. II B 3. From the viewpoint of the whole manifold, these experiments probe local Lorentz and diffeomorphism invariance. In the linearized description, however, this reduces to studying the analogues in approximately flat spacetime of Minkowski-spacetime Lorentz transformations and translations, which can mix local Lorentz transformations and diffeomorphisms as shown in Sec. II B 3. The spacetime symmetries of a given theory on the manifold therefore can correspond nontrivially to spacetime symmetries of its linearized limit. For example, a Lorentz transformation in experiments searching for sidereal or annual variations involves changes both of the velocity in the local frame and of the spacetime position, so even backgrounds k having only spacetime indices can generate Lorentz vi-

olation in experiments. In this subsection, we consider some aspects of this correspondence.

In the limit of weak gravitational fields in approximately flat spacetime, the action (22) is linearized to S^{L} according to

$$S = \int d^4x \, e\mathcal{L} \quad \to \quad S^{\mathrm{L}} = \int d^4x \, \mathcal{L}^{\mathrm{L}}, \qquad (25)$$

where the linearized Lagrange density $\mathcal{L}^{\mathrm{L}}$ incorporates relevant contributions from the linearization of the vierbein determinant e. The pure-gravity sector of $\mathcal{L}^{\mathrm{L}}$ is understood to contain terms up to second order in the fluctuations h and χ, which permits exploration of effects on gravitational waves and graviton propagation, except that contributions from the cosmological-constant term are kept only to first order in h. For the matter-gravity sector, $\mathcal{L}^{\mathrm{L}}$ is restricted to contain terms at first order in h and χ but to include other fields at all orders. These choices are the usual ones adopted for the linearization procedure in GR coupled to matter.

In the linearized limit, three kinds of spacetime transformations in approximately flat spacetime are of interest: Lorentz transformations, gauge transformations, and translations. Their definitions and basic properties are presented in Sec. II B 3. The three symmetries can be broken spontaneously or explicitly.

Consider first Lorentz transformations in the linearized theory. These are combinations of special local Lorentz transformations and diffeomorphisms, so theories with either local Lorentz violation or diffeomorphism violation typically have linearized limits violating Lorentz invariance. In some special scenarios, however, the backgrounds on the manifold reduce in the linearized limit to combinations of the Lorentz-group tensors $\eta_{\mu\nu}$ and $\epsilon_{\kappa\lambda\mu\nu}$. Lorentz invariance is then preserved in the linearized theory despite the presence of local Lorentz violation or diffeomorphism violation in the original theory.

Gauge transformations of linearized gravitational fields in approximately flat spacetime are given by Eq. (13), while for nongravitational fields they are given by the linearized diffeomorphisms (14). In the linearized limit, the Riemann curvature $R_{\mu\nu\rho\sigma}$ and its contractions $R_{\mu\nu}$ and R are gauge invariant at first order, while the combination eR is gauge invariant at second order. These features insure gauge invariance of the linearized action for GR coupled to the SM. Gauge invariance also holds for other theories with unbroken local Lorentz and diffeomorphism invariance, provided any dynamical backgrounds $k(x)$ without indices are treated like matter scalar fields in the linearization procedure. Note that terms with nondynamical backgrounds $\overline{k}(x)$ break diffeomorphism invariance and may lead to gauge violations in the linearized limit, irrespective of their spacetime- or local-index structures.

With the above understandings of the linearization procedure and gauge transformations, calculation shows that when a theory is diffeomorphism invariant (DI) then its linearized limit is gauge invariant (GI). This holds

TABLE III. Examples of terms in the linearized Lagrange density with different transformation properties.

Lorentz transformations	Gauge transformations	Translations	Examples
invariance (LI)	invariance (GI)	invariance (TI)	GR
		violation (TV)	none
	violation (GV)	invariance (TI)	$k^{\mu\nu}\mathcal{O}'_{\mu\nu}, \ k^{\mu\nu} = \eta^{\mu\nu}$
		violation (TV)	none
violation (LV)	invariance (GI)	invariance (TI)	$k^a\mathcal{O}_a, \ k^a$ constant
		violation (TV)	$k^\mu\mathcal{O}_\mu, k^\mu$ nonconstant
	violation (GV)	invariance (TI)	$k^\mu\mathcal{O}'_\mu, \ k^\mu$ constant
		violation (TV)	$k^a\mathcal{O}'_a, \ k^a$ nonconstant

irrespective of the properties of the theory under local Lorentz transformations. It implies a linearized theory with gauge violation (GV) comes from a theory with diffeomorphism violation (DV),

$$\text{DI} \Rightarrow \text{GI}, \quad \text{GV} \Rightarrow \text{DV}. \tag{26}$$

Note that the converses are false. For example, a term in $\mathcal{L}$ of the form $\mathcal{L} \supset k^{\alpha\beta\gamma\delta\kappa\lambda\mu\nu}R_{\alpha\beta\gamma\delta}R_{\kappa\lambda\mu\nu}$ violates diffeomorphism invariance on the manifold but preserves gauge invariance in the linearized theory $\mathcal{L}^{\text{L}}$, so DV$\not\Rightarrow$GV.

Translations in approximately flat spacetime are special cases of linearized diffeomorphisms for which the displacements $\xi^\mu(x)$ of spacetime points are independent of spacetime position. Therefore, if a theory is DI then its linearized limit is also translation invariant (TI), and hence if a linearized theory has translation violation (TV) then the full theory is DV,

$$\text{DI} \Rightarrow \text{TI}, \quad \text{TV} \Rightarrow \text{DV}. \tag{27}$$

As before, the converses are false.

Translation violation also implies the existence of at least one nonconstant background k. Since nonzero derivatives of k at a spacetime point determine preferred directions at that point, translation violation in a linearized theory is accompanied by Lorentz violation (LV). The contrapositive insures that Lorentz invariance (LI) in the linearized theory implies translation invariance. We thus have

$$\text{LI} \Rightarrow \text{TI}, \quad \text{TV} \Rightarrow \text{LV}. \tag{28}$$

Again, the converses are false, as a constant background can violate Lorentz symmetry while preserving translation invariance.

Given the three kinds of transformations at the linearized level, each of which allows the two options of invariance or violation, one might expect to classify any term in a linearized theory as one of eight types. However, the relation (28) implies that two of these eight classes must be empty. Table III displays the eight possibilities and provides examples of terms in the linearized

Lagrange density $\mathcal{L}^{\text{L}}$. The first three columns list the possible properties under Lorentz transformations, gauge transformations, and translations, which we denote by the abbreviations shown in parentheses. The final column shows representative terms in $\mathcal{L}^{\text{L}}$ for each of the five nonempty classes. The backgrounds k are assumed to be generic unless otherwise indicated. Some entries involve the comparatively simple single-index backgrounds listed in the set (19), and some contain two-index backgrounds determined by the Minkowski and Levi-Civita tensors. In this table, gauge-invariant operators are denoted by $\mathcal{O}$ while gauge-violating ones are denoted by $\mathcal{O}'$. The operators are taken to have the same basic properties as those adopted for Table II, except that any gravitational field they contain is linearized. For example, the gauge-invariant operators displayed could include linearizations of products of the scalar curvature and the Ricci tensor, which are gauge invariant in $\mathcal{L}^{\text{L}}$ because each factor is separately gauge invariant at first order in the fluctuations. The entries in the final column are only representative, and other simple examples exist. Note also that combinations of terms in a given model can produce more complicated combinations of effects.

The correspondence between the spacetime symmetries of a given term in the Lagrange density of a theory and the spacetime symmetries in the linearized limit is depicted schematically in Fig. 1. Each of the six boxes with a white background represents one of the classes of terms in Table II, labeled according to its properties under local Lorentz transformations and diffeomorphisms using the abbreviations shown in the table. Note that for simplicity we omit from the figure the two classes SLLV-EDV and ELLV-SDV that involve mixed backgrounds. Each of the six boxes with a gray background represents one of the six nonempty classes of terms in Table III, labeled by its properties under Lorentz transformations, gauge transformations, and translations. The lines specify the classes of linearized terms that can arise from a given term in the original theory. For example, a term with spacetime symmetries in the SLLV-SDV class can produce terms at the linearized level lying in one of the

TABLE IV. Symmetry properties of sample terms and their linearizations.

Theory	Linearization					
	LI, GI, TI	LV, GV, TI	LI, GV, TI	LV, GV, TV	LV, GI, TV	LV, GI, TI
LLI, DI	GR	none	none	none	none	none
SLLV, SDV	none	$\langle k \rangle^\mu \mathcal{O}_\mu$, $\langle k \rangle^\mu$ constant	none	$\langle k \rangle^\mu \mathcal{O}_\mu$, $\langle k \rangle^\mu$ nonconstant	$\langle k \rangle^{\mu\nu} R_{\mu\nu} R$, $\langle k \rangle^{\mu\nu}$ nonconstant	$\langle k \rangle^{\mu\nu} R_{\mu\nu} R$, $\langle k \rangle^{\mu\nu}$ constant
LLI, SDV	$\langle k \rangle^{\mu\nu} R_{\mu\nu} R$, $\langle k \rangle^{\mu\nu} = \eta^{\mu\nu}$	none	$\langle k \rangle^{\mu\nu} \mathcal{O}_{\mu\nu}$, $\langle k \rangle^{\mu\nu} = \eta^{\mu\nu}$	none	none	none
ELLV, DI	none	none	none	none	none	$\overline{k}^a \mathcal{O}_a$, $\overline{k}^a$ constant
LLI, EDV	$\overline{k}^{\mu\nu} R_{\mu\nu} R$, $\overline{k}^{\mu\nu} = \eta^{\mu\nu}$	$\overline{k}^\mu \mathcal{O}_\mu$, $\overline{k}^\mu$ constant	$\overline{k}^{\mu\nu} \mathcal{O}_{\mu\nu}$, $\overline{k}^{\mu\nu} = \eta^{\mu\nu}$	$\overline{k}^\mu \mathcal{O}_\mu$, $\overline{k}^\mu$ nonconstant	$\overline{k}^{\mu\nu} R_{\mu\nu} R$, $\overline{k}^{\mu\nu}$ nonconstant	$\overline{k}^{\mu\nu} R_{\mu\nu} R$, $\overline{k}^{\mu\nu}$ constant
ELLV, EDV	$\overline{k}^{\mu a} e^\nu{}_a R_{\mu\nu} R$, $\overline{k}^{\mu a} = \eta^{\mu a}$	$\overline{k}^{\mu a} \mathcal{O}_{\mu a}$, $\overline{k}^{\mu a}$ constant	$\overline{k}^{\mu a} \mathcal{O}_{\mu a}$, $\overline{k}^{\mu a} = \eta^{\mu a}$	$\overline{k}^a \mathcal{O}_a$, $\overline{k}^a$ nonconstant	$\overline{k}^{\mu a} e^\nu{}_a R_{\mu\nu} R$, $\overline{k}^{\mu a}$ nonconstant	$\overline{k}^{\mu a} e^\nu{}_a R_{\mu\nu} R$, $\overline{k}^{\mu a}$ constant

four classes LV-GV-TI, LV-GV-TV, LV-GI-TV, and LV-GI-TI. The figure applies to single terms in the original theory, as combinations of terms can be associated with different classes. It also assumes the term in the original theory contributes in the linearized limit, hence excluding certain possible operators such as cubic products of curvatures.
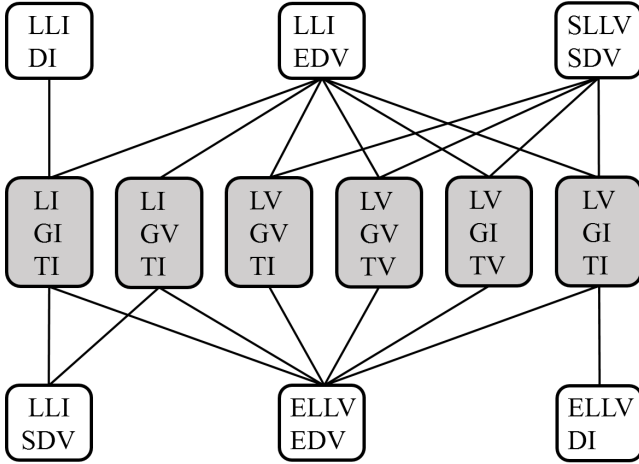

FIG. 1. Relationships between different types of effective terms and their linearizations.

Table IV provides specific examples of the connections displayed in Fig. 1 between different types of terms and their linearizations. The six entries in the first column list the symmetry properties of the six types of terms represented by the boxes with a white background in the figure. The other six columns in the table are labeled with the symmetry properties of the six classes of linearized terms denoted by the boxes with a gray background in the figure. Each of the 36 entries contained in these five columns matches a particular line in the figure

and thereby provides a specific example of a term with the corresponding symmetries, except for the 16 entries for which no such term exists. These 16 cases are excluded by the constraints (21), (26), (27), and (28). The backgrounds k are generic unless otherwise indicated and the operators $\mathcal{O}$ represent appropriate dyamical fields. The entries are only representative examples, and additional possibilities exist.

The existence of certain invariances in a theory has implications for the observability of the fluctuations $h_{\mu\nu}$ and $\chi_{\mu a}$ introduced in Eq. (10). Consider first a theory with local Lorentz invariance described by a Lagrange density $\mathcal{L}$. Under an infinitesimal local Lorentz transformation with $\Lambda^a{}_b(x) = \eta^a{}_b + \epsilon^a{}_b(x)$, the fluctuation $h_{\mu\nu}$ is invariant while $\chi_{\mu a}$ changes at leading order,

$$
\begin{aligned}
h_{\mu\nu}(x) &\to h_{\mu\nu}(x), \\
\chi_{\mu a}(x) &\to \chi_{\mu a}(x) - \epsilon_{\mu a}(x).
\end{aligned}
\tag{29}
$$

Since $\chi_{\mu a}$ and $\epsilon_{\mu a}$ are both antisymmetric, it follows that the $\chi_{\mu a}$ modes can be transformed to zero using a suitable local Lorentz transformation. Other dynamical fields change to new expressions under this transformation. For example, the spinor field $\psi(x)$ changes according to

$$
\psi(x) \to \psi'(x) = \exp[\tfrac{1}{4} i \epsilon_{ab}(x) \sigma^{ab}] \psi(x).
\tag{30}
$$

Writing the theory in terms of transformed fields amounts to implementing field redefinitions and so produces a Lagrange density equivalent to $\mathcal{L}$ but lacking the $\chi_{\mu a}$ modes. This confirms that the $\chi_{\mu a}$ modes are unobservable in a theory with local Lorentz invariance [36]. In particular, it implies that the $\chi_{\mu a}$ modes play no physical role in theories of the LLI-DI, LLI-SDV, and LLI-EDV types shown in Fig. 1 and in the first, third, and fifth rows of Table IV. Note, however, that the $\chi_{\mu a}$ modes can be physical in theories with Lorentz invariance in approximately

flat spacetime, as these can arise as linearized limits of theories with local Lorentz violation.

Similarly, some components of the metric and vierbein are unobservable in theories with gauge invariance in approximately flat spacetime. Under an infinitesimal gauge transformation with $x^\mu \to x^\mu + \xi^\mu(x)$, the fluctuations $h_{\mu\nu}$ and $\chi_{\mu a}$ change according to Eq. (13). It follows that four degrees of freedom in the metric and vierbein are associated with gauge transformations and hence are unobservable in theories with gauge invariance. As in the case of the $\chi_{\mu a}$ modes discussed above, field redefinitions can remove these ξ_μ modes by generating a physically equivalent Lagrange density in which they are absent. This establishes that the ξ_μ modes are unphysical in linearized theories of the LI-GI-TI, LV-GI-TV, and LV-GI-TI classes shown in Fig. 1 and in the corresponding columns of Table IV.

F. No-go constraints

In any model based on Riemann geometry or its extensions to include torsion and nonmetricity, the fields must satisfy the Bianchi identities, which are intrinsically imposed by the geometric structure [80, 96]. The Bianchi identities hold both on and off shell, and their compatibility with the variational principle imposes constraints that must be satisfied for consistency of the model. In GR, for example, the Bianchi identity implies the on-shell conservation of the energy-momentum tensor, $D_\mu T^{\mu\nu} = 0$, which is compatible with the dynamics and symmetries of the theory obtained by variation of the action. Similarly, in a model with spontaneous violation of one or more spacetime symmetries, compatibility with the Bianchi identities is maintained because the variational procedure is standard. However, explicit violation of a spacetime symmetry requires the presence in the action of one or more nondynamical background fields $\overline{k}^{\mu\cdots}{}_{\nu\cdots}{}^{a\cdots}(x)$, which behave unconventionally under variations. The variational results can then become incompatible with implications from the Bianchi identities and hence can render problematic a model containing explicit violation [2]. This can, for instance, induce outright inconsistencies in the model or impose unnatural requirements such as fine tuning of the explicit background. The potential constraints on a model with explicit violation of spacetime symmetries are called no-go constraints. Their role has been the subject of extensive recent investigation by Bluhm and collaborators [75–78].

As described in Sec. II C, a spontaneous background $\langle k \rangle$ can be viewed as a nondynamical background $\overline{k}$ that is on shell and that comes with dynamical fluctuations including Nambu-Goldstone and massive modes. In this context, incompatibilities in a model with explicit breaking can be interpreted as due to the absence of dynamical fluctuations [2]. Within the Stückelberg approach [97, 98], the missing Nambu-Goldstone modes correspond to extra dynamical scalar fields that can be added to re-store the explicitly broken symmetry [78]. For explicit diffeomorphism violation, the no-go constraints can be identified with the Noether identities [99] arising from the requirement of general coordinate invariance of the model [75]. Under suitable circumstances, the constraints can be satisfied by appropriately fixing the ξ_μ modes. Similar results hold for local Lorentz violation and the corresponding $\chi_{\mu a}$ modes [77], as well as in the presence of matter-gravity couplings [78]. However, in some scenarios the no-go constraints cannot be satisfied for any choice of the ξ_μ or $\chi_{\mu a}$ modes [75].

The present subsection contributes to the ongoing discussion of this topic by demonstrating that an explicit background $\overline{k}^{\mu\cdots}{}_{\nu\cdots}{}^{a\cdots}(x)$ typically cannot satisfy the no-go constraints in a model that modifies the physics only perturbatively. This result holds in the matter-gravity sector as well as in the pure-gravity one, and it thus extends the practical impact of the no-go constraints on the space of possible realistic models that seek to describe small deviations from known physics. Some comments about models based on non-Riemann geometry are also presented.

Consider first a model with an explicit background $\overline{k}_{\mu_1\cdots\mu_n}$ carrying n covariant spacetime indices. The associated current can be defined as usual by variation of the action, $J^{\mu_1\cdots\mu_n} \equiv \delta S/\delta\overline{k}_{\mu_1\cdots\mu_n}$. Following the calculational procedure in Ref. [2] reveals that the Bianchi identity implies

$$
\begin{aligned}
D_\mu T^\mu{}_\nu &= J^{\mu_1\cdots\mu_n} D_\nu \overline{k}_{\mu_1\cdots\mu_n} - D_{\mu_1}(J^{\mu_1\cdots\mu_n}\overline{k}_{\nu\mu_2\cdots\mu_n}) \\
&\quad - \ldots - D_{\mu_n}(J^{\mu_1\cdots\mu_n}\overline{k}_{\mu_1\cdots\mu_{n-1}\nu}) \\
&= 0,
\end{aligned}
\tag{31}
$$

where the covariant derivatives are combinations of partial derivatives and Levi-Civita connections. This result matches Eq. (27) of Ref. [77], and it reduces to Eq. (A.17) of Ref. [78] in the limit of a single spacetime index. It represents four no-go constraints that must be obeyed by the model for internal consistency.

In practical applications relevant for laboratory and solar-system experiments, gravity is weak and the spacetime is approximately flat. The metric can then be expanded as in Eq. (10), with the dynamics determined by the metric fluctuation $h_{\mu\nu}$. Linearizing Eq. (31) in $h_{\mu\nu}$ produces an equation of the schematic form

$$
J(\partial k + k\partial h) + (\partial J + J\partial h)k = 0,
\tag{32}
$$

where we suppress all indices and factors. The no-go constraints thus correspond to conditions of the schematic form

$$
\frac{\partial \overline{k}}{\overline{k}} + \frac{\partial J}{J} + \partial h = 0.
\tag{33}
$$

Note that the metric fluctuation h appearing in this expression includes both the conventional modes appearing in GR and the ξ_μ modes. The latter are physical in some scenarios but are unobservable in GR and models with

gauge invariance, as discussed at the end of the previous subsection.

We can consider the implications of the no-go constraints (31) and the schematic condition (33) for different models with an explicit background $\overline{k}_{\mu_1\cdots\mu_n}$. Suppose first that the background $\overline{k}_{\mu_1\cdots\mu_n}$ appears only in the pure-gravity sector of a model, so that the current $J^{\mu_1\cdots\mu_n}$ is composed of $h_{\mu\nu}$. The no-go constraints can then be impossible to satisfy. A simple example is a model with a fixed but nonconstant cosmological term $\overline{\Lambda}(x)$. This is incompatible with the condition (31) required by the Bianchi identity, which demands $\partial_\mu\overline{\Lambda} = 0$ [75]. Another example is the LLI-EDV model with action

$$S \propto \int d^4x \; e(R + \overline{k}^{\mu\nu}g_{\mu\nu}), \qquad (34)$$

containing a two-index symmetric prescribed background $\overline{k}^{\mu\nu}$. At zeroth order in $h_{\mu\nu}$, the condition (31) reduces to the constraint

$$\partial_\mu\overline{k}^{\mu\nu} + \tfrac{1}{2}\eta_{\alpha\beta}\eta^{\mu\nu}\partial_\nu\overline{k}^{\alpha\beta} = 0, \qquad (35)$$

showing that only special choices of backgrounds $\overline{k}^{\mu\nu}$ can be admissible. Generic backgrounds $\overline{k}^{\mu\nu}$ in this model are therefore perturbatively incompatible with the no-go constraints independently of the behavior of $h_{\mu\nu}$ and its ξ_μ modes, and so arbitrary explicit diffeomorphism violation is excluded. Moreover, at first order in h the schematic condition (33) in this model reduces to $\partial\overline{k}/\overline{k} \sim \partial h/h$. Since $\partial h/h$ is tiny near the Earth and since $\overline{k}$ is perturbative by construction, it follows that $\overline{k}$ must be almost constant. Thus, even a restricted background $\overline{k}^{\mu\nu}$ satisfying (35) must have a fine-tuned structure to satisfy the no-go constraints.

In the above models, the no-go constraints generate direct restrictions on explicit backgrounds without involvement of $h_{\mu\nu}$ or ξ_μ at leading order because the background terms in the action are linear in $h_{\mu\nu}$. However, backgrounds in more involved models typically also conflict with the no-go constraints and the perturbative assumption. Consider, for example, a model with action

$$S \propto \int d^4x \; e(R + \overline{k}(x)h_{\mu\nu}h^{\mu\nu}), \qquad (36)$$

which can be viewed as incorporating a constant two-index background $\overline{\eta}_{\mu\nu}$ such that $h_{\mu\nu} = g_{\mu\nu} - \overline{\eta}_{\mu\nu}$ and $h^{\mu\nu} \equiv \overline{\eta}^{\mu\alpha}\overline{\eta}^{\nu\beta}h_{\alpha\beta}$, along with a background function $\overline{k}(x)$. At leading order in small quantities, the no-go constraints (31) for this model reduce to the schematic form $\partial\overline{k}/\overline{k} \sim \partial h/h$, in accordance with the result (33). As before, this shows that the structure of $\overline{k}(x)$ must be fine tuned to be nearly constant in the vicinity of the Earth for the perturbative assumption to be valid. In the particular special case that $\overline{k}(x) = m^2/2$ is a positive constant, the action (36) describes a simple model for massive gravity. At leading order, the no-go constraints then collapse to $\partial_\mu h^{\mu\nu} = 0$, which is analogous to enforcing a particular gauge-fixing condition and imposes

a corresponding form for the ξ_μ modes. In this limit, the model (36) is therefore compatible with the no-go constraints at least to first order in $h_{\mu\nu}$. However, even constant backgrounds may be insufficient to insure compatibility with the no-go constraints in many models. For example, no useful post-Newton expansion exists in pure-gravity models with constant-background $d = 4$ terms of the form $\overline{s}^{\mu\nu}R_{\mu\nu}$ or $\overline{t}^{\kappa\lambda\mu\nu}R_{\kappa\lambda\mu\nu}$ because the ξ_μ modes decouple at leading order and hence cannot be used to satisfy the no-go constraints, leading to severe restrictions on acceptable linearized curvatures [75]. As before, the schematic constraint (33) for these models shows that the only potential backgrounds that are admissible must be almost constant, so generic explicit backgrounds are excluded.

Next, suppose instead that the explicit background $\overline{k}$ appears in the matter-gravity sector, so that the current J includes matter fields. If we insist that $\overline{k}$ modifies the physics only perturbatively, as required in an effective field theory, then we can show that the model is typically incompatible with the no-go constraints. Consider, for example, a matter sector involving a spinor field ψ. The fermion bilinears can involve Dirac matrices and associated couplings to the vierbein $e_\mu{}^a$, which in approximately flat spacetime can be expanded according to Eq. (10). Linearizing the current J in the metric and vierbein fluctuations produces an expression of the schematic form

$$J \sim \overline{\psi}\psi + \mathcal{O}(h)\overline{\psi}\psi + \mathcal{O}(\chi)\overline{\psi}\psi + \dots, \qquad (37)$$

where for simplicity we suppress all indices, factors, and structures involving Dirac matrices. At leading order, we thus find $\partial J/J \sim \overline{\psi}\partial\psi/\overline{\psi}\psi$. However, the size of $\overline{\psi}\partial\psi/\overline{\psi}\psi$ in laboratory experiments is much larger than the unperturbed GR contributions from ∂h, which are dominated by the local gravitational acceleration. For instance, neutrons bound in the Earth's gravitational field have values of $\overline{\psi}\partial\psi/\overline{\psi}\psi$ of order 10^{-12} GeV, whereas the gravitational acceleration on the Earth's surface is of order 10^{-32} GeV. The condition (33) from the no-go constraints thus typically cannot be satisfied unless any physical effects from the ξ_μ modes or from $\partial\overline{k}/\overline{k}$ are also much larger than the gravitational acceleration, both of which are excluded for perturbative modifications to GR.

This line of reasoning can be applied to most models with explicit backgrounds in the matter-gravity sector. The argument can be evaded for certain cases in which the background $\overline{k}$ is constant or nearly so, $\partial\overline{k} \sim 0$, and the current J happens to have a special form that is conserved or almost conserved, $\partial J \sim 0$, since then both $\overline{k}$ and the ξ_μ modes can yield perturbative contributions to the usual gravitational acceleration while still satisfying the condition (33) from the no-go constraints. For example, one-loop radiative corrections in certain models of massive gravity yield a $\overline{c}$-type coefficient in the matter-gravity sector [78, 100], which amounts to a background $\overline{k}$ with $\partial\overline{k} \sim 0$ and a current J with $\partial J \sim 0$. As another example, consider a term of the form $\mathcal{L} \supset -\overline{a}_\mu e^\mu{}_a \overline{\psi}\gamma^a\psi$

involving an explicit background $\overline{a}_\mu$, which can produce nontrivial physical effects when $\overline{a}_\mu$ differs from a gradient, $\overline{a}_\mu \neq \partial_\mu \overline{f}$ [2]. This term produces a current J that is conserved, $\partial J = 0$, by virtue of a global U(1) symmetry. The condition (33) then reduces to $\partial \overline{a}/\overline{a} \sim \partial h$, which is compatible with perturbative behavior. In this instance, further insight can be gleaned from the analogous term for spontaneous symmetry breaking, $\mathcal{L} \supset -a_\mu e^\mu{}_a \overline{\psi} \gamma^a \psi$, where a_μ now contains a background $\langle a \rangle_\mu$ along with dynamical fluctuations insuring compatibility between the Bianchi identity and the variational procedure. This situation is comparatively simple because $\partial J = 0$ and so the condition (33) implies $\partial a \sim a \partial h$, which is congruent with the solution (87) for the fluctuation modes given in Ref. [36]. For a more complicated matter-gravity term with a spontaneous background k that lacks a conservation law for J, satisfying (33) requires solving $\partial k \sim k \partial J / J$. This typically is challenging to perform in detail and remains an open problem of definite interest. In contrast, for most explicit backgrounds, nonperturbative solutions that are compatible with the no-go constraints may be possible in principle but typically are incompatible with existing experiments.

Consider next a model with an explicit background $\overline{k}^{\mu_1 \cdots \mu_n}$ carrying n contravariant spacetime indices. This faces challenges similar to those for the background $\overline{k}_{\mu_1 \cdots \mu_n}$. For this case, the no-go constraints take the form

$$D_\mu T^\mu{}_\nu = J_{\mu_1 \cdots \mu_n} D_\nu \overline{k}^{\mu_1 \cdots \mu_n} + D_{\mu_1}(\overline{k}^{\mu_1 \cdots \mu_n} J_{\nu \mu_2 \cdots \mu_n})$$
$$+ \ldots + D_{\mu_n}(\overline{k}^{\mu_1 \cdots \mu_n} J_{\mu_1 \cdots \mu_{n-1} \nu})$$
$$= 0. \tag{38}$$

The solution to this equation has the same schematic form as Eq. (33) and so exhibits the same problems, with only fine-tuned backgrounds offering the potential for consistency. While nonperturbative solutions of the no-go constraints may exist for arbitrary backgrounds, they typically are excluded by existing experiments.

Finally, we discuss a model containing a background $\overline{k}_a{}^{b_1 \cdots b_n}$ with $n + 1$ local indices. The corresponding no-go constraints now involve the antisymmetric part of the energy-momentum tensor. They take the form

$$T^{\mu\nu} - T^{\nu\mu}$$
$$= -e^{\mu a} e^{\nu b} [(J_{ac_1 \cdots c_n} \overline{k}_b{}^{c_1 \cdots c_n} + J_{c_1 a c_2 \cdots c_n} \overline{k}^{c_1}{}_b{}^{c_2 \cdots c_n}$$
$$+ \ldots + J_{c_1 \cdots c_n a} \overline{k}^{c_1 \cdots c_n}{}_b) - (a \leftrightarrow b)]$$
$$= 0, \tag{39}$$

where $J_{ab_1 \cdots b_n} \equiv \delta S / \delta \overline{k}^{ab_1 \cdots b_n}$ is the relevant current. These constraints amount to the requirement of the vanishing of a generalized cross product between the current and the background. As before, the obstacle for this case can be understood in the perturbative limit, where the modes of relevance are $h_{\mu\nu}$ and $\chi_{\mu a}$.

Consider first a background $\overline{k}_a{}^{b_1 \cdots b_n}$ appearing in the matter-gravity sector, with the current J expanded as in Eq. (37). Since J cannot be significantly modified by perturbative $h_{\mu\nu}$ or $\chi_{\mu a}$, the no-go constraints (39) can be seen to require the vanishing of a linear combination of the currents determined by the background. This represents a strong restriction on the structure of the background $\overline{k}_a{}^{b_1 \cdots b_n}$, with generic backgrounds being inadmissible.

Next, suppose $\overline{k}_a{}^{b_1 \cdots b_n}$ appears instead in the pure-gravity sector. For most terms in the corresponding Lagrange density, including ones containing factors of the curvature, the contribution to the current at leading order in small quantities cannot contain the $\chi_{\mu a}$ modes. This is because the local indices on the background must ultimately be contracted with factors of the vierbein, which contains the $\chi_{\mu a}$ modes only at subleading order and which has vanishing covariant derivative. The no-go constraints (39) then reduce to the requirement that a certain linear combination of derivatives of $h_{\mu\nu}$ must vanish, which is problematic in the Newton limit. Typical backgrounds obeying the no-go constraints are thus incompatible with known perturbative physics.

A comparatively simple example illustrating a few of these features is given by the term $\mathcal{L} = \overline{k}^a{}_\mu e^\mu{}_a$, for which the constraints (39) reduce to $\overline{k}^a{}_\mu e^{\mu b} - \overline{k}^b{}_\mu e^{\mu a} = 0$ and thus exclude an arbitrary explicit background $\overline{k}^a{}_\mu$. A suitable nonperturbative $\overline{k}^a{}_\mu$ can nonetheless yield a condition determining $\chi_{\mu a}$ that provides consistency with the no-go constraints. For example, demanding that $\overline{k}^a{}_\mu$ takes the form of a background vierbein yields an equation for $\chi_{\mu a}$ analogous to a gauge condition [77]. Overall, the above discussions confirm that the no-go constraints strongly restrict models with a generic background $\overline{k}_a{}^{b_1 \cdots b_n}$.

Note that the extension of Riemann geometry to include torsion and nonmetricity typically cannot satisfy the no-go constraints either. If the torsion and nonmetricity are dynamical, then they are fully determined by the equations of motion, and so no extra degrees of freedom are available to insure compatibility of the geometry with the variational principle. The situation with dynamical torsion is discussed in Ref. [2]. If instead the torsion and nonmetricity are nondynamical, then they are predetermined and hence can be interpreted as fixed background fields. They therefore are also subject to no-go constraints, and the same arguments apply. Consistent solutions to the equations of motion typically are nonperturbative and hence incompatible with existing experiments.

The no-go constraints thus imply that a generic perturbative model with explicit violation of spacetime symmetries cannot be based on Riemann geometry or its extensions to include torsion and nonmetricity. Conceivably, such a model might be formulated instead within the context of some other geometry, or it might even be a nongeometric theory lacking a smooth manifold. Investigations of these possibilities are of definite interest but lie beyond our present scope. However, it must be possible to approximate the infrared limit of any complete and consistent realistic model using an effective field theory

based on GR and the SM. The framework studied in this work or its extension to include torsion and nonmetricity includes all possible backgrounds in the Lagrange density, so it suffices as a low-energy approximation of any complete and realistic model. Although this approximation may well violate the no-go constraints from Riemann geometry, the complete model must satisfy any corresponding constraints arising from the underlying geometry.

One option for a geometry compatible with explicit breaking is Finsler geometry, which can be viewed as a generalization of Riemann geometry with the role of the metric in determining geometric features supplemented by other quantities prescribed on the manifold [101, 102]. With the latter quantities identified as explicit background fields, Finsler geometry has been conjectured as a possible route to escape the no-go constraints in Riemann geometry [2]. Investigating this conjecture in detail is hampered by the lack of a satisfactory definition for Lorentz-Finsler geometry, which is currently the subject of active research [103–116]. Support for the conjecture includes the demonstration that the trajectory of a fermion or scalar particle in the present of explicit backgrounds corresponds to a geodesic in a Riemann-Finsler space [103, 104, 117–120].

Figure 2 provides a pictorial representation of the different options for theories with backgrounds, omitting as before the two classes SLLV-EDV and ELLV-SDV that involve mixed backgrounds. Terms within the hexagon are built on Riemann geometry. Each of the six triangles in the hexagon corresponds to one of the six classes of theories contained in the rows of Table IV. Theories in the LLI-DI, SLLV-SDV, and LLI-SDV classes satisfy the no-go constraints and are compatible with Riemann geometry. Theories in the LLI-EDV, ELLV-DI, and ELLV-EDV classes listed in the last three rows of Table IV all involve explicit violation and are depicted by shaded triangles. When they incorporate only perturbative deviations from GR coupled to the SM, these models typically are inconsistent or incompatible with experiments. Theories with explicit violation that represent perturbative deviations from GR coupled to the SM generically lie outside the hexagon, so they must be constructed from some other geometry such as Finsler geometry or have a nongeometrical basis. Attempting to express them in terms of effective field theory based on Riemann geometry is an approximation, and it typically implies incompatibilities with the no-go constraints from Riemann geometry.

III. EFFECTIVE FIELD THEORY

In this section, we develop a methodology for the construction of a realistic effective field theory involving gravity and matter in the presence of arbitrary backgrounds. This enables the explicit derivation of all desired terms in the action, including ones in the pure-gravity sector and those involving matter-gravity couplings to gauge fields, fermions, and scalars. It also yields
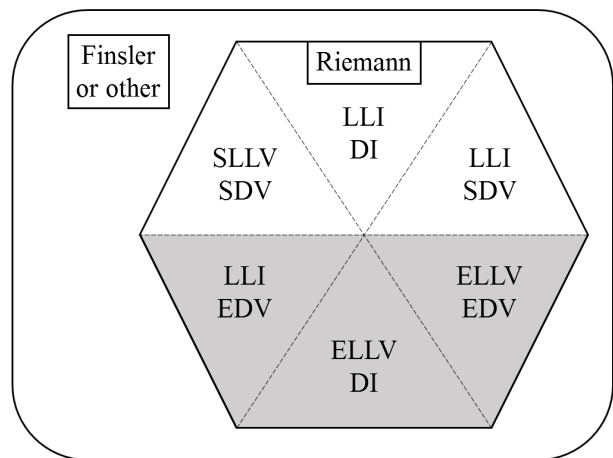

FIG. 2. Pictorial classification of background terms.

the terms describing the dynamics of the background.

The methodology is initiated in Sec. III A, which describes the procedure for building dynamical operators with appropriate spacetime and gauge properties. It is convenient to separate the action (22) into four sectors,

$$S = \int d^4x \ e(\mathcal{L}_g + \mathcal{L}_A + \mathcal{L}_\psi + \mathcal{L}_\phi), \qquad (40)$$

where $\mathcal{L}_g$ contains pure-gravity terms and any background dynamics, $\mathcal{L}_A$ describes gauge fields and their gravity couplings, $\mathcal{L}_\psi$ involves fermions including their gravity and gauge couplings, and $\mathcal{L}_\phi$ contains all terms with scalars. The pure-gravity sector is addressed in Sec. III B, along with the dynamics of the background. We consider the matter-gravity sector in Sec. III C, starting with the pure-gauge sector and then adding fermion and scalar terms. For definiteness, we work in Sec. III C with a Dirac fermion or complex scalar in a single representation of a gauge group. Applications involving more general types of fermions and scalars in realistic scenarios, including the construction of the effective field theory based on GR coupled to the SM, are provided in Sec. IV.

Any single effective term in the Lagrange density $\mathcal{L}$ takes the form of a contraction between a dynamical operator $\mathcal{O}$ and a background k or its derivatives, as illustrated in Eq. (17). A specific operator $\mathcal{O}$ may be contracted directly to one or more backgrounds k or their derivatives, or may be contracted instead via combinations of the vierbein, metric, and Levi-Civita tensor. It is convenient to adopt a compact notation for these various types of backgrounds and contractions, thereby simplifying expressions in the Lagrange density. The idea is to introduce a quantity $\check{k}^{\mu\cdots}{}_{\nu\ldots}{}^{a\cdots}$ that is a linear combination of all terms formed from background fields, vierbeins, metrics, and the Levi-Civita tensor. Note that multiple vierbein and metric factors may appear in a given term, but at most one Levi-Civita factor is needed because products of the Levi-Civita tensor reduce to products of vierbeins or metrics. Contracting the combination

$\breve{k}^{\mu\cdots}{}_{\nu\ldots}{}^{a\cdots}$, with any specific operator $\mathcal{O}_{\mu\ldots}{}^{\nu\cdots}{}_{a\ldots}$ then produces a single expression in the Lagrange density $\mathcal{L}$ of the form $\mathcal{L} \supset \breve{k}^{\mu\cdots}{}_{\nu\ldots}{}^{a\cdots}(x)\,\mathcal{O}_{\mu\ldots}{}^{\nu\cdots}{}_{a\ldots}(x)$, which expands into many individual terms of the form (17). Terms involving contractions between dynamical operators and derivatives of backgrounds can also be combined in this way by using derivatives of $\breve{k}^{\mu\cdots}{}_{\nu\ldots}{}^{a\cdots}$.

As an example, consider the simplest case where $\breve{k}$ has no indices, so that the corresponding term $\mathcal{L} \supset \breve{k}\mathcal{O}$ in the Lagrange density involves a dynamical operator $\mathcal{O}$ without indices. We can expand the quantity $\breve{k}$ to display the component backgrounds,

$$\breve{k} = k + k^{\mu\nu}g_{\mu\nu} + k_{\mu\nu}g^{\mu\nu} + k_\mu{}^a e^\mu{}_a + k^{\mu a}e_{\mu a} + \ldots$$
$$+ k^{\kappa\lambda\mu\nu}g_{\kappa\lambda}g_{\mu\nu} + \ldots + k^{\kappa\lambda\mu\nu}\epsilon_{\kappa\lambda\mu\nu} + \ldots, \qquad (41)$$

which explicitly reveals the dependence on the gravitational degrees of freedom and illustrates the compactness of the expression $\breve{k}\mathcal{O}$. This expansion is also appropriate for terms such as $\mathcal{L} \supset (D_\mu\breve{k})\mathcal{O}^\mu$, which involve the contraction of a dynamical operator with the derivative of a combination of backgrounds. As another example, consider the case where $\breve{k}^\mu$ has a single contravariant spacetime index. A term $\mathcal{L} \supset \breve{k}^\mu\mathcal{O}_\mu$ in the Lagrange density can then be expanded using

$$\breve{k}^\mu = k^\mu + k_\nu g^{\mu\nu} + k^a e^\mu{}_a + \ldots. \qquad (42)$$

Similarly, the term $\mathcal{L} \supset \breve{k}^a\mathcal{O}_a$ involving a dynamical operator with a local index can be expanded using

$$\breve{k}^a = k^a + k^\mu e_\mu{}^a + k_\mu e^{\mu a} + \ldots. \qquad (43)$$

Depending on the hypotheses of a specific theory, the various backgrounds $k^{\mu\cdots}{}_{\nu\ldots}{}^{a\cdots}(x)$ combined in the above expansions may be partially or wholly related to each other. For instance, a given theory containing only a single background k^μ might nonetheless have a Lagrange density with terms involving a two-index background $k^{\mu\nu}$ formed as $k^{\mu\nu} \propto k^\mu k^\nu$. A given quantity $\breve{k}^{\mu\cdots}{}_{\nu\ldots}{}^{a\cdots}$ may therefore be nonlinear in the backgrounds $k^{\mu\cdots}{}_{\nu\ldots}{}^{a\cdots}$ that specify a particular theory. Since the mass dimensionality of each $k^{\mu\cdots}{}_{\nu\ldots}{}^{a\cdots}$ is determined by the operator structure of the Lagrange density, any nonlinear relationships may also involve dimensionful factors that insure a definite mass dimensionality for $\breve{k}^{\mu\cdots}{}_{\nu\ldots}{}^{a\cdots}$.

Expressed using the above notation, the Lagrange density for the effective field theory can be used for spontaneous or explicit violation of spacetime symmetries, with the various combinations of backgrounds $k^{\mu\cdots}{}_{\nu\ldots}{}^{a\cdots}$ contained in the quantities $\breve{k}^{\mu\cdots}{}_{\nu\ldots}{}^{a\cdots}$ correspondingly understood to be spontaneous or explicit. For spontaneous breaking, the backgrounds $\langle k\rangle^{\mu\cdots}{}_{\nu\ldots}{}^{a\cdots}$ are understood to come with concomitant dynamical fluctuations, as described in Sec. II C. For explicit breaking, the backgrounds $\overline{k}^{\mu\cdots}{}_{\nu\ldots}{}^{a\cdots}$ are nondynamical, and typically the underlying theory cannot be based on Riemann geometry for reasons outlined in Sec. II F.

A. Dynamical operators

To construct terms in the Lagrange density $\mathcal{L}$, we require a procedure to build suitable dynamical operators $\mathcal{O}_{\mu\ldots}{}^{\nu\cdots}{}_{a\ldots}$. For effective field theory based on GR and gauge theory, the terms must be independent of observer general coordinate transformations and be locally gauge invariant. In Minkowski spacetime, a procedure to construct gauge-covariant operators has recently been developed, from which all gauge-invariant terms in the action can be built [74]. In this subsection, we extend this procedure to gauge-covariant spacetime-tensor operators, which permits the construction of all terms with both gauge invariance and observer independence in curved spacetime.

For the gauge symmetry, consider first the scenario in Minkowski spacetime with a Dirac fermion ψ in a representation U of the gauge group [74]. Then, $\psi \to U\psi$ under a gauge transformation, while the Dirac conjugate transforms as $\overline{\psi} \to \overline{\psi}U^\dagger$. The gauge-covariant derivative acting on ψ can be written as $D_\mu\psi = \partial_\mu\psi - igA_\mu\psi$, where g is the gauge coupling and A_μ is the gauge field in the U representation, and it transforms as $D_\mu \to UD_\mu U^\dagger$. The gauge field strength $F_{\mu\nu}$ in the U representation is generated by the commutator $[D_\mu, D_\nu] = -igF_{\mu\nu}$. By definition, an operator $\mathcal{O}$ formed from gauge fields is called gauge covariant if $\mathcal{O} \to U\mathcal{O}U^\dagger$. Given gauge-covariant operators $\mathcal{O}$ and $\mathcal{O}'$, two kinds of gauge-invariant operators can be constructed, $\mathrm{tr}(\mathcal{O})$ and $(\overline{\mathcal{O}\psi})\Gamma\mathcal{O}'\psi$, where Γ represents the 16 matrices $\{1, i\gamma_5, \gamma_\mu, \gamma_5\gamma_\mu, \sigma_{\mu\nu}/2\}$ spanning the spinor space. These gauge-invariant operators are the desired objects from which to build terms in the Lagrange density for the effective field theory in Minkowski spacetime. More details about this construction and its implications can be found in Sec. II of Ref. [74].

To generalize this construction to curved spacetime, we can work with spacetime-tensor fields and covariant derivatives extended to include an appropriate connection. Relevant spacetime-tensor fields include the metric $g_{\mu\nu}$, the curvature tensor $R_{\kappa\lambda\mu\nu}$, the gauge field strength $F_{\mu\nu}$, the spinor bilinears $\overline{\psi}\Gamma\psi$, scalars ϕ, and combinations. For this purpose, the quantities γ_μ appearing in Γ are now understood as $\gamma_\mu = e_{\mu a}\gamma^a$, where the Dirac matrices γ^a are defined in the local frame. In combinations, gauge-invariant operators are placed inside fermion bilinears. All these spacetime-tensor operators are also gauge covariant. The covariant derivative D_μ acting on ψ or on $DD\cdots D\psi$ can be expressed as

$$D_\mu = \overset{\star}{D}_\mu + \tfrac{1}{4}\omega_\mu{}^{ab}\sigma_{ab} - igA_\mu, \qquad (44)$$

where $\overset{\star}{D}_\mu$ is the usual covariant derivative of GR containing the partial derivative ∂_μ and the appropriate connection term formed using Christoffel symbols, and where $\omega_\mu{}^{ab}$ is the spin connection. For explicit derivations, we adopt the conventions of Ref. [2]. Direct calculation shows that any mixture of covariant derivatives and the

spacetime-tensor operators also forms a gauge-covariant spacetime-tensor operator.

Building the Lagrange density $\mathcal{L}$ using all possible gauge-covariant spacetime-tensor operators would introduce many redundancies due to relationships between various mixtures of operators. It is therefore useful to work instead with a standard basis set that has no or controlled redundancies. The key result here is that any mixture of g, R, F, Γ, and D can be written in the standard form

$$[(D_{(n_1)}R)\cdots(D_{(n_t)}R)][(D_{(m_1)}F)\cdots(D_{(m_s)}F)]\Gamma D_{(l)},\tag{45}$$

where all indices on spacetime-tensor fields are omitted for simplicity. In this expression, we introduce the notation

$$D_{(n)} \equiv \frac{1}{n!}\sum D_{\alpha_1}D_{\alpha_2}\cdots D_{\alpha_n}\tag{46}$$

as a symmetric sum over the n indices. Note that explicit factors of the metric tensor $g_{\mu\nu}$ can safely be disregarded in the form (45), as $g_{\mu\nu}$ commutes with covariant derivatives and all other operators.

To prove the result (45), we follow a path similar to the proof of Eq. (2) in Ref. [74]. It suffices to consider the case that the operator (45) acts on a Dirac fermion field ψ, as other cases are both similar and simpler. Using the product rule for covariant derivatives, any mixture of D, R, F, and Γ can be expressed in block form as

$$\mathcal{O} = \sum (DD\cdots DR)\cdots(DD\cdots DR)$$
$$\times (DD\cdots DF)\cdots(DD\cdots DF)\Gamma DD\cdots D,\tag{47}$$

where we use $D\Gamma = 0$, which follows from Eq. (5). It therefore suffices to prove that operators of the form $D_{\alpha_1}D_{\alpha_2}\cdots D_{\alpha_n}$ can be expressed as linear combinations of the basis (45). This can be achieved by mathematical induction. The case $n = 1$ follows directly from the definition of $D_{(n)}$. Suppose the proposition holds for $n \leq k$. Then, we can decompose $D_{\alpha_1}D_{\alpha_2}\cdots D_{\alpha_{k+1}}$ using Young tableaux,

$$\boxed{1}\,\boxed{2}\,\boxed{\cdots}\,\boxed{k}\otimes\boxed{k{+}1} = \boxed{1}\,\boxed{2}\,\boxed{\cdots}\,\boxed{k}\,\boxed{k{+}1} \oplus \begin{array}{c}\boxed{1}\,\boxed{2}\,\boxed{\cdots}\,\boxed{k}\\[-2pt]\boxed{k{+}1}\end{array}.\tag{48}$$

The first term on the right-hand side is of the form $D_{(k+1)}$. The second term contains at least one commutator of two covariant derivatives, as proved in Ref. [74]. Direct calculation shows that

$$[D_\mu, D_\nu] = [\overset{\star}{D}_\mu, \overset{\star}{D}_\nu] + \tfrac{1}{4}iR_{\kappa\lambda\mu\nu}e^{\kappa a}e^{\lambda b}\sigma_{ab} - igF_{\mu\nu},\tag{49}$$

where the commutator on the right-hand side generates a combination of curvature tensors, so the commutator of covariant derivatives yields factors of R, F, and Γ. The remaining part of the second Young tableau (48) thus contains at most $k - 1 < k$ covariant derivatives. The proposition therefore holds for $n = k + 1$, concluding the proof.

Next, we consider the linear independence of the operators (45). Note first that the operators $D_{(n_i)}R$ and $D_{(n_j)}R$ are linearly independent when $n_i \neq n_j$ because they have different mass dimensions. Since $D_{(n_i)}R$ commutes with $D_{(n_j)}R$, we can impose $n_1 \leq \cdots \leq n_t$ on the basis (45). Similarly, $D_{(n_i)}F$ and $D_{(n_j)}F$ are linearly independent when $n_i \neq n_j$, but they commute only for abelian gauge field theory. In the abelian case, we can therefore choose the linearly independent standard basis as

$$\left\{[(D_{(n_1)}R)\cdots(D_{(n_t)}R)][(D_{(m_1)}F)\cdots(D_{(m_s)}F)]\Gamma D_{(l)}\right.$$
$$\left.|n_1 \leq \cdots \leq n_t, m_1 \leq \cdots \leq m_s\right\}.\tag{50}$$

However, in a nonabelian gauge theory, we choose instead the basis

$$\left\{[(D_{(n_1)}R)\cdots(D_{(n_t)}R)][(D_{(m_1)}F)\cdots(D_{(m_s)}F)]\Gamma D_{(l)}\right.$$
$$\left.|n_1 \leq \cdots \leq n_t\right\}.\tag{51}$$

This basis is linearly independent in some cases and is almost linearly dependent in others, depending on the structure of the gauge group [74].

In building a hermitian Lagrange density, it is useful to have explicit expressions for the hermitian conjugations of the operators (45). Including fermions to form bilinears, we find

$$[(\overline{D_{(m_0)}\psi})(D_{(m_1)}F_{\beta_1\gamma_1})\cdots(D_{(m_s)}F_{\beta_s\gamma_s})\Gamma(D_{(m_{s+1})}\psi)]^\dagger$$
$$= (\overline{D_{(m_{s+1})}\psi})(D_{(m_s)}F_{\beta_s\gamma_s})\cdots(D_{(m_1)}F_{\beta_1\gamma_1})\Gamma(D_{(m_0)}\psi).\tag{52}$$

Any pieces involving $D_{(n)}R$ can be omitted because they are independently real and commute with all other operators. The result (52) takes the same form as Eq. (13) in Ref. [74], but the covariant derivatives now include also terms involving the spacetime and spin connections. The proof of Eq. (52) follows the path given in Ref. [74].

B. Pure-gravity and background sector

With the construction of generic gauge-invariant spacetime-tensor operators in hand, we can address specific sectors of the effective field theory in turn. In this subsection, we consider operators involving pure-gravity fields.

For the pure-gravity sector, it is convenient to distinguish terms in the Lagrange density $\mathcal{L}_g$ according to mass dimension. We therefore write

$$\mathcal{L}_g = \frac{1}{2\kappa}(\mathcal{L}_{g0}+\mathcal{L}_g^{(2)}+\mathcal{L}_g^{(3)}+\mathcal{L}_g^{(4)}+\mathcal{L}_g^{(5)}+\mathcal{L}_g^{(6)}+\ldots),\tag{53}$$

where $1/2\kappa \equiv 1/16\pi G_N \simeq 3 \times 10^{36}$ GeV^2 is the gravitational coupling constant formed from the Newton gravitational constant G_N. The term $\mathcal{L}_{g0} = R - 2\Lambda$ is the

TABLE V. Schematic structure of terms with $d \leq 6$ in the Lagrange density $\mathcal{L}_g$.

Component	Expression
$\mathcal{L}_g^{(2)}$	$\check{k}$
$\mathcal{L}_g^{(3)}$	$\mathcal{L}_{g,\delta}^{(3)} + \check{k}(Dk)$
$\mathcal{L}_g^{(4)}$	$\check{k}R + \check{k}(Dk)(Dk)$
$\mathcal{L}_g^{(5)}$	$\check{k}DR + \mathcal{L}_{g,\delta}^{(5)} + \check{k}(Dk)R + \check{k}(Dk)(Dk)(Dk) + \check{k}(Dk)(D_{(2)}k)$
$\mathcal{L}_g^{(6)}$	$\check{k}RR + \check{k}D_{(2)}R + \check{k}(Dk)(Dk)R + \check{k}(D_{(2)}k)R + \check{k}(Dk)(Dk)(Dk)(Dk) + \check{k}(Dk)(Dk)(D_{(2)}k) + \check{k}(D_{(2)}k)(D_{(2)}k)$

conventional Einstein-Hilbert expression with cosmological constant, while the terms $\mathcal{L}_g^{(d)}$ represent contributions to the effective field theory. Note that each individual component $\mathcal{L}_g^{(d)}$ has mass dimension two, but by convention the superscript d represents the mass dimension of the dynamical operator in $\mathcal{L}_g^{(d)}$ including the factor of the gravitational coupling constant. For example, $\mathcal{L}_g^{(4)}$ includes terms with the Riemann tensor as dynamical operator, which in this convention is of mass dimension two.

Table V displays all terms in $\mathcal{L}_g^{(d)}$ with $d \leq 6$ in schematic form. The first column lists each component Lagrange density given in Eq. (53), while the second column specifies the corresponding operator structures that can appear. In this schematic notation, each instance of k can represent a distinct background even when occurring in a single term, and the various quantities $\check{k}$ may also be distinct. In each row, terms without background derivatives Dk are listed first. All terms are invariant under general coordinate transformations except for $\mathcal{L}_{g,\delta}^{(d)}$ with $d = 3$ or 5, which transform into a total derivative and hence maintain invariance of the action rather than the Lagrange density.

Throughout Table V, all indices and any numerical or dimensionful factors are omitted for simplicity. In each term, we can absorb into $\check{k}$ any metric, vierbein, or Levi-Civita factors, so all indices on $D_{(n)}\overline{k}$ and $D_{(n)}R$ can be assumed distinct and contracted with indices on $\check{k}$. To maintain hermiticity of the Lagrange density, all backgrounds are taken to be real. The mass dimensionality of a given background k or $\check{k}$ is fixed by the operator structure of the Lagrange density. As examples, the quantity $\check{k}$ in $\mathcal{L}_g^{(2)}$ has mass dimension $d = 2$, while $\check{k}$ in the term $\check{k}R$ is dimensionless. The mass dimension of k may be different in different theories, so for convenience and simplicity we adopt the convention that the mass dimension of k is disregarded in assigning a given term to a particular component $\mathcal{L}_g^{(d)}$. For instance, the term $\check{k}(Dk)$ is assigned to $\mathcal{L}_g^{(3)}$ irrespective of the actual mass dimension of k. The number of derivatives D is instead the relevant factor in defining the mass dimension of terms involving background derivatives.

Since the commutator of two covariant derivatives gen-

erates curvature factors, and since the product of two backgrounds can schematically be viewed as a single background, certain terms can be omitted from the table without loss of generality. For example, the term $\check{k}[D,D]k$ is schematically equivalent to the term $\check{k}R$, which itself is typically included in $\check{k}(Dk)(Dk)$ up to a surface term. The only exception arises when the background is index free, with $\check{k}R$ representing the term $k(x)g^{\alpha\gamma}g^{\beta\delta}R_{\alpha\beta\gamma\delta}$.

More insight can be obtained from the explicit forms of the components of the Lagrange density $\mathcal{L}_g$. First, consider terms without background derivatives Dk. The curvature plays a central role in most of these terms. Table VI displays the explicit form of all terms in $\mathcal{L}_g$ for $d \leq 8$. In this table, unlike others in this section, we include operators with $d = 7$ and 8 because they are comparatively few in number and because certain $d = 8$ backgrounds have recently been constrained by observational data [60, 70]. Each row of the table lists a component of $\mathcal{L}_g$ constructed from operators of a given mass dimension, followed by the explicit form of the corresponding terms. In each term, the combination $\check{k}$ of backgrounds is understood to be real and to inherit from the dynamical operator the appropriate symmetry under permutation of the indices. Most of the operators can be constructed by inspection from Table V. The components $\mathcal{L}_{g,\delta}^{(d)}$ with $d = 3$ and 5 appearing in Table V, which are general coordinate invariant only up to a surface term, are represented in Table VI by the three terms with backgrounds $(\check{k}_{\Gamma}^{(3)})^{\mu}$, $(\check{k}_{\mathrm{CS},1}^{(5)})_{\mu}$, and $(\check{k}_{\mathrm{CS},2}^{(5)})_{\mu}$. The operators associated with the latter two are Chern-Simons terms expressed using the vierbein and spin connection. To preserve general coordinate invariance of the action, these three backgrounds must obey

$$D_{\mu}(\check{k}_{\Gamma}^{(3)})^{\mu} = 0,$$
$$D_{\mu}(\check{k}_{\mathrm{CS},1}^{(5)})_{\nu} - D_{\nu}(\check{k}_{\mathrm{CS},1}^{(5)})_{\mu} = 0,$$
$$D_{\mu}(\check{k}_{\mathrm{CS},2}^{(5)})_{\nu} - D_{\nu}(\check{k}_{\mathrm{CS},2}^{(5)})_{\mu} = 0. \qquad (54)$$

The first equation implies $\partial_{\mu}[(\check{k}_{\Gamma}^{(3)})^{\mu}/e] = 0$ in any coordinate frame. The second equation implies $\partial_{\mu}(\check{k}_{\mathrm{CS},1}^{(5)})_{\nu} - \partial_{\nu}(\check{k}_{\mathrm{CS},1}^{(5)})_{\mu} = 0$, so if the topology is trivial then we can write $(\check{k}_{\mathrm{CS},1}^{(5)})_{\mu} = \partial_{\mu}k$ for some scalar background $k(x)$.

TABLE VI. Terms with $d \leq 8$ and without background derivatives in the Lagrange density $\mathcal{L}_g$.

Component	Expression
$\mathcal{L}_{g0}$	$R - 2\Lambda$
$\mathcal{L}_g^{(2)}$	$\breve{k}^{(2)}$
$\mathcal{L}_g^{(3)}$	$(\breve{k}_\Gamma^{(3)})^\mu \Gamma^\alpha{}_{\mu\alpha}$
$\mathcal{L}_g^{(4)}$	$(\breve{k}_R^{(4)})^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta}$
$\mathcal{L}_g^{(5)}$	$(\breve{k}_D^{(5)})^{\alpha\beta\gamma\delta\kappa} D_\kappa R_{\alpha\beta\gamma\delta} + (\breve{k}_{\mathrm{CS},1}^{(5)})_\kappa \epsilon^{\kappa\lambda\mu\nu} \eta_{ac}\eta_{bd}(\omega_\lambda{}^{ab}\partial_\mu\omega_\nu{}^{cd} + \frac{2}{3}\omega_\lambda{}^{ab}\omega_\mu{}^{ce}\omega_{\nu e}{}^d)$
	$\quad + (\breve{k}_{\mathrm{CS},2}^{(5)})_\kappa \epsilon^{\kappa\lambda\mu\nu} \epsilon_{abcd}(\omega_\lambda{}^{ab}\partial_\mu\omega_\nu{}^{cd} + \frac{2}{3}\omega_\lambda{}^{ab}\omega_\mu{}^{ce}\omega_{\nu e}{}^d)$
$\mathcal{L}_g^{(6)}$	$(\breve{k}_D^{(6)})^{\alpha\beta\gamma\delta\kappa\lambda} D_{(\kappa}D_{\lambda)}R_{\alpha\beta\gamma\delta} + (\breve{k}_R^{(6)})^{\alpha_1\beta_1\gamma_1\delta_1\alpha_2\beta_2\gamma_2\delta_2} R_{\alpha_1\beta_1\gamma_1\delta_1} R_{\alpha_2\beta_2\gamma_2\delta_2}$
$\mathcal{L}_g^{(7)}$	$(\breve{k}_D^{(7)})^{\alpha\beta\gamma\delta\kappa\lambda\mu} D_{(\kappa}D_\lambda D_{\mu)}R_{\alpha\beta\gamma\delta} + (\breve{k}_{DR}^{(7)})^{\alpha_1\beta_1\gamma_1\delta_1\alpha_2\beta_2\gamma_2\delta_2\kappa} R_{\alpha_1\beta_1\gamma_1\delta_1} D_\kappa R_{\alpha_2\beta_2\gamma_2\delta_2}$
$\mathcal{L}_g^{(8)}$	$(\breve{k}_D^{(8)})^{\alpha\beta\gamma\delta\kappa\lambda\mu\nu} D_{(\kappa}D_\lambda D_\mu D_{\nu)}R_{\alpha\beta\gamma\delta} + (\breve{k}_{DR,1}^{(8)})^{\alpha_1\beta_1\gamma_1\delta_1\alpha_2\beta_2\gamma_2\delta_2\kappa\lambda} R_{\alpha_1\beta_1\gamma_1\delta_1} D_{(\kappa}D_{\lambda)}R_{\alpha_2\beta_2\gamma_2\delta_2}$
	$\quad + (\breve{k}_{DR,2}^{(8)})^{\alpha_1\beta_1\gamma_1\delta_1\alpha_2\beta_2\gamma_2\delta_2\kappa\lambda}(D_\kappa R_{\alpha_1\beta_1\gamma_1\delta_1})(D_\lambda R_{\alpha_2\beta_2\gamma_2\delta_2})$
	$\quad + (\breve{k}_R^{(8)})^{\alpha_1\beta_1\gamma_1\delta_1\alpha_2\beta_2\gamma_2\delta_2\alpha_3\beta_3\gamma_3\delta_3} R_{\alpha_1\beta_1\gamma_1\delta_1} R_{\alpha_2\beta_2\gamma_2\delta_2} R_{\alpha_3\beta_3\gamma_3\delta_3}$

A similar result holds for the third equation.

Next, consider the terms in Table V involving one or more factors of Dk that cannot be moved onto the dynamical operators via integration by parts. Since different backgrounds and hence distinct Dk factors can appear in a given term, an explicit listing of all such terms is impractical in the general case. We can, however, gain useful insight about generic terms with background derivatives Dk by considering ones constructed explicitly using a one-index background because any given background $k^{\mu\cdots}{}_{\nu\ldots}{}^{a\cdots}$ can be expressed as a linear combination of products of backgrounds with only one index. For example, a two-index background $k^{\mu\nu}$ can be viewed as a linear combination of products of eight one-index backgrounds as follows. Given $k^{\mu\nu}$ in a specific coordinate system, we can define four one-index backgrounds in these coordinates via $(k_\mu)^\nu \equiv k^{\mu\nu}$ and another four via $(k'_\mu)^\nu \equiv \delta_\mu{}^\nu$. In the chosen coordinate system, it then follows that $k^{\mu\nu} = \sum_\lambda (k_\lambda)^\nu (k'_\lambda)^\mu$. Provided we define the eight one-index backgrounds to transform as 4-vectors under general coordinate transformations, this expression is a tensor expression and hence is valid in any coordinate system, as required.

Taking advantage of this result, we can consider a scenario involving a single one-index background k_μ as an example to gain insight. Table VII displays possible operators of mass dimension $d \leq 6$ involving derivatives of a background k_μ with one covariant spacetime index. The first column lists $\mathcal{L}_k^{(d)}$ for different values of d, while the second column displays the corresponding possible terms involving $D_\mu k_\nu$. As before, each combination $\breve{k}$ of backgrounds is taken as real and inherits the symmetry under index permutation of the operator, and each operator can be constructed using Table V as a guide. Following the convention described above, the assigned value of the mass dimension d of a particular term is defined for con-

TABLE VII. Terms with $d \leq 6$ involving derivatives of a single background k_μ.

Component	Expression
$\mathcal{L}_k^{(3)}$	$(\breve{k}_{Dk}^{(3)})^{\kappa\lambda}(D_\kappa k_\lambda)$
$\mathcal{L}_k^{(4)}$	$(\breve{k}_{Dk}^{(4)})^{\kappa\lambda\mu\nu}(D_\kappa k_\lambda)(D_\mu k_\nu)$
$\mathcal{L}_k^{(5)}$	$(\breve{k}_{Dk}^{(5)})^{\kappa\lambda\mu\nu\rho\sigma}(D_\kappa k_\lambda)(D_\mu k_\nu)(D_\rho k_\sigma)$,
	$(\breve{k}_{DDk}^{(5)})^{\kappa\lambda\mu\nu\rho}(D_\kappa k_\lambda)(D_{(\mu}D_{\nu)}k_\rho)$,
	$(\breve{k}_{DkR}^{(5)})^{\kappa\lambda\mu\nu\rho\sigma}(D_\kappa k_\lambda)R_{\mu\nu\rho\sigma}$
$\mathcal{L}_k^{(6)}$	$(\breve{k}_{Dk}^{(6)})^{\kappa\lambda\mu\nu\rho\sigma\tau\upsilon}(D_\kappa k_\lambda)(D_\mu k_\nu)(D_\rho k_\sigma)(D_\tau k_\upsilon)$,
	$(\breve{k}_{DDk}^{(6)})^{\kappa\lambda\mu\nu\rho\sigma\tau}(D_\kappa k_\lambda)(D_\mu k_\nu)(D_{(\rho}D_{\sigma)}k_\tau)$,
	$(\breve{k}_{DDkDDk}^{(6)})^{\kappa\lambda\mu\nu\rho\sigma}(D_{(\kappa}D_{\lambda)}k_\mu)(D_{(\nu}D_{\rho)}k_\sigma)$,
	$(\breve{k}_{DkR}^{(6)})^{\kappa\lambda\mu\nu\rho\sigma\tau\upsilon}(D_\kappa k_\lambda)(D_\mu k_\nu)R_{\rho\sigma\tau\upsilon}$,
	$(\breve{k}_{DDkR}^{(6)})^{\kappa\lambda\mu\nu\rho\sigma\tau}(D_{(\kappa}D_{\lambda)}k_\mu)R_{\nu\rho\sigma\tau}$

venience by the number of derivatives D involved. As a result, the various combinations $\breve{k}^{(d)}$ at given d may have different mass dimensions and may incorporate different powers of k_μ. Notice also that the combinations $\breve{k}^{(d)}$ with even numbers of indices cannot contain terms linear in k_μ. For example, the leading-order k_μ-dependent term in $(\breve{k}_{Dk}^{(3)})^{\kappa\lambda}$ is $(\breve{k}_{Dk}^{(3)})^{\kappa\lambda} \propto k_\mu k_\nu g^{\mu\kappa} g^{\nu\lambda}$, which produces a cubic coupling in k_μ for the component $\mathcal{L}_k^{(3)}$.

If the breaking is spontaneous, the type of index carried by the background has no effect on the physics, as discussed in Sec. II C. In this scenario, Table VII also encompasses operators of mass dimension $d \leq 6$ constructed from a single background k^μ or k^a, provided each instance of k_μ in the table is replaced with $g_{\mu\nu}k^\nu$ or $e_{\mu a}k^a$, respectively. However, if the breaking is explicit,

TABLE VIII. Schematic structure of terms with $d \leq 6$ in the gauge Lagrange density $\mathcal{L}_A$.

Component	Expression
$\mathcal{L}_A^{(1)}$	$\mathcal{L}_{A,\delta}^{(1)}$
$\mathcal{L}_A^{(3)}$	$\mathcal{L}_{A,\delta}^{(3)} + \check{k}(Dk)\mathrm{tr}(F)$
$\mathcal{L}_A^{(4)}$	$\check{k}\mathrm{tr}(FF) + \check{k}\mathrm{tr}(D_{(2)}F) + \check{k}R\mathrm{tr}(F) + \check{k}(Dk)(Dk)\mathrm{tr}(F) + \check{k}(D_{(2)}k)\mathrm{tr}(F)$
$\mathcal{L}_A^{(5)}$	$\check{k}\mathrm{tr}(FDF) + \check{k}\mathrm{tr}(D_{(3)}F) + \check{k}R\mathrm{tr}(DF) + \check{k}(DR)\mathrm{tr}(F)$
	$\quad + \check{k}(Dk)R\mathrm{tr}(F) + \check{k}(Dk)(Dk)(Dk)\mathrm{tr}(F) + \check{k}(Dk)(D_{(2)}k)\mathrm{tr}(F) + \check{k}(D_{(3)}k)\mathrm{tr}(F)$
$\mathcal{L}_A^{(6)}$	$\check{k}\mathrm{tr}(FD_{(2)}F) + \check{k}\mathrm{tr}((DF)(DF)) + \check{k}\mathrm{tr}(D_{(4)}F) + [\check{k}\mathrm{tr}(FFF) + \mathrm{h.c.}]$
	$\quad + \check{k}R\mathrm{tr}(FF) + \check{k}R\mathrm{tr}(D_{(2)}F) + \check{k}(D_{(2)}R)\mathrm{tr}(F) + \check{k}(DR)\mathrm{tr}(DF) + \check{k}RR\mathrm{tr}(F)$
	$\quad + \check{k}(Dk)\mathrm{tr}(FDF) + \check{k}(Dk)(Dk)\mathrm{tr}(FF) + \check{k}(D_{(2)}k)\mathrm{tr}(FF) + \check{k}(D_{(4)}k)\mathrm{tr}(F)$
	$\quad + \check{k}(Dk)(Dk)(Dk)(Dk)\mathrm{tr}(F) + \check{k}(Dk)(Dk)(D_{(2)}k)\mathrm{tr}(F) + \check{k}(D_{(2)}k)(D_{(2)}k)\mathrm{tr}(F) + \check{k}(Dk)(D_{(3)}k)\mathrm{tr}(F)$
	$\quad + \check{k}(Dk)(DR)\mathrm{tr}(F) + \check{k}(Dk)(Dk)R\mathrm{tr}(F) + \check{k}(D_{(2)}k)R\mathrm{tr}(F)$

then each of the three types of single-index background k_μ, k^μ, and k^a can lead to different physics. If all three are present, then we can write

$$\mathcal{L}_k^{(3)} = (\check{k}_{Dk,1}^{(3)})^{\mu\nu}D_\mu\overline{k}_\nu + (\check{k}_{Dk,2}^{(3)})^\mu{}_\nu D_\mu\overline{k}^\nu + (\check{k}_{Dk,3}^{(3)})^{\mu a}D_\mu\overline{k}_a, \tag{55}$$

with distinct combinations $\check{k}$ for each term.

Taken together, the results in this subsection provide the explicit form of all terms at $d \leq 6$ in the expansion (57) of Ref. [2] in the zero-torsion limit. Our construction covers both the pure-gravity sector and any background dynamics, allowing for either spontaneous or explicit violations of spacetime symmetries. It opens the way to broadening theoretical explorations of many issues including, for example, the t puzzle [10, 50], the relevance of spontaneous violation [121], and the influence of gravitational effects on the matter sector [122]. It also has potential applications to the numerous ongoing experimental searches for backgrounds.

C. Matter sector

Next, we turn attention to the matter sector, including pure matter, matter-gravity, and matter-background terms. It is convenient to separate the discussion according to the type of matter involved. In what follows, we first consider the inclusion of gauge fields, then Dirac spinors, and finally complex scalar fields.

1. Gauge fields

In the gauge sector, we consider for definiteness a Lagrange density $\mathcal{L}_A$ determining the behavior of a gauge field A_μ with gauge field strength $F_{\mu\nu}$ in the U representation of the gauge group. This follows the setup adopted in Sec. II of Ref. [74] and summarized in Sec. III A above.

Couplings of A_μ to gravity and backgrounds are included in this sector. It is convenient to split $\mathcal{L}_A$ into pieces according to the mass dimension d of the dynamical operator,

$$\mathcal{L}_A = \mathcal{L}_{A0} + \mathcal{L}_A^{(1)} + \mathcal{L}_A^{(2)} + \mathcal{L}_A^{(3)} + \mathcal{L}_A^{(4)} + \mathcal{L}_A^{(5)} + \dots \tag{56}$$

The first piece $\mathcal{L}_{A0} = \mathrm{tr}(F_{\mu\nu}F^{\mu\nu})/2$ is the usual Yang-Mills term in curved spacetime expressed with a trace taken in the gauge space. On each of the other pieces, the superscript indicates the value of d.

Table VIII lists the schematic form of all terms in $\mathcal{L}_A^{(d)}$ with mass dimension $d \leq 6$. The first entry in each row shows a component of $\mathcal{L}_A$ with fixed d, while the second entry lists the schematic forms of the corresponding operators. In this notation, each occurrence of k in a given term can represent a distinct background, and the combinations $\check{k}$ appearing in different terms can also be distinct. Terms in the pure gauge sector are shown first, followed by terms with gauge-curvature couplings, and then by terms with background derivatives Dk. All terms in the Lagrange density are invariant under gauge transformations except $\mathcal{L}_{A,\delta}^{(d)}$ with $d = 1$ and 3, which become total derivatives and thus leave the action invariant instead. The table omits all indices and factors, and all backgrounds can be taken as real except for the one involving the operator $\mathrm{tr}(FFF)$. Note that the components $\mathcal{L}_A^{(d)}$ have mass dimension four, unlike the components $\mathcal{L}_g^{(d)}$ in Eq. (53). Other properties of the backgrounds k and the combinations $\check{k}$ follow those described for Table V.

The explicit form of terms in $\mathcal{L}_A$ without background derivatives are displayed in Table IX for $d \leq 6$. The structure of the table follows that of Table VIII, and the various expressions can be derived using the latter as a guide. Each combination $\check{k}$ appearing in Table IX is understood to have index symmetry determined by the index structure of the corresponding dynamical operator.

TABLE IX. Terms with $d \leq 6$ and without background derivatives in the gauge Lagrange density $\mathcal{L}_A$.

Component	Expression
$\mathcal{L}_{A0}$	$-\frac{1}{2}\mathrm{tr}(F_{\mu\nu}F^{\mu\nu})$
$\mathcal{L}_A^{(1)}$	$-(\breve{k}^{(1)})^\mu \mathrm{tr}(A_\mu)$
$\mathcal{L}_A^{(3)}$	$(\breve{k}_{\mathrm{CS}}^{(3)})_\kappa \epsilon^{\kappa\lambda\mu\nu}\mathrm{tr}(A_\lambda F_{\mu\nu} + \frac{2}{3}igA_\lambda A_\mu A_\nu) + (\breve{k}_{DF}^{(3)})^{\alpha\mu\nu}\mathrm{tr}(D_\alpha F_{\mu\nu})$
$\mathcal{L}_A^{(4)}$	$-\frac{1}{2}(\breve{k}_F^{(4)})^{\kappa\lambda\mu\nu}\mathrm{tr}(F_{\kappa\lambda}F_{\mu\nu}) - (\breve{k}_{DF}^{(4)})^{\alpha\beta\mu\nu}\mathrm{tr}(D_{(\alpha}D_{\beta)}F_{\mu\nu}) - (\breve{k}_{RF}^{(4)})^{\alpha\beta\gamma\delta\mu\nu}R_{\alpha\beta\gamma\delta}\mathrm{tr}(F_{\mu\nu})$
$\mathcal{L}_A^{(5)}$	$\frac{1}{2}(\breve{k}_D^{(5)})^{\alpha\kappa\lambda\mu\nu}\mathrm{tr}(F_{\kappa\lambda}D_\alpha F_{\mu\nu}) + (\breve{k}_{DF}^{(5)})^{\alpha\beta\gamma\mu\nu}\mathrm{tr}(D_{(\alpha}D_\beta D_{\gamma)}F_{\mu\nu})$
	$+(\breve{k}_{RDF}^{(5)})^{\alpha\beta\gamma\delta\epsilon\mu\nu}R_{\alpha\beta\gamma\delta}\mathrm{tr}(D_\epsilon F_{\mu\nu}) + (\breve{k}_{DRF}^{(5)})^{\alpha\beta\gamma\delta\epsilon\mu\nu}(D_\epsilon R_{\alpha\beta\gamma\delta})\mathrm{tr}(F_{\mu\nu})$
$\mathcal{L}_A^{(6)}$	$-\frac{1}{2}(\breve{k}_D^{(6)})^{\alpha\beta\kappa\lambda\mu\nu}\mathrm{tr}(F_{\kappa\lambda}D_{(\alpha}D_{\beta)}F_{\mu\nu}) - \frac{1}{2}(\breve{k}_{DFDF}^{(6)})^{\alpha\beta\kappa\lambda\mu\nu}\mathrm{tr}((D_\alpha F_{\kappa\lambda})(D_\beta F_{\mu\nu}))$
	$-(\breve{k}_{DF}^{(6)})^{\alpha\beta\gamma\delta\mu\nu}\mathrm{tr}(D_{(\alpha}D_\beta D_\gamma D_{\delta)}F_{\mu\nu}) - \frac{1}{12}[(\breve{k}_F^{(6)})^{\kappa\lambda\mu\nu\rho\sigma}\mathrm{tr}(F_{\kappa\lambda}F_{\mu\nu}F_{\rho\sigma}) + \mathrm{h.c.}]$
	$-\frac{1}{2}(\breve{k}_{RFF}^{(6)})^{\alpha\beta\gamma\delta\kappa\lambda\mu\nu}R_{\alpha\beta\gamma\delta}\mathrm{tr}(F_{\kappa\lambda}F_{\mu\nu}) - (\breve{k}_{RDDF}^{(6)})^{\alpha\beta\gamma\delta\epsilon\zeta\mu\nu}R_{\alpha\beta\gamma\delta}\mathrm{tr}(D_{(\epsilon}D_{\zeta)}F_{\mu\nu})$
	$-(\breve{k}_{DDRF}^{(6)})^{\alpha\beta\gamma\delta\epsilon\zeta\mu\nu}(D_{(\epsilon}D_{\zeta)}R_{\alpha\beta\gamma\delta})\mathrm{tr}(F_{\mu\nu}) - (\breve{k}_{DRDF}^{(6)})^{\alpha\beta\gamma\delta\epsilon\zeta\mu\nu}(D_\epsilon R_{\alpha\beta\gamma\delta})\mathrm{tr}(D_\zeta F_{\mu\nu})$
	$-(\breve{k}_{RRF}^{(6)})^{\alpha_1\beta_1\gamma_1\delta_1\alpha_2\beta_2\gamma_2\delta_2\mu\nu}R_{\alpha_1\beta_1\gamma_1\delta_1}R_{\alpha_2\beta_2\gamma_2\delta_2}\mathrm{tr}(F_{\mu\nu})$

All combinations $\breve{k}$ are real except for the one controlling the dynamical operator $\mathrm{tr}(F_{\kappa\lambda}F_{\mu\nu}F_{\rho\sigma})$. The terms $\mathcal{L}_{A,\delta}^{(1)}$ and $\mathcal{L}_{A,\delta}^{(3)}$ in Table VIII are explicitly given in Table IX as the terms with backgrounds $(\breve{k}^{(1)})^\mu$ and $(\breve{k}_{\mathrm{CS}}^{(3)})_\kappa$, with the latter governing the nonabelian Chern-Simons operator. To maintain gauge symmetry, these backgrounds must satisfy the conditions

$$D_\mu(\breve{k}^{(1)})^\mu = 0,$$
$$D_\mu(\breve{k}_{\mathrm{CS}}^{(3)})_\nu - D_\nu(\breve{k}_{\mathrm{CS}}^{(3)})_\mu = 0. \qquad (57)$$

The first of these expressions implies $\partial_\mu[(\breve{k}^{(1)})^\mu/e] = 0$, while the second implies $\partial_\mu(\breve{k}_{\mathrm{CS}}^{(3)})_\nu - \partial_\nu(\breve{k}_{\mathrm{CS}}^{(3)})_\nu = 0$. If the topology is trivial, a scalar background field $k(x)$ can be found such that $(\breve{k}_{\mathrm{CS}}^{(3)})_\mu = \partial_\mu k$, in which case its contribution to $\mathcal{L}_A^{(3)}$ is equivalent under partial integration to a $d = 4$ term $k\epsilon^{\kappa\lambda\mu\nu}F_{\kappa\lambda}F_{\mu\nu}$. If the topology is nontrivial, however, the Chern-Simons contribution to $\mathcal{L}_A^{(3)}$ can produce independent physical effects. Note that the putative $d = 2$ term $\breve{k}^{\mu\nu}\mathrm{tr}(F_{\mu\nu}) = \breve{k}^{\mu\nu}\mathrm{tr}(\partial_\mu A_\nu - \partial_\nu A_\mu)$ involves a combination of partial derivatives of A_μ, so it is included in $\mathcal{L}_A^{(1)}$ up to a surface term. We also omit terms such as $(\mathrm{tr}F)(\mathrm{tr}(\cdots F))$ because these terms either vanish if the gauge group is SU(N) or are incorporated in terms like $\mathrm{tr}(F \cdots F)$ if the gauge group is U(1), but their inclusion may be appropriate for other gauge groups.

2. Fermions

Consider next the fermion sector, including couplings to gravity and to gauge fields. For definiteness, we adopt the scenario described in Sec. II of Ref. [74] and outlined in Sec. III A above, with a Dirac fermion ψ of mass m_ψ lying in the U representation of the gauge group. The Lagrange density $\mathcal{L}_\psi$ can be split into components containing dynamical operators with definite mass dimension d,

$$\mathcal{L}_\psi = \mathcal{L}_{\psi 0} + \mathcal{L}_\psi^{(3)} + \mathcal{L}_\psi^{(4)} + \mathcal{L}_\psi^{(5)} + \mathcal{L}_\psi^{(6)} + \ldots. \qquad (58)$$

The term $\mathcal{L}_{\psi 0}$ is the usual Dirac Lagrange density $\mathcal{L}_{\psi 0} = \overline{\psi}(e^\mu{}_a\gamma^a iD_\mu - m_\psi)\psi/2 + \mathrm{h.c.}$, which includes minimal couplings to gravity and to the nonabelian field. All components $\mathcal{L}_\psi^{(d)}$ have overall mass dimension four, with the superscript d indicating the mass dimension of the associated dynamical operator.

Table X provides the schematic structure of terms in $\mathcal{L}_\psi^{(d)}$ of mass dimension $d \leq 6$. The first column lists the components of $\mathcal{L}_\psi$, while the second column displays the corresponding operators. In this schematic context, the backgrounds k or combinations $\breve{k}$ can all be distinct, even within a single term. In a given row, terms involving fermions and covariant derivatives are listed first, then ones involving curvature couplings and gauge couplings, and finally ones with background derivatives Dk. All indices and constants are omitted for simplicity, and all structures involving hermitian combinations of Dirac matrices are represented by Γ. The factor h.c. appearing at the end of most expressions for a given component $\mathcal{L}_\psi^{(d)}$ represents the addition of the hermitian conjugate of all terms explicitly written for that component. In principle, any background k or combination $\breve{k}$ can be taken as complex, but its imaginary part is contained up to surface terms in the real parts of other backgrounds. For example, the imaginary part of $\breve{k}$ in $\breve{k}\overline{\psi}\Gamma iD\psi$ is contained in the real part of $\breve{k}(D\overline{k})\overline{\psi}\Gamma\psi$, and the imaginary part of $\breve{k}$ in $\breve{k}R\overline{\psi}\Gamma iD\psi$ is contained in the real parts of $\breve{k}(DR)\overline{\psi}\Gamma\psi$ and $\breve{k}(D\overline{k})R\overline{\psi}\Gamma\psi$. To avoid redundancy, all background fields in this schematic notation can therefore be taken

TABLE X. Schematic structure of terms with $d \leq 6$ in the fermion Lagrange density $\mathcal{L}_\psi$.

Component	Expression
$\mathcal{L}_\psi^{(3)}$	$\check{k}\overline{\psi}\Gamma\psi$
$\mathcal{L}_\psi^{(4)}$	$\check{k}\overline{\psi}\Gamma iD\psi + \check{k}(Dk)\overline{\psi}\Gamma\psi + \text{h.c.}$
$\mathcal{L}_\psi^{(5)}$	$\check{k}\overline{\psi}\Gamma i^2 D_{(2)}\psi + \check{k}\overline{\psi}\Gamma F\psi + \check{k}R\overline{\psi}\Gamma\psi + \check{k}(Dk)\overline{\psi}\Gamma iD\psi + \check{k}(Dk)(Dk)\overline{\psi}\Gamma\psi + \check{k}(D_{(2)}k)\overline{\psi}\Gamma\psi + \text{h.c.}$
$\mathcal{L}_\psi^{(6)}$	$\check{k}\overline{\psi}\Gamma i^3 D_{(3)}\psi + \check{k}R\overline{\psi}\Gamma iD\psi + \check{k}(DR)\overline{\psi}\Gamma\psi + \check{k}\overline{\psi}\Gamma FiD\psi + \check{k}\overline{\psi}\Gamma(DF)\psi + \check{k}(\overline{\psi}\Gamma\psi)(\overline{\psi}\Gamma\psi)$
	$+\check{k}(Dk)\overline{\psi}\Gamma i^2 D_{(2)}\psi + \check{k}(Dk)R\overline{\psi}\Gamma\psi + \check{k}(Dk)\overline{\psi}\Gamma F\psi + \check{k}(D_{(2)}k)\overline{\psi}\Gamma iD\psi + \check{k}(Dk)(Dk)\overline{\psi}\Gamma iD\psi$
	$+\check{k}(Dk)(D_{(2)}k)\overline{\psi}\Gamma\psi + \check{k}(Dk)(Dk)(Dk)\overline{\psi}\Gamma\psi + \check{k}(D_{(3)}k)\overline{\psi}\Gamma\psi + \text{h.c.}$

as real. Further properties of the backgrounds k and the combinations $\check{k}$ discussed for Table V also hold here.

To offer more insight into the content of $\mathcal{L}_\psi$, we provide in Table XI the explicit form of all terms in $\mathcal{L}_\psi$ with operators of mass dimension $d \leq 6$ and without background derivatives, omitting total-derivative terms. The structure of this table matches that of Table X. The notation for the combinations $\check{k}$ appearing in this table is chosen to match standard conventions in the literature, with different symbols distinguishing the spin and CPT properties of the various dynamical operators as usual. All backgrounds can be taken real. The index symmetry of each background $\check{k}$ is understood to match the index symmetry of the associated dynamical operator. The position of the indices is also chosen to match conventions in the literature. In particular, the notation in $\mathcal{L}_\psi^{(3)}$ and $\mathcal{L}_{\psi D}^{(4)}$ involves covariant indices on the backgrounds in agreement with Ref. [2] and reducing in Minkowski spacetime to Ref. [81], while the usage of contravariant background indices in $\mathcal{L}^{(d)}$ with $d \geq 5$ is compatible with that in published works discussing the nonminimal fermion sector, such as Refs. [74, 123].

The reader is reminded that for spontaneous breaking the index position on the background has no effect on the physics, as discussed in Sec. II C, while for explicit breaking the choice of index position establishes a definition of the corresponding physical effects. For example, in explicit breaking the physical effects of the two backgrounds $\overline{b}_\mu$ and $\overline{b}'^\mu$ can be different, with these two possibilities contained among others in $\check{b}_\mu$ as $\overline{b}_\mu$ and $g_{\mu\nu}\overline{b}'^\nu$. To achieve an unambiguous statement, experiments measuring a given background component must therefore report results on the background using a specific convention. A separate confusion can arise because raising or lowering indices on a given background component can introduce spurious signs and coordinate dependences. This issue exists already in Minkowski spacetime but can be particularly acute in gravitational experiments. It could, for example, be problematic to report results for an quantity $\overline{k}^\mu$ defined as $\overline{k}^\mu \equiv g^{\mu\nu}\overline{b}_\nu$ using data from an experiment sensitive to a background $\overline{b}_\mu$, given that $g^{\mu\nu}$ itself is a nontrivial object.

For any d, the structure of the couplings involving gravity, gauge, and fermion fields can be written in schematic form as

$$\mathcal{L}' \supset \check{k}(Dk)\cdots(Dk)(DR)\cdots(DR)$$
$$\times (\text{tr}[(DF)\cdots(DF)])\cdots(\text{tr}[(DF)\cdots(DF)])$$
$$\times [(\overline{D\psi})(DF)\cdots(DF)\Gamma(D\psi)]\cdots$$
$$\times \cdots[(\overline{D\psi})(DF)\cdots(DF)\Gamma(D\psi)] + \text{h.c.}, \quad (59)$$

where D here denotes any symmetric combination of derivatives. Inspection reveals that the only terms of this form absent from Table XI involve the operators $\text{tr}(F)\overline{\psi}\Gamma\psi$, $\text{tr}(F)\overline{\psi}\Gamma iD\psi$, and $\text{tr}(DF)\overline{\psi}\Gamma\psi$. These vanish for gauge groups with traceless adjoint generators, including $SU(N)$, but they reduce to terms in Table XI if the gauge group is abelian. Otherwise, the operators of lowest mass dimension that involve traces of powers of F take the schematic form $\text{tr}(FF)\overline{\psi}\Gamma\psi$, which have $d = 7$. Although this value of d lies outside the range of Table XI, the unique structure of these terms may be of interest for theoretical and experimental studies.

3. Scalars

In the scalar sector, we consider first a scenario with a scalar field ϕ lying in any representation of the gauge group. Depending on the model of interest and the gauge group involved, the representation may be complex, real, or pseudoreal, and it may be different from the representations of the fermion and gauge fields. The Lagrange density $\mathcal{L}_\phi$ involves all couplings of ϕ, including to gravity, gauge, and fermion fields as well as self couplings. It can be decomposed into a sum of pieces,

$$\mathcal{L}_\phi = \mathcal{L}_{\phi 0} + \mathcal{L}_\phi^{(2)} + \mathcal{L}_\phi^{(3)} + \mathcal{L}_\phi^{(4)} + \mathcal{L}_\phi^{(5)} + \mathcal{L}_\phi^{(6)} + \dots, \quad (60)$$

where $\mathcal{L}_{\phi 0}$ is a conventional renormalizable Lagrange density for ϕ that is invariant under gauge transformations, local Lorentz transformations, and diffeomorphisms. Typically, $\mathcal{L}_{\phi 0}$ incorporates a kinetic term quadratic in the covariant derivative of ϕ along with a potential term involving a globally stable polynomial in ϕ. According to its sign, the term quadratic in ϕ may represent a scalar mass or may trigger spontaneous breaking

TABLE XI. Terms with $d \leq 6$ and without background derivatives in the fermion Lagrange density $\mathcal{L}_\psi$.

Component	Expression
$\mathcal{L}_{\psi 0}$	$\frac{1}{2}\overline{\psi}(e^\mu{}_a\gamma^a iD_\mu - m_\psi)\psi + \text{h.c.}$
$\mathcal{L}_\psi^{(3)}$	$-\breve{m}'\overline{\psi}\psi - i\breve{m}_5\overline{\psi}\gamma_5\psi - \breve{a}_\kappa e^\kappa{}_a\overline{\psi}\gamma^a\psi - \breve{b}_\kappa e^\kappa{}_a\overline{\psi}\gamma_5\gamma^a\psi - \frac{1}{2}\breve{H}_{\kappa\lambda}e^\kappa{}_a e^\lambda{}_b\overline{\psi}\sigma^{ab}\psi$
$\mathcal{L}_{\psi D}^{(4)}$	$-\frac{1}{2}\breve{c}_{\kappa\mu}e^\kappa{}_a e^\mu{}_b\overline{\psi}\gamma^a iD_\nu\psi - \frac{1}{2}\breve{d}_{\kappa\mu}e^\kappa{}_a e^\mu{}_b{}^{\nu b}\overline{\psi}\gamma_5\gamma^a iD_\nu\psi$
	$\quad -\frac{1}{2}\breve{e}_\mu e^\mu{}_b{}^{\nu b}\overline{\psi}iD_\nu\psi - \frac{1}{2}i\breve{f}_\mu e^\mu{}_b{}^{\nu b}\overline{\psi}\gamma_5 iD_\mu\psi - \frac{1}{4}\breve{g}_{\kappa\lambda\mu}e^\kappa{}_a e^\lambda{}_b e^\mu{}_b{}^{\nu b}\overline{\psi}\sigma^{ab}iD_\nu\psi + \text{h.c.}$
$\mathcal{L}_{\psi D}^{(5)}$	$-\frac{1}{2}(\breve{m}^{(5)})^{\mu\nu}\overline{\psi}iD_{(\mu}iD_{\nu)}\psi - \frac{1}{2}i(\breve{m}_5^{(5)})^{\mu\nu}\overline{\psi}\gamma_5 iD_{(\mu}iD_{\nu)}\psi$
	$\quad +\frac{1}{2}(\breve{a}^{(5)})^{\kappa\mu\nu}e_{\kappa a}\overline{\psi}\gamma^a iD_{(\mu}iD_{\nu)}\psi + \frac{1}{2}(\breve{b}^{(5)})^{\kappa\mu\nu}e_{\kappa a}\overline{\psi}\gamma_5\gamma^a iD_{(\mu}iD_{\nu)}\psi$
	$\quad -\frac{1}{4}(\breve{H}^{(5)})^{\kappa\lambda\mu\nu}e_{\kappa a}e_{\lambda b}\overline{\psi}\sigma^{ab}iD_{(\mu}iD_{\nu)}\psi + \text{h.c.}$
$\mathcal{L}_{\psi R}^{(5)}$	$-(\breve{m}_R^{(5)})^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}\overline{\psi}\psi - i(\breve{m}_{5R}^{(5)})^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}\overline{\psi}\gamma_5\psi$
	$\quad -(\breve{a}_R^{(5)})^{\kappa\mu\nu\rho\sigma}e_{\kappa a}R_{\mu\nu\rho\sigma}\overline{\psi}\gamma^a\psi - (\breve{b}_R^{(5)})^{\kappa\mu\nu\rho\sigma}e_{\kappa a}R_{\mu\nu\rho\sigma}\overline{\psi}\gamma_5\gamma^a\psi - \frac{1}{2}(\breve{H}_R^{(5)})^{\kappa\lambda\mu\nu\rho\sigma}e_{\kappa a}e_{\lambda b}R_{\mu\nu\rho\sigma}\overline{\psi}\sigma^{ab}\psi$
$\mathcal{L}_{\psi F}^{(5)}$	$-\frac{1}{2}(\breve{m}_F^{(5)})^{\mu\nu}\overline{\psi}F_{\mu\nu}\psi - \frac{1}{2}i(\breve{m}_{5F}^{(5)})^{\mu\nu}\overline{\psi}\gamma_5 F_{\mu\nu}\psi$
	$\quad +\frac{1}{2}(\breve{a}_F^{(5)})^{\kappa\mu\nu}e_{\kappa a}\overline{\psi}\gamma^a F_{\mu\nu}\psi + \frac{1}{2}(\breve{b}_F^{(5)})^{\kappa\mu\nu}e_{\kappa a}\overline{\psi}\gamma_5\gamma^a F_{\mu\nu}\psi - \frac{1}{4}(\breve{H}_F^{(5)})^{\kappa\lambda\mu\nu}e_{\kappa a}e_{\lambda b}\overline{\psi}\sigma^{ab}F_{\mu\nu}\psi$
$\mathcal{L}_{\psi D}^{(6)}$	$-\frac{1}{2}(\breve{c}^{(6)})^{\kappa\mu\nu\rho}e_{\kappa a}\overline{\psi}\gamma^a iD_{(\mu}iD_\nu iD_{\rho)}\psi - \frac{1}{2}(\breve{d}^{(6)})^{\kappa\mu\nu\rho}e_{\kappa a}\overline{\psi}\gamma_5\gamma^a iD_{(\mu}iD_\nu iD_{\rho)}\psi$
	$\quad +\frac{1}{2}(\breve{e}^{(6)})^{\mu\nu\rho}\overline{\psi}iD_{(\mu}iD_\nu iD_{\rho)}\psi + \frac{1}{2}i(\breve{f}^{(6)})^{\mu\nu\rho}\overline{\psi}\gamma_5 iD_{(\mu}iD_\nu iD_{\rho)}\psi$
	$\quad +\frac{1}{4}(\breve{g}^{(6)})^{\kappa\lambda\mu\nu\rho}e_{\kappa a}e_{\lambda b}\overline{\psi}\sigma^{ab}iD_{(\mu}iD_\nu iD_{\rho)}\psi + \text{h.c.}$
$\mathcal{L}_{\psi R}^{(6)}$	$-\frac{1}{2}(\breve{c}_R^{(6)})^{\kappa\mu\nu\rho\sigma\tau}e_{\kappa a}R_{\nu\rho\sigma\tau}\overline{\psi}\gamma^a iD_\mu\psi - \frac{1}{2}(\breve{d}_R^{(6)})^{\kappa\mu\nu\rho\sigma\tau}e_{\kappa a}R_{\nu\rho\sigma\tau}\overline{\psi}\gamma_5\gamma^a iD_\mu\psi$
	$\quad -\frac{1}{2}(\breve{e}_R^{(6)})^{\mu\nu\rho\sigma\tau}R_{\nu\rho\sigma\tau}\overline{\psi}iD_\mu\psi - \frac{1}{2}i(\breve{f}_R^{(6)})^{\mu\nu\rho\sigma\tau}R_{\nu\rho\sigma\tau}\overline{\psi}\gamma_5 iD_\mu\psi$
	$\quad -\frac{1}{4}(\breve{g}_R^{(6)})^{\kappa\lambda\mu\nu\rho\sigma\tau}e_{\kappa a}e_{\lambda b}R_{\nu\rho\sigma\tau}\overline{\psi}\sigma^{ab}iD_\mu\psi + \text{h.c.}$
$\mathcal{L}_{\psi DR}^{(6)}$	$-(\breve{m}_{DR}^{(6)})^{\mu\nu\rho\sigma\tau}(D_\mu R_{\nu\rho\sigma\tau})\overline{\psi}\psi - i(\breve{m}_{5DR}^{(6)})^{\mu\nu\rho\sigma\tau}(D_\mu R_{\nu\rho\sigma\tau})\overline{\psi}\gamma_5\psi$
	$\quad -(\breve{a}_{DR}^{(6)})^{\kappa\mu\nu\rho\sigma\tau}e_{\kappa a}(D_\mu R_{\nu\rho\sigma\tau})\overline{\psi}\gamma^a\psi - (\breve{b}_{DR}^{(6)})^{\kappa\mu\nu\rho\sigma\tau}e_{\kappa a}(D_\mu R_{\nu\rho\sigma\tau})\overline{\psi}\gamma_5\gamma^a\psi$
	$\quad -\frac{1}{2}(\breve{H}_{DR}^{(6)})^{\kappa\lambda\mu\nu\rho\sigma\tau}e_{\kappa a}e_{\lambda b}(D_\mu R_{\nu\rho\sigma\tau})\overline{\psi}\sigma^{ab}\psi$
$\mathcal{L}_{\psi F}^{(6)}$	$-\frac{1}{4}(\breve{c}_F^{(6)})^{\kappa\mu\nu\rho}e_{\kappa a}\overline{\psi}\gamma^a F_{\nu\rho}iD_\mu\psi - \frac{1}{4}(\breve{d}_F^{(6)})^{\kappa\mu\nu\rho}e_{\kappa a}\overline{\psi}\gamma_5\gamma^a F_{\nu\rho}iD_\mu\psi$
	$\quad +\frac{1}{4}(\breve{e}_F^{(6)})^{\mu\nu\rho}\overline{\psi}F_{\nu\rho}iD_\mu\psi + \frac{1}{4}i(\breve{f}_F^{(6)})^{\mu\nu\rho}\overline{\psi}\gamma_5 F_{\nu\rho}iD_\mu\psi + \frac{1}{8}(\breve{g}_F^{(6)})^{\kappa\lambda\mu\nu\rho}e_{\kappa a}e_{\lambda b}\overline{\psi}\sigma^{ab}F_{\nu\rho}iD_\mu\psi + \text{h.c.}$
$\mathcal{L}_{\psi DF}^{(6)}$	$-\frac{1}{2}(\breve{m}_{DF}^{(6)})^{\mu\nu\rho}\overline{\psi}(D_\mu F_{\nu\rho})\psi - \frac{1}{2}i(\breve{m}_{5DF}^{(6)})^{\mu\nu\rho}\overline{\psi}\gamma_5(D_\mu F_{\nu\rho})\psi$
	$\quad +\frac{1}{2}(\breve{a}_{DF}^{(6)})^{\kappa\mu\nu\rho}e_{\kappa a}\overline{\psi}\gamma^a(D_\mu F_{\nu\rho})\psi + \frac{1}{2}(\breve{b}_{DF}^{(6)})^{\kappa\mu\nu\rho}e_{\kappa a}\overline{\psi}\gamma_5\gamma^a(D_\mu F_{\nu\rho})\psi$
	$\quad -\frac{1}{4}(\breve{H}_{DF}^{(6)})^{\kappa\lambda\mu\nu\rho}e_{\kappa a}e_{\lambda b}\overline{\psi}\sigma^{ab}(D_\mu F_{\nu\rho})\psi$
$\mathcal{L}_{\psi\psi}^{(6)}$	$\breve{k}_{SS}(\overline{\psi}\psi)(\overline{\psi}\psi) - \breve{k}_{PP}(\overline{\psi}\gamma_5\psi)(\overline{\psi}\gamma_5\psi) + i\breve{k}_{SP}(\overline{\psi}\psi)(\overline{\psi}\gamma_5\psi)$
	$\quad -(\breve{k}_{SV})^\kappa e_{\kappa a}(\overline{\psi}\psi)(\overline{\psi}\gamma^a\psi) - (\breve{k}_{SA})^\kappa e_{\kappa a}(\overline{\psi}\psi)(\overline{\psi}\gamma_5\gamma^a\psi) + \frac{1}{2}(\breve{k}_{ST})^{\kappa\lambda}e_{\kappa a}e_{\lambda b}(\overline{\psi}\psi)(\overline{\psi}\sigma^{ab}\psi)$
	$\quad -i(\breve{k}_{PV})^\kappa e_{\kappa a}(\overline{\psi}\gamma_5\psi)(\overline{\psi}\gamma^a\psi) - i(\breve{k}_{PA})^\kappa e_{\kappa a}(\overline{\psi}\gamma_5\psi)(\overline{\psi}\gamma_5\gamma^a\psi) + \frac{1}{2}i(\breve{k}_{PT})^{\kappa\lambda}e_{\kappa a}e_{\lambda b}(\overline{\psi}\gamma_5\psi)(\overline{\psi}\sigma^{ab}\psi)$
	$\quad +\frac{1}{2}(\breve{k}_{VV})^{\kappa\lambda}e_{\kappa a}e_{\lambda b}(\overline{\psi}\gamma^a\psi)(\overline{\psi}\gamma^b\psi) + \frac{1}{2}(\breve{k}_{AA})^{\kappa\lambda}e_{\kappa a}e_{\lambda b}(\overline{\psi}\gamma_5\gamma^a\psi)(\overline{\psi}\gamma_5\gamma^b\psi)$
	$\quad -\frac{1}{2}(\breve{k}_{VT})^{\kappa\lambda\mu}e_{\kappa a}e_{\lambda b}e_{\mu c}(\overline{\psi}\gamma^a\psi)(\overline{\psi}\sigma^{bc}\psi) - \frac{1}{2}(\breve{k}_{AT})^{\kappa\lambda\mu}e_{\kappa a}e_{\lambda b}e_{\mu c}(\overline{\psi}\gamma_5\gamma^a\psi)(\overline{\psi}\sigma^{bc}\psi)$
	$\quad +(\breve{k}_{VA})^{\kappa\lambda}e_{\kappa a}e_{\lambda b}(\overline{\psi}\gamma^a\psi)(\overline{\psi}\gamma_5\gamma^b\psi) + \frac{1}{8}(\breve{k}_{TT})^{\kappa\lambda\mu\nu}e_{\kappa a}e_{\lambda b}e_{\mu c}e_{\nu d}(\overline{\psi}\sigma^{ab}\psi)(\overline{\psi}\sigma^{cd}\psi)$

of the gauge symmetry. Where compatible with gauge invariance, $\mathcal{L}_{\phi 0}$ also includes conventional couplings to other sectors. The components $\mathcal{L}_\phi^{(d)}$ have overall mass dimension four, and they represent corrections in the effective field theory involving dynamical operators of mass dimension d.

In constructing possible contributions to the terms $\mathcal{L}_\phi^{(d)}$

in the effective field theory, we seek gauge-invariant operators that are polynomials in ϕ and in covariant derivatives of ϕ. Any given scalar operator of this type can be characterized by its mass dimension d and assigned to a corresponding set $V^{(d)}$. For instance, $V^{(1)}$ contains the operator ϕ and any independent conjugates, $V^{(2)}$ contains $\phi\phi$, $D_\mu\phi$, and various conjugates, and $V^{(3)}$ contains

TABLE XII. Schematic structure of terms with $d \leq 6$ in the scalar Lagrange density $\mathcal{L}_\phi$.

Component	Expression
$\mathcal{L}_\phi^{(2)}$	$\check{k}V^{(2)} + \check{k}(Dk)V^{(1)}$
$\mathcal{L}_\phi^{(3)}$	$\check{k}V^{(3)} + \check{k}RV^{(1)} + \check{k}FV^{(1)} + \check{k}(Dk)V^{(2)} + \check{k}(Dk)(Dk)V^{(1)} + \check{k}(D_{(2)}k)V^{(1)}$
$\mathcal{L}_\phi^{(4)}$	$\check{k}V^{(4)} + \check{k}RV^{(2)} + \check{k}FV^{(2)} + \check{k}(DR)V^{(1)} + \check{k}(DF)V^{(1)} + \check{k}(Dk)V^{(3)} + \check{k}(Dk)RV^{(1)} + \check{k}(Dk)FV^{(1)}$
	$+\check{k}(Dk)(Dk)V^{(2)} + \check{k}(D_{(2)}k)V^{(2)} + \check{k}(Dk)(Dk)(Dk)V^{(1)} + \check{k}(Dk)(D_{(2)}k)V^{(1)} + \check{k}(D_{(3)}k)V^{(1)}$
$\mathcal{L}_\phi^{(5)}$	$\check{k}V^{(5)} + \check{k}RV^{(3)} + \check{k}FV^{(3)} + \check{k}(DR)V^{(2)} + \check{k}(DF)V^{(2)} + \check{k}(\overline{\psi}\Gamma\psi)V^{(2)} + \check{k}(D_{(2)}R)V^{(1)} + \check{k}(D_{(2)}F)V^{(1)}$
	$+\check{k}RRV^{(1)} + \check{k}FFV^{(1)} + \check{k}RFV^{(1)} + \check{k}[(\overline{\psi}\Gamma D\psi)V^{(1)} + \text{h.c.}] + \check{k}(Dk)V^{(4)} + \check{k}(Dk)RV^{(2)}$
	$+\check{k}(Dk)FV^{(2)} + \check{k}(Dk)(DR)V^{(1)} + \check{k}(Dk)(DF)V^{(1)} + \check{k}(Dk)(Dk)V^{(3)} + \check{k}(Dk)(Dk)RV^{(1)}$
	$+\check{k}(Dk)(Dk)FV^{(1)} + \check{k}(D_{(2)}k)V^{(3)} + \check{k}(D_{(2)}k)RV^{(1)} + \check{k}(D_{(2)}k)FV^{(1)} + \check{k}(Dk)(Dk)(Dk)V^{(2)}$
	$+\check{k}(Dk)(D_{(2)}k)V^{(2)} + \check{k}(D_{(3)}k)V^{(2)} + \check{k}(Dk)(Dk)(Dk)(Dk)V^{(1)} + \check{k}(Dk)(Dk)(D_{(2)}k)V^{(1)}$
	$+\check{k}(D_{(2)}k)(D_{(2)}k)V^{(1)} + \check{k}(Dk)(D_{(3)}k)V^{(1)} + \check{k}(D_{(4)}k)V^{(1)}$
$\mathcal{L}_\phi^{(6)}$	$\check{k}V^{(6)} + \check{k}RV^{(4)} + \check{k}FV^{(4)} + \check{k}(DR)V^{(3)} + \check{k}(DF)V^{(3)} + \check{k}(\overline{\psi}\Gamma\psi)V^{(3)} + \check{k}(D_{(2)}R)V^{(2)} + \check{k}(D_{(2)}F)V^{(2)}$
	$+\check{k}RRV^{(2)} + \check{k}FFV^{(2)} + \check{k}RFV^{(2)} + \check{k}[(\overline{\psi}\Gamma D\psi)V^{(2)} + \text{h.c.}] + \check{k}(D_{(3)}R)V^{(1)} + \check{k}(D_{(3)}F)V^{(1)}$
	$+\check{k}R(D_{(1)}R)V^{(1)} + \check{k}F(D_{(1)}R)V^{(1)} + \check{k}F(D_{(1)}F)V^{(1)} + \check{k}R(D_{(1)}F)V^{(1)} + \check{k}R(\overline{\psi}\Gamma\psi)V^{(1)} + \check{k}F(\overline{\psi}\Gamma\psi)V^{(1)}$
	$+\check{k}[(\overline{\psi}\Gamma D_{(2)}\psi)V^{(1)} + \text{h.c.}] + \check{k}(\overline{D\psi}\Gamma D\psi)V^{(1)} + \check{k}(Dk)V^{(5)} + \check{k}(Dk)RV^{(3)} + \check{k}(Dk)FV^{(3)}$
	$+\check{k}(Dk)(DR)V^{(2)} + \check{k}(Dk)(DF)V^{(2)} + \check{k}(Dk)(\overline{\psi}\Gamma\psi)V^{(2)} + \check{k}(Dk)(D_{(2)}R)V^{(1)} + \check{k}(Dk)(D_{(2)}F)V^{(1)}$
	$+\check{k}(Dk)RRV^{(1)} + \check{k}(Dk)FFV^{(1)} + \check{k}(Dk)RFV^{(1)} + \check{k}(Dk)[(\overline{\psi}\Gamma D\psi)V^{(1)} + \text{h.c.}] + \check{k}(Dk)(Dk)V^{(4)}$
	$+\check{k}(Dk)(Dk)RV^{(2)} + \check{k}(Dk)(Dk)FV^{(2)} + \check{k}(Dk)(Dk)(DR)V^{(1)} + \check{k}(Dk)(Dk)(DF)V^{(1)} + \check{k}(D_{(2)}k)V^{(4)}$
	$+\check{k}(D_{(2)}k)RV^{(2)} + \check{k}(D_{(2)}k)FV^{(2)} + \check{k}(D_{(2)}k)(DR)V^{(1)} + \check{k}(D_{(2)}k)(DF)V^{(1)} + \check{k}(Dk)(Dk)(Dk)V^{(3)}$
	$+\check{k}(Dk)(Dk)(Dk)RV^{(1)} + \check{k}(Dk)(Dk)(Dk)FV^{(1)} + \check{k}(Dk)(D_{(2)}k)V^{(3)} + \check{k}(Dk)(D_{(2)}k)RV^{(1)}$
	$+\check{k}(Dk)(D_{(2)}k)FV^{(1)} + \check{k}(D_{(3)}k)V^{(3)} + \check{k}(D_{(3)}k)RV^{(1)} + \check{k}(D_{(3)}k)FV^{(1)} + \check{k}(Dk)(Dk)(Dk)(Dk)V^{(2)}$
	$+\check{k}(Dk)(Dk)(D_{(2)}k)V^{(2)} + \check{k}(D_{(2)}k)(D_{(2)}k)V^{(2)} + \check{k}(Dk)(D_{(3)}k)V^{(2)} + \check{k}(D_{(4)}k)V^{(2)}$
	$+\check{k}(Dk)(Dk)(Dk)(Dk)(Dk)V^{(1)} + \check{k}(Dk)(Dk)(Dk)(D_{(2)}k)V^{(1)} + \check{k}(Dk)(Dk)(D_{(3)}k)V^{(1)}$
	$+\check{k}(Dk)(D_{(2)}k)(D_{(2)}k)V^{(1)} + \check{k}(Dk)(D_{(4)}k)V^{(1)} + \check{k}(D_{(2)}k)(D_{(3)}k)V^{(1)} + \check{k}(D_{(5)}k)V^{(1)}$

$\phi\phi$, $\phi D_\mu\phi$, $D_\mu D_\nu\phi$, and various conjugates. Note that any given operator in the set $V^{(d)}$ may carry up to $d-1$ spacetime indices. The properties of operators quadratic in ϕ at any d are studied in Ref. [104].

Table XII displays the terms in $\mathcal{L}_\phi^{(d)}$ with $d \leq 6$ in schematic form. For each $\mathcal{L}_\phi^{(d)}$ with given d shown in the first column, the possible corresponding operator structures are presented in the second column. In this notation, a factor $V^{(d)}$ represents any element of the associated set of scalar operators, so an expression containing $V^{(d)}$ is a compact description of several distinct terms. Operators without background derivatives are listed first, with pure-scalar terms preceding ones containing gravity, gauge, and fermion couplings. These are followed by operators involving increasing numbers of background derivatives Dk. Indices and all numerical or dimensional factors are omitted, and any appearance of a hermitian combination of Dirac matrices is represented by Γ. A

few terms appear with hermitian conjugates, which are denoted by h.c. in the table. All appearances of the backgrounds k and combinations $\check{k}$ can be distinct, even within a single term, and the backgrounds are assumed real. Since all terms are required to be gauge invariant, the choice of representation for ϕ restricts the appearance of some operator structures. For example, the term $\check{k}(Dk)V^{(1)}$ in $\mathcal{L}_\phi^{(2)}$ is gauge invariant only when the scalar is in the singlet representation.

To offer further insight, the explicit terms with $d \leq 6$ in the Lagrange density for a special model are provided in Table XIII. For simplicity and definiteness, the complex scalar ϕ in this model is assumed to be in the same U representation of the gauge group as the fermion field and the gauge field strength, only terms invariant by virtue of the combination $U^\dagger U = I$ are listed, and total-derivative terms are omitted. This excludes, for instance, singlet terms arising from products of U representations that are specific to a particular gauge group. For example,

TABLE XIII. Terms with $d \leq 6$ and without background derivatives in the scalar Lagrange density $\mathcal{L}_\phi$.

Component	Expression
$\mathcal{L}_{\phi 0}$	$-(D_\mu \phi)^\dagger D^\mu \phi \pm \mu^2 \phi^\dagger \phi - \frac{1}{6}\lambda(\phi^\dagger \phi)^2$
$\mathcal{L}_\phi^{(2)}$	$\check{k}^{(2)}\phi^\dagger \phi$
$\mathcal{L}_\phi^{(3)}$	$-\frac{1}{2}(\check{k}^{(3)})^\mu \phi^\dagger i D_\mu \phi + \text{h.c.}$
$\mathcal{L}_\phi^{(4)}$	$[\frac{1}{2}(\check{k}^{(4)})^{\mu\nu}\phi^\dagger i D_{(\mu}i D_{\nu)}\phi + \text{h.c.}] - (\check{k}_{R\phi}^{(4)})^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}\phi^\dagger\phi - \frac{1}{2}(\check{k}_{F\phi}^{(4)})^{\mu\nu}\phi^\dagger F_{\mu\nu}\phi - \frac{1}{6}\check{k}_{\phi\phi}^{(4)}(\phi^\dagger\phi)^2$
$\mathcal{L}_\phi^{(5)}$	$[-\frac{1}{2}(\check{k}^{(5)})^{\mu\nu\rho}\phi^\dagger i D_{(\mu}i D_\nu i D_{\rho)}\phi + \frac{1}{2}(\check{k}_{RD\phi}^{(5)})^{\mu\nu\rho\sigma\tau}R_{\nu\rho\sigma\tau}\phi^\dagger i D_\mu \phi + \frac{1}{4}(\check{k}_{FD\phi}^{(5)})^{\mu\nu\rho}\phi^\dagger F_{\nu\rho}i D_\mu \phi$
	$\quad + \frac{1}{2}(\check{k}_{\phi D\phi}^{(5)})^\mu(\phi^\dagger\phi)(\phi^\dagger i D_\mu \phi) + \text{h.c.}] + \frac{1}{2}(\check{k}_{DR\phi}^{(5)})^{\mu\nu\rho\sigma\tau}(D_\mu R_{\nu\rho\sigma\tau})\phi^\dagger\phi + \frac{1}{2}(\check{k}_{DF\phi}^{(5)})^{\mu\nu\rho}\phi^\dagger(D_\mu F_{\nu\rho})\phi$
	$\quad + \check{k}_{S\phi}^{(5)}(\overline{\psi}\psi)(\phi^\dagger\phi) + i\check{k}_{P\phi}^{(5)}(\overline{\psi}\gamma_5\psi)(\phi^\dagger\phi) + (\check{k}_{V\phi}^{(5)})^\kappa e_{\kappa a}(\overline{\psi}\gamma^a\psi)(\phi^\dagger\phi) + (\check{k}_{A\phi}^{(5)})^\kappa e_{\kappa a}(\overline{\psi}\gamma_5\gamma^a\psi)(\phi^\dagger\phi)$
	$\quad + \frac{1}{2}(\check{k}_{T\phi}^{(5)})^{\kappa\lambda}e_{\kappa a}e_{\lambda b}(\overline{\psi}\sigma^{ab}\psi)(\phi^\dagger\phi)$
$\mathcal{L}_\phi^{(6)}$	$[\frac{1}{2}(\check{k}^{(6)})^{\mu\nu\rho\sigma}\phi^\dagger i D_{(\mu}i D_\nu i D_\rho i D_{\sigma)}\phi + \frac{1}{2}(\check{k}_{RDD\phi}^{(6)})^{\mu\nu\rho\sigma\tau\upsilon}R_{\rho\sigma\tau\upsilon}\phi^\dagger i D_{(\mu}i D_{\nu)}\phi + \frac{1}{4}(\check{k}_{FDD\phi}^{(6)})^{\mu\nu\rho\sigma}\phi^\dagger F_{\rho\sigma}i D_{(\mu}i D_{\nu)}\phi$
	$\quad + \frac{1}{2}(\check{k}_{DRD\phi}^{(6)})^{\mu\nu\rho\sigma\tau\upsilon}(D_\nu R_{\rho\sigma\tau\upsilon})\phi^\dagger i D_\mu \phi + \frac{1}{4}(\check{k}_{DFD\phi}^{(6)})^{\mu\nu\rho\sigma}\phi^\dagger(D_\nu F_{\rho\sigma})i D_\mu \phi + \frac{1}{2}(\check{k}_{\phi DD\phi}^{(6)})^{\mu\nu}(\phi^\dagger\phi)(\phi^\dagger i D_{(\mu}i D_{\nu)}\phi)$
	$\quad + \frac{1}{2}(\check{k}_{D\phi D\phi}^{(6)})^{\mu\nu}(\phi^\dagger i D_\mu \phi)(\phi^\dagger i D_\nu \phi) + \frac{1}{2}(\check{k}_{D\phi^\dagger D\phi}^{(6)})^{\mu\nu}((i D_\mu \phi)^\dagger \phi)(\phi^\dagger i D_\nu \phi) + \frac{1}{2}(\check{k}_{SD\phi}^{(6)})^\mu(\overline{\psi}\psi)(\phi^\dagger i D_\mu \phi)$
	$\quad + \frac{1}{2}i(\check{k}_{PD\phi}^{(6)})^\mu(\overline{\psi}\gamma^5\psi)(\phi^\dagger i D_\mu \phi) + \frac{1}{2}(\check{k}_{VD\phi}^{(6)})^{\kappa\mu}e_{\kappa a}(\overline{\psi}\gamma^a\psi)(\phi^\dagger i D_\mu \phi) + \frac{1}{2}(\check{k}_{AD\phi}^{(6)})^{\kappa\mu}e_{\kappa a}(\overline{\psi}\gamma_5\gamma^a\psi)(\phi^\dagger i D_\mu \phi)$
	$\quad + \frac{1}{4}(\check{k}_{TD\phi}^{(6)})^{\kappa\lambda\mu}e_{\kappa a}e_{\lambda b}(\overline{\psi}\sigma^{ab}\psi)(\phi^\dagger i D_\mu \phi) + \frac{1}{2}(\check{k}_{DS\phi}^{(6)})^\mu(\overline{\psi}i D_\mu \psi)(\phi^\dagger\phi) + \frac{1}{2}i(\check{k}_{DP\phi}^{(6)})^\mu(\overline{\psi}\gamma^5 i D_\mu \psi)(\phi^\dagger\phi)$
	$\quad + \frac{1}{2}(\check{k}_{DV\phi}^{(6)})^{\kappa\mu}e_{\kappa a}(\overline{\psi}\gamma^a i D_\mu \psi)(\phi^\dagger\phi) + \frac{1}{2}(\check{k}_{DA\phi}^{(6)})^{\kappa\mu}e_{\kappa a}(\overline{\psi}\gamma_5\gamma^a i D_\mu \psi)(\phi^\dagger\phi)$
	$\quad + \frac{1}{4}(\check{k}_{DT\phi}^{(6)})^{\kappa\lambda\mu}e_{\kappa a}e_{\lambda b}(\overline{\psi}\sigma^{ab}i D_\mu \psi)(\phi^\dagger\phi) + \text{h.c.}] + (\check{k}_{DDR\phi}^{(6)})^{\mu\nu\rho\sigma\tau\upsilon}(D_{(\mu}D_{\nu)}R_{\rho\sigma\tau\upsilon})\phi^\dagger\phi$
	$\quad + \frac{1}{2}(\check{k}_{DDF\phi}^{(6)})^{\mu\nu\rho\sigma}\phi^\dagger(D_{(\mu}D_{\nu)}F_{\rho\sigma})\phi + (\check{k}_{RR\phi}^{(6)})^{\mu\nu\rho\sigma\tau\upsilon\chi\omega}R_{\mu\nu\rho\sigma}R_{\tau\upsilon\chi\omega}\phi^\dagger\phi + \frac{1}{2}(\check{k}_{RF\phi}^{(6)})^{\mu\nu\rho\sigma\tau\upsilon}R_{\rho\sigma\tau\upsilon}\phi^\dagger F_{\mu\nu}\phi$
	$\quad + \frac{1}{4}(\check{k}_{FF\phi,1}^{(6)})^{\mu\nu\rho\sigma}\phi^\dagger F_{\mu\nu}F_{\rho\sigma}\phi + \frac{1}{4}(\check{k}_{FF\phi,2}^{(6)})^{\mu\nu\rho\sigma}\text{tr}(F_{\mu\nu}F_{\rho\sigma})\phi^\dagger\phi + \frac{1}{4}(\check{k}_{R\phi\phi}^{(6)})^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}(\phi^\dagger\phi)^2$
	$\quad + \frac{1}{4}(\check{k}_{F\phi\phi}^{(6)})^{\mu\nu}(\phi^\dagger\phi)(\phi^\dagger F_{\mu\nu}\phi) + \frac{1}{120}\check{k}_{\phi\phi\phi}^{(6)}(\phi^\dagger\phi)^3$

when the gauge group is SU(3) and the scalar is in the octet representation, the Lagrange density $\mathcal{L}_\phi^{(6)}$ contains an additional term

$$\mathcal{L}_\phi^{(6)} \supset (\check{k}_{D\phi DD\phi}^{(6)})^{\mu\nu\rho}\epsilon_{abc}\phi_a(i D_\mu \phi)_b(i D_{(\mu}i D_{\nu)}\phi)_c + \text{h.c.},\tag{61}$$

where a, b, c are the gauge indices in the adjoint representation.

The format of Table XIII parallels that of Table XII, and the contents of the former can be deduced by inspection of the latter. In Table XIII, the mass parameter $\pm\mu^2$ in the conventional piece $\mathcal{L}_{\phi 0}$ of the Lagrange density can take either sign, with the lower sign triggering spontaneous breaking of the gauge symmetry. In other terms, each background $\check{k}$ is understood to have index symmetry inherited from the index symmetry of the corresponding operator. All backgrounds can be assumed real. The results in the table emphasize the rapid growth with d of terms involving scalar couplings already visible in the schematic notation of Table XII, even in the context of a specific and comparatively simple model.

IV. APPLICATIONS

In this section, we consider some applications of the formalism to several cases of practical relevance. These include effective field theories extending Einstein-Maxwell theory, GR coupled to the SM, and conventional models involving only scalar coupling constants.

A. Einstein-Maxwell theories

As a first application, we consider effective field theories based on the usual Einstein-Maxwell theory coupling gravity to electrodynamics and possibly additional matter fields. In any specific scenario, the Einstein-Maxwell Lagrange density $\mathcal{L}_{\text{EM}}$ can be separated as

$$\mathcal{L}_{\text{EM}} = \mathcal{L}_g + \mathcal{L}_A + \ldots,\tag{62}$$

where $\mathcal{L}_g$ contains terms in the pure-gravity and background sector, $\mathcal{L}_A$ describes the photon sector including gravitational couplings, and the ellipsis indicates any other component Lagrange densities for couplings to fermions and scalars.

The general features of the Lagrange density $\mathcal{L}_g$ for the gravity sector are treated in Sec. III B. The discussion applies directly to the Einstein-Maxwell case. In

TABLE XIV. Terms with $d \leq 6$ in the photon sector of the Einstein-Maxwell effective field theory.

Component	Expression
$\mathcal{L}_{A0}$	$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$
$\mathcal{L}_A^{(1)}$	$-(\check{k}_A)_\mu A^\mu$
$\mathcal{L}_A^{(3)}$	$\frac{1}{2}(\check{k}_{AF})^\kappa \epsilon_{\kappa\lambda\mu\nu}A^\lambda F^{\mu\nu} + \frac{1}{2}(\check{k}_{DF}^{(3)})^{\alpha\mu\nu}D_\alpha F_{\mu\nu}$
$\mathcal{L}_A^{(4)}$	$-\frac{1}{4}(\check{k}_F)_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu} - \frac{1}{2}(\check{k}_{DF}^{(4)})^{\alpha\beta\mu\nu}D_{(\alpha}D_{\beta)}F_{\mu\nu} - \frac{1}{2}(\check{k}_{RF})^{\alpha\beta\gamma\delta\mu\nu}R_{\alpha\beta\gamma\delta}F_{\mu\nu}$
$\mathcal{L}_A^{(5)}$	$\frac{1}{4}(\check{k}_D^{(5)})^{\alpha\kappa\lambda\mu\nu}F_{\kappa\lambda}D_\alpha F_{\mu\nu} + \frac{1}{2}(\check{k}_{DF}^{(5)})^{\alpha\beta\gamma\mu\nu}D_{(\alpha}D_\beta D_{\gamma)}F_{\mu\nu}$
	$\quad + \frac{1}{2}(\check{k}_{RDF}^{(5)})^{\alpha\beta\gamma\delta\epsilon\mu\nu}R_{\alpha\beta\gamma\delta}D_\epsilon F_{\mu\nu} + \frac{1}{2}(\check{k}_{DRF}^{(5)})^{\alpha\beta\gamma\delta\epsilon\mu\nu}(D_\epsilon R_{\alpha\beta\gamma\delta})F_{\mu\nu}$
$\mathcal{L}_A^{(6)}$	$-\frac{1}{4}(\check{k}_D^{(6)})^{\alpha\beta\kappa\lambda\mu\nu}F_{\kappa\lambda}D_{(\alpha}D_{\beta)}F_{\mu\nu} - \frac{1}{2}(\check{k}_{DF}^{(6)})^{\alpha\beta\gamma\delta\mu\nu}D_{(\alpha}D_\beta D_\gamma D_{\delta)}F_{\mu\nu} - \frac{1}{4}(\check{k}_{DFDF}^{(6)})^{\alpha\beta\kappa\lambda\mu\nu}(D_\alpha F_{\kappa\lambda})(D_\beta F_{\mu\nu})$
	$\quad -\frac{1}{12}(\check{k}_F^{(6)})^{\kappa\lambda\mu\nu\rho\sigma}F_{\kappa\lambda}F_{\mu\nu}F_{\rho\sigma} - \frac{1}{4}(\check{k}_{RFF}^{(6)})^{\alpha\beta\gamma\delta\kappa\lambda\mu\nu}R_{\alpha\beta\gamma\delta}F_{\kappa\lambda}F_{\mu\nu} - \frac{1}{2}(\check{k}_{RDDF}^{(6)})^{\alpha\beta\gamma\delta\epsilon\zeta\mu\nu}R_{\alpha\beta\gamma\delta}D_{(\epsilon}D_{\zeta)}F_{\mu\nu}$
	$\quad -\frac{1}{2}(\check{k}_{DDRF}^{(6)})^{\alpha\beta\gamma\delta\epsilon\zeta\mu\nu}(D_{(\epsilon}D_{\zeta)}R_{\alpha\beta\gamma\delta})F_{\mu\nu} - \frac{1}{2}(\check{k}_{DRDF}^{(6)})^{\alpha\beta\gamma\delta\epsilon\zeta\mu\nu}(D_\epsilon R_{\alpha\beta\gamma\delta})D_\zeta F_{\mu\nu}$
	$\quad -\frac{1}{2}(\check{k}_{RRF}^{(6)})^{\alpha_1\beta_1\gamma_1\delta_1\alpha_2\beta_2\gamma_2\delta_2\mu\nu}R_{\alpha_1\beta_1\gamma_1\delta_1}R_{\alpha_2\beta_2\gamma_2\delta_2}F_{\mu\nu}$

TABLE XV. Terms with $d \leq 6$ for an uncharged scalar coupled to the Einstein-Maxwell effective field theory.

Component	Expression
$\mathcal{L}_{\phi 0}$	$-\frac{1}{2}(D_\mu\phi)D^\mu\phi \pm \frac{1}{2}\mu^2\phi^2 + \frac{1}{6}g\phi^3 - \frac{1}{24}\lambda\phi^4$
$\mathcal{L}_\phi^{(2)}$	$\frac{1}{2}\check{k}^{(2)}\phi^2$
$\mathcal{L}_\phi^{(3)}$	$\frac{1}{6}\check{k}_\phi^{(3)}\phi^3 + (\check{k}_{R\phi}^{(3)})^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}\phi + \frac{1}{2}(\check{k}_{F\phi}^{(3)})^{\mu\nu}F_{\mu\nu}\phi$
$\mathcal{L}_\phi^{(4)}$	$\frac{1}{2}(\check{k}^{(4)})^{\mu\nu}\phi i D_{(\mu}i D_{\nu)}\phi - \frac{1}{24}\check{k}_\phi^{(4)}\phi^4 - \frac{1}{2}(\check{k}_{R\phi}^{(4)})^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}\phi^2 - \frac{1}{4}(\check{k}_{F\phi}^{(4)})^{\mu\nu}F_{\mu\nu}\phi^2 - (\check{k}_{DR\phi}^{(4)})^{\mu\nu\rho\sigma\tau}(D_\mu R_{\nu\rho\sigma\tau})\phi$
	$\quad -\frac{1}{2}(\check{k}_{DF\phi}^{(4)})^{\mu\nu\rho}(D_\mu F_{\nu\rho})\phi$
$\mathcal{L}_\phi^{(5)}$	$\frac{1}{6}(\check{k}_{DD\phi}^{(5)})^{\mu\nu}\phi^2 i D_{(\mu}i D_{\nu)}\phi + \frac{1}{6}(\check{k}_{D\phi D\phi}^{(5)})^{\mu\nu}\phi(i D_\mu\phi)(i D_\nu\phi) + \frac{1}{120}\check{k}_\phi^{(5)}\phi^5 + \frac{1}{6}(\check{k}_{R\phi}^{(5)})^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}\phi^3$
	$\quad +\frac{1}{2}(\check{k}_{DR\phi}^{(5)})^{\mu\nu\rho\sigma\tau}(D_\mu R_{\nu\rho\sigma\tau})\phi^2 + (\check{k}_{DDR\phi}^{(5)})^{\mu\nu\rho\sigma\tau\upsilon}(D_{(\mu}D_{\nu)}R_{\rho\sigma\tau\upsilon})\phi + \frac{1}{12}(\check{k}_{F\phi}^{(5)})^{\mu\nu}F_{\mu\nu}\phi^3$
	$\quad +\frac{1}{4}(\check{k}_{DF\phi}^{(5)})^{\mu\nu\rho}(D_\mu F_{\nu\rho})\phi^2 + \frac{1}{2}(\check{k}_{DDF\phi}^{(5)})^{\mu\nu\rho\sigma}(D_{(\mu}D_{\nu)}F_{\rho\sigma})\phi + (\check{k}_{RR\phi}^{(5)})^{\mu\nu\rho\sigma\tau\upsilon\chi\omega}R_{\mu\nu\rho\sigma}R_{\tau\upsilon\chi\omega}\phi$
	$\quad +\frac{1}{4}(\check{k}_{FF\phi}^{(5)})^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\phi + \frac{1}{2}(\check{k}_{RF\phi}^{(5)})^{\mu\nu\rho\sigma\tau\upsilon}R_{\mu\nu\rho\sigma}F_{\tau\upsilon}\phi$
$\mathcal{L}_\phi^{(6)}$	$\frac{1}{2}(\check{k}^{(6)})^{\mu\nu\rho\sigma}\phi i D_{(\mu}i D_\nu i D_\rho i D_{\sigma)}\phi + \frac{1}{24}(\check{k}_{DD\phi}^{(6)})^{\mu\nu}\phi^3 i D_{(\mu}i D_{\nu)}\phi + \frac{1}{24}(\check{k}_{D\phi D\phi}^{(6)})^{\mu\nu}\phi^2(i D_\mu\phi)(i D_\nu\phi) + \frac{1}{720}\check{k}_\phi^{(6)}\phi^6$
	$\quad +\frac{1}{24}(\check{k}_{R\phi}^{(6)})^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}\phi^4 + \frac{1}{48}(\check{k}_{F\phi}^{(6)})^{\mu\nu}F_{\mu\nu}\phi^4 + \frac{1}{2}(\check{k}_{RDD\phi}^{(6)})^{\mu\nu\rho\sigma\tau\upsilon}R_{\rho\sigma\tau\upsilon}\phi i D_{(\mu}i D_{\nu)}\phi$
	$\quad +\frac{1}{4}(\check{k}_{FDD\phi}^{(6)})^{\mu\nu\rho\sigma}F_{\rho\sigma}\phi i D_{(\mu}i D_{\nu)}\phi + \frac{1}{6}(\check{k}_{DR\phi}^{(6)})^{\mu\nu\rho\sigma\tau}(D_\mu R_{\nu\rho\sigma\tau})\phi^3 + \frac{1}{12}(\check{k}_{DF\phi}^{(6)})^{\mu\nu\rho}(D_\mu F_{\nu\rho})\phi^3$
	$\quad +\frac{1}{2}(\check{k}_{DDR\phi}^{(6)})^{\mu\nu\rho\sigma\tau\upsilon}(D_{(\mu}D_{\nu)}R_{\rho\sigma\tau\upsilon})\phi^2 + \frac{1}{4}(\check{k}_{DDF\phi}^{(6)})^{\mu\nu\rho\sigma}(D_{(\mu}D_{\nu)}F_{\rho\sigma})\phi^2 + \frac{1}{2}(\check{k}_{RR\phi}^{(6)})^{\mu\nu\rho\sigma\tau\upsilon\chi\omega}R_{\mu\nu\rho\sigma}R_{\tau\upsilon\chi\omega}\phi^2$
	$\quad +\frac{1}{4}(\check{k}_{RF\phi}^{(6)})^{\mu\nu\rho\sigma\tau\upsilon}R_{\mu\nu\rho\sigma}F_{\tau\upsilon}\phi^2 + \frac{1}{8}(\check{k}_{FF\phi}^{(6)})^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\phi^2 + (\check{k}_{DDDR\phi}^{(6)})^{\mu\nu\rho\sigma\tau\upsilon\chi}(D_{(\mu}D_\nu D_{\rho)}R_{\sigma\tau\upsilon\chi})\phi$
	$\quad +\frac{1}{2}(\check{k}_{DDDF\phi}^{(6)})^{\mu\nu\rho\sigma\tau}(D_{(\mu}D_\nu D_{\rho)}F_{\sigma\tau})\phi + (\check{k}_{RDR\phi}^{(6)})^{\lambda\mu\nu\rho\sigma\tau\upsilon\chi\omega}R_{\mu\nu\rho\sigma}(D_\lambda R_{\tau\upsilon\chi\omega})\phi$
	$\quad +\frac{1}{2}(\check{k}_{RDF\phi}^{(6)})^{\mu\nu\rho\sigma\tau\upsilon\chi}R_{\nu\rho\sigma\tau}(D_\mu F_{\upsilon\chi})\phi + \frac{1}{2}(\check{k}_{DRF\phi}^{(6)})^{\mu\nu\rho\sigma\tau\upsilon\chi}(D_\mu R_{\nu\rho\sigma\tau})F_{\upsilon\chi}\phi + \frac{1}{4}(\check{k}_{FDF\phi}^{(6)})^{\mu\nu\rho\sigma\tau}F_{\nu\rho}(D_\mu F_{\sigma\tau})\phi$

particular, the explicit forms of the terms in $\mathcal{L}_g$ without background derivatives are listed in Table VI.

For the photon sector, the terms in $\mathcal{L}_A$ can be extracted from the results obtained for nonabelian gauge theories in Sec. III C 1, and in particular from Table IX. However, the abelian nature of the gauge group simplifies the structure of certain terms, while some terms that are distinct in the nonabelian case merge in the abelian limit. For clarity, we display in Table XIV all terms in $\mathcal{L}_A$ with operators of mass dimension $d \leq 6$ without background

derivatives. The first column lists the components $\mathcal{L}_A^{(d)}$ of $\mathcal{L}_A$, while the second shows the terms they incorporate. In this table, each background can be taken to be real and to have index symmetry inherited from that of the corresponding operator. The index positions and the labels on the backgrounds match existing conventions in the literature.

Augmenting the Einstein-Maxwell theory with matter fields implies a corresponding extension of the effective field theory. The inclusion of a single Dirac field, which

TABLE XVI. Terms with $d \leq 6$ in the gravity and gauge sectors $\mathcal{L}_{\text{gravity}}$ and $\mathcal{L}_{\text{gauge}}$.

Component	Expression
$\mathcal{L}_{\text{gravity},0} + \mathcal{L}_{\text{gauge},0}$	$\frac{1}{2\kappa}(R - 2\Lambda) - \frac{1}{2}\text{tr}(G_{\mu\nu}G^{\mu\nu}) - \frac{1}{2}\text{tr}(W_{\mu\nu}W^{\mu\nu}) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$
$\mathcal{L}_{\text{gravity}}^{(2)}$	$\frac{1}{2\kappa}\breve{k}^{(2)}$
$\mathcal{L}_{\text{gravity}}^{(3)}$	$\frac{1}{2\kappa}(\breve{k}_\Gamma^{(3)})^\mu \Gamma^\alpha{}_{\mu\alpha}$
$\mathcal{L}_{\text{gravity}}^{(4)}$	$\frac{1}{2\kappa}(\breve{k}_R^{(4)})^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta}$
$\mathcal{L}_{\text{gravity}}^{(5)}$	$\frac{1}{2\kappa}\big[(\breve{k}_D^{(5)})^{\alpha\beta\gamma\delta\kappa} D_\kappa R_{\alpha\beta\gamma\delta} + (\breve{k}_{\text{CS},1}^{(5)})_\kappa \epsilon^{\kappa\lambda\mu\nu} \eta_{ac}\eta_{bd}(\omega_\lambda{}^{ab}\partial_\mu\omega_\nu{}^{cd} + \frac{2}{3}\omega_\lambda{}^{ab}\omega_\mu{}^{ce}\omega_{\nu e}{}^d)$
	$\qquad + (\breve{k}_{\text{CS},2}^{(5)})_\kappa \epsilon^{\kappa\lambda\mu\nu} \epsilon_{abcd}(\omega_\lambda{}^{ab}\partial_\mu\omega_\nu{}^{cd} + \frac{2}{3}\omega_\lambda{}^{ab}\omega_\mu{}^{ce}\omega_{\nu e}{}^d)\big]$
$\mathcal{L}_{\text{gravity}}^{(6)}$	$\frac{1}{2\kappa}\big[(\breve{k}_D^{(6)})^{\alpha\beta\gamma\delta\kappa\lambda} D_{(\kappa}D_{\lambda)}R_{\alpha\beta\gamma\delta} + (\breve{k}_R^{(6)})^{\alpha_1\beta_1\gamma_1\delta_1\alpha_2\beta_2\gamma_2\delta_2} R_{\alpha_1\beta_1\gamma_1\delta_1}R_{\alpha_2\beta_2\gamma_2\delta_2}\big]$
$\mathcal{L}_{\text{gauge}}^{(1)}$	$(\breve{k}_0)_\kappa B_\kappa$
$\mathcal{L}_{\text{gauge}}^{(3)}$	$(\breve{k}_3)_\kappa \epsilon^{\kappa\lambda\mu\nu}\text{tr}(G_\lambda G_{\mu\nu} + \frac{2}{3}ig_3 G_\lambda G_\mu G_\nu) + (\breve{k}_2)_\kappa \epsilon^{\kappa\lambda\mu\nu}\text{tr}(W_\lambda W_{\mu\nu} + \frac{2}{3}ig W_\lambda W_\mu W_\nu) + (\breve{k}_1)_\kappa \epsilon^{\kappa\lambda\mu\nu}B_\lambda B_{\mu\nu}$
	$\qquad + \frac{1}{2}(\breve{k}_{DB}^{(3)})^{\alpha\kappa\lambda}D_\alpha B_{\kappa\lambda}$
$\mathcal{L}_{\text{gauge}}^{(4)}$	$-\frac{1}{2}(\breve{k}_G)_{\kappa\lambda\mu\nu}\text{tr}(G^{\kappa\lambda}G^{\mu\nu}) - \frac{1}{2}(\breve{k}_W)_{\kappa\lambda\mu\nu}\text{tr}(W^{\kappa\lambda}W^{\mu\nu}) - \frac{1}{4}(\breve{k}_B)_{\kappa\lambda\mu\nu}B^{\kappa\lambda}B^{\mu\nu} - \frac{1}{2}(\breve{k}_{RB})^{\kappa\lambda\mu\nu\rho\sigma}R_{\kappa\lambda\mu\nu}B_{\rho\sigma}$
	$\qquad - \frac{1}{2}(\breve{k}_{DB}^{(4)})^{\alpha\beta\kappa\lambda}D_{(\alpha}D_{\beta)}B_{\kappa\lambda}$
$\mathcal{L}_{\text{gauge}}^{(5)}$	$-\frac{1}{2}(\breve{k}_{DG}^{(5)})^{\alpha\kappa\lambda\mu\nu}\text{tr}(G_{\kappa\lambda}D_\alpha G_{\mu\nu}) - \frac{1}{2}(\breve{k}_{DW}^{(5)})^{\alpha\kappa\lambda\mu\nu}\text{tr}(W_{\kappa\lambda}D_\alpha W_{\mu\nu}) - \frac{1}{4}(\breve{k}_{DB}^{(5)})^{\alpha\kappa\lambda\mu\nu}B_{\kappa\lambda}D_\alpha B_{\mu\nu}$
	$\qquad - \frac{1}{2}(\breve{k}_{DRB}^{(5)})^{\alpha\kappa\lambda\mu\nu\rho\sigma}(D_\alpha R_{\kappa\lambda\mu\nu})B_{\rho\sigma} - \frac{1}{2}(\breve{k}_{RDB}^{(5)})^{\alpha\kappa\lambda\mu\nu\rho\sigma}R_{\kappa\lambda\mu\nu}D_\alpha B_{\rho\sigma} - \frac{1}{2}(\breve{k}_{DB}^{(5)})^{\alpha\beta\gamma\kappa\lambda}D_{(\alpha}D_\beta D_{\gamma)}B_{\kappa\lambda}$
$\mathcal{L}_{\text{gauge}}^{(6)}$	$-\frac{1}{2}(\breve{k}_{DDG}^{(6)})^{\alpha\beta\kappa\lambda\mu\nu}\text{tr}(G_{\kappa\lambda}D_{(\alpha}D_{\beta)}G_{\mu\nu}) - \frac{1}{2}(\breve{k}_{DDW}^{(6)})^{\alpha\beta\kappa\lambda\mu\nu}\text{tr}(W_{\kappa\lambda}D_{(\alpha}D_{\beta)}W_{\mu\nu})$
	$\qquad - \frac{1}{4}(\breve{k}_{DDB}^{(6)})^{\alpha\beta\kappa\lambda\mu\nu}B_{\kappa\lambda}D_{(\alpha}D_{\beta)}B_{\mu\nu} - \frac{1}{2}(\breve{k}_{DGDG}^{(6)})^{\alpha\beta\kappa\lambda\mu\nu}\text{tr}\big((D_\alpha G_{\kappa\lambda})(D_\beta G_{\mu\nu})\big)$
	$\qquad - \frac{1}{2}(\breve{k}_{DWDW}^{(6)})^{\alpha\beta\kappa\lambda\mu\nu}\text{tr}\big((D_\alpha W_{\kappa\lambda})(D_\beta W_{\mu\nu})\big) - \frac{1}{4}(\breve{k}_{DBDB}^{(6)})^{\alpha\beta\kappa\lambda\mu\nu}(D_\alpha B_{\kappa\lambda})(D_\beta B_{\mu\nu})$
	$\qquad - \frac{1}{12}\big[(\breve{k}_G^{(6)})^{\kappa\lambda\mu\nu\rho\sigma}\text{tr}(G_{\kappa\lambda}G_{\mu\nu}G_{\rho\sigma}) + \text{h.c.}\big] - \frac{1}{12}\big[(\breve{k}_W^{(6)})^{\kappa\lambda\mu\nu\rho\sigma}\text{tr}(W_{\kappa\lambda}W_{\mu\nu}W_{\rho\sigma}) + \text{h.c.}\big]$
	$\qquad - \frac{1}{2}(\breve{k}_{RDDB}^{(6)})^{\alpha\beta\gamma\delta\epsilon\zeta\mu\nu}R_{\alpha\beta\gamma\delta}D_{(\epsilon}D_{\zeta)}B_{\mu\nu} - \frac{1}{2}(\breve{k}_{DDRB}^{(6)})^{\alpha\beta\gamma\delta\epsilon\zeta\mu\nu}(D_{(\epsilon}D_{\zeta)}R_{\alpha\beta\gamma\delta})B_{\mu\nu}$
	$\qquad - \frac{1}{2}(\breve{k}_{DRDB}^{(6)})^{\alpha\beta\gamma\delta\epsilon\zeta\mu\nu}(D_\epsilon R_{\alpha\beta\gamma\delta})D_\zeta B_{\mu\nu} - \frac{1}{12}(\breve{k}_B^{(6)})^{\kappa\lambda\mu\nu\rho\sigma}B_{\kappa\lambda}B_{\mu\nu}B_{\rho\sigma}$
	$\qquad - \frac{1}{4}(\breve{k}_{BGG}^{(6)})^{\kappa\lambda\mu\nu\rho\sigma}B_{\kappa\lambda}\text{tr}(G_{\mu\nu}G_{\rho\sigma}) - \frac{1}{4}(\breve{k}_{BWW}^{(6)})^{\kappa\lambda\mu\nu\rho\sigma}B_{\kappa\lambda}\text{tr}(W_{\mu\nu}W_{\rho\sigma})$
	$\qquad - \frac{1}{2}(\breve{k}_{RGG}^{(6)})^{\alpha\beta\gamma\delta\kappa\lambda\mu\nu}R_{\alpha\beta\gamma\delta}\text{tr}(G_{\kappa\lambda}G_{\mu\nu}) - \frac{1}{2}(\breve{k}_{RWW}^{(6)})^{\alpha\beta\gamma\delta\kappa\lambda\mu\nu}R_{\alpha\beta\gamma\delta}\text{tr}(W_{\kappa\lambda}W_{\mu\nu})$
	$\qquad - \frac{1}{4}(\breve{k}_{RBB}^{(6)})^{\alpha\beta\gamma\delta\kappa\lambda\mu\nu}R_{\alpha\beta\gamma\delta}B_{\kappa\lambda}B_{\mu\nu} - \frac{1}{2}(\breve{k}_{RRB}^{(6)})^{\alpha_1\beta_1\gamma_1\delta_1\alpha_2\beta_2\gamma_2\delta_2\kappa\lambda}R_{\alpha_1\beta_1\gamma_1\delta_1}R_{\alpha_2\beta_2\gamma_2\delta_2}B_{\kappa\lambda}$

may be charged under the U(1) gauge group, yields an additional component $\mathcal{L}_\psi$ in $\mathcal{L}_{\text{EM}}$. The terms in $\mathcal{L}_\psi$ involving operators of mass dimension $d \leq 6$ and without background derivatives can be extracted directly from Table XI in Sec. III C 2, with the gauge field strength $F_{\mu\nu}$ taken as abelian. Similarly, adding a complex scalar with U(1) charge generates a component $\mathcal{L}_\phi$ in $\mathcal{L}_{\text{EM}}$. All contributions to $\mathcal{L}_\phi$ containing operators of mass dimension $d \leq 6$ without background derivatives are given by Table XIII in Sec. III C 3, with $F_{\mu\nu}$ understood to be abelian.

The inclusion of an uncharged real scalar is more involved because additional terms can be constructed. For this case, all terms in $\mathcal{L}_\phi$ with operators of mass dimension $d \leq 6$ are presented in Table XV. The format of the table matches that of Table XIII, and most of the comments in Sec. III C 3 apply. In some of the terms unique to this scenario, a factor of i has been inserted to keep all backgrounds and combinations real.

B. General Relativity and the Standard Model

Next, we turn attention to the effective field theory constructed from GR coupled to the SM, the gravitational Standard-Model Extension (SME). The framework for this scenario is described in Ref. [2], which explicitly presents all minimal operators allowing for nonzero torsion in Riemann-Cartan spacetime. Here, we extend these results to include operators of mass dimension $d \leq 6$, restricting attention to the zero-torsion limit and to terms without background derivatives.

The Lagrange density $\mathcal{L}_{\text{SME}}$ for this theory can be decomposed into a sum of terms assigned to sectors for each type of field,

$$\mathcal{L}_{\text{SME}} = \mathcal{L}_{\text{gravity}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{lepton}} + \mathcal{L}_{\text{quark}}$$
$$+ \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}. \tag{63}$$

The gauge invariance of the corresponding action is the SM group SU(3)×SU(2)×U(1). In what follows, we

TABLE XVII. Terms with $d \leq 6$ in the fermion sector $\mathcal{L}_{\text{fermion}} = \mathcal{L}_{\text{lepton}} + \mathcal{L}_{\text{quark}}$.

Component	Expression
$\mathcal{L}_{\text{fermion},0}$	$\frac{1}{2}e^{\mu}{}_{a}\overline{L}_{A}\gamma^{a}iD_{\mu}L_{A} + \frac{1}{2}e^{\mu}{}_{a}\overline{R}_{A}\gamma^{a}iD_{\mu}R_{A} + \frac{1}{2}e^{\mu}{}_{a}\overline{Q}_{A}\gamma^{a}iD_{\mu}Q_{A} + \frac{1}{2}e^{\mu}{}_{a}\overline{U}_{A}\gamma^{a}iD_{\mu}U_{A} + \frac{1}{2}e^{\mu}{}_{a}\overline{D}_{A}\gamma^{a}iD_{\mu}D_{A} + \text{h.c.}$
$\mathcal{L}_{\text{lepton}}^{(3)}$	$-(\breve{a}_{L})_{\kappa AB}e^{\kappa}{}_{a}\overline{L}_{A}\gamma^{a}L_{B} - (\breve{a}_{R})_{\kappa AB}e^{\kappa}{}_{a}\overline{R}_{A}\gamma^{a}R_{B}$
$\mathcal{L}_{\text{lepton}}^{(4)}$	$-\frac{1}{2}(\breve{c}_{L})_{\kappa\mu AB}e^{\kappa}{}_{a}e^{\mu}{}_{b}e^{\nu b}\overline{L}_{A}\gamma^{a}iD_{\nu}L_{B} - \frac{1}{2}(\breve{c}_{R})_{\kappa\mu AB}e^{\kappa}{}_{a}e^{\mu}{}_{b}e^{\nu b}\overline{R}_{A}\gamma^{a}iD_{\nu}R_{B} + \text{h.c.}$
$\mathcal{L}_{\text{lepton},D}^{(5)}$	$\frac{1}{2}(\breve{a}_{L}^{(5)})_{AB}^{\kappa\mu\nu}e_{\kappa a}\overline{L}_{A}\gamma^{a}iD_{(\mu}iD_{\nu)}L_{B} + \frac{1}{2}(\breve{a}_{R}^{(5)})_{AB}^{\kappa\mu\nu}e_{\kappa a}\overline{R}_{A}\gamma^{a}iD_{(\mu}iD_{\nu)}R_{B} + \text{h.c.}$
$\mathcal{L}_{\text{lepton},R}^{(5)}$	$(\breve{a}_{R,L}^{(5)})_{AB}^{\kappa\mu\nu\rho\sigma}e_{\kappa a}R_{\mu\nu\rho\sigma}\overline{L}_{A}\gamma^{a}L_{B} + (\breve{a}_{R,R}^{(5)})_{AB}^{\kappa\mu\nu\rho\sigma}e_{\kappa a}R_{\mu\nu\rho\sigma}\overline{R}_{A}\gamma^{a}R_{B}$
$\mathcal{L}_{\text{lepton},W}^{(5)}$	$\frac{1}{2}(\breve{a}_{W,L}^{(5)})_{AB}^{\kappa\mu\nu}e_{\kappa a}\overline{L}_{A}W_{\mu\nu}\gamma^{a}L_{B}$
$\mathcal{L}_{\text{lepton},B}^{(5)}$	$\frac{1}{2}(\breve{a}_{B,L}^{(5)})_{AB}^{\kappa\mu\nu}e_{\kappa a}B_{\mu\nu}\overline{L}_{A}\gamma^{a}L_{B} + \frac{1}{2}(\breve{a}_{B,R}^{(5)})_{AB}^{\kappa\mu\nu}e_{\kappa a}B_{\mu\nu}\overline{R}_{A}\gamma^{a}R_{B}$
$\mathcal{L}_{\text{lepton},D}^{(6)}$	$-\frac{1}{2}(\breve{c}_{L}^{(6)})_{AB}^{\kappa\mu\nu\rho}e_{\kappa a}\overline{L}_{A}\gamma^{a}iD_{(\mu}iD_{\nu}iD_{\rho)}L_{B} - \frac{1}{2}(\breve{c}_{R}^{(6)})_{AB}^{\kappa\mu\nu\rho}e_{\kappa a}\overline{R}_{A}\gamma^{a}iD_{(\mu}iD_{\nu}iD_{\rho)}R_{B} + \text{h.c.}$
$\mathcal{L}_{\text{lepton},R}^{(6)}$	$-\frac{1}{2}(\breve{c}_{R,L}^{(6)})_{AB}^{\kappa\mu\nu\rho\sigma\tau}e_{\kappa a}R_{\nu\rho\sigma\tau}\overline{L}_{A}\gamma^{a}iD_{\mu}L_{B} - \frac{1}{2}(\breve{c}_{R,R}^{(6)})_{AB}^{\kappa\mu\nu\rho\sigma\tau}e_{\kappa a}R_{\nu\rho\sigma\tau}\overline{R}_{A}\gamma^{a}iD_{\mu}R_{B} + \text{h.c.}$
$\mathcal{L}_{\text{lepton},DR}^{(6)}$	$(\breve{c}_{DR,L}^{(6)})_{AB}^{\kappa\mu\nu\rho\sigma\tau}e_{\kappa a}(D_{\mu}R_{\nu\rho\sigma\tau})\overline{L}_{A}\gamma^{a}L_{B} + (\breve{c}_{DR,R}^{(6)})_{AB}^{\kappa\mu\nu\rho\sigma\tau}e_{\kappa a}(D_{\mu}R_{\nu\rho\sigma\tau})\overline{R}_{A}\gamma^{a}R_{B}$
$\mathcal{L}_{\text{lepton},W}^{(6)}$	$-\frac{1}{4}(\breve{c}_{W,L}^{(6)})_{AB}^{\kappa\mu\nu\rho}e_{\kappa a}\overline{L}_{A}\gamma^{a}W_{\nu\rho}iD_{\mu}L_{B} + \text{h.c.}$
$\mathcal{L}_{\text{lepton},DW}^{(6)}$	$\frac{1}{2}(\breve{c}_{DW,L}^{(6)})_{AB}^{\kappa\mu\nu\rho}e_{\kappa a}\overline{L}_{A}\gamma^{a}(D_{\mu}W_{\nu\rho})L_{B}$
$\mathcal{L}_{\text{lepton},B}^{(6)}$	$-\frac{1}{4}(\breve{c}_{B,L}^{(6)})_{AB}^{\kappa\mu\nu\rho}e_{\kappa a}B_{\nu\rho}\overline{L}_{A}\gamma^{a}iD_{\mu}L_{B} - \frac{1}{4}(\breve{c}_{B,R}^{(6)})_{AB}^{\kappa\mu\nu\rho}e_{\kappa a}B_{\nu\rho}\overline{R}_{A}\gamma^{a}iD_{\mu}R_{B} + \text{h.c.}$
$\mathcal{L}_{\text{lepton},DB}^{(6)}$	$\frac{1}{2}(\breve{c}_{DB,L}^{(6)})_{AB}^{\kappa\mu\nu\rho}e_{\kappa a}(D_{\mu}B_{\nu\rho})\overline{L}_{A}\gamma^{a}L_{B} + \frac{1}{2}(\breve{c}_{DB,R}^{(6)})_{AB}^{\kappa\mu\nu\rho}e_{\kappa a}(D_{\mu}B_{\nu\rho})\overline{R}_{A}\gamma^{a}R_{B}$
$\mathcal{L}_{\text{lepton},2}^{(6)}$	$\frac{1}{2}(\breve{k}_{LL}^{(6)})_{ABCD}^{\kappa\lambda}e_{\kappa a}e_{\lambda b}(\overline{L}_{A}\gamma^{a}L_{B})(\overline{L}_{C}\gamma^{b}L_{D}) + \frac{1}{2}(\breve{k}_{RR}^{(6)})_{ABCD}^{\kappa\lambda}e_{\kappa a}e_{\lambda b}(\overline{R}_{A}\gamma^{a}R_{B})(\overline{R}_{C}\gamma^{b}R_{D})$
	$\quad + (\breve{k}_{LR}^{(6)})_{ABCD}^{\kappa\lambda}e_{\kappa a}e_{\lambda b}(\overline{L}_{A}\gamma^{a}L_{B})(\overline{R}_{C}\gamma^{b}R_{D})$
$\mathcal{L}_{\text{quark}}^{(3)}$	$-(\breve{a}_{Q})_{\kappa AB}e^{\kappa}{}_{a}\overline{Q}_{A}\gamma^{a}Q_{B} - (\breve{a}_{U})_{\kappa AB}e^{\kappa}{}_{a}\overline{U}_{A}\gamma^{a}U_{B} - (\breve{a}_{D})_{\kappa AB}e^{\kappa}{}_{a}\overline{D}_{A}\gamma^{a}D_{B}$
$\mathcal{L}_{\text{quark}}^{(4)}$	$-\frac{1}{2}(\breve{c}_{Q})_{\kappa\mu AB}e^{\kappa}{}_{a}\overline{Q}_{A}\gamma^{a}iD^{\mu}Q_{B} - \frac{1}{2}(\breve{c}_{U})_{\kappa\mu AB}e^{\kappa}{}_{a}\overline{U}_{A}\gamma^{a}iD^{\mu}U_{B} - \frac{1}{2}(\breve{c}_{D})_{\kappa\mu AB}e^{\kappa}{}_{a}\overline{D}_{A}\gamma^{a}iD^{\mu}D_{B} + \text{h.c.}$
$\mathcal{L}_{\text{quark},D}^{(5)}$	$\frac{1}{2}(\breve{a}_{Q}^{(5)})_{AB}^{\kappa\mu\nu}e_{\kappa a}\overline{Q}_{A}\gamma^{a}iD_{(\mu}iD_{\nu)}Q_{B} + \frac{1}{2}(\breve{a}_{U}^{(5)})_{AB}^{\kappa\mu\nu}e_{\kappa a}\overline{U}_{A}\gamma^{a}iD_{(\mu}iD_{\nu)}U_{B} + \frac{1}{2}(\breve{a}_{D}^{(5)})_{AB}^{\kappa\mu\nu}e_{\kappa a}\overline{D}_{A}\gamma^{a}iD_{(\mu}iD_{\nu)}D_{B} + \text{h.c.}$
$\mathcal{L}_{\text{quark},R}^{(5)}$	$(\breve{a}_{R,Q}^{(5)})_{AB}^{\kappa\mu\nu\rho\sigma}e_{\kappa a}R_{\mu\nu\rho\sigma}\overline{Q}_{A}\gamma^{a}Q_{B} + (\breve{a}_{R,U}^{(5)})_{AB}^{\kappa\mu\nu\rho\sigma}e_{\kappa a}R_{\mu\nu\rho\sigma}\overline{U}_{A}\gamma^{a}U_{B} + (\breve{a}_{R,D}^{(5)})_{AB}^{\kappa\mu\nu\rho\sigma}e_{\kappa a}R_{\mu\nu\rho\sigma}\overline{D}_{A}\gamma^{a}D_{B}$
$\mathcal{L}_{\text{quark},G}^{(5)}$	$\frac{1}{2}(\breve{a}_{G,Q}^{(5)})_{AB}^{\kappa\mu\nu}e_{\kappa a}\overline{Q}_{A}G_{\mu\nu}\gamma^{a}Q_{B} + \frac{1}{2}(\breve{a}_{G,U}^{(5)})_{AB}^{\kappa\mu\nu}e_{\kappa a}\overline{U}_{A}G_{\mu\nu}\gamma^{a}U_{B} + \frac{1}{2}(\breve{a}_{G,D}^{(5)})_{AB}^{\kappa\mu\nu}e_{\kappa a}\overline{D}_{A}G_{\mu\nu}\gamma^{a}D_{B}$
$\mathcal{L}_{\text{quark},W}^{(5)}$	$\frac{1}{2}(\breve{a}_{W,Q}^{(5)})_{AB}^{\kappa\mu\nu}e_{\kappa a}\overline{Q}_{A}W_{\mu\nu}\gamma^{a}Q_{B}$
$\mathcal{L}_{\text{quark},B}^{(5)}$	$\frac{1}{2}(\breve{a}_{B,Q}^{(5)})_{AB}^{\kappa\mu\nu}e_{\kappa a}B_{\mu\nu}\overline{Q}_{A}\gamma^{a}Q_{B} + \frac{1}{2}(\breve{a}_{B,U}^{(5)})_{AB}^{\kappa\mu\nu}e_{\kappa a}B_{\mu\nu}\overline{U}_{A}\gamma^{a}U_{B} + \frac{1}{2}(\breve{a}_{B,D}^{(5)})_{AB}^{\kappa\mu\nu}e_{\kappa a}B_{\mu\nu}\overline{D}_{A}\gamma^{a}D_{B}$
$\mathcal{L}_{\text{quark},D}^{(6)}$	$-\frac{1}{2}(\breve{c}_{Q}^{(6)})_{AB}^{\kappa\mu\nu\rho}e_{\kappa a}\overline{Q}_{A}\gamma^{a}iD_{(\mu}iD_{\nu}iD_{\rho)}Q_{B} - \frac{1}{2}(\breve{c}_{U}^{(6)})_{AB}^{\kappa\mu\nu\rho}e_{\kappa a}\overline{U}_{A}\gamma^{a}iD_{(\mu}iD_{\nu}iD_{\rho)}U_{B}$
	$\quad -\frac{1}{2}(\breve{c}_{D}^{(6)})_{AB}^{\kappa\mu\nu\rho}e_{\kappa a}\overline{D}_{A}\gamma^{a}iD_{(\mu}iD_{\nu}iD_{\rho)}D_{B} + \text{h.c.}$
$\mathcal{L}_{\text{quark},R}^{(6)}$	$-\frac{1}{2}(\breve{c}_{R,Q}^{(6)})_{AB}^{\kappa\mu\nu\rho\sigma\tau}e_{\kappa a}R_{\nu\rho\sigma\tau}\overline{Q}_{A}\gamma^{a}iD_{\mu}Q_{B} - \frac{1}{2}(\breve{c}_{R,U}^{(6)})_{AB}^{\kappa\mu\nu\rho\sigma\tau}e_{\kappa a}R_{\nu\rho\sigma\tau}\overline{U}_{A}\gamma^{a}iD_{\mu}U_{B}$
	$\quad -\frac{1}{2}(\breve{c}_{R,D}^{(6)})_{AB}^{\kappa\mu\nu\rho\sigma\tau}e_{\kappa a}R_{\nu\rho\sigma\tau}\overline{D}_{A}\gamma^{a}iD_{\mu}D_{B} + \text{h.c.}$
$\mathcal{L}_{\text{quark},DR}^{(6)}$	$(\breve{c}_{DR,Q}^{(6)})_{AB}^{\kappa\mu\nu\rho\sigma\tau}e_{\kappa a}(D_{\mu}R_{\nu\rho\sigma\tau})\overline{Q}_{A}\gamma^{a}Q_{B} + (\breve{c}_{DR,U}^{(6)})_{AB}^{\kappa\mu\nu\rho\sigma\tau}e_{\kappa a}(D_{\mu}R_{\nu\rho\sigma\tau})\overline{U}_{A}\gamma^{a}U_{B}$
	$\quad + (\breve{c}_{DR,D}^{(6)})_{AB}^{\kappa\mu\nu\rho\sigma\tau}e_{\kappa a}(D_{\mu}R_{\nu\rho\sigma\tau})\overline{D}_{A}\gamma^{a}D_{B}$
$\mathcal{L}_{\text{quark},G}^{(6)}$	$-\frac{1}{4}(\breve{c}_{G,Q}^{(6)})_{AB}^{\kappa\mu\nu\rho}e_{\kappa a}\overline{Q}_{A}\gamma^{a}G_{\nu\rho}iD_{\mu}Q_{B} - \frac{1}{4}(\breve{c}_{G,U}^{(6)})_{AB}^{\kappa\mu\nu\rho}e_{\kappa a}\overline{U}_{A}\gamma^{a}G_{\nu\rho}iD_{\mu}U_{B} - \frac{1}{4}(\breve{c}_{G,D}^{(6)})_{AB}^{\kappa\mu\nu\rho}e_{\kappa a}\overline{D}_{A}\gamma^{a}G_{\nu\rho}iD_{\mu}D_{B} + \text{h.c.}$
$\mathcal{L}_{\text{quark},DG}^{(6)}$	$\frac{1}{2}(\breve{c}_{DG,Q}^{(6)})_{AB}^{\kappa\mu\nu\rho}e_{\kappa a}\overline{Q}_{A}\gamma^{a}(D_{\mu}G_{\nu\rho})Q_{B} + \frac{1}{2}(\breve{c}_{DG,U}^{(6)})_{AB}^{\kappa\mu\nu\rho}e_{\kappa a}\overline{U}_{A}\gamma^{a}(D_{\mu}G_{\nu\rho})U_{B} + \frac{1}{2}(\breve{c}_{DG,D}^{(6)})_{AB}^{\kappa\mu\nu\rho}e_{\kappa a}\overline{D}_{A}\gamma^{a}(D_{\mu}G_{\nu\rho})D_{B}$
$\mathcal{L}_{\text{quark},W}^{(6)}$	$-\frac{1}{4}(\breve{c}_{W,Q}^{(6)})_{AB}^{\kappa\mu\nu\rho}e_{\kappa a}\overline{Q}_{A}\gamma^{a}W_{\nu\rho}iD_{\mu}Q_{B} + \text{h.c.}$
$\mathcal{L}_{\text{quark},DW}^{(6)}$	$\frac{1}{2}(\breve{c}_{DW,Q}^{(6)})_{AB}^{\kappa\mu\nu\rho}e_{\kappa a}\overline{Q}_{A}\gamma^{a}(D_{\mu}W_{\nu\rho})Q_{B}$
$\mathcal{L}_{\text{quark},B}^{(6)}$	$-\frac{1}{4}(\breve{c}_{B,Q}^{(6)})_{AB}^{\kappa\mu\nu\rho}e_{\kappa a}B_{\nu\rho}\overline{Q}_{A}\gamma^{a}iD_{\mu}Q_{B} - \frac{1}{4}(\breve{c}_{B,U}^{(6)})_{AB}^{\kappa\mu\nu\rho}e_{\kappa a}B_{\nu\rho}\overline{U}_{A}\gamma^{a}iD_{\mu}U_{B} - \frac{1}{4}(\breve{c}_{B,D}^{(6)})_{AB}^{\kappa\mu\nu\rho}e_{\kappa a}B_{\nu\rho}\overline{D}_{A}\gamma^{a}iD_{\mu}D_{B} + \text{h.c.}$
$\mathcal{L}_{\text{quark},DB}^{(6)}$	$\frac{1}{2}(\breve{c}_{DB,Q}^{(6)})_{AB}^{\kappa\mu\nu\rho}e_{\kappa a}(D_{\mu}B_{\nu\rho})\overline{Q}_{A}\gamma^{a}Q_{B} + \frac{1}{2}(\breve{c}_{DB,U}^{(6)})_{AB}^{\kappa\mu\nu\rho}e_{\kappa a}(D_{\mu}B_{\nu\rho})\overline{U}_{A}\gamma^{a}U_{B} + \frac{1}{2}(\breve{c}_{DB,D}^{(6)})_{AB}^{\kappa\mu\nu\rho}e_{\kappa a}(D_{\mu}B_{\nu\rho})\overline{D}_{A}\gamma^{a}D_{B}$
$\mathcal{L}_{\text{quark},2}^{(6)}$	$\frac{1}{2}(\breve{k}_{QQ}^{(6)})_{ABCD}^{\kappa\lambda}e_{\kappa a}e_{\lambda b}(\overline{Q}_{A}\gamma^{a}Q_{B})(\overline{Q}_{C}\gamma^{b}Q_{D}) + \frac{1}{2}(\breve{k}_{UU}^{(6)})_{ABCD}^{\kappa\lambda}e_{\kappa a}e_{\lambda b}(\overline{U}_{A}\gamma^{a}U_{B})(\overline{U}_{C}\gamma^{b}U_{D})$
	$\quad + \frac{1}{2}(\breve{k}_{DD}^{(6)})_{ABCD}^{\kappa\lambda}e_{\kappa a}e_{\lambda b}(\overline{D}_{A}\gamma^{a}D_{B})(\overline{D}_{C}\gamma^{b}D_{D}) + (\breve{k}_{QU}^{(6)})_{ABCD}^{\kappa\lambda}e_{\kappa a}e_{\lambda b}(\overline{Q}_{A}\gamma^{a}Q_{B})(\overline{U}_{C}\gamma^{b}U_{D})$
	$\quad + (\breve{k}_{QD}^{(6)})_{ABCD}^{\kappa\lambda}e_{\kappa a}e_{\lambda b}(\overline{Q}_{A}\gamma^{a}Q_{B})(\overline{D}_{C}\gamma^{b}D_{D}) + (\breve{k}_{UD}^{(6)})_{ABCD}^{\kappa\lambda}e_{\kappa a}e_{\lambda b}(\overline{U}_{A}\gamma^{a}U_{B})(\overline{D}_{C}\gamma^{b}D_{D})$
$\mathcal{L}_{\text{quark,lepton}}^{(6)}$	$(\breve{k}_{QL}^{(6)})_{ABCD}^{\kappa\lambda}e_{\kappa a}e_{\lambda b}(\overline{Q}_{A}\gamma^{a}Q_{B})(\overline{L}_{C}\gamma^{b}L_{D}) + (\breve{k}_{QR}^{(6)})_{ABCD}^{\kappa\lambda}e_{\kappa a}e_{\lambda b}(\overline{Q}_{A}\gamma^{a}Q_{B})(\overline{R}_{C}\gamma^{b}R_{D})$
	$\quad + (\breve{k}_{UL}^{(6)})_{ABCD}^{\kappa\lambda}e_{\kappa a}e_{\lambda b}(\overline{U}_{A}\gamma^{a}U_{B})(\overline{L}_{C}\gamma^{b}L_{D}) + (\breve{k}_{UR}^{(6)})_{ABCD}^{\kappa\lambda}e_{\kappa a}e_{\lambda b}(\overline{U}_{A}\gamma^{a}U_{B})(\overline{R}_{C}\gamma^{b}R_{D})$
	$\quad + (\breve{k}_{DL}^{(6)})_{ABCD}^{\kappa\lambda}e_{\kappa a}e_{\lambda b}(\overline{D}_{A}\gamma^{a}D_{B})(\overline{L}_{C}\gamma^{b}L_{D}) + (\breve{k}_{DR}^{(6)})_{ABCD}^{\kappa\lambda}e_{\kappa a}e_{\lambda b}(\overline{D}_{A}\gamma^{a}D_{B})(\overline{R}_{C}\gamma^{b}R_{D})$

TABLE XVIII. Terms with $d \leq 6$ in the Higgs sector $\mathcal{L}_{\text{Higgs}}$.

Component	Expression
$\mathcal{L}_{\text{Higgs},0}$	$-(D_\mu \phi)^\dagger D^\mu \phi + \mu^2 \phi^\dagger \phi - \frac{1}{6}\lambda(\phi^\dagger \phi)^2$
$\mathcal{L}_{\text{Higgs}}^{(2)}$	$\check{k}_\phi^{(2)} \phi^\dagger \phi$
$\mathcal{L}_{\text{Higgs}}^{(3)}$	$(\check{k}_\phi)^\mu \phi^\dagger i D_\mu \phi + \text{h.c.}$
$\mathcal{L}_{\text{Higgs}}^{(4)}$	$[\frac{1}{2}(\check{k}_{\phi\phi})^{\mu\nu}(D_\mu \phi)^\dagger D_\nu \phi + \text{h.c.}] - (\check{k}_{\phi R})^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} \phi^\dagger \phi - \frac{1}{2}(\check{k}_{\phi W})^{\mu\nu} \phi^\dagger W_{\mu\nu} \phi - \frac{1}{2}(\check{k}_{\phi B})^{\mu\nu} B_{\mu\nu} \phi^\dagger \phi - \frac{1}{6}\check{k}_{\phi\phi}^{(4)}(\phi^\dagger \phi)^2$
$\mathcal{L}_{\text{Higgs}}^{(5)}$	$[\frac{1}{2}(\check{k}^{(5)})^{\mu\nu\rho} \phi^\dagger i D_{(\mu} i D_\nu i D_{\rho)} \phi + \frac{1}{2}(\check{k}_{RD\phi}^{(5)})^{\mu\nu\rho\sigma\tau} R_{\nu\rho\sigma\tau} \phi^\dagger i D_\mu \phi + \frac{1}{4}(\check{k}_{WD\phi}^{(5)})^{\mu\nu\rho} \phi^\dagger W_{\nu\rho} i D_\mu \phi + \frac{1}{4}(\check{k}_{BD\phi}^{(5)})^{\mu\nu\rho} B_{\nu\rho} \phi^\dagger i D_\mu \phi$
	$+ \frac{1}{2}(\check{k}_{\phi D\phi}^{(5)})^{\mu}(\phi^\dagger \phi)(\phi^\dagger i D_\mu \phi) + \text{h.c.}] + (\check{k}_{DR\phi}^{(5)})^{\mu\nu\rho\sigma\tau}(D_\mu R_{\nu\rho\sigma\tau})\phi^\dagger \phi + \frac{1}{2}(\check{k}_{DW\phi}^{(5)})^{\mu\nu\rho} \phi^\dagger (D_\mu W_{\nu\rho})\phi$
	$+ \frac{1}{2}(\check{k}_{DB\phi}^{(5)})^{\mu\nu\rho}(D_\mu B_{\nu\rho})\phi^\dagger \phi$
$\mathcal{L}_{\text{Higgs}}^{(6)}$	$[\frac{1}{2}(\check{k}^{(6)})^{\mu\nu\rho\sigma} \phi^\dagger i D_{(\mu} i D_\nu i D_\rho i D_{\sigma)} \phi + \frac{1}{2}(\check{k}_{RDD\phi}^{(6)})^{\mu\nu\rho\sigma\tau\upsilon} R_{\rho\sigma\tau\upsilon} \phi^\dagger i D_{(\mu} i D_{\nu)} \phi + \frac{1}{4}(\check{k}_{WDD\phi}^{(6)})^{\mu\nu\rho\sigma} \phi^\dagger W_{\rho\sigma} i D_{(\mu} i D_{\nu)} \phi$
	$+ \frac{1}{4}(\check{k}_{BDD\phi}^{(6)})^{\mu\nu\rho\sigma} B_{\rho\sigma} \phi^\dagger i D_{(\mu} i D_{\nu)} \phi + \frac{1}{2}(\check{k}_{DRD\phi}^{(6)})^{\mu\nu\rho\sigma\tau\upsilon}(D_\nu R_{\rho\sigma\tau\upsilon})\phi^\dagger i D_\mu \phi + \frac{1}{4}(\check{k}_{DWD\phi}^{(6)})^{\mu\nu\rho\sigma} \phi^\dagger (D_\nu W_{\rho\sigma}) i D_\mu \phi$
	$+ \frac{1}{4}(\check{k}_{DBD\phi}^{(6)})^{\mu\nu\rho\sigma}(D_\nu B_{\rho\sigma})\phi^\dagger i D_\mu \phi + \frac{1}{2}(\check{k}_{\phi DD\phi}^{(6)})^{\mu\nu}(\phi^\dagger \phi)(\phi^\dagger i D_{(\mu} i D_{\nu)} \phi) + \frac{1}{2}(\check{k}_{D\phi D\phi}^{(6)})^{\mu\nu}(\phi^\dagger i D_\mu \phi)(\phi^\dagger i D_\nu \phi)$
	$+ \frac{1}{2}(\check{k}_{D\phi^\dagger D\phi}^{(6)})^{\mu\nu}((i D_\mu \phi)^\dagger \phi)(\phi^\dagger i D_\nu \phi) + \text{h.c.}] + (\check{k}_{DDR\phi}^{(6)})^{\mu\nu\rho\sigma\tau\upsilon}(D_{(\mu} D_{\nu)} R_{\rho\sigma\tau\upsilon})\phi^\dagger \phi$
	$+ \frac{1}{2}(\check{k}_{DDW\phi}^{(6)})^{\mu\nu\rho\sigma} \phi^\dagger (D_{(\mu} D_{\nu)} W_{\rho\sigma})\phi + \frac{1}{2}(\check{k}_{DDB\phi}^{(6)})^{\mu\nu\rho\sigma}(D_{(\mu} D_{\nu)} B_{\rho\sigma})\phi^\dagger \phi + (\check{k}_{RR\phi}^{(6)})^{\mu\nu\rho\sigma\tau\upsilon\chi\omega} R_{\mu\nu\rho\sigma} R_{\tau\upsilon\chi\omega} \phi^\dagger \phi$
	$+ (\check{k}_{RW\phi}^{(6)})^{\mu\nu\rho\sigma\tau\upsilon} R_{\rho\sigma\tau\upsilon} \phi^\dagger W_{\mu\nu} \phi + \frac{1}{2}(\check{k}_{RB\phi}^{(6)})^{\mu\nu\rho\sigma\tau\upsilon} R_{\rho\sigma\tau\upsilon} B_{\mu\nu} \phi^\dagger \phi + \frac{1}{4}(\check{k}_{WW\phi,1}^{(6)})^{\mu\nu\rho\sigma} \phi^\dagger W_{\mu\nu} W_{\rho\sigma} \phi$
	$+ \frac{1}{4}(\check{k}_{WW\phi,2}^{(6)})^{\mu\nu\rho\sigma} \text{tr}(W_{\mu\nu} W_{\rho\sigma})\phi^\dagger \phi + \frac{1}{4}(\check{k}_{BB\phi}^{(6)})^{\mu\nu\rho\sigma} B_{\mu\nu} B_{\rho\sigma} \phi^\dagger \phi + \frac{1}{4}(\check{k}_{BW\phi}^{(6)})^{\mu\nu\rho\sigma} B_{\mu\nu} \phi^\dagger W_{\rho\sigma} \phi$
	$+ \frac{1}{4}(\check{k}_{R\phi\phi}^{(6)})^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}(\phi^\dagger \phi)^2 + \frac{1}{4}(\check{k}_{W\phi\phi}^{(6)})^{\mu\nu}(\phi^\dagger \phi)(\phi^\dagger W_{\mu\nu} \phi) + \frac{1}{4}(\check{k}_{B\phi\phi}^{(6)})^{\mu\nu} B_{\mu\nu}(\phi^\dagger \phi)^2 + \frac{1}{120}\check{k}_{\phi\phi\phi}^{(6)}(\phi^\dagger \phi)^3$

present explicit terms in the effective field theory prior to the usual spontaneous breaking of the gauge group to SU(3)×U(1), adopting the conventions of Ref. [2]. In particular, the covariant derivative D_μ is both spacetime and SU(3)×SU(2)×U(1) covariant. The coupling constants for SU(3), SU(2), and U(1) are denoted by g_3, g, and g', respectively. They are related to the charge q for the electromagnetic U(1) group and to the angle θ_W via $q = g \sin\theta_W = g' \cos\theta_W$.

The piece $\mathcal{L}_{\text{gravity}}$ can be identified with the Lagrange density $\mathcal{L}_g$ discussed in Sec. III B, $\mathcal{L}_{\text{gravity}} = \mathcal{L}_g$, which has terms listed in Table VI. In the gauge sector, which includes gauge couplings to gravitational fields, the gauge fields for the strong interaction are described by the hermitian SU(3) adjoint matrix G_μ, while the SU(2) gauge fields form a hermitian adjoint matrix W_μ. The hermitian singlet hypercharge gauge field is denoted B_μ. The field strengths corresponding to these gauge fields are $G_{\mu\nu}$, $W_{\mu\nu}$, and $B_{\mu\nu}$. The corresponding Lagrange density $\mathcal{L}_{\text{gauge}}$ can be constructed using the techniques and results presented in Sec. III C 1 and Table IX, along with the U(1) limit provided in Table XIV.

Table XVI displays the terms in $\mathcal{L}_{\text{gravity}}$ and $\mathcal{L}_{\text{gauge}}$ without background derivatives and containing operators of mass dimension $d \leq 6$. The first row of the table provides the usual Lagrange density in the absence of backgrounds. For the remainder of the table, the first column lists the component $\mathcal{L}_{\text{gravity}}^{(d)}$ or $\mathcal{L}_{\text{gauge}}^{(d)}$, while the second column contains the corresponding terms. The notation for the background combinations $\check{k}$ is introduced in Sec.

III. Each combination $\check{k}$ is real and has index symmetry derived from that of its associated dynamical operator. For the terms with $d = 3$ and 4, the index positions and identifying subscripts on the combinations $\check{k}$ match those adopted in Ref. [2]. For generality, we include in this table all terms with dynamical operators that are total derivatives. Note that the meaning of the trace operation $\text{tr}(\mathcal{O})$ for an operator $\mathcal{O}$ in the gauge sector depends on the context, being taken in the appropriate representation for $\mathcal{O}$. Since $\text{tr}G_\mu$ and $\text{tr}W_\mu$ both vanish, only traces linear in the U(1) gauge fields appear in the table.

In the fermion sector, the generations are distinguished by an index $A = 1, 2, 3$. The three charged leptons are denoted $l_A \equiv (e, \mu, \tau)$, the three neutrinos are $\nu_A \equiv (\nu_e, \nu_\mu, \nu_\tau)$. For simplicity, we consider the minimal SM with massless neutrinos. The general effects of masses and combinations of Dirac and Majorana couplings for operators of arbitrary dimension affecting neutrino propagation are discussed in Ref. [90]. The six quark flavors are $u_A \equiv (u, c, t)$, $d_A \equiv (d, s, b)$, with the color index suppressed. Left- and right-handed spinor components are defined as usual by $\psi_L \equiv \frac{1}{2}(1 - \gamma_5)\psi$, $\psi_R \equiv \frac{1}{2}(1 + \gamma_5)\psi$. The right-handed leptons and quarks are SU(2) singlets, $R_A = (l_A)_R$, $U_A = (u_A)_R$, $D_A = (d_A)_R$, while the left-handed leptons and quarks form SU(2) doublets, $L_A = ((\nu_A)_L, (l_A)_L)^T$, $Q_A = ((u_A)_L, (d_A)_L)^T$.

Terms involving operators of mass dimension $d \leq 6$ in the Lagrange density $\mathcal{L}_{\text{fermion}} = \mathcal{L}_{\text{lepton}} + \mathcal{L}_{\text{quark}}$ are listed in Table XVII, including all couplings to the gravity and gauge sectors. For simplicity, this table is restricted to terms without background derivatives, and operators

TABLE XIX. Terms with $d \leq 6$ in the Yukawa sector $\mathcal{L}_{\text{Yukawa}}$.

Component	Expression
$\mathcal{L}_{\text{Yukawa},0}$	$-(G_L)_{AB}\overline{L}_A\phi R_B - (G_U)_{AB}\overline{Q}_A\phi^c U_B - (G_D)_{AB}\overline{Q}_A\phi D_B + \text{h.c.}$
$\mathcal{L}_{\text{Yukawa}}^{(4)}$	$-(\breve{G}_L^{(4)})_{AB}\overline{L}_A\phi R_B - (\breve{G}_U^{(4)})_{AB}\overline{Q}_A\phi^c U_B - (\breve{G}_D^{(4)})_{AB}\overline{Q}_A\phi D_B - \frac{1}{2}(\breve{H}_L)_{\kappa\lambda AB}e^{\kappa}{}_a e^{\lambda}{}_b\overline{L}_A\phi\sigma^{ab}R_B$
	$\quad -\frac{1}{2}(\breve{H}_U)_{\kappa\lambda AB}e^{\kappa}{}_a e^{\lambda}{}_b\overline{Q}_A\phi\sigma^{ab}U_B - \frac{1}{2}(\breve{H}_D)_{\kappa\lambda AB}e^{\kappa}{}_a e^{\lambda}{}_b\overline{Q}_A\phi\sigma^{ab}D_B + \text{h.c.}$
$\mathcal{L}_{\text{Yukawa}}^{(5)}$	$-(\breve{G}_L^{(5)})^{\mu}_{AB}\overline{L}_A\phi iD_\mu R_B - (\breve{G}_U^{(5)})^{\mu}_{AB}\overline{Q}_A\phi^c iD_\mu U_B - (\breve{G}_D^{(5)})^{\mu}_{AB}\overline{Q}_A\phi iD_\mu D_B - \frac{1}{2}(\breve{H}_L^{(5)})^{\kappa\lambda\mu}_{AB}e_{\kappa a}e_{\lambda b}\overline{L}_A\phi\sigma^{ab}iD_\mu R_B$
	$\quad -\frac{1}{2}(\breve{H}_U^{(5)})^{\kappa\lambda\mu}_{AB}e_{\kappa a}e_{\lambda b}\overline{Q}_A\phi^c\sigma^{ab}iD_\mu U_B - \frac{1}{2}(\breve{H}_D^{(5)})^{\kappa\lambda\mu}_{AB}e_{\kappa a}e_{\lambda b}\overline{U}_A\phi\sigma^{ab}iD_\mu D_B - (\breve{G}_{D\phi,L}^{(5)})^{\mu}_{AB}\overline{L}_A(iD_\mu\phi)R_B$
	$\quad -(\breve{G}_{D\phi,U}^{(5)})^{\mu}_{AB}\overline{Q}_A(iD_\mu\phi)^c U_B - (\breve{G}_{D\phi,D}^{(5)})^{\mu}_{AB}\overline{Q}_A(iD_\mu\phi)D_B - \frac{1}{2}(\breve{H}_{D\phi,L}^{(5)})^{\kappa\lambda\mu}_{AB}e_{\kappa a}e_{\lambda b}\overline{L}_A(iD_\mu\phi)\sigma^{ab}R_B$
	$\quad -\frac{1}{2}(\breve{H}_{D\phi,U}^{(5)})^{\kappa\lambda\mu}_{AB}e_{\kappa a}e_{\lambda b}\overline{Q}_A(iD_\mu\phi)^c\sigma^{ab}U_B - \frac{1}{2}(\breve{H}_{D\phi,D}^{(5)})^{\kappa\lambda\mu}_{AB}e_{\kappa a}e_{\lambda b}\overline{U}_A(iD_\mu\phi)\sigma^{ab}D_B + \text{h.c.}$
$\mathcal{L}_{\text{Yukawa},\psi\phi}^{(5)}$	$(\breve{k}_{L\phi}^{(5)})^{\kappa}_{AB}e_{\kappa a}(\overline{L}_A\gamma^a L_B)(\phi^\dagger\phi) + (\breve{k}_{R\phi}^{(5)})^{\kappa}_{AB}e_{\kappa a}(\overline{R}_A\gamma^a R_B)(\phi^\dagger\phi) + (\breve{k}_{Q\phi}^{(5)})^{\kappa}_{AB}e_{\kappa a}(\overline{Q}_A\gamma^a Q_B)(\phi^\dagger\phi)$
	$\quad +(\breve{k}_{U\phi}^{(5)})^{\kappa}_{AB}e_{\kappa a}(\overline{U}_A\gamma^a U_B)(\phi^\dagger\phi) + (\breve{k}_{D\phi}^{(5)})^{\kappa}_{AB}e_{\kappa a}(\overline{D}_A\gamma^a D_B)(\phi^\dagger\phi) + \text{h.c.}$
$\mathcal{L}_{\text{Yukawa},D}^{(6)}$	$-(\breve{G}_L^{(6)})^{\mu\nu}_{AB}\overline{L}_A\phi iD_{(\mu}iD_{\nu)}R_B - (\breve{G}_U^{(6)})^{\mu\nu}_{AB}\overline{Q}_A\phi^c iD_{(\mu}iD_{\nu)}U_B - (\breve{G}_D^{(6)})^{\mu\nu}_{AB}\overline{Q}_A\phi iD_{(\mu}iD_{\nu)}D_B$
	$\quad -\frac{1}{2}(\breve{H}_L^{(6)})^{\kappa\lambda\mu\nu}_{AB}e_{\kappa a}e_{\lambda b}\overline{L}_A\phi\sigma^{ab}iD_{(\mu}iD_{\nu)}R_B - \frac{1}{2}(\breve{H}_U^{(6)})^{\kappa\lambda\mu\nu}_{AB}e_{\kappa a}e_{\lambda b}\overline{Q}_A\phi^c\sigma^{ab}iD_{(\mu}iD_{\nu)}U_B$
	$\quad -\frac{1}{2}(\breve{H}_D^{(6)})^{\kappa\lambda\mu\nu}_{AB}e_{\kappa a}e_{\lambda b}\overline{Q}_A\phi\sigma^{ab}iD_{(\mu}iD_{\nu)}D_B - (\breve{G}_{DD\phi,L}^{(6)})^{\mu\nu}_{AB}\overline{L}_A(iD_{(\mu}iD_{\nu)}\phi)R_B$
	$\quad -(\breve{G}_{DD\phi,U}^{(6)})^{\mu\nu}_{AB}\overline{Q}_A(iD_{(\mu}iD_{\nu)}\phi)^c U_B - (\breve{G}_{DD\phi,D}^{(6)})^{\mu\nu}_{AB}\overline{Q}_A(iD_{(\mu}iD_{\nu)}\phi)D_B$
	$\quad -\frac{1}{2}(\breve{H}_{DD\phi,L}^{(6)})^{\kappa\lambda\mu\nu}_{AB}e_{\kappa a}e_{\lambda b}\overline{L}_A(iD_{(\mu}iD_{\nu)}\phi)\sigma^{ab}R_B - \frac{1}{2}(\breve{H}_{DD\phi,U}^{(6)})^{\kappa\lambda\mu\nu}_{AB}e_{\kappa a}e_{\lambda b}\overline{Q}_A(iD_{(\mu}iD_{\nu)}\phi)^c\sigma^{ab}U_B$
	$\quad -\frac{1}{2}(\breve{H}_{DD\phi,D}^{(6)})^{\kappa\lambda\mu\nu}_{AB}e_{\kappa a}e_{\lambda b}\overline{Q}_A(iD_{(\mu}iD_{\nu)}\phi)\sigma^{ab}D_B - (\breve{G}_{D\phi,L}^{(6)})^{\mu\nu}_{AB}\overline{L}_A(iD_\mu\phi)iD_\nu R_B$
	$\quad -(\breve{G}_{D\phi,U}^{(6)})^{\mu\nu}_{AB}\overline{Q}_A(iD_\mu\phi)^c iD_\nu U_B - (\breve{G}_{D\phi,D}^{(6)})^{\mu\nu}_{AB}\overline{Q}_A(iD_\mu\phi)iD_\nu D_B - \frac{1}{2}(\breve{H}_{D\phi,L}^{(6)})^{\kappa\lambda\mu\nu}_{AB}e_{\kappa a}e_{\lambda b}\overline{L}_A(iD_\mu\phi)\sigma^{ab}iD_\nu R_B$
	$\quad -\frac{1}{2}(\breve{H}_{D\phi,U}^{(6)})^{\kappa\lambda\mu\nu}_{AB}e_{\kappa a}e_{\lambda b}\overline{Q}_A(iD_\mu\phi)^c\sigma^{ab}iD_\nu U_B - \frac{1}{2}(\breve{H}_{D\phi,D}^{(6)})^{\kappa\lambda\mu\nu}_{AB}e_{\kappa a}e_{\lambda b}\overline{Q}_A(iD_\mu\phi)\sigma^{ab}iD_\nu D_B + \text{h.c.}$
$\mathcal{L}_{\text{Yukawa},R}^{(6)}$	$-(\breve{G}_{R,L}^{(6)})^{\mu\nu\rho\sigma}_{AB}R_{\mu\nu\rho\sigma}\overline{L}_A\phi R_B - (\breve{G}_{R,U}^{(6)})^{\mu\nu\rho\sigma}_{AB}R_{\mu\nu\rho\sigma}\overline{Q}_A\phi^c U_B - (\breve{G}_{R,D}^{(6)})^{\mu\nu\rho\sigma}_{AB}R_{\mu\nu\rho\sigma}\overline{Q}_A\phi D_B$
	$\quad -\frac{1}{2}(\breve{H}_L^{(6)})^{\kappa\lambda\mu\nu\rho\sigma}_{AB}e_{\kappa a}e_{\lambda b}R_{\mu\nu\rho\sigma}\overline{L}_A\phi\sigma^{ab}R_B - \frac{1}{2}(\breve{H}_U^{(6)})^{\kappa\lambda\mu\nu\rho\sigma}_{AB}e_{\kappa a}e_{\lambda b}R_{\mu\nu\rho\sigma}\overline{Q}_A\phi^c\sigma^{ab}U_B$
	$\quad -\frac{1}{2}(\breve{H}_D^{(6)})^{\kappa\lambda\mu\nu\rho\sigma}_{AB}e_{\kappa a}e_{\lambda b}R_{\mu\nu\rho\sigma}\overline{Q}_A\phi\sigma^{ab}D_B + \text{h.c.}$
$\mathcal{L}_{\text{Yukawa},G}^{(6)}$	$-(\breve{G}_{G,U}^{(6)})^{\mu\nu}_{AB}\overline{Q}_A G_{\mu\nu}\phi^c U_B - (\breve{G}_{G,D}^{(6)})^{\mu\nu}_{AB}\overline{Q}_A G_{\mu\nu}\phi D_B - \frac{1}{2}(\breve{H}_{G,U}^{(6)})^{\kappa\lambda\mu\nu}_{AB}e_{\kappa a}e_{\lambda b}\overline{Q}_A G_{\mu\nu}\phi^c\sigma^{ab}U_B$
	$\quad -\frac{1}{2}(\breve{H}_{G,D}^{(6)})^{\kappa\lambda\mu\nu}_{AB}e_{\kappa a}e_{\lambda b}\overline{Q}_A G_{\mu\nu}\phi\sigma^{ab}D_B + \text{h.c.}$
$\mathcal{L}_{\text{Yukawa},W}^{(6)}$	$-(\breve{G}_{W,L}^{(6)})^{\mu\nu}_{AB}\overline{L}_A W_{\mu\nu}\phi R_B - (\breve{G}_{W,U}^{(6)})^{\mu\nu}_{AB}\overline{Q}_A W_{\mu\nu}\phi^c U_B - (\breve{G}_{W,D}^{(6)})^{\mu\nu}_{AB}\overline{Q}_A W_{\mu\nu}\phi D_B$
	$\quad -\frac{1}{2}(\breve{H}_{W,L}^{(6)})^{\kappa\lambda\mu\nu}_{AB}e_{\kappa a}e_{\lambda b}\overline{L}_A W_{\mu\nu}\phi\sigma^{ab}R_B - \frac{1}{2}(\breve{H}_{W,U}^{(6)})^{\kappa\lambda\mu\nu}_{AB}e_{\kappa a}e_{\lambda b}\overline{Q}_A W_{\mu\nu}\phi^c\sigma^{ab}U_B$
	$\quad -\frac{1}{2}(\breve{H}_{W,D}^{(6)})^{\kappa\lambda\mu\nu}_{AB}e_{\kappa a}e_{\lambda b}\overline{Q}_A W_{\mu\nu}\phi\sigma^{ab}D_B + \text{h.c.}$
$\mathcal{L}_{\text{Yukawa},B}^{(6)}$	$-(\breve{G}_{B,L}^{(6)})^{\mu\nu}_{AB}B_{\mu\nu}\overline{L}_A\phi R_B - (\breve{G}_{B,U}^{(6)})^{\mu\nu}_{AB}B_{\mu\nu}\overline{Q}_A\phi^c U_B - (\breve{G}_{B,D}^{(6)})^{\mu\nu}_{AB}B_{\mu\nu}\overline{Q}_A\phi D_B$
	$\quad -\frac{1}{2}(\breve{H}_{B,L}^{(6)})^{\kappa\lambda\mu\nu}_{AB}e_{\kappa a}e_{\lambda b}B_{\mu\nu}\overline{L}_A\phi\sigma^{ab}R_B - \frac{1}{2}(\breve{H}_{B,U}^{(6)})^{\kappa\lambda\mu\nu}_{AB}e_{\kappa a}e_{\lambda b}B_{\mu\nu}\overline{Q}_A\phi^c\sigma^{ab}U_B$
	$\quad -\frac{1}{2}(\breve{H}_{B,D}^{(6)})^{\kappa\lambda\mu\nu}_{AB}e_{\kappa a}e_{\lambda b}B_{\mu\nu}\overline{Q}_A\phi\sigma^{ab}D_B + \text{h.c.}$
$\mathcal{L}_{\text{Yukawa},\phi}^{(6)}$	$-(\breve{G}_{\phi,L}^{(6)})_{AB}(\phi^\dagger\phi)\overline{L}_A\phi R_B - (\breve{G}_{\phi,U}^{(6)})_{AB}(\phi^\dagger\phi)\overline{Q}_A\phi^c U_B - (\breve{G}_{\phi,D}^{(6)})_{AB}(\phi^\dagger\phi)\overline{Q}_A\phi D_B$
	$\quad -\frac{1}{2}(\breve{H}_{\phi,L}^{(6)})^{\kappa\lambda}_{AB}e_{\kappa a}e_{\lambda b}(\phi^\dagger\phi)\overline{L}_A\phi\sigma^{ab}R_B - \frac{1}{2}(\breve{H}_{\phi,U}^{(6)})^{\kappa\lambda}_{AB}e_{\kappa a}e_{\lambda b}(\phi^\dagger\phi)\overline{Q}_A\phi^c\sigma^{ab}U_B$
	$\quad -\frac{1}{2}(\breve{H}_{\phi,D}^{(6)})^{\kappa\lambda}_{AB}e_{\kappa a}e_{\lambda b}(\phi^\dagger\phi)\overline{Q}_A\phi\sigma^{ab}D_B + \text{h.c.}$
$\mathcal{L}_{\text{Yukawa},\psi\phi}^{(6)}$	$\frac{1}{2}(\breve{k}_{LD\phi}^{(6)})^{\kappa\mu}_{AB}e_{\kappa a}(\overline{L}_A\gamma^a L_B)(\phi^\dagger iD_\mu\phi) + \frac{1}{2}(\breve{k}_{RD\phi}^{(6)})^{\kappa\mu}_{AB}e_{\kappa a}(\overline{R}_A\gamma^a R_B)(\phi^\dagger iD_\mu\phi)$
	$\quad +\frac{1}{2}(\breve{k}_{QD\phi}^{(6)})^{\kappa\mu}_{AB}e_{\kappa a}(\overline{Q}_A\gamma^a Q_B)(\phi^\dagger iD_\mu\phi) + \frac{1}{2}(\breve{k}_{UD\phi}^{(6)})^{\kappa\mu}_{AB}e_{\kappa a}(\overline{U}_A\gamma^a U_B)(\phi^\dagger iD_\mu\phi)$
	$\quad +\frac{1}{2}(\breve{k}_{DD\phi,1}^{(6)})^{\kappa\mu}_{AB}e_{\kappa a}(\overline{D}_A\gamma^a D_B)(\phi^\dagger iD_\mu\phi) + \frac{1}{2}(\breve{k}_{DL\phi}^{(6)})^{\kappa\mu}_{AB}e_{\kappa a}(\overline{L}_A\gamma^a iD_\mu L_B)(\phi^\dagger\phi)$
	$\quad +\frac{1}{2}(\breve{k}_{DR\phi}^{(6)})^{\kappa\mu}_{AB}e_{\kappa a}(\overline{R}_A\gamma^a iD_\mu R_B)(\phi^\dagger\phi) + \frac{1}{2}(\breve{k}_{DQ\phi}^{(6)})^{\kappa\mu}_{AB}e_{\kappa a}(\overline{Q}_A\gamma^a iD_\mu Q_B)(\phi^\dagger\phi)$
	$\quad +\frac{1}{2}(\breve{k}_{DU\phi}^{(6)})^{\kappa\mu}_{AB}e_{\kappa a}(\overline{U}_A\gamma^a iD_\mu U_B)(\phi^\dagger\phi) + \frac{1}{2}(\breve{k}_{DD\phi,2}^{(6)})^{\kappa\mu}_{AB}e_{\kappa a}(\overline{D}_A\gamma^a iD_\mu D_B)(\phi^\dagger\phi) + \text{h.c.}$

that are total derivatives are omitted. Except for the latter restriction, the format of the table follows that of Table XVI. The conventions adopted in Table XVII agree with standard usage in the literature. In particular, backgrounds with different spin and CPT properties are represented by different symbols in the usual way, and the index positions on the backgrounds are the standard ones. The index symmetry of a given background is determined by that of the corresponding dynamical operator. Each occurrence of the symbol h.c. for the hermitian conjugate applies to all terms in the particular component of the Lagrange density. In rows without the symbol h.c., the backgrounds can be assumed hermitian in generation space.

In the Higgs sector, we write the Higgs doublet ϕ in unitary gauge in the form $\phi = (0, r_\phi)^T/\sqrt{2}$. The conjugate Higgs doublet is denoted ϕ^c. Table XVIII provides the explicit form of the components $\mathcal{L}^{(d)}_{\text{Higgs}}$ with $d \leq 6$ of the Lagrange density in the Higgs sector, excluding terms with background derivatives and ones involving total derivatives of the dynamical operators. The structure of the table follows that of Table XVII, and it includes all couplings to the gravity and gauge sectors. In the table, each background $\breve{k}$ has indices with symmetry matching that of the associated dynamical operator, and all backgrounds can be taken as real. Note that a few terms have backgrounds $\breve{k}$ without spacetime indices. For example, a term with $d = 4$ proportional to $\breve{k}^{(4)}_{\phi\phi}(\phi^\dagger\phi)^2$ appears in the table. These types of expressions incorporate both scalar coupling constants and position-dependent effects. The component terms of this form in $\mathcal{L}^{(2)}_{\text{Higgs}}$ and $\mathcal{L}^{(4)}_{\text{Higgs}}$ replicate conventional SM terms in the first row of the table, so they can either be omitted or understood as renormalizations of the conventional terms having no physical effects. A similar comment applies to the first term in $\mathcal{L}^{(4)}_{\text{Higgs}}$, where the trace piece of the constant component of the background $(\breve{k})_{\phi\phi}{}^{\mu\nu}$ can be viewed as a renormalization of the usual Higgs kinetic term.

Finally, we present in Table XIX all terms involving operators with $d \leq 6$ that couple the fermions to the Higgs boson, restricting attention for simplicity to terms without background derivatives and disregarding total derivatives of dynamical operators. The terms in this table represent generalizations of the Yukawa couplings in the SM, which are listed in the first row. Note that the first three entries for $\mathcal{L}^{(4)}_{\text{Yukawa}}$ contain pieces that can be understood as physically irrelevant renormalizations of the usual SM Yukawa couplings, along with other nontrivial background effects. For entries already discussed in the literature, the notation in the table follows existing conventions. Backgrounds in terms written without an accompanying hermitian conjugate h.c. can be taken as hermitian in generation space. All backgrounds have indices with symmetries constrained according to the structure of the corresponding operators.

C. Constant scalar backgrounds

All the examples discussed above contain special cases with constant scalar backgrounds, which can be viewed as scalar coupling constants. In particular, the corresponding limits of the effective field theories built on the Einstein-Maxwell theories and on GR coupled to the SM can be extracted from the tables provided in the previous subsections. For a given theory, the explicit terms of this type are obtained as appropriate restrictions of the backgrounds $\breve{k}$ to maintain local Lorentz and diffeomorphism invariance.

As discussed in Sec. II C, a background k transforms as a spacetime tensor under observer local Lorentz and general coordinate transformations, but it remains invariant under all particle transformations including both local Lorentz transformations and diffeomorphisms. This behavior is compatible with local Lorentz and diffeomorphism invariance of the Lagrange density only if k carries no indices and is constant, in which case it acts as a conventional coupling constant. We can therefore identify all contributions that produce scalar coupling constants in a given effective Lagrange density by keeping only terms involving components of each combination $\breve{k}^{\mu\cdots}{}_{\nu\cdots}{}^{a\cdots}$ that are proportional to products of the vierbein $e_\mu{}^a$, metric $g_{\mu\nu}$, and Levi-Civita tensor $\epsilon_{\kappa\lambda\mu\nu}$ and then fixing these components to be spacetime constants. Note that this implies discarding all terms involving nonzero background derivatives Dk.

With this procedure in hand, it is straightforward to extract the limits of the various theories discussed above that have only scalar coupling constants. As an illustration, we provide in Table XX a listing of terms with $d \leq 6$ having only scalar coupling constants that is obtained from the generic Lagrange densities presented in Tables VI, IX, XI, and XIII. The first column of Table XX specifies the sector, the second column fixes the value of d, and the third column displays the corresponding terms with scalar coupling constants. The complete Lagrange density of this type is obtained by multiplying each operator displayed by a coupling constant and adding all the resulting terms. Note that the term $\epsilon^{\kappa\lambda\mu\nu}R^\alpha{}_{\beta\kappa\lambda}R^\beta{}_{\alpha\mu\nu}$ and the combination

$$\epsilon^{\alpha\beta\gamma\delta}\epsilon^{\kappa\lambda\mu\nu}R_{\alpha\beta\kappa\lambda}R_{\gamma\delta\mu\nu}$$
$$= 4(R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2) \quad (64)$$

can be expressed as total derivatives. For simplicity, other total-derivative operators are omitted from the table.

In the limit of scalar coupling constants, all indices on the dynamical operators must be contracted. The curvature, the gauge field strength, and the scalar field all have even numbers of indices and appear in dynamical combinations that have even mass dimension d. Since the vierbein, metric, and Levi-Civita tensors also have even numbers of indices, no terms with odd d can appear in the gravity or gauge sectors. Since the quadratic combination

TABLE XX. Terms with $d \leq 6$ involving scalar coupling constants.

Sector	d	Terms
Gravity	4	$\frac{1}{2\kappa}R$
	6	$\frac{1}{2\kappa}R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$, $\frac{1}{2\kappa}R^{\alpha\beta}R_{\alpha\beta}$, $\frac{1}{2\kappa}R^2$, $\frac{1}{2\kappa}\epsilon^{\kappa\lambda\mu\nu}R^{\alpha}{}_{\beta\kappa\lambda}R^{\beta}{}_{\alpha\mu\nu}$
Gauge	4	$\mathrm{tr}(F_{\mu\nu}F^{\mu\nu})$, $\mathrm{tr}(F_{\mu\nu}\widetilde{F}^{\mu\nu})$
	6	$\mathrm{tr}(F_{\mu\nu}D^{\alpha}D_{\alpha}F^{\mu\nu})$, $\mathrm{tr}(F_{\mu\nu}D^{\alpha}D_{\alpha}\widetilde{F}^{\mu\nu})$, $\mathrm{tr}(F^{\mu\nu}D_{(\mu}D_{\rho)}F^{\rho}{}_{\nu})$, $\mathrm{tr}(F^{\mu\nu}D_{(\mu}D_{\rho)}\widetilde{F}^{\rho}{}_{\nu})$,
		$R^{\kappa\lambda\mu\nu}\mathrm{tr}(F_{\kappa\lambda}F_{\mu\nu})$, $R^{\kappa\lambda\mu\nu}\mathrm{tr}(F_{\kappa\lambda}\widetilde{F}_{\mu\nu})$, $R^{\nu\rho}\mathrm{tr}(F_{\mu\nu}F^{\mu}{}_{\rho})$, $R^{\nu\rho}\mathrm{tr}(F_{\mu\nu}\widetilde{F}^{\mu}{}_{\rho})$, $R\,\mathrm{tr}(F_{\mu\nu}F^{\mu\nu})$, $R\,\mathrm{tr}(F_{\mu\nu}\widetilde{F}^{\mu\nu})$,
		$R^{\alpha\beta}\mathrm{tr}(D_{(\alpha}D_{\gamma)}F_{\beta}{}^{\gamma})$, $R^{\alpha\beta}\mathrm{tr}(D_{(\alpha}D_{\gamma)}\widetilde{F}_{\beta}{}^{\gamma})$, $i\,\mathrm{tr}(F^{\mu}{}_{\nu}F^{\nu}{}_{\rho}F^{\rho}{}_{\mu})$, $i\,\mathrm{tr}(F^{\mu}{}_{\nu}F^{\nu}{}_{\rho}\widetilde{F}^{\rho}{}_{\mu})$
Fermion	3	$\overline{\psi}\psi$, $i\overline{\psi}\gamma_5\psi$
	4	$\frac{1}{2}e^{\mu}{}_a\overline{\psi}\gamma^a iD_{\mu}\psi + \mathrm{h.c.}$, $\frac{1}{2}e^{\mu}{}_a\overline{\psi}\gamma_5\gamma^a iD_{\mu}\psi + \mathrm{h.c.}$
	5	$\frac{1}{2}\overline{\psi}iD^{\mu}iD_{\mu}\psi + \mathrm{h.c.}$, $\frac{1}{2}i\overline{\psi}\gamma_5 iD^{\mu}iD_{\mu}\psi + \mathrm{h.c.}$, $R\overline{\psi}\psi$, $iR\overline{\psi}\gamma_5\psi$, $e^{\mu}{}_a e^{\nu}{}_b\overline{\psi}\sigma^{ab}F_{\mu\nu}\psi$, $e^{\mu}{}_a e^{\nu}{}_b\overline{\psi}\sigma^{ab}\widetilde{F}_{\mu\nu}\psi$,
	6	$\frac{1}{2}g^{\nu\rho}e^{\mu}{}_a\overline{\psi}\gamma^a iD_{(\mu}iD_{\nu}iD_{\rho)}\psi + \mathrm{h.c.}$, $\frac{1}{2}g^{\nu\rho}e^{\mu}{}_a\overline{\psi}\gamma_5\gamma^a iD_{(\mu}iD_{\nu}iD_{\rho)}\psi + \mathrm{h.c.}$,
		$\frac{1}{2}e_{\kappa a}R^{\kappa\mu}\overline{\psi}\gamma^a iD_{\mu}\psi + \mathrm{h.c.}$, $\frac{1}{2}e_{\kappa a}R^{\kappa\mu}\overline{\psi}\gamma_5\gamma^a iD_{\mu}\psi + \mathrm{h.c.}$, $\frac{1}{2}e^{\mu}{}_a R\overline{\psi}\gamma^a iD_{\mu}\psi + \mathrm{h.c.}$, $\frac{1}{2}e^{\mu}{}_a R\overline{\psi}\gamma_5\gamma^a iD_{\mu}\psi + \mathrm{h.c.}$,
		$e_{\nu a}(D_{\mu}R^{\mu\nu})\overline{\psi}\gamma^a\psi$, $e_{\nu a}(D_{\mu}R^{\mu\nu})\overline{\psi}\gamma_5\gamma^a\psi$, $e^{\mu}{}_a(D_{\mu}R)\overline{\psi}\gamma^a\psi$, $e^{\mu}{}_a(D_{\mu}R)\overline{\psi}\gamma_5\gamma^a\psi$,
		$e_{\kappa a}\overline{\psi}\gamma^a(D_{\mu}F^{\kappa\mu})\psi$, $e_{\kappa a}\overline{\psi}\gamma_5\gamma^a(D_{\mu}F^{\kappa\mu})\psi$, $\frac{1}{2}e_{\kappa a}\overline{\psi}\gamma^a F^{\kappa\mu}iD_{\mu}\psi + \mathrm{h.c.}$, $\frac{1}{2}e_{\kappa a}\overline{\psi}\gamma_5\gamma^a F^{\kappa\mu}iD_{\mu}\psi + \mathrm{h.c.}$,
		$\frac{1}{2}e_{\kappa a}\overline{\psi}\gamma^a \widetilde{F}^{\kappa\mu}iD_{\mu}\psi + \mathrm{h.c.}$, $\frac{1}{2}e_{\kappa a}\overline{\psi}\gamma_5\gamma^a \widetilde{F}^{\kappa\mu}iD_{\mu}\psi + \mathrm{h.c.}$,
		$(\overline{\psi}\psi)(\overline{\psi}\psi)$, $i(\overline{\psi}\psi)(\overline{\psi}\gamma_5\psi)$, $-(\overline{\psi}\gamma_5\psi)(\overline{\psi}\gamma_5\psi)$, $(\overline{\psi}\gamma^a\psi)(\overline{\psi}\gamma_a\psi)$, $(\overline{\psi}\gamma^a\psi)(\overline{\psi}\gamma_5\gamma_a\psi)$,
		$(\overline{\psi}\gamma_5\gamma^a\psi)(\overline{\psi}\gamma_5\gamma_a\psi)$, $\frac{1}{4}(\overline{\psi}\sigma^{ab}\psi)(\overline{\psi}\sigma_{ab}\psi)$, $\frac{1}{4}\epsilon_{abcd}(\overline{\psi}\sigma^{ab}\psi)(\overline{\psi}\sigma^{cd}\psi)$
Scalar	2	$\phi^{\dagger}\phi$
	4	$\frac{1}{2}\phi^{\dagger}iD^{\mu}iD_{\mu}\phi + \mathrm{h.c.}$, $R\phi^{\dagger}\phi$, $(\phi^{\dagger}\phi)^2$
	5	$(\overline{\psi}\psi)(\phi^{\dagger}\phi)$, $i(\overline{\psi}\gamma_5\psi)(\phi^{\dagger}\phi)$
	6	$\frac{1}{2}g^{\mu\nu}g^{\rho\sigma}\phi^{\dagger}iD_{(\mu}iD_{\nu}iD_{\rho}iD_{\sigma)}\phi + \mathrm{h.c.}$, $\frac{1}{2}R^{\mu\nu}\phi^{\dagger}iD_{(\mu}iD_{\nu)}\phi + \mathrm{h.c.}$, $\frac{1}{2}R\phi^{\dagger}iD^{\mu}iD_{\mu}\phi + \mathrm{h.c.}$,
		$\frac{1}{2}(D_{\mu}R^{\mu\nu})\phi^{\dagger}iD_{\nu}\phi + \mathrm{h.c.}$, $\frac{1}{2}(D^{\mu}R)\phi^{\dagger}iD_{\mu}\phi + \mathrm{h.c.}$, $\frac{1}{2}\phi^{\dagger}(D_{\mu}F^{\mu\nu})iD_{\nu}\phi + \mathrm{h.c.}$, $\frac{1}{2}(\phi^{\dagger}\phi)(\phi^{\dagger}iD^{\mu}iD_{\mu}\phi) + \mathrm{h.c.}$,
		$\frac{1}{2}(\phi^{\dagger}iD^{\mu}\phi)(\phi^{\dagger}iD_{\mu}\phi) + \mathrm{h.c.}$, $\frac{1}{2}((iD^{\mu}\phi)^{\dagger}\phi)(\phi^{\dagger}iD_{\mu}\phi) + \mathrm{h.c.}$, $\frac{1}{2}e^{\mu}{}_a(\overline{\psi}\gamma^a\psi)(\phi^{\dagger}iD_{\mu}\phi) + \mathrm{h.c.}$,
		$\frac{1}{2}e^{\mu}{}_a(\overline{\psi}\gamma_5\gamma^a\psi)(\phi^{\dagger}iD_{\mu}\phi) + \mathrm{h.c.}$, $\frac{1}{2}e^{\mu}{}_a(\overline{\psi}\gamma^a iD_{\mu}\psi)(\phi^{\dagger}\phi) + \mathrm{h.c.}$, $\frac{1}{2}e^{\mu}{}_a(\overline{\psi}\gamma_5\gamma^a iD_{\mu}\psi)(\phi^{\dagger}\phi) + \mathrm{h.c.}$,
		$(D_{\mu}D_{\nu}R^{\mu\nu})\phi^{\dagger}\phi$, $(D^{\mu}D_{\mu}R)\phi^{\dagger}\phi$, $R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}(\phi^{\dagger}\phi)$, $R^{\alpha\beta}R_{\alpha\beta}(\phi^{\dagger}\phi)$, $R^2(\phi^{\dagger}\phi)$, $\epsilon^{\kappa\lambda\mu\nu}R^{\alpha}{}_{\beta\kappa\lambda}R^{\beta}{}_{\alpha\mu\nu}(\phi^{\dagger}\phi)$,
		$\phi^{\dagger}F_{\mu\nu}F^{\mu\nu}\phi$, $\phi^{\dagger}F_{\mu\nu}\widetilde{F}^{\mu\nu}\phi$, $\mathrm{tr}(F_{\mu\nu}F^{\mu\nu})(\phi^{\dagger}\phi)$, $\mathrm{tr}(F_{\mu\nu}\widetilde{F}^{\mu\nu})(\phi^{\dagger}\phi)$, $R(\phi^{\dagger}\phi)^2$, $(\phi^{\dagger}\phi)^3$

of fermions has odd mass dimension $d = 3$, terms with odd d can appear in the fermion sector and in fermion couplings to scalars. These features are reflected in the result in Table XX.

The gravity sector of Table XX consists of terms of dimension $d \leq 6$ in the effective field theory for GR that involve only scalar coupling constants. Various combinations of the sectors in the table form other effective field theories of this type including, for example, ones based on Einstein-Yang-Mills and Einstein-Maxwell theory, on nonabelian gauge theory and Maxwell electrodynamics in Minkowski spacetime, and theories with fermions and scalars. Using the same procedure, the effective field theory for GR coupled to the SM can be obtained directly from Tables XVI, XVII, XVIII, and XIX. In Minkowski spacetime, this reduces to the SM effective field theory

with $d \leq 6$. The operators for arbitrary d involving only scalar coupling constants in these various realistic theories have been presented elsewhere in the literature, including ones for arbitrary d in the graviton sector [59], the neutrino sector [90], the photon sector [124], and the fermion sector [125].

V. SUMMARY

In this work, we develop the framework for gravitational effective field theory in the presence of backgrounds and provide a methodology for constructing operators of arbitrary mass dimension d in the Lagrange density. Explicit terms with $d \leq 6$ are obtained for several theories, including the realistic cases of GR coupled

to the SM and some of its limits. The results presented here are achieved through a combination of conceptual developments and technical results.

The underpinnings of the framework are discussed in Sec. II. Table I summarizes the relevant spacetime transformations in both curved and approximately flat spacetimes. The various types of backgrounds and their implications for violations of symmetries in curved spacetimes are described, with examples provided in Table II. Linearizing in approximately flat spacetimes produces limiting cases of the spacetime transformations, listed in Table III. The links between properties of terms in the full and linearized Lagrange densities are schematically displayed in Fig. 1, and the symmetry properties of the various cases are illustrated with examples in Table IV. We also revisit the no-go constraints arising from the compatibility of the variational procedure with the Bianchi identities, showing that a large class of potential perturbative terms in the effective Lagrange density cannot arise from a Riemann geometry or its extensions but instead must have an alternative geometric or nongeometric origin in the underlying theory. These results are illustrated pictorially in Fig. 2.

Using this framework, the methodology for constructing terms in a generic effective Lagrange density is presented in Sec. III. The use of compact notation for backgrounds and technical results for construction of gauge-covariant operators permit the enumeration and classification of terms in the Lagrange density. Tables V, VI,

and VII provide explicit results for the pure-gravity and background sector involving operators with $d \leq 6$. Analogous forms for terms in the matter-gravity sector are obtained. Tables VIII and IX consider operators containing gauge fields, Tables X and XI present results for Dirac fermions, and Tables XII and XIII treat scalars.

Applications of the methodology to cases of practical importance are considered in Sec. IV. For Einstein-Maxwell effective field theories, terms in the matter-gravity Lagrange density with $d \leq 6$ are provided for photons in Table XIV and for uncharged scalars in Table XV. For the realistic effective field theory consisting of GR coupled to the SM, we present all terms with $d \leq 6$ in the matter-gravity sector involving gauge, lepton, quark, Higgs, and Yukawa couplings in Tables XVI, XVII, XVIII, and XIX. Terms for the limiting case with backgrounds acting only as scalar coupling constants are displayed in Table XX.

The results obtained in this work establish the foundation for further investigations of gravitational effective field theories. The explicit characterization of terms in realistic scenarios provided here opens the way for future phenomenological and experimental searches, with promising potential for detecting observable signals from the Planck scale.

VI.   ACKNOWLEDGMENTS

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